IMVI
Open Mathematical Education Notes
Vol. 8(2018), 61-67
www.imvibl.org / JOURNALS / IMVI OMEN
DOI: 10.7251/OMEN1802061H

ISSN (p) 2303-4882, ISSN 1840-4383

Comparative analysis of mathematics curricula for 5th grades in Bosnia and Herzegovina, Croatia, Montenegro and Serbia Middle Schools

Amor Hasić
Mixed High School,
Petnjica, Montenegro
e-mail: amorhasic@gmail.com

Daniel A. Romano
International Mathematical Virtual Institute
Banja Luka, Bosnia and Herzegovina
e-mail: bato49@hotmail.com

Abstract. In this article, the authors offer a comparative analysis of mathematics curricula for the 5th grades of Middle Schools in Bosnia and Herzegovina, Montenegro, Croatia and Serbia by comparing teaching plans (annual number of hours) and elements of curricula (teaching programs). In this qualitative analysis of curricula, we rely on the experiences of other authors. Our intention in this principled orientation was to enable the international academic community of researchers in mathematics education to have a deeper and more reliable insight into the 5th Middle School grade curricula of mathematics of the aforementioned socio-political communities. In addition, special attention is devoted to the observed similarities and noticeable differences.

AMS Subject Classification (2010): **97B70, 97B20, 97D30**

ZDM Didactic classification (2010): B70, B20, C30, C20, D30

Key words and phrases: Comparative study of mathematics education, mathematics curriculum, 5th grade, Bosnia and Herzegovina, Croatia, Montenegro and Serbia

1. Introduction

Why is it important to compare mathematics curricula within two or more educational systems by looking at similarities and differences? It is quite a justifiable question that the reader can ask. What scientific information is displayed when comparing curricula (teaching plan and teaching programs) for 5th grade in Middle School[1] in Bosnia and Herzegovina [B&H], Croatia [CR], Montenegro [MNE] and Serbia [SR]? Comparative studies may be based on different assumptions - ranging from presumably universal models of mathematical competency to in-depth comparisons of local practices in two or more specific contexts. Didactics of mathematics is the science that takes, primarily, the specific knowledge about its object. This discipline is extremely specific and does not resemble any other specific school didactics since it contains very specific categorical notions, concepts and processes specific to school mathematics only. In the didactics of mathematics, several theoretical frameworks are available to organize and, indeed, make possible, different kinds and levels of research (e.g. [10], [14], [19]).

[1] The elementary school system in the observed societies is comprised of 8 (or 9) grades. Formally and declaratively they are divided into three triads, but the practice is realized in two forms: the Primary part (for children from 6/7 to 10 years of age) and the Middle part (for children from 11 to 15 years of age). In what follows we will use the term 'Middle School' to designate this second part of the Elementary school.

Under the term 'research in mathematics education' we mean activities in accordance with the second author's previously published paper [16]. After the introduction, in the second part of that paper, titled by "Determination of problem" the author showed some peculiarity of mathematics education research within the Stockes' perspective [24] on research domain. In the third part in that paper following ideas of Anna Sierpinska and Jeremy Kilpatrick, expressed in the editorial part of famous book "*Mathematics education as a research Domain: A Search for identity*" [23], the author presented five questions on research of mathematics education. In the fourth part of that article [16] the author surveys the history of research on theories in mathematics education.

In the present qualitative study the authors focused on comparative analysis of 5th grade of Middle School mathematics in four neighboring countries. The general and specific goals of teaching mathematics relying on Bloom's taxonomy[2] are compared. As is customary in Research in Mathematical Education, under the term 'Affective domain' in Mathematics education it will understand the following clusters: Beliefs, Attitudes, Valuation (and among other things, so called: Socio-mathematical norms, [28]) and Emotions and Feelings (including various forms of mathematical thinking, [15], [17]). Of course, the term 'proficiency' is used in accordance with the well-known book [11]: '*Adding it up: Helping children learn mathematics*'

Our goals of this article are the following:

- Study the mathematics curriculum systems of the 5th grade of the mentioned countries;
- Recognize the similarities and differences among them in the context of their socio-economic and cultural backgrounds.
- Identify general trends as well as the unique features of their problems in these areas.

This research topic is among the most important areas of interest of the academic community of researchers in mathematics education. The ERME Association at each of its meetings has a working theme 'Comparative study of mathematics education'. For example, the reader can look at the proceedings of the conferences from CERME 5 (2007) to CERME 10 (2017). The importance of this topic is confirmed by recent completion of doctoral dissertations in this area: for example, [9], [12] and [29]. Moreover, in the importance of comparative studies of mathematics, a reader can be convinced by looking at books [7] and [22]. Also, there are many comparative studies on mathematics education for Primary School teachers ([1], [3], [6], [10], [25]), but also about the education of Middle School mathematics teachers ([2], [4], [8], [21], [25], [26], [29]).

2. Our Findings

Our intention with this article is to open a dialogue between the designers of mathematical curricula. In the school systems of B&H, Croatia, Montenegro and Serbia, the term 'teaching plan' refers to the planned annual number of hours of mathematics. The term 'teaching program' refers to the objectives of teaching mathematics (general and individual), teaching contents, planned outcomes of mathematics and didactic instruction for teaching realizes. In B&H and Montenegro, Elementary school, the basic education system - primary grades (or children aged 6 to 10 years) and Middle School's grades (for children from 11 to 15 years of age) - lasts nine years. Therefore, the corresponding classes are: 6th (B&H), 5th (CR), 6th (MNE) and 5th (SR).

Information on mathematics teaching plans for the observed grade in Middle Schools in mentioned countries is presented in the following table:

State	Number of weekly classes	Number of annually classes	Grades
B&H	4	140	6th
CR	4	140	5th
MNE	4	136	6th
SR	4	144	5th

Table 1: Teaching plans of mathematics for V/VI grade

[2] It is assumed that the categories of Bloom's taxonomy are widely known to all researchers in mathematics education, and in this report we will not list the sources for them.

The teaching contents of mathematics for the observed grade are the following

Thematic content	**B&H**		**CR**	**MNE**	**SR**
	RS	FB&H			
Sets	+(16)	+(20)			+(15)
Sets of points	+(14)		+	+(7)	+(10)
Angle	+(22)		+	+(11)	+(20)
Circle line, circle, angle		+(20)			
Triangle			+		
Natural numbers		+(20)			
Numbers divisibility	+(16)	+(20)	+	+(26)	+(14)
Fractions	+(52)	+(30)	+	+(65)	+(61)
Decimal numbers			+		
Isometric transformation	+(24)				
Axial symmetry			+		+(16)
Axial and central symmetry				+(11)	
Written assignments with corrections					+(8)
Measurement of volume				+(12)	

Table 2: Teaching program of mathematics for 5^{th} /6^{th} grade

Legend: B&H is a very specific socio-political creation. This community is under the protectorate by so-called the international community. In B&H there exists two entities (The Republic of Srpska [RS] - 49% of the territory and the Federation of Bosnia and Herzegovina [FB&H] - 51% of the territory) and the 'Brčko District' in which only three so-called constitutive people enjoy almost equal rights. Entities and the District have a significant number of elements of statehood. The FB&H consists of 10 almost independent cantons (quasi-states). Education is in the full competencies of the RS, the District and each individual canton. In such a political-economic environment, collective education of all levels is realized. Therefore, all kinds of education are under the political and financial (only for public institutions) coverage of the government's administrations of the RS, the District and the cantons.

The official document of the Ministry of Education of the *Republic of Srpska* lists 12 general aims of teaching mathematics for the 6^{th} grade of the Middle School. For the sake of illustration we pointed out some of the first goals taken from the official document:

- Developing the ability of logical thinking (rules of formal logic);
- Developing basic mental operations: abstraction, comparing, ordering and generalization;
- Developing socio-affective goals, value orientations and positive relationship to science;
- Acquiring mathematical knowledge and abilities necessary for understanding quantitative and spatial relations and laws in nature and society;
- Developing students' ability to reason correctly and logically conclude, mathematically describe and model simpler phenomena and processes.

In FB&H's official document, the Ministry of Education in the *Central Bosnia Canton* stated 39 general aims of teaching mathematics for Middle School 6^{th} grade. As in the previous case, we will only list one of them (FB&H]: 'Knowledge and use of mathematical symbols' because we can classify all others into a cluster of knowledge or memory within the Cognitive domain according with Bloom's (revised) taxonomy.

In the official document of the Institute for Education of *Montenegro*, the goals of mathematics teaching are common for all nine grades of primary school. For the sake of illustration, we will cite all of them:

Through cognitive goals, students should:

- Master the calculation techniques;
- Understand mathematical rules and assertions;
- Adopts mathematical symbolism;

- Understand the mathematical language;
- Master the mathematical modeling techniques in solving textual tasks.

Through the process goals students should develop:
- Ability of logical thinking, conclusion, generalization and mathematical proof;
- The skills and ability to formulate problems;
- The ability to solve problems;
- The skills of interpreting the data presented in diagrams, tables or charts of different types;
- The skill of using geometric tools and measuring instruments;
- Ability to recognize situations in everyday life in which mathematical knowledge can be applied;
- Innovation and creative thinking;
- The ability of critical thinking;
- Cultural, ethical, aesthetic and work habits, criteria and abilities.

The official presentation of the National Council for Education of Serbia stands the aim of teaching mathematics in elementary school is:
- That students adopt the elementary mathematical knowledge needed to understand phenomena and dependencies in life and society;
- To enable students to apply mathematical knowledge in solving various tasks from real life, to successfully continue mathematical education and self-education;
- As well as to contribute to the development of mental abilities, the formation of a scientific view of the world and the comprehensive development of student personality.

Then, this document lists the tasks of teaching mathematics (12 items) and operational tasks (14 items).

3. Observation and Comments

We will not engage in commenting on planned teaching content in the curricula of various socio-political societies, because they are the choices of their designers. We will comment:
- On the highlighted goals of teaching mathematics in these curricula;
- The teaching tasks that these goals have for achieving;
- As well as didactic instructions on the basis of which mathematics teachers can reach these planned goals; and
- How math teachers can determine if they have achieved the planned goals.

Analyzing mathematics curricula for the 5^{th} (or 6^{th}) grades of the Middle School in B&H, Croatia, Montenegro and Serbia, it is not uncommon for designers of these programs to use words that have some other meanings in the domain of Research in Mathematics Education, but creators of these programs attributed to them. For example, the Montenegrian program mentions 'process goals'. In each of the observed programs, the wording 'tasks of teaching mathematics' is used with the meanings that we, as researchers in mathematics education, are not familiar with. Among the goals of teaching mathematics in all observed programs, one can notice the use of terms that associate with some of the cognitive domain clusters in accordance with Bloom's taxonomy. In the section 'Mathematics teaching tasks', terms and phrases are often used, such as the possibilities, abilities and skills. It is not unusual for program designers to use terms such as 'abstract thinking', and 'logical conclusions' (for example, Montenegro and Croatia, Bosnia and Herzegovina), 'critical thinking' (Serbia). In deep conviction of this article's authors, the program designers do not distinguish between the phrases 'geometric thinking' and 'spatial thinking'. In the use of the term "proficiencies", in the observed programs, it is possible to notice the terms that recognize as "conceptual understanding" and "procedural fluency" (cited according to [11]). In none of the observed programs, we were unable to recognize words that covered meanings such as "strategic competence", "adaptive reasoning" and "productive disposition" (also cited according to [11]). It was obvious that creators of the program do not use words that can be recognized as belonging to van Hiele's level of understanding geometry ([15], [17]) or some of the elements of arithmetic thinking, arithmetic-early-algebraic thinking, and algebraic thinking ([5], [18]).

It is not necessary to enter into the assessment of the justification of the teaching material for the mathematical program for the 5th or 6th grade in the Middle School in the mentioned socio-political

communities. These choices are the legitimate choices of their designers. However, it can evaluate how these programs are designed. One can evaluate the goals of mathematics teaching within the various clusters in the cognitive domain according the Bloom's taxonomy. Also, one can compare our conviction about the need for the existence of exact objectives of teaching mathematics within the affective domain and what we have managed to recognize in the offered programs. The authors will look at our observations in the form of the following questions:

- What is the principled-philosophical orientation of the policy makers in designing such programs of mathematics education for 5^{th} (or 6^{th}) Middle School grade?
- What theories of mathematics education and learning are incorporated in these curricula?
- Why is it not the possible to use the so-called 'differentiated approach' in teaching and assessing student success?
- In what way would a mathematics teacher identify what teaching tasks he or she should insist on in order to achieve the goals of teaching mathematics related to development of the set-relational thinking?
- What elements of logical thinking is it desirable to develop among students of 5^{th} (or 6^{th}) Middle School grade?
- What level of understanding of geometry, according to van Hiele's classification, the teacher can expect that his pupils recognize, accept in apply to solve the planned geometric tasks for this grade? Should the teacher in the construction of cognitive levels in his pupils also point out some level 2 elements other than element of level 0 and 1?
- What elements of early-algebraic thinking and which elements of algebraic thinking a teacher should develop in their students by teaching them divisibility in semi-ring N of natural numbers?
- What students' mathematical proficiencies will develop, apart from conceptual and procedural, when their teacher teaches them fractions and decimals?
-

4. Conclusion

Comparison of the structure of the Mathematics curriculum for the 5th Middle School grade in B&H, Croatia, Montenegro and Serbia shows that the characteristics of the program in the aforementioned socio-political communities are characterized by different assignments of their creators of education policies and designers of mathematical curricula. In addition, there are significant differences in curricula. In addition, there are significant differences in curricula between Croatia and Macedonia, on the one hand, and B&H, Montenegro and Serbia on the other hand. While Macedonia has taken the curriculum from professional program creators, Croatia has its own Croatian education system as a result of their specific orientation. In B&H, Montenegro and Serbia use very similar programs inherited from the school system of the former Yugoslavia. By our deep conviction, these programs are more consequences of the hasty political decisions on the independence and autonomy of the newly emerging social communities than the result of rigorous scientific thoughts.

References

[1] Andrews, P., Sayers, J. and Marschall, G. (2015). Developing foundational number sense: Number line examples from Poland and Russia. In: K. Krainer & N. Vondrová (Eds.). *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (CERME 9, February 4 – 8, 2015) (pp. 1681-1687). Prague, Czech Republic: Charles University in Prague, Faculty of Education and ERME.

[2] Back, J., Sayers, J. and Andrews, P. (2013). The development of foundational number sense in England and Hungary: a case study comparison. B. Ubuz, C. Haser & M. A. Mariotti (Eds.). *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education* (CERME 8, February 6 - 10, 2013) (pp. 1735-1844). Ankara, Turkey: Middle East Technical University and ERME

[3] Bjarnadóttir, K. (2007). Development of the mathematics education system in Iceland in the 1960s in comparison to three neighbouring countries. In: D. Pitta-Pantazi & C. Philippou (Eds.). *Proceedings of the*

Fifth Congress of the European Society for Research in Mathematics Education (CERME 5, February 22 - 26, 2007) (pp. 2403-2412). Larnaca, Cyprus: University of Cyprus and ERME.

[4] Cabassut, R. (2010). Examples of comparative methods in the teaching of mathematics in France and in Germany. In: V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello (Eds.). *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education* (CERME 6, January 28 – February 1, 2009) (pp. 2423-2432). Lyon, France: Institut National de Recherche Pédagogique and ERME.

[5] Crvenković, S., Milovanović, M. and Romano, D. A. (2012). Comparative analysis of the nature of mathematical knowledge used and constructed in the classroom. [Упоредна анализа природе математичких знања које се користи и конструише у учионици]. *Norma (Sombor),* **17**(2): 133-154 (In Serbian).

[6] Jablonka, E., Andrews, P., Clarke, D. and Xenofontos, C. (2017). Comparative Studies in Mathematics Education. In: T. Dooley & G. Gueudet (Eds.). *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (CERME10, February 1 – 5, 2017) (pp. TWG 11). Dublin, Ireland: DCU Institute of Education & ERME.

[7] G. Kaiser (2017). *Proceedings of the 13th International Congress on Mathematical Education ICME-13*. Springer

[8] Kastrati, J. K., Sajka, M. and Vula, E. (2017). Comparison of Kosovan and Polish pre-service teachers' knowledge of fractions. In: T. Dooley & G. Gueudet (Eds.). *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (CERME 10, February 1 – 5, 2017) (TWG 11: 1618-1625). Dublin, Ireland: DCU Institute of Education & ERME.

[9] Lu, Y. (2016). *A comparison of mathematics teaching and learning in China and the United States -: Problem solving skills in Geometry of Chinese and U.S. Students*. Ph.D. Dissertation. The Ohio State University.

[10] Milovanović, M., Romano, D. A. and Todić, V. (2012). Different theoretical aspects of the study of mathematics teaching [Различити теоријски аспекти истраживања наставе математике]. *Nova škola (Bijeljina),* **9**/**10**: 89-109.

[11] National Research Council (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatric, J. Swafford and B. Findell (Eds). Mathematics Learning Study Committee, Centre or education, Division of behavioral and social sciences and education. Washington, DC: National Academy Press.

[12] Pepin, B., Jablonka, E. and Cabassut, R. (2007). Comparative studies in mathematics education – comparing the incomparable? In: D. Pitta-Pantazi & C. Philippou (Eds.). *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (CERME 5, February 22 - 26, 2007) (pp. 2399-2402). Larnaca, Cyprus: University of Cyprus and ERME.

[13] T. L. Powell (2014). *A comparative analysis of the Singapore Math Curriculum and the everyday Mathematics Curriculum on fifth grade achievement in a large Northeastern Urban Public School District*. Ph.D. Dissertation. Seton Hall University.

[14] Romano, D. A. (2007). Thinking about mathematics education [*Razmišljanje o matematičkom obrazovanju*]*;* IV Symposium "*Technology, Informatics and Education for Learning and Knowledge Society*", Novi Sad, 26-27.01.2007. (pp. 82-90). Beograd: Institut za pedagoška istraživanja, Novi Sad: Centar za primenu nauke, tehnologije i informatike, Novi Sad: Prirodno-matematički fakultet. (In Serbian).

[15] Romano, D. A. (2008). On motives for research in mathematical thinking. *Nastava matematike (Beograd),* **53**(3-4): 1-11 (In Serbian)

[16] Romano, D. A. (2009). Research in mathematics education [Istraživanje u matematičkom obrazovanju]. *IMO-Istraživanje mateatičkog obrazovanja*, **1**(1): 1-10 (In Serbian)

[17] Romano, D. A. (2009a). On geometrical thinking [О геометријском мишљењу]. *Nastava matematike* (Beograd), **54**(2-3): 1-11 (In Serbian)

[18] Romano, D. A. (2009b). What is the algebraic thinking? [Šta je algebarsko mišljenje?]. *MAT-KOL (Banja Luka),* **15**(2): 19-29 (In Serbian)

[19] Romano, D. A. (2010). Methodology of mathematics – a scientific discipline? [Metodika matematike – naučna disciplima ?] V Symposium "*Technology, Informatics and Education for Learning and Knowledge Society*", Novi Sad, 19-20.06.2009. (Book 1: pp. 455-468). Novi Sad: Centar za primenu nauke, tehnologije i informatike. (In Serbian).

[20] Romano, D. A. (2010). Mathematics, Didactic of mathematics and Research in Mathematical Education – three related but different domains [Matematike, Metodika matematike i Istraživanje u matematičkom obrazovanju – tri srodna ali različita domena]. IMO-*Istraživanje mateatičkog obrazovanja*, **2**(2) 2, 3-10 (In Serbian)

[21] Saeki, A., Matszaki, A., Kawakami, T. and Lamb, J. (2015). Examining the heart of the dual modelling cycle: Japanese and Australian students advance this approach. In: K. Krainer & N. Vondrová (Eds.) . *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (CERME 9, February 4 – 8, 2015) (pp. 1745-1751). Prague, Czech Republic: Charles University in Prague, Faculty of Education and ERME.

[22] Schmidt, W., Blömeke, S., Tatto, M. T., Hsieh, F.-J., Cogan, L. S., Houang, R. T. , Bankov, K., Santillan, M., Cedillo, T., Han, S.-I., Paine, M. C., L. and Schwille, J. (2011). *Teacher Education Matters: A Study of Middle School Mathematics Teacher Preparation in Six Countries.* New York: Teacher College Press

[23] Sierpinska, A. and Kilpatrick, J. (1998). *Mathematics education as a research domain: A search for identity* (vols. 1 & 2). Kluwer Academic Publishers.

[24] Stokes, D. E. (1997). *Pasteur's quadrant: Basic science and technological innovation*. New York: Brookings Institution Press.

[25] Tchoshanov, M. Quinones, M. C., Shakirova, K., Ibragimova, E. and Shakirova, L. (2017). Cross-national study of lower secondary mathematics teachers' content knowledge in the USA and Russia. In: T. Dooley, & G. Gueudet (Eds.). *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (CERME 10, February 1 – 5, 2017) (TWG 11: pp. 1658-1665). Dublin, Ireland: DCU Institute of Education & ERME.

[26] Vula, E. Kingji-Kastrati, J. and Podvorica, F. (2015). A comparative analysis of mathematics textbooks from Kosovo and Albania based on the topic of fractions. In: Krainer, K., & Vondrová, N. (Eds.). *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (CERME 9, February 4 – 8, 2015) (pp. 1759-1765). Prague, Czech Republic: Charles University in Prague, Faculty of Education and ERME.

[27] Xenofontos C. and Andrews, P. (2017). Explanations as tools for evaluating content knowledge for teaching: A cross-national pilot study in Cyprus and Greece. In: T. Dooley & G. Gueudet (Eds.) (2017). *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (CERME 10, February 1 – 5, 2017) (TWG 11: pp. 1594-1601). Dublin, Ireland: DCU Institute of Education & ERME.

[28] Yackel, E. and Cobb, P. (1996). Socio-mathematical norms, argumentation, and autonomy in Mathematics. *Journal for Research in Mathematics Education*, **27**(4): 458-477.

[29] Zou, H. (2014). *U.S. and Chinese Middle School Mathematics teachers' pedagogical content knowledge: The case of functions*. Ph.D. Dissertation. Arizona State University.