E-SUPER ARITHMETIC GRACEFUL LABELLING OF SOME STANDARD CLASSES OF CUBIC GRAPHS RELATED TO CYCLES

Anubala Sekar and Ramachandran Varatharajaperumal

Abstract. We introduced a new concept called E-super arithmetic graceful labelling of graphs. A \((p, q)\)-graph \(G\) is said to be E-super arithmetic graceful if there exists a bijection \(f\) from \(V(G) \cup E(G)\) to \(\{1, 2, \ldots, p+q\}\) such that \(f(E(G)) = \{1, 2, \ldots, q\}\), \(f(V(G)) = \{q+1, q+2, \ldots, p+q\}\) and the induced mapping \(f^*\) given by \(f^*(uv) = f(u) + f(v) - f(uv)\) for \(uv \in E(G)\) has the range \(\{p+q+1, p+q+2, \ldots, p+2q\}\).

In this paper we prove that the complete graph, flower snarks and its related graphs, the cubic graphs \(F(3)(C_n)\), generalised Petersen graphs \(P(n, 2)\), the Petersen graph which has chromatic number 3, Desargues graph and Heawood graph are E-super arithmetic graceful.

1. Introduction

Rosa [9] in 1967, called a function \(f\) a \(\beta\) - valuation of a graph \(G\) with \(q\) edges if \(f\) is an injection from the vertices of \(G\) to the set \(\{0, 1, \ldots, q\}\) such that when each edge \(xy\) is assigned the label \(|f(x) - f(y)|\), the resulting edge labels are distinct. Golomb[3] subsequently called such labelling graceful. Acharya and Hedge [1] have defined \((k, d)\) – arithmetic graphs. Let \(G\) be a graph with \(q\) edges and let \(k\) and \(d\) be positive integers. A labelling \(f\) of \(G\) is said to be \((k, d)\) – arithmetic if the vertex labels are distinct nonnegative integers and the edge labels induced by \(f(x) + f(y)\) for each edge \(xy\) are \(k, k+d, k+2d, \ldots, k+(q-1)d\). The case where \(k = 1\) and \(d = 1\) was called additively graceful by Hedge [4].


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V. Ramachandran and C Sekar \[8\] introduced \((1,N)\)-arithmetic labelling.

In 1970 Kotzig and Rosa \[5\] defined a magic valuation of a graph \(G(V, E)\) as a bijection \(f\) from \(V \cup E\) to \([1, 2, ..., |V \cup E|]\) such that for all edges \(xy\), \(f(x) + f(y) + f(xy)\) is constant. Ringel and Llado in 1996 called this labelling edge - magic. If the vertex labels are 1 to \(|V|\), it is called Super edge - magic total labelling.

MacDougall, Slamin, Miller and Wallis \[6\] introduced the notion of a vertex - magic total labelling in 1999. For a graph \(G(V, E)\) an injective mapping \(f\) from \(V \cup E\) to the set \([1, 2, ..., |V| + |E|]\) is a vertex - magic total labelling if there is a constant \(k\), called the magic constant such that for every vertex \(v\), \(f(u) + \sum f(vu) = k\) where the sum is taken over all vertices \(u\) adjacent to \(v\).

A vertex magic total labelling of \(G(V, E)\) is said to be E-super if \(f(E(G)) = \{1, 2, 3, ..., |E(G)|\}\).

A labelling of \(G(V, E)\) is said to be E-super if \(f(E(G)) = \{1, 2, 3, ..., |E(G)|\}\).

Marimuthu and Balakrishnan \[7\] defined a graph \(G(V, E)\) to be edge magic graceful if there exists a bijection \(f\) from \(V(G) \cup E(G)\) to \([1, 2, ..., p + q]\) such that \(|f(u) + f(v) - f(uv)|\) is a constant for all edges \(uv\) of \(G\).

We introduced a new concept called E-super arithmetic graceful labelling of graphs \[10\]. We define a graph \(G(p, q)\) to be \textbf{E-super arithmetic graceful} if there exists a bijection \(f\) from \(V(G) \cup E(G)\) to \([1, 2, ..., p + q]\) such that \(f(E(G)) = \{1, 2, ..., q\}\), \(f(V(G)) = \{q + 1, q + 2, ..., q + p\}\) and the induced mapping \(f^*\) given by \(f^*(uv) = f(u) + f(v) - f(uv)\) for \(uv \in E(G)\) has the range \([p + q + 1, p + q + 2, ..., p + 2q]\).

In the field of graph theory, the flower snarks form an infinite family of snarks introduced by Rufao Issac in 1975. As snarks, the flower snarks are connected bridgeless cubic graphs with chromatic index equal to 4. The flower snarks are nonplanar and non-hamiltonian.

In this paper we prove that the complete graphs, flower snarks and its related graphs, the cubic graphs \(F^{(3)}(C_n)\), generalised Petersen graphs \(P(n, 2)\), the Petersen graph which has chromatic number 3, Desargues graph and the Heawood graphs are E-super arithmetic graceful.

\[2.\text{ Preliminaries}\]

\textbf{Definition 2.1.} For \(n = 3, 4\), the graph related to flower snark, denoted by \(F_n\), is a cubical graph with vertex set \(V(F_n) = \{a_i, i = 0, 1, 2, ..., n - 1\} \cup \{b_i, i = 0, 1, 2, ..., n - 1\} \cup \{c_i, i = 0, 1, 2, ..., 2n - 1\}\) and edge set \(E(F_n) = \{a_i a_{i+1 (\mod n)}, 0 \leq i \leq n - 1\} \cup \{a_i b_i, 0 \leq i \leq n - 1\} \cup \{b_i c_i, 0 \leq i \leq n - 1\} \cup \{b_i c_{i+1 (\mod 2n)}, 0 \leq i \leq 2n - 1\}\). \(F_n\) has \(4n\) vertices and \(6n\) edges. For odd \(n \geq 5\) similar graphs are called flower snarks.
Fig 2.1

**Definition 2.2.** \( F^{(3)}(C_n) \) for \( n \geq 3 \) denotes a cubic graph with vertex set \( V = \{ a_i, 0 \leq i \leq n - 1 \} \cup \{ b_i, 0 \leq i \leq n - 1 \} \cup \{ c_i, 0 \leq i \leq 2n - 1 \} \), and edge set

\[
E = \{ a_i a_{i+1}, \mod n \}, 0 \leq i \leq n - 1 \} \cup \{ a_i b_i, 0 \leq i \leq n - 1 \}
\cup \{ b_i c_i, 0 \leq i \leq n - 1 \} \cup \{ b_i c_{n+i}, 0 \leq i \leq n - 1 \}
\cup \{ c_i c_{i+1}, \mod n \}, 0 \leq i \leq n - 1 \} \cup \{ c_{n+i} c_{n+i+1}, 0 \leq i \leq n - 2 \} \cup \{ c_{2n-1} c_n \}
\]

\( F^{(3)}(C_n) \) has 4n vertices and 6n edges.

\( F^{(3)}(C_4) \):

Fig 2.2
Definition 2.3. The generalized Peterson graph $P(n,k)$, $n \geq 5$, $k \geq 2$ is the graph with vertex set \{u_1, u_2, \ldots, u_n\} \cup \{v_1, v_2, \ldots, v_n\} and edge set
\\{u_iu_{i+1} \mid i = 1, 2, \ldots, n \text{ where } u_{n+1} = u_1\} \cup \{u_iv_i \mid i = 1, 2, \ldots, n\} \\
\cup \{v_iv_{i+k} \mid i = 1, 2, \ldots, n \text{ where } v_{n+j} = v_j\}

The usual Peterson graph is $P(5,2)$.

$P(n,2)$ for all $n \geq 5$ is a cubic graph related to cycle $C_n$.

Generalised Petersen graph $P(7,2)$:

![Diagram of Petersen graph](image)

Definition 2.4. Let $G$ be the graph having vertices $u_0, u_1, u_2, \ldots, u_9$ and edge set \{u_0u_1, u_0u_4, u_0u_7\} \cup \{u_iu_{i+1} \mid i = 1, 2, \ldots, 9 \text{ where } u_{10} = u_1\} \cup \{u_2u_6, u_3u_8, u_5u_9\}.

This graph $G$ is a cubic graph called Petersen graph which has chromatic number 3 which is given in the adjoined figure.

![Diagram of Petersen graph](image)
Definition 2.5. The Desargues graph is a distance transitive cubic graph with 20 vertices and 30 edges. The graph is as shown in the figure.

Definition 2.6. Heawood graph is a cubic graph with 14 vertices and 21 edges as given in the adjoined figure.

3. Main results

Theorem 3.1. The graphs $F_n$ for $n = 3$ and even $n \geq 4$ related to flower snarks and the flower snarks $F_n$ for odd $n \geq 5$ are $E$-super arithmetic graceful.

Proof. We give a common labelling for the graphs related to flower snarks and flower snarks.
Consider $F_n$, $n \geq 3$.
Let \( \{a_i, i = 0, 1, 2, \ldots, n-1\} \cup \{b_i, i = 0, 1, \ldots, n-1\} \cup \{c_i, i = 0, 1, 2, \ldots, 2n-1\} \) be the vertices of $F_n$.

$F_n$ has $4n$ vertices and $6n$ edges.
Define $f : V(F_n) \cup E(F_n) \rightarrow \{1, 2, \ldots, 10n\}$ as follows:
\[
\begin{align*}
    f(a_i) &= 6n + 1 + i, \quad \text{for } i = 0, 1, \ldots, n-1 \\
n\quad f(b_i) &= 8n - i, \quad \text{for } i = 0, 1, \ldots, n-1 \\
n\quad f(c_i) &= 9n + 1 + i, \quad \text{for } i = 0, 1, \ldots, n-1 \\
n\quad f(c_{n+i}) &= 8n + 2 + i, \quad \text{for } i = 0, 1, \ldots, n-2 \\
n\quad f(c_{2n-1}) &= 8n + 1. \\
n\quad f(a_{n+i}(\text{mod } 2n)) &= n + 1 + i, \quad \text{for } i = 0, 1, \ldots, n-1 \\
n\quad f(a_i b_i) &= 3n + 1 + i, \quad \text{for } i = 0, 1, \ldots, n-1 \\
n\quad f(b_i c_i) &= 2n + 1 + i, \quad \text{for } i = 0, 1, \ldots, n-1 \\
n\quad f(b_i c_{n+i}) &= 2 + i, \quad \text{for } i = 0, 1, \ldots, n-2 \\
n\quad f(b_{n-i}c_{n+i}) &= 1. \\
n\quad f(c_{0}c_{2n-1}) &= 5n + 1. \\
n\quad f(c_{i}c_{n+i}) &= 5n + 2 + i, \quad \text{for } i = 0, 1, 2, \ldots, n-2 \\
n\quad f(c_{n-1}c_{n+i}) &= 4n + 2. \\
n\quad f(c_{n+i}c_{n+1+i}) &= 4n + 3 + i, \quad \text{for } i = 0, 1, 2, \ldots, n-3 \\
n\quad f(c_{2n-1}c_{2n-1}) &= 4n + 1.
\end{align*}
\]
Clearly $f$ is a bijection.
$f(E(F_n)) = \{1, 2, \ldots, 6n\}$.

\[
\begin{align*}
\{f^*(a_i a_{i+1}(\text{mod } n)) \mid 0 \leq i \leq n-1\} &= \{11n + 1 + i \mid 0 \leq i \leq n-1\} \\
&= \{11n + 1, 11n + 2, \ldots, 12n\} \\
n\quad \{f^*(a_i b_i) \mid 0 \leq i \leq n-1\} &= \{11n - i \mid 0 \leq i \leq n-1\} = \{10n + 1, 10n + 2, \ldots, 11n\} \\
n\quad \{f^*(b_i c_i) \mid 0 \leq i \leq n-1\} &= \{15n + i \mid 0 \leq i \leq n-1\} = \{14n + 1, 14n + 2, \ldots, 15n\} \\
n\quad \{f^*(b_i c_{n+i}) \mid 0 \leq i \leq n-2\} &= \{16n - i \mid 0 \leq i \leq n-2\} = \{15n + 2, 15n + 3, \ldots, 16n\} \\
n\quad f^*(b_{n-1} c_{2n-1}) &= 15n + 1 \\
\quad \{f^*(c_i c_{i+1}) \mid 0 \leq i \leq n-2\} &= \{13n + 1 + i \mid 0 \leq i \leq n-2\} \\
&= \{13n + 1, 13n + 2, \ldots, 14n - 1\} \\
n\quad f^*(c_{n-1} c_n) &= 14n \\
\quad \{f^*(c_{n+i} c_{n+i+1}) \mid 0 \leq i \leq n-3\} &= \{12n + 2 + i \mid 0 \leq i \leq n-3\} \\
&= \{12n + 2, 12n + 3, \ldots, 13n - 1\} \\
n\quad f^*(c_{2n-2} c_{2n-1}) &= 13n \quad f^*(c_{0} c_{2n-1}) = 12n + 1
\end{align*}
\]
Therefore $f^*(E(F_n)) = \{10n + 1, 10n + 2, \ldots, 16n\}$.
Thus $F_n$ is E-super arithmetic graceful for all $n \geq 3$. \qed
Example 3.1. E-super arithmetic graceful labelling of flower snark $F_7$.

Theorem 3.2. $F^{(3)}(C_n)$ is E-super arithmetic graceful for all $n \geq 3$.

Proof. Let $\{a_i, 0 \leq i \leq n - 1\} \cup \{b_i, 0 \leq i \leq n - 1\} \cup \{c_i, 0 \leq i \leq 2n - 1\}$ be the vertices of $F^{(3)}(C_n)$.

$F^{(3)}(C_n)$ has 4n vertices and 6n edges.

Define $f : V \cup E \rightarrow \{1, 2, 3, ..., 10n\}$ as follows:

$f(a_i) = 8n - i$, \quad $i = 0, 1, ..., n - 1$
Clearly $f$ is a bijection.

Therefore $f^*(E(F^{(3)}(C_n))) = \{10n + 1, 10n + 2, ..., 16n\}$.

Thus $F^{(3)}(C_n)$ is E-super arithmetic graceful.
THEOREM 3.3. $P(n, 2)$ for $n \geq 5$ is E-super arithmetic graceful.

PROOF. The generalized Petersen graph $P(n, 2)$ for $n \geq 5$ has $2n$ vertices and $3n$ edges. Define $f : V \cup E \rightarrow \{1, 2, \ldots, 5n\}$ as follows:

- $f(u_i) = 3n + i$, $i = 1, 2, \ldots, n$
- $f(v_i) = 4n + i$, $i = 1, 2, \ldots, n$
- $f(u_iu_{i+1}) = n + i$, $i = 1, 2, \ldots, n$ where $u_{n+1} = u_1$
- $f(u_iv_i) = i$, $i = 1, 2, \ldots, n$
- $f(v_iv_{i+2}) = 2n + i$, $i = 1, 2, \ldots, n$ where $v_{n+1} = v_1$ and $v_{n+2} = v_2$

Clearly $f$ is a bijection.

- $f(E(P(n, 2))) = \{1, 2, \ldots, 3n\}$
- $f^*(E(P(n, 2))) = \{5n + 1, 5n + 2, \ldots, 8n\}$

\[
\{f^*(u_iu_{i+1}) \mid i = 1, 2, \ldots, n - 1\} = \{5n + 1 + i \mid i = 1, 2, \ldots, n - 1\} = \{5n + 2, 5n + 3, \ldots, 6n\}
\]

- $f^*(u_nv_1) = 5n + 1$
- $f^*(u_iv_i) = \{7n + i \mid i = 1, 2, \ldots, n\} = \{7n + 1, 7n + 2, \ldots, 8n\}$

\[
\{f^*(v_iv_{i+2}) \mid i = 1, 2, \ldots, n - 2\} = \{6n + i + 2 \mid i = 1, 2, \ldots, n - 2\} = \{6n + 3, 6n + 4, \ldots, 7n\}
\]

Therefore $P(n, 2)$ for $n \geq 5$ is E-super arithmetic graceful for all $n \geq 5$. \hfill \Box

EXAMPLE 3.4. E-super arithmetic graceful labelling of $P(7, 2)$.

\[\text{Fig 3.4}\]
A particular labelling:
E-super arithmetic graceful labelling of the Petersen graph which has chromatic number 3 is given below:

![Petersen Graph with Labelling](image)

\[ f : V(G) \cup E(G) \to \{1, 2, \ldots, 50\} \]

**Theorem 3.4.** The Desargues graph is E-super arithmetic graceful.

**Proof.** Let G be the Desargues graph with 20 vertices and 30 edges. Let \( V(G) = \{v_1, v_2, \ldots, v_{20}\} \). The edge set

\[ E(G) = \{v_i v_{i+1} | 1 \leq i \leq 9\} \cup \{v_1 v_{10}\} \cup \{v_i v_{i+1} | 1 \leq i \leq 10\} \]

\[ \cup \{v_i v_{i+3} | 11 \leq i \leq 17\} \cup \{v_i v_{i+7} | 11 \leq i \leq 13\} \]

Define \( f : V(G) \cup E(G) \to \{1, 2, \ldots, 50\} \) as given below.

\( f(v_1) = 30 + i, \quad i = 1, 2, \ldots, 20 \)

\( f(v_i v_{i+1}) = 10 + i, \quad i = 1, 2, \ldots, 9 \)

\( f(v_1 v_{10}) = 20 \)

\( f(v_i v_{i+10}) = i, \quad i = 1, 2, \ldots, 10 \)

\( f(v_i v_{i+3}) = 10 + i, \quad 11 \leq i \leq 17 \)

\( f(v_i v_{i+7}) = 17 + i, \quad 11 \leq i \leq 13 \)

Clearly \( f(E(G)) = \{1, 2, \ldots, 30\} \)
\( f(V(G)) = \{31, 32, \ldots, 50\} \)
\( f^*(E(G)) = \{51, 52, \ldots, 80\} \).

Therefore G is E-super arithmetic graceful.
Example 3.5. E-super arithmetic graceful labelling of Desargues Graph.

**Theorem 3.5.** Heawood graph is E-super arithmetic graceful.

**Proof.** Let $G$ be the Heawood graph with 14 vertices and 21 edges. Let $V(G) = \{v_1, v_2, \ldots, v_{14}\}$. The edge set

$$E(G) = \{v_i v_{i+1} \mid i = 1, 2, \ldots, 13\} \cup \{v_1 v_{14}\} \cup \{v_i v_{i+5} \mid i = 1, 3, 5, 7, 9\} \cup \{v_2 v_{11}, v_4 v_{13}\}$$

Define $f : V(G) \cup E(G) \to \{1, 2, \ldots, 35\}$ as follows:

- $f(v_i) = 22 + i$, $i = 1, 2, \ldots, 13$
- $f(v_{14}) = 22$
- $f(v_i v_{i+1}) = 2 + i$, $i = 1, 2, \ldots, 12$
- $f(v_3 v_{14}) = 1$
- $f(v_1 v_{14}) = 2$
- $f(v_i v_{i+5}) = 14 + i$, $i = 1, 3, 5, 7$
- $f(v_9 v_{14}) = 16$
- $f(v_{11} v_{14}) = 18$
- $f(v_4 v_{13}) = 20$

Clearly $f(E(G)) = \{1, 2, \ldots, 21\}$ and $f(V(G)) = \{22, 23, \ldots, 35\}$.

Thus $f^*(E(G)) = \{36, 37, \ldots, 56\}$.

Therefore $G$ is E-super arithmetic graceful. □
Example 3.6. E-super arithmetic graceful labelling of Heawood graph.

Remark 3.1. The complete graph $K_4$, a cubic graph related to cycle is also E-super arithmetic graceful. It is shown as a particular case in the following generalised result.

Theorem 3.6. Complete graphs $K_n$, $n \geq 3$ are E-super arithmetic graceful.

Proof. $K_n$ has $n$ vertices and $\frac{n(n-1)}{2}$ edges.

Case:(i) $n = 3$.
Let $u_1, u_2, u_3$ be the vertices of $K_3$.
E-super arithmetic graceful labelling of $K_3$ is given below.

Clearly $f^*(E(K_3)) = \{7, 8, 9\}$
Case (ii) \( n = 4 \).
Let \( u_1, u_2, u_3, u_4 \) be the vertices of \( K_4 \).
E-super arithmetic graceful labelling of \( K_4 \) is given below.

![Fig 3.9](image)

Clearly \( f^*(E(K_4)) = \{11, 12, 13, 14, 15, 16\} \)

Case (iii) \( n \geq 5 \), \( n \) is odd.
Let \( u_1, u_2, \ldots, u_n \) be the vertices of \( K_n \).
Define \( f : V(K_n) \cup E(K_n) \rightarrow \left\{1, 2, \ldots, \frac{n(n+1)}{2}\right\} \) as follows:

\[
f(u_i) = \frac{n(n-1)}{2} + i, \quad \text{for} \quad i = 1, 2, \ldots, n
\]

\[
f(u_iu_{i+1}) = \frac{n(n-3)}{2} + i, \quad \text{for} \quad i = 1, 2, \ldots, n
\]

where \( u_{n+1} = u_1\).
For \( 2 \leq k \leq \frac{n-1}{2} \),
define \( f(u_iu_{i+k}) = (k-2)n + i \), \( \text{for} \quad i = 1, 2, \ldots, n \)
where \( u_{n+k} = u_k \) for all \( k \).
Clearly \( f(E(K_n)) = \left\{1, 2, \ldots, \frac{n(n-1)}{2}\right\} \)

\[
\{f^*(u_iu_{i+1}) \mid 1 \leq i \leq n-1\} = \left\{\frac{n(n+1)}{2} + i + 1 \mid 1 \leq i \leq n-1\right\}
\]

\[
= \left\{\frac{n(n+1)}{2} + 2, \ldots, \frac{n(n+3)}{2}\right\}
\]

\[
f^*(u_nu_1) = \frac{n(n+1)}{2} + 1
\]

\[
\left\{f^*(u_iu_{i+k}) \mid 1 \leq i \leq n, \quad 2 \leq k \leq \frac{n-1}{2}\right\} =
\]

\[
= \left\{(n-k)(n-1) + 2n + i \mid 1 \leq i \leq n, \quad 2 \leq k \leq \frac{n-1}{2}\right\} = \left\{\frac{n(n+3)}{2} + 1, \ldots, n^2\right\}.
\]

Therefore \( f^*(E(K_n)) = \left\{\frac{n(n+1)}{2} + 1, \frac{n(n+1)}{2} + 2, \ldots, n^2\right\} \)

Thus \( K_n \) is E-super arithmetic graceful

Case (iv) \( n \geq 6 \), \( n \) is even.
Let \( u_1, u_2, \ldots, u_n \) be the vertices of \( K_n \).
Define \( f : V(K_n) \cup E(K_n) \rightarrow \left\{1, 2, \ldots, \frac{n(n+1)}{2}\right\} \) as follows:
$f(u_i) = \frac{n(n-1)}{2} + i, \text{ for } i = 1, 2, ..., n$

$f(u_{i+1}u_{i+1}) = \frac{n(n-3)}{2} + i, \text{ for } i = 1, 2, ..., n$

where $u_{n+1} = u_1$

For $2 \leq k \leq \frac{n}{2} - 1$,

define $f(u_{i+1}u_{i+k}) = (k-2)n + i$, for $i = 1, 2, ..., n$

where $u_{n+k} = u_k$ for all $k$.

Define $f(u_{i+1}u_{\frac{3}{2}}) = \frac{n(n-4)}{2} + i, \text{ for } i = 1, 2, ..., \frac{n}{2}$

As in the above case, $f^*(E(K_n)) = \left\{ \frac{n(n+1)}{2} + 1, \frac{n(n+1)}{2} + 2, ..., n^2 \right\}$

Thus $K_n$ is $E$-super arithmetic graceful.

\[
\begin{align*}
\text{Example 3.7.} & \quad \text{E-super arithmetic graceful labelling of } K_7. \\
\text{Fig 3.10} \\
\end{align*}
\]

\[
\begin{align*}
\text{Example 3.8.} & \quad \text{E-super arithmetic graceful labelling of } K_6. \\
\text{Fig 3.11} \\
\end{align*}
\]
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Anubala Sekar, Assistant Professor, Department of Mathematics, Sri Kaliswari College (Autonomous), Sivakasi, Tamil Nadu, India

Email address: anubala.ias@gmail.com

Ramachandran Varatharajaperumal, Assistant Professor, Department of Mathematics, Mannar Thirumalai Naicker College, Madurai, Tamil Nadu, India

Email address: me.ram11@gmail.com