

Notes on Geometrical and Pedagogical Knowledge of Prospective Primary School Teachers in the Republic of Srpska

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Abstract. The goal of this paper is to present to the international academic community some specificity in mathematics education of the Republic of Srpska [RS] primary school teachers. The paper draws on some previously published texts on the mathematical and methodical literacy of the RS prospective primary school teachers. The observations on this segment of mathematical education in the RS are the final comments of the author within the project „Research of mathematical and pedagogical literacy of prospective primary school teachers” which is realized by the Scientific Society of Mathematicians Banja Luka. The author deeply believes that accepting these observations and assessments can be a very useful argument to our academic community in negotiating with the RS social-political community on rising the quality of mathematics education. With this commitment, the author has revealed several examples of failures in the education of prospective elementary school teachers in the following domains: (a) specific geometric knowledge necessary for teachers; (b) specific methodological-geometric knowledge necessary for teachers, and (c) understanding of the process of teaching and the process of student's learning of geometry. These omissions in mathematics education of elementary school teachers are the systemic failures and cannot be eliminated within the existing educational paradigm by stronger efforts of the teachers themselves according to the author's deep belief.

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1. What is in this paper?

As a university teacher who, for more than 15 years, taught to prospective teachers mathematical-methodical knowledge and skills, the author gained beliefs about interrelationship between the principled-philosophical declarative orientations of the RS social community on the one side, the actual operationalization of these orientations through the school system on the other side, and common sense knowledge, abilities and skills of teachers of mathematical content in primary schools on the third side. The author gained an impression that the RS social community is trying to show that it is concerned with the mathematical education of elementary school pupils by providing them with school books and handbooks (but not free of charge), while, on the other hand, it can only identify social concern for students' teachers only with the principle-philosophical attitudes declared in mathematics curricula for grades 1-5 and with supervisory services.

Pupils' teaching of mathematical content in our primary schools is still realized into the traditional way. It seems that the mathematics education policy-makers in our political society (the Ministry of Education, the Republic Pedagogical Institute) and in the academic society (mathematics departments and chairs of mathematics didactics at public universities, the Mathematical Society of the Republic of Srpska) have not yet heard about the extremely strong interest of the international community in rising the quality of mathematics education in Elementary (Primary and Middle) schools. For example, a differentiated approach in teaching

mathematics is still not in use in our school system. The legislators (the National Assembly of the Republic of Srpska, the Ministry of Education, the Republic Pedagogical Institute, the managements of elementary schools) did not allow in any way for the realization of mathematics education process in a different way except for the traditional aspect. In the RS, education of primary school teachers is realized in two public universities (Banja Luka University and East Sarajevo University) and in two private universities (Independent University at Banja Luka and European University at Brčko District). Unfortunately, at all these institutions, only two university teacher candidates (Bijeljina teacher faculty at the East Sarajevo University and Teacher Department at Banja Luka Independent University) have a formal choice to be a lecturer of the didactic of mathematics.

All working mathematicians at universities are convinced that they are fully competent to be lecturers at all courses of the didactic of mathematics teaching. With these attitudes about their own competences, they forget that they have been educated thirty years ago. Although they have the ability to quickly self-educate in the didactic of mathematics education domain, they do not do that. Thus, their teaching of prospective teachers focuses on acquiring procedural knowledge both in mathematics and in the methodology of teaching mathematics. Thus, their students are deprived of knowledge in the philosophy of mathematical education and knowledge of contemporary theories of mathematics education. For example, they do not meet the elements of the 'Theory of didactic situations' and the 'Theory of realistic mathematics education'. Furthermore, their students are not subject to the requirements to recognize, understand and know how to apply the elements of Bloom's taxonomy [4], [20] to the goals of teaching mathematics. This last remark has a special value when speaking of the clusters of affective goals of teaching mathematics [17], [34], especially when it comes to socio-mathematical norms [36]. Therefore, their students are not familiar with the need to understand and to accept contemporary academic attitudes about mathematical thinking, mathematical values and socio-mathematical norms. From here on, we deduce that their students are not able to recognize the elements of various forms of mathematical thinking. Moreover, they are not able to select incentives that facilitate the development of set-relational, arithmetic, arithmetic-early-algebraic, algebraic, geometric and other forms of mathematical thinking. In connection with the aforementioned, they will not be able to assess the success of their future students; they will not be able to recognize types of mathematical tasks in order to reach the intended goals of teaching mathematics; for example, according to SOLO [8] or MATH taxonomy [32]. Based on the previously presented conclusions, prospective teachers educated by such instructors will be deprived of possessing significant methodical knowledge.

For the sake of illustration, of what the curriculum of the course "How to Teach Mathematics" is about, something that the author has, lectured for many years, the reader can see in the text [18]. That was a required course focusing on the following topics:

Theme 1: Mathematics and Teaching Mathematics;

Theme 2: Mathematics teaching methodology;

2.1. General socio-cultural conditions for teaching mathematics;

2.2. Law of the Primary Education in the RS;

2.3. Law of the Secondary Education in the RS;

2.4. Goals of teaching mathematics / Blum taxonomy;

2.5. Tasks of Teaching Mathematics;

2.6. Mathematical proficiencies;

2.7. Paradigms of mathematical education:

- School Mathematics (Domain of Mathematical Knowledge),
- Mathematical knowledge necessary for mathematics teaching,
- Methodology of mathematics teaching (mathematical-humanistic domain of knowledge),
- Understanding of mathematical education (humanistic-mathematical domain of knowledge).

Theme 3: Theories of Mathematical Education;

3.0. Traditional approach;

3.1. Constructive approach;

3.2. The theory of didactic situations;

3.3. Socio-cultural approach;

3.4. Theory of realistic mathematical education.

Theme 4: Mathematical thinking:

- 4.1. Motives for the study of mathematical thinking;
- 4.2. The Origin of Mathematical Thinking (Uri Leron's Theory);
- 4.3. Arithmetic thinking (Stanislav Dahne's research);
- 4.4. Early-algebraic and Algebraic thinking (Sheelie Kregler);
- 4.5. Geometric thinking (Geometric paradigms and the theory of van Hiele's level of understanding geometry).

Theme 5: Planning the teaching of mathematics:

- 5.0. Types of plans;
- 5.1. Linear planning;
- 5.2. Spiral planning;
- 5.3. MEP Project (Centre for Innovation in Mathematics Teaching, University of Plymouth).

Theme 6: Organization and realization of primary school mathematics teaching.

Theme 7: Mathematical Tasks (Classification of Mathematical Tasks).

Theme 8: Assessment of the success of mathematics teaching.

Theme 9: Professional Advancement (Research in Mathematical Education).

It is not unusual in our universities that significant participation in mathematical-methodical education for future teachers have persons without any mathematical and methodical competences. For example, a common case in practice is that the methodology of teaching mathematics is taught by a pedagogue without any formal mathematical competences other than high school mathematics only. Another such example that we can identify at our universities is that the course 'Elementary mathematical concepts' is taught by an elementary school teacher or a pedagogue. By solving the problem of the lack of staff at the departments for the preparation of the elementary school teachers at our universities in such an incorrect way, how can one correctly respond to the following questions

- What kind of categorical mathematical and methodical concepts will these lecturers use when they interpret basic geometric concepts such as points, lines and planes in accordance with the levels of van Hiele's theory [35] of understanding geometry?

- How can one distinguish classroom teaching in the second and fourth grade when teaching students the geometric concept of 'angle'?

- How can one identify student geometric knowledge about triangle and quadrangle on 'level 0' (Visualization level) and on 'level 1' (Analysis level) of van Hiele's theory?

The difficulties highlighting the last two issues deal with the fact that the aforementioned persons are not familiar, at all with van Hiele's theory of understanding geometry.

It is not difficult to estimate what kind of misunderstandings create elementary school teachers in their work, when being educated by such inadequate university lecturers. The described situation is present in our elementary school system since 1994. Now, at the end of 2017, when this paper is written, the school system in the RS is completely designed and it contains a lot of weak places in mathematics education. Some of these places are described above.

Assessing the level of geometric competencies of future teachers is a very important research topic ([1], [5], [7-8], [14-15], [17-18], [20-22], [21], [33]). The author has long been engaged in the evaluation of geometric literacy of pre-service primary school teachers (see, for example, [23-25]).

2. Some disturbing examples of RS teachers' mathematical skills quality

We remind the reader that a teacher should recognize, distinguish and be able to understand and accept differences in the following domains ([2], [3]):

- School mathematics (Collective mathematical knowledge of high school students);
- Mathematical knowledge necessary for teachers to teach mathematics (Specific mathematical knowledge necessary for teachers);
- Methodological knowledge necessary for teaching mathematics;
- Understanding the process for teaching mathematics and understanding the process of students' learning;

- Mathematics curriculum for the appropriate grade;
- An awareness of the mathematical horizon \ the frontiers of mathematics knowledge.

In addition, we remind the reader that a lecturer of elementary school mathematical contents should be familiar with categorical terms describing mathematical proficiencies in accordance with [19]. Therefore a primary school teacher should be able to recognize, understand, accept, and correctly apply the following categorical concept-ideas in mathematics classroom: conceptual knowledge, procedural skills, strategic abilities, and adaptive reasoning.

Why is this paper designed in that way? What does the author want to achieve with this paper? What is the purpose of its content? Who are the people whom the author would like to be introduced to the content of this paper? - Those are the questions to which the author will offer observations and reflections in the sections that follows as acceptable answers to them.

2.1 The first set of examples: *segment line*

In this set of examples, the author will present his observations on the problems encountered by prospective elementary school teachers mathematical and methodical knowledge of the geometric concept of 'segment line'. About this the author wrote in [26].

In the written part of the exam, students were asked to provide answers, among others, to the following questions:

- (a) Describe the concept of segment line;
- (b) How many points does a segment line have?
- (c) Offer arguments for the assertion: "*There is a limited set of points on a segment line.*"
- (d) Describe the relationship between a point at a segment line and the segment line itself; and
- (e) Points on a segment line are in some type of order. Describe that order.

The population consists of 137 students of the third year of a teacher education program in Bosnia and Herzegovina [B&H]. 77 of them agreed to be tested. Table 1.1 presents students' performance on question (a).

Quality of responses	0	1	2	Σ
number	36	31	10	77
%	46.75	40.26	12.99	100.0

Table 1.1: Distribution of students' success of question (a) (N = 77).

Codes: Code 0 means that student did not even try to respond to the question or that the information offered was completely unacceptable; 1 – acceptable answer; 2 – correct answer.

The distribution of the evaluation of student reflections to question (b) is shown in Table 1.2.

Quality of responses	0	1	2	Σ
number	57	13	7	77
%	74.03	16.88	9.09	100.0

Table 1.2: Distribution of students' success of question (b) (N = 77).

Codes: 0 – the student did not offer any answer or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

Comments regarding questions (a) and (b): Student responses can be divided into clusters taking into account the nature of the geometric paradigm. We also identified several types of answers to questions (a) and (b) about the segment line. This allows us to highlight the main types of students' approaches:

First, the answers based on the analytical approach to the concept of segment line are very rare - we identified that the number of such students is less than 10%. We identified that a smaller group of the tested students has intuitive knowledge that points at a segment line have infinitely many, but they do not distinguish between potential and actual infinity. For the purpose of illustration, the cluster of acceptable responses

contains, inter alia, the following answer: ‘The segment line has an unlimited number of points.’ It is surprising that a very large number of tested students do not have an intuitive idea that a segment line has infinitely many points. Difficulties with understanding the 'amount of points that make segment line' are very clear represented by students' answers. From the offered range of answers, the conclusion is that the interviewed students apparently use the (mathematics) noun 'infinitely' as the verbal phrase 'to prolong the process unlimited', although it is not clear to them which concept is hidden behind the term 'unlimited'.

The distribution of the evaluation of student reflections to the third question (c) is shown in Table 1.3.

Quality of responses	0	1	2	Σ
number	61	15	1	77
%	79.22	19.48	1.3	100.0

Table 1.3: Distribution of students' success of question (c) (N = 77).

Codes: 0 – the student did not offer any answers or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

Comments on question (c): Tested students do not have knowledge and understanding about the mathematical concept of being *limited*. Their notion of unlimited is without any relation to the concept of the order relation among the points on the line.

The distribution of the evaluation of student reflections to the fourth question (d) is shown in the Table 1.4.

Quality of responses	0	1	2	Укупно
number	74	3	0	77
%	96.1	3.9	0.00	100.0

Table 1.4: Distribution of students' success of question (d) (N = 77).

Codes: 0 – the student did not offer any answers or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

The distribution of the evaluation of student reflections to the fifth question (e) is shown in Table 1.5.

Quality of responses	0	1	2	Σ
number	77	0	0	77
%	100.0	0.00	0.00	100.0

Table 1.5: Distribution of students' success of question (e) (N = 77).

Codes: 0 – the student did not offer any answers or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

Comments regarding question (d) and (e): The term segment line (as well as the notion of the straight line preceding it) is introduced into mathematics classes in the second grade of the elementary school at an intuitive level (i.e., at 'level 0' in the van Hiele classification), that is, it is not defined. By that we mean that the mathematics teaching 'shows' the model of a straight line (physical model, model in nature, model drawn on a board or computer screen) and tells the students the name of that object. The term segment line, in a methodological sense for teachers and students, is a term that follows from the notion of straight line. Descriptively, in the first approximation (that is, roughly speaking), a segment line is a part of the line between two points on that line. So, the terms that precede the term segment line are: 'straight line', 'line', the 'points on the straight line', and the 'relationship between the points'. These points have the function of 'borderline points on the line'. This way of looking at the term segment line from the intuitive aspect in itself implies the need by apriori acceptance of the concept of 'boundary' points. Methodologically, this procedure has many omissions. The concept of 'boundary point' is necessary in the further construction of geometric

knowledge because they are used in the analysis of many other geometric figures. In these analyses, at 'level 1' (in the van Hiele's classification), edge / border features are crucial in understanding the interrelations of the elements of these analysed figures. So, the disposition of points at a line is important for recognizing, understanding and accepting the concept of boundary points. The concept of the relationship between the points on the line is indicated by the word *before* as one of the basic categorical terms. To the author's surprise, the tested students have shown that they have a total absence of the perception of this categorical term. In addition, this statement also is suggested to us by our empirical experience shown in the Table 1.5.

2.2. The second set of examples – relationship between points, lines and planes

In interaction with students of the third and fourth year of the teacher education program, the author got the impression that these students have omissions in their previous geometric education. In order to evaluate the extent to which the basic geometric knowledge necessary for mathematics teaching and the actual geometric knowledge of these students, the author tested the population of students in 2011 by asking them questions about the interaction of points, lines and planes. Some of the collected experiences are presented in [28].

The first series of questions

The first question was: Q1. *Determine the relationship between the point and the line.* Table 2.1 shows the distribution of student responses to question Q1 (N = 63):

Quality of responses	0	1	2	Σ
number	35	15	13	63
percentage	55.56	23.81	20.63	100.0

Table 2.1: Distribution of students' responses to question Q1 (N = 63)

Codes: 0 - the student did not offer any answer or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

The next task was: Q2. *Determine the possible relationships between the two lines.* Table 2.2 shows the distribution of student responses to question Q2 (N = 63):

Quality of response	0	1	2	Σ
number	29	16	18	63
percentage	46.03	25.4	28.57	100.0

Table 2.2: Distribution of students' responses to question Q2 (N = 63)

Codes: 0 - the student did not offer any answer or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

The third question was: Q3. *Determine the relationship between a point and a plane.* Table 2.3 shows the distribution of student responses to question Q3 (N = 64):

Quality of responses	0	1	2	Σ
number	41	16	7	64
percentage	64.06	25.0	10.94	100.0

Table 2.3: Distribution of students' responses to question Q3 (N = 64)

Codes: 0 - the student did not offer any answer or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

The fourth question was: Q4. *Determine the relationships between a line and a plane.* Table 2.4 shows the distribution of student responses to question Q4 (N = 64):

Quality of responses	0	1	2	Σ
number	47	13	4	64
percentage	73.44	20.31	6.25	100.0

Table 2.4: Distribution of students’ responses to question Q4 (N = 64)

Codes: 0 - the student did not offer any answer or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

The last question in the first series of tasks was: Q5. *Determine the relationships between two planes.* Table 2.5 shows the distribution of student responses to question Q5 (N = 62):

Quality of responses	0	1	2	Σ
number	53	6	3	62
percentage	85.48	9.68	4.84	100.0

Table 2.5: Distribution of students’ responses to question Q5 (N = 62)

Codes: 0 - the student did not offer any answers or the student offered a response that cannot be classified or unacceptable answer; 1 – acceptable answer; 2 – correct answer.

The second series of questions

What level of understanding of geometry is induced to students by teaching them links about connecting among these basic geometric concepts?

The distribution of students' recognition of the cognitive levels of the geometric thinking developed by answering Question 1 is shown in the Table 2.6.

Quality of responses	0	1:0 Level 0	1:0-1 Level 1	2 Level 2	Σ
number	19	44	17	5	63
percentage	30.16	69.84	26.98	7.94	

Table 2.6: Distribution of students’ responses to question (N = 63)

In this table, displaying numbers is not the same as in the previous tables. Numbers 17 (column 1: 0-1, Level 1) and number 5 (column 2, Level 2) are contained in number 44 (column 1: 0, Level 0) and number 5 is contained in number 17.

Codes: 0 - the student did not offer any answer or the student offered a response that cannot be classified or is an unacceptable answer; 1:0 – acceptable answer - student recognizes level 0; 1:0-1 – acceptable answer - student recognizes levels 0 and 1; 2 – correct answer - student recognizes levels 0, 1 and 2.

The third series of questions

What are the tools of logical thinking that our students understand and accept when we teach them the relationship between these basic geometric concepts?

Students’ identification of logical tools [TND] and non-contradiction [nonK] is presented in Table 2.7.

Quality of responses	0	1	2	Σ
number	48	13	2	63
percentage	76.19	20.63	3.17	100.0

Table 2.7: Distribution of students’ responses to the question (N = 63)

Codes: 0 - the student did not offer any answers or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer - student recognizes TND; 2 – correct answer - students recognizes TND and nonK.

2.3. The third set of examples: *the concept of line*

In 2014, the author interviewed 57 students of the teacher education program on the topic of how to teach primary school students teaching to geometric concept of line. The participants were offered the following question:

How to teach primary school students (grade 2-5) geometric concept of line?

The results of this interview with students are presented below [12].

Quality of responses	0	1	2	Σ
number	54	3	0	57
percentage	94.74	5.26	.0	100.0

Table 3.1: Distribution of students respond to the question (N = 62)

Codes: 0 - the student did not offer any answers or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

The tested students do not distinguish between the straight line (which is a basic concept in the teaching methodology and a teacher has an obligation to teach his/her students about this object at an intuitive level in the second grade mathematics) and the geometric concept of line as an abstracted object (a teacher must offer more clear explanation on this concept in the fourth grade mathematics).

2.4. The fourth set of examples: *the concept of ray* [29]

The tested students were asked to show their perception of the geometric object ray. In addition, they were also asked to design a method by primary school pupils could be introduced to this geometric object. Students in our school system meet the geometric concept of ray in the second and fourth grades. This encounter should take place in two different ways (van Hile's classification) in the second grade - at level 0 and in the fourth grade - at level 1.

More specifically, the following questions were asked:

Question 1: Describe the geometric object of a ray. Which of mathematical concepts is preceding this description?

Question 2: How do elementary school teachers teach this geometric concept to their students?

(1) at ‘level 0’? and

(2) at ‘level 1’?

So, from the students we expected that in addition to the near-definition, they also to would offer their thoughts on the categories they used in designing their description. If students offered an acceptable description, we classified it into cluster 1; if at least the list of previous terms was offered, we classified it into cluster 2. The distribution of the evaluation of student reflections to the first question is shown in Table 4.1.

Quality of responses	0	1	2	Σ
number	82	44	18	144
%	56.94	30.56	12.5	100.0

Table 4.1: Distribution of students respond on the Question 1 (N = 144).

Codes: 0 – the student did not offer any answers or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

The part (a) of the second question required from students to demonstrate their knowledge of geometric teaching methodology. They were expected to show that they know which requirements and their fulfillment are covered by the term 'level 0'. The distribution of the evaluation of student reflections to part (a) of the second question is shown in Table 4.2.

Quality of responses	0	1	2	Σ
number	23	95	26	144
%	15.97	65.97	18.06	100.0

Table 4.2: Distribution of students' success of part (a) of question 2 (N = 144).

Codes: 0 – the student did not offer any answers or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

Part (b) of the second question required the tested population to offer their thoughts about the requirements that pupils of the fourth grade of elementary school should fulfill in order to assess the elementary school students' knowledge as level 1. The distribution of the evaluation of student reflections to part (b) of the second question is shown in Table 4.3.

Quality of responses	0	1	2	Σ
number	40	97	7	144
%	27.78	67.36	4.86	100.0

Table 4.3: Distribution of students' success of part (b) of question 2 (N = 144).

Codes: 0 – the student did not offer any answers or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

2.5. The fifth set of examples: *the concept of angle* [9, 10]

In articles [9] and [10], authors offer their experiences gained during the interview with students of two teacher education program on the topic of angle in the primary school. During the final assessment of success, on two occasions, students were asked to explain their view of introducing the term 'angle' in lower grades of the elementary school.

Task 1: Offer an explanation at level 1 of the term 'angle'.

Table 4.3 shows the student's success on the Task 1:

Quality of responses	0	1	2	Σ
Test 1	80.70%	8.77%	10.53%	100%
Test 2	79.69%	9.38%	10.93%	100%

Table 4.3: Distribution of students' success during Test1 and Test 2

Codes: 0 – the student did not offer any answers or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

Task 2: How do we distinguish the 'inner domain' of an angle and 'outer area' of same angle?

The notion and the word 'angle' are introduced into mathematics of the second grade depending on the intuitive ability of students. This is mainly done by showing to students one (or more) models of angle. In order to recognize the student's knowledge about angle at 'level 0', it is enough for a student to recognize the angle, know how to distinguish it from other figures and know its name. Teaching elementary school students to the geometric concept of the angle at level 1 is cognitively demanding. Students should be able to

- recognize the elements of any angle,
- know the names of these elements,
- describe these elements as independent geometric figures and

- recognize the interaction between the elements of the observed angle.

Elements of a single angle are the angular line, the apex, the inner region and the outer region of that corner. Table 4.3 shows the student's success on the questions asked,

Quality of responses	0	1	2	Σ
Test 1	90.79%	9.21%	.0%	100%
Test 2	65.15%	28.79%	6.06%	100%

Table 4.3: Distribution of students' success during Test1 and Test 2

Codes: 0 – the student did not offer any answers or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

The aim of research presented in [9] and [10] was to access the achievement of one generation of students through a qualitative analysis on two occasions on their methodical knowledge in connection with ‘angle’ as a geometric object. In these texts, the authors intention was to point out some of the dilemmas encountered by prospective elementary school teachers in mastering the knowledge and skills of teaching their future students about the term "geometric notion of angle" at 'level 1': How to introduce the notion of the angle in the lower grades of the primary school, so that the students construct mathematically correct knowledge about this abstract geometric concept.

The analysis of the collected student reflections on these requirements suggested to us that the tested population of students has an extremely poor knowledge of this geometric figure. In both tests, 80% of the tested population did not offer an acceptable description of angle at the analytical level. However, there was a significant difference in the quality of student reflections on the methodical question, Task 2, in two interviews. It is explained by the authors’ assumption that students were significantly better prepar for second testing.

Regardless of the previous assumption, the findings of the author of the present research are alarming: 80% of the tested population have no ability to describe the concept of 'angle' at the analytical level. On the other hand, slightly more than one-third of the tested students have knowledge of how to teach students about the concept of angle at analytic level, although they do not know how to do it effectively.

2.6. The sixth set of examples: *the concept of triangle*

The students discussed the following issues:

Describe the triangle at the '0' level. Which concepts precede this description?

Give a description of the triangle at 'level 1'.

In [11], the authors present their observations of prospective elementary school teachers' knowledge about triangle. The results of the research showed that students' knowledge and understanding of the geometric concept of triangle are not satisfactory. 24 out of 65 students did not provide any information on the questions asked. The other 41 students offered their knowledge and understanding about this figure on estimation. 62.5% of the tested population showed that they do not have consolidated knowledge and ability at level 1 of understanding of the concept of triangle.

2.7. The seventh set of examples: *the concept of quadrilateral*

In [30], the author offers the collected students' reflections on the following two research questions:

Which cognitive plane is predominantly present in the tested population of 63 prospective primary school teachers in connection of the concept of quadrilateral?

Have the tested population consolidated methodical knowledge on how to teach students of lower grades of elementary school about the aforementioned geometric concept?

For this purpose, students were asked: What is a quadrilateral? The distribution of students’ responses to this question is shown in Table 6.1.

Quality of responses	0	1	2	Σ
number	33	21	9	63
%	52.38	33.33	14.29	100.0

Table 6.1: Distribution of students' success to the question (N = 63).

Codes: 0 – the student did not offer any answers or the student offered a response that cannot be classified or is an unacceptable answer; 1 – acceptable answer; 2 – correct answer.

Quadrilateral is a polygon with (only) four sides. A quadrilateral's elements are the vertices, sides and inner angles. The tested students recognized a quadrilateral but they were not precise in its description and even less precise in defining it. Thus, a significant number of the tested students show that their cognitive plane in which they recognize quadrilateral is at the 'level 0' (according to van Hiele's classification). 16 teacher candidates (or 25.4% of the total number) did not offer any answer to the question, while 17 teacher candidates (26.98%) offered an unacceptable answer. Thus, for 33 future teachers (or more than half of the tested teacher candidates), in this test, they were found not to be able to describe quadrilateral by using acceptable geometric categories. 30 teacher candidates (47.62%) have developed the ability to see and describe quadrilateral, while only 9 teacher candidates (14.29%) offered a precise definition. How to understand the information obtained that half of the tested population is not able to describe quadrilateral, using acceptable categories of the geometric language? Of course, we cannot be satisfied with the statement that, in this case, is imposed on itself: *The tested students do not know how to describe quadrilateral in an acceptable way within the geometric language.* The author is more inclined to accept the following assertion:

The tested population of future teachers in their previous mathematical education was not often in position to formulate their understanding of these questions.

2.8. The eight set of examples: the concept of rectangle

Finally, in paper [27], the author pointed out that prospective primary school teachers do not have consolidated knowledge of the geometric figure of rectangle. In that paper, the author offered some of the collected data of pre-service primary school teachers' perception of the geometric concept of rectangle. In order to establish what students think about the concept of a “rectangle” and how they understand it, the author tested 65 students of the Faculty of Education in Bijeljina and Bihać within the multi-year research project “Establishing the Level of Mathematical Literacy” realized by the Scientific Society of Mathematicians Banja Luka. The testing was carried out in the period February - March 2013. Within the framework of the test, teacher candidates were asked questions about rectangle, how they understand it and how it should be presented to students in lower grades of the primary school:

- Q1. Describe a rectangle;
- Q2. Give the precise definition of rectangle; and
- Q3. How do teachers teach the primary school students about the concept of rectangle:
 - (a) In the second grade?
 - (b) In the fourth grade?

The results of the students' responses to Questions 1 and 2 are shown in Table 8.1.

Required concept	0	1	2	Σ
Q1	5	28	32	65
%	7.69	43.08	49.23	100.00
Q2	10	48	7	65
%	15.38	73.85	10.77	100.00

Table 8.1: Distribution of students' success on questions Q1 and Q2 (N = 65).

Codes: 0 – the student did not offer any answers; 1 - the student offered a response that cannot be classified or is an unacceptable answer; 2 – acceptable answer.

To conclude this section, we offer the following two questions:

What information on rectangles will the tested students pass on to their future students in the fourth grade of our primary school?

What kind of social-mathematical norms in relation to rectangle will be adopted by future students of the tested teacher candidates?

3. Conclusion / Personal Observations

This research is aimed at gaining an insight into the perceptions of the Faculty of Education students about some geometric concepts and processes associated with them. The author(s) tested third and fourth year students of two faculties of education in B&H. Based on the feedback, the author is inclined to conclude that the tested population has significant difficulties with determining the basic geometric concepts. This initial study suggests that the misconceptions of the lower primary school students regarding these geometric figures are the result of their teachers' misconceptions. Therefore, the problems of teaching geometry in lower grades of the primary school should be resolved by shifting the focus from pupils to their teachers.

This paper aims to share the author's reflection on collected information on cognitive structures constructed with the prospective primary school teachers according to mathematical and their personal concept of several geometric objects. The study was conducted with the students of two pedagogical faculties in B&H in the framework of multi-year research project 'Research of mathematical and methodical literacy of pedagogical faculties students' implemented by the Scientific Society of Mathematicians Banja Luka. The research results suggest to us that students, even though they have the correct intuitive picture of what the studied geometric figures are, only with considerable difficulties they form acceptable description of these geometric figures. These difficulties are not the result of students' poorly developed necessary mathematical proficiencies. The author inclined to accept the hypothesis that these difficulties, mainly, are a consequence of the design of mathematical education in our school system. The aforementioned assessments are based on analysis of tested students' collected reflections on questions related to the concepts of the mentioned figures.

If someone asks the question:

What should our socio-political community do in the near future regarding the uplifting of the quality of primary school mathematics education?

one of the answers could be as following: Our socio-political community should accept the following obvious conclusions:

- (1) The traditional approach to mathematical education does not work.
- (2) The orientation about the principle-philosophical foundations of mathematics education should be dealing with the educational constructivism.
- (3) Accept that the theory of mathematics teacher education may be the Theory of Didactic Situations or the Theory of Realistic Mathematics Education.
- (4) Develop and adopt a new law on primary education that would enable the introduction of differentiated teaching of mathematics and require the acceptance of the Standards of mathematics education.
- (5) Build and adopt a modernized curricula of mathematics education.
- (6) Prepare, adopt and apply instructions for the realization of mathematics teaching that would be much more detailed than it had been so far: (i) forms of global and operational annual plans for the realization of mathematics education; (ii) forms of standard assignments for students' assessing of mathematical literacy; (iii) algorithms for evaluating students' mathematical proficiencies and etc.
- (7) Accept the obligation of yearly external and anonymous testing of students at the beginning and at the end of each school year.
- (8) Accept the obligation to spend 30% of school time in the plans for the operationalization of teaching mathematics used for repetition and internal student testing.

As the last observation, we offer our conviction that primary school teachers bear less than 5% of total responsibility for the quality of mathematics education in the RS according to my approximate assessment of the responsibility based on the decades-long experience of teaching at high schools and universities.

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