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The use of mathematical tasks design to establish development of student's mathematical thinking by an admission exam at Faculty of Mechanical Engineering of the Banja Luka University

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Abstract. In this article we describe some principles of tasks design by the classification of entrance examination at University of Banja Luka Faculty of Mechanical Engineering. Through the analysis of those tasks we identify some delicate points in the design of examination tasks. We offer a hypothesis that teacher's understanding of the designing process of examination tasks opens a possibility to enhance students' thinking when solving such tasks.

Key words and phrases: *task design, problem solving, students' success*

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AMS Math. Subject Classification (2010): **B40, C30, D60**

1. Introduction

Analyzing success in solving mathematical tasks by the candidates applying to the Faculty of Mechanical Engineering in Banja Luka is the opportunity to determine the mathematical proficiencies of high school graduates in our educational system. This, of course, is not our scientific interest. Our research interest is to design tasks for the testing and the feedback parameters of such designed tasks. Also, our research interest is to empower our hypothesis that there is a gap between the proclaimed goals of mathematics education in our community and established the mathematical proficiencies of the tested candidates. Our research goal of this analysis is to understand the factors that lead to it. Leaving aside the competence of mathematics educators who teach the tested students, the primary objective of this analysis is to understand why the applied didactic lead to this gap.

Methodology applied in the choice and selection and/or design questions and tasks for the mathematical test of the qualifying exam at the Faculty of Mechanical Engineering is in the accordance with the determination of the elements of mathematical proficiencies such as *understanding concepts, procedural fluency, strategic competence, adaptive reasoning,* and *productive disposition.* [15] The adoption of mathematical concepts and their fluent use in procedures and also encouragement the development of various forms of mathematical thinking should be the main goals of school mathematics education. Thus, tasks were selected and designed to enable the establishment of the existence of the elements of mathematical thinking (i.e., logical, arithmetic, early-algebraic, geometric and algebraic thinking, etc) in cognitive planes of tested candidates at the basic and advanced levels.

In the literature, there are several taxonomies of mathematical tasks relatively close to each other. Sangwin [21] offers a reflection on the milieu in which we should design the test tasks. His opinion is that the meaning of objects and concepts and their processes with them should be based on the analysis of what is really required of students to do.

The problem of designing teaching materials and, in particular, the design of mathematical tasks to determine the performance of students in mastering the mathematics proficiency is attracting insufficient attention of the scientific and academic communities. We have the viewpoint that every teacher before embarking on understanding the paradigms of designing mathematical tasks to test his students should be aware of the following issues.

(A) Identification of the mathematical problem that will be placed before students through the task;

(B) Identification of the goals of mathematics teaching could be accomplished by a student solving specifically selected or designed task;

(C) Identification of principled and philosophical orientation of teacher's teaching and student's learning that are involved in specifically selected or designed task;

(D) Identifying the level of communication in the didactic triangle 'teacher - task - student' by the expected outcomes and the complexity of the specifically chosen or designed task.

This article is basically writen for our academic society and the management of the Faculty of Mechanical Engineering in Banja Luka. We believe that it can be useful for students of the second and third cycles of study groups for education of mathematics teachers at all levels of education.

2. Motivation

In the Republic of Srpska, an entity of Bosnia and Herzegovina, education had a centralized system. Our experience is that many teachers have university reluctance to carefully choosing mathematical tasks for entrance exam in technical faculties at our public universities. In most cases, they choose questions from available collection of mathematics tasks. In all of that, they mainly judge candidates' algorithmic skills. Our conception is different. We always take this change to learn something more on candidates' mathematical proficiencies than the official information from their certificates. Different approach has the potential to perceive not only the cognitive parameters, but also some affective parameters.

3. Structured problem solving

Almost every teaching lesson in mathematics follows a certain form, which Stigler and Hiebert [22] identified as 'structured problem solving'. In our teaching practice, almost always the activity simplifies focusing on the solving of one or more non-linear complex tasks (or within SOLO - taxonomy, multi-structural tasks). Activities in solving such tasks are usually broken down into the following four phases:

1. Presentation of the problem;
2. The student problem solving;
3. Comparing of the obtained solutions and assessing their eligibility;
4. Teaching sublimation.

The phrase 'representation problem' typically involves the teacher addressing students in order to help them understand the context of the task and what is expected as an acceptable solution to this task. It does not include any instructions on how the task can be solved. Instead, the students are expected to work independently of each other trying to find a solution of the given task within a real-time. During that time, at least some of the students should construct a solution to the problem.

This concept assumes that students during independent work offer a variety of procedures to solve the given task. In the third phase, methods and task solutions are compared. So, in order to support all students to understand mathematical concepts and used and applied procedure, teacher encourages the development of elements of mathematical thinking. To stimulate the development of thinking should be the task understandable for the students with minimal teacher intervention and the task should be solvable.

In the fourth phase, the teacher can say something about why the chosen strategy is sophisticated and which are mathematical and education values of the task.

Resat and Strässer [20] identified the student mathematical activities as one of Vygotsky's concept of instrumental acts [23], whereby the student interaction with mathematical ideas is implemented by mathematical tasks. It seems to be extremely important to notice the functional connections of mathematical tasks with stimulation of student learning and teacher's efforts to teach students. In the context to the previous one, the mentioned researchers in mathematics education reconceptualized well-known didactic triangle (teacher - student - Mathematics) in the socio - didactic tetrahedron whose vertexes are teacher (T), student (S), Mathematics (M) and the mathematical task (MT). This reconceptualization of didactic relationship is to point out existence of multi-layer connectivity than previously presented, in this case, the sides of the tetrahedron.

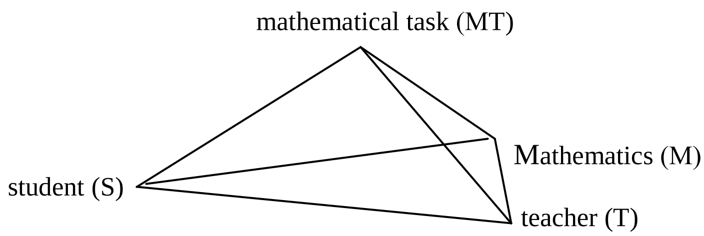

Figure 1. Socio - didactic tetrahedron

It can be said that each of the parties, in this case, each of these triangles, point out the existence of specific aspects of looking at mathematical tasks within the mathematics education of students.

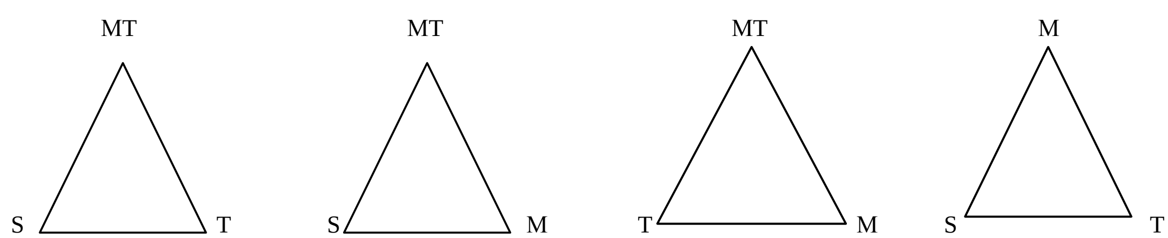

Figure 2. Didactic triangles of the socio - didactic tetrahedron

Didactic role of the teacher is very well described as the orchestration of student activities through the triangle *teacher* - *task* - *student.* Triangle *student* - *task* - *Math* represents their activities necessary for learning math, and the triangle *teacher* - *task* - *math* is a good didactic activity for teachers in the mathematics teaching (so-called construction of a fundamental situation within the Theory of didactic situations in mathematics education [7]).

Pointon and Sangwin [18] have developed taxonomy of mathematical questions with the intention to cover the classification of mathematical tasks:

1. Describe the facts;

2. Exposure of routine calculations or working with algorithms;
3. Classification of mathematical objects;
4. Interpretation of the situation or the reply;
5. Proving, showing the estimate (general argument);
6. Extending the concepts;
7. Construction of the examples;
8. Analysis of errors.

This aspect of the problem solving implies the specific requirements when designing such tasks.

4. Designing of mathematical tasks

Activities involved in the formation of hypothesis can be categorized based on whether they are related to (1) curriculum, (2) students, (3) mathematics, or (4) tasks. However, determining the student's performance focuses mainly on whether the selected tasks are in accordance with the objectives of the research. Researchers in mathematics education make a distinction between *learning how to solve tasks* and *learn mathematics through problem solving*. This implies that students are introduced to a large number of mathematical concepts and their fluent use to focus on one to three tasks of the same type. If tasks are carefully selected, they allow the students to understand new mathematical ideas in these tasks by using new mathematical objects, processes, and practicing procedures with them.

Students' independent work in problem solving stimulates the development of their *strategic competence, adaptive reasoning,* and *productive disposition*.

We have the following principles of task design [11], [12]:

1. The suitability and mathematical significance of the term used in accordance with the goals of the task;
2. Task meaningfulness of interest to the students;
3. Task complexity gave to bi in line with the task aim;
4. Asking questions in the task enable to find different strategies to solve it;
5. Strategies applicable in the task are applicable for solving other mathematical tasks or real life problems;
6. The task has the potential to be ongoing to resolve this notice, and may accept some desirable social and socio-mathematical norms.

In [13] one can find appear to be following the principles of tasks design:

Principle 1*: There is a real embedding of the concept of task in the students' real world.*

Principle 2*: There is a possibility of identification and specification mathematically important questions of general statements.*

Principle 3*: Formulation of at least one algorithm to solve the task is feasible knowledge and skills available to the student at this level of education with the construction assumptions and gathering the necessary data given.*

Principle 4*: The mathematical solution set of basic problems is in a good correlation with the knowledge and skills of students.*[1]

Principle 5*:* The *procedure for assessing the success of the mathematical correctness, and the acceptability of the decision with respect to with the context of the task requires.*

[1] Put another way, more comfortable speaking, within the school system of mathematics education, Principle 3 and Principle 4 score mandatory solid high correlation curricula in mathematics education of students with assigned tasks.

Didactical principle: *The problem may be structured into sequential questions that retain the integrity of the real situation.*[2]

Doig, Groves and Fujii [9], [10] emphasize the following four types of tasks that are commonly used:

1. Tasks that are directly related to a mathematical concept;
2. Tasks which encourage fluency of mathematical procedures;
3. Tasks that precisely examine the possibilities;
4. Tasks that establish an understanding or misunderstanding of the concepts and processes with them.

5. An example of design tasks with analysis of students' outcome

This section describes the design of tasks in the case of the classification of mathematics examination at the Faculty of Mechanical Engineering, University of Banja Luka. Candidates previously had the opportunity to meet with similar tasks and solutions in testing implemented in previous years. Through the analysis of the tasks we indentify sensitive points in the task design.

The tasks are evaluated in a way that in determining student performance enables the application of technology 'chunk-by-chunk' analysis. Expected responses to the questions in the tasks are divided into independent parts that can no longer be interpreted.

1. The code $\varnothing$ (empty set) means that a candidate offered no information as an answer to the nominated question.
2. The code 0 (zero) means that the information which candidate offered as an answer to the question is completely unacceptable.

Task 1 (5 points) (Establishing arithmetic thinking)

Given: 2, -2, 0, $\frac{3}{7}$, e, α, π, $\sqrt{2}$, $7-4i$, $\sqrt[3]{7}$, $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$. *Which of the givens are natural, integral, rational, irrational and complex numbers?*

Task goals: Task is of type (1): Recognition of mathematical objects (2, -2, 0, $\frac{3}{7}$, $\sqrt{2}$, $7-4i$, $\sqrt[3]{7}$, e, π) *and concepts* (π, $\mathrm{e} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$).

Successful distribution:

Number of points	$\varnothing$	**0**	**1**	**2**	**3**	**4**	**5**	**Σ**
Number of candidates	3	9	12	9	5	4	3	**45**
%	6.67	20.0	26.67	20.0	11.11	8.89	6.67	**100.0**

Reflection: 53.33% of the tested candidates did not offer any or offered almost unacceptable answers. Thus, more than half of the tested candidates do not recognize arithmetic objects - natural and whole numbers, even while 26.67% of the candidates do not recognize the natural numbers. The 'fractions' are not recognized by 73.33% of the candidates. Recognition of irrational numbers and concepts is very low - only 26.67% of the candidates.

Task 2 (3 points) (Establishing of the arithmetical - early algebra thinking)

Determine the accuracy / inaccuracy of the following statements:

2.1. *A natural number is an even number if it can be presented as a sum of two equal natural numbers.*

2.2. *An even number is an integer that is divisible by 2.*

[2] This can be implemented as appropriate incentives by teachers, or by teaching intervention using the technology of 'scaffolding- a'.

2.3. *A prime number is a natural number that is divisible by itself and number 1.*

Task goals: The task is the type (4): Recognizing and understanding the concept of an even number (Question 2.1., and Question 2.2.) and (mis-)understanding of the concept of prime number (Question 2.3.). Understanding the representation of these concepts classifies this task in a category of stimulating of arithmetic – early-algebraic thinking.

Comments: (1) The concepts of even and odd natural numbers are extremely important. Insisting on student's understanding of these concepts and their properties encourages the development of the following elements of mathematical thinking: the principle of the excluded middle *(An integer is either even or odd.)* and principle of non-contradiction *(An integer cannot be simultaneously even and odd.)*
(2) The concept of prime number is a fundamental mathematical categorical term and it cannot be reduced to simpler terms.

Successful distribution:

Number of points	∅	0	1	2	3	**Σ**
Number of candidates	1	5	5	28	6	**45**
%	2.22	11.11	11.11	62.22	13.33	**100.0**

Reflection: As we can see from the above table only 6 (or 13.33%) candidates recognize the concept of prime number. Most candidates did not recognize the correct definition of the concept of 'odd number' but it is identified (as applicable) with its fundamental description: *A natural number is an even number if and only if it is divisible by 2.*

Task 3 (6 points) (Establishing of the algebraic thinking elements)
Show that:
3.1. *If the integer n is even, then its square n^2 is also even. (Write algebraically this implication)*
3.2. *If n is an odd, then its square n^2 is also odd number. (Write algebraically this implication)*

Comment: The task is of type (2): Developing fluency by working with representations. This task encourages the development of algebraic thinking because there is a request for 'recognition of the representation of even number and work with this representation'.

Successful distribution:

Number of points	∅	0	1	2	3	4	5	6	**Σ**
Number of candidates	7	28	0	1	4	0	1	4	**45**
%	15.56	62.22	0.0	2.22	8.89	0.0	2.22	8.89	**100.0**

Reflection: A significant number of candidates (36 of 45, or 80%) do not have skills to work with representation of even numbers. None of the tested candidates offered answers that correctly use the above mentioned implications.

Task 4 (8 points) (Logical thinking)
Show that:
4.1. *If the square n^2 of an integer n is an odd number, then n is an odd number as well. (Write algebraically this implication)*
4.2. *If the square n^2 of an integer n is an even, then n is also an even number. (Write algebraically this implication?)*

Task goals: Establishing of understanding and correct use of logical tools 'Principle of excluded middle', 'Principle of no contradiction', 'Principle of double negation', 'Contraposition' and 'Modus Ponens'.

Comment: This task is of type (3): Understanding the process of indirect evidence and its application to the example.

Successful distribution:

Number of points	∅	0	1	2	3	4	5	6	7	8	Σ
Number of candidates	13	31	1	0	0	0	0	0	0	0	**45**
%	28.89	68.89	2.22	0	0	0	0	0	0	0	**100.0**

Reflection: A complete lack of logical thinking tools in the tested candidates raises many questions about appropriateness of mathematics education in our school system. It seems that this suggests that it is reasonable to form a hypothesis: *"The vast majority of high school graduates do not have in its common-sense vocabulary the elementary tools of logical thinking."* [3] To conclude, note that the overall tested population does not have any knowledge about the concepts of direct and indirect evidence and their associated logical terms.

Task 5 (9 points) (Establishing advanced algebraic thinking)
Draw the graphs of the following functions:
5.1 $f : \boldsymbol{N} \ni x \rightarrow 2x - 3 \in \boldsymbol{N}$;
5.2 $g : \boldsymbol{Z} \ni x \rightarrow 2x - 3 \in \boldsymbol{Z}$;
5.3 $h : \boldsymbol{R} \ni x \rightarrow 2x - 3 \in \boldsymbol{R}$ **.**

Task goals: This is a classic task through which we establish the level of understanding of the concept of function in different environments (in the semi-ring ***N*** of natural numbers, the ring ***Z*** of integers and the field ***R*** of real numbers): Do students understand 'the concept of function as a rule' or they 'understand its as a relational between the objects'?

Comments: (1) The task is of type (4). The concept of function can be viewed as a minimal threshold of the candidates' literacy.
(2) A thorough understanding of the functions and work with them in a variety of mathematical environments allow candidates to imagine them as actions, processes and objects in their uniqueness. The concept of 'function' is one of the central ideas of calculus (mathematical courses, which are implemented in technical colleges in our school systems) and the basic tools of a significant number of other areas of mathematics. Still, Vinner [24] takes into account the difference in the understanding of the concept of function. Vinner [24] has made *a distinction between the concept of the definition of* functions, on the one hand, and *the concept of the rules of accession*, on the other hand. He observed that the students' construction of the graphs of functions is not consistent with the mathematical definition of the function.

Successful distribution:

Number of points	∅	0	1	2	3	4	5	6	7	8	**9**	Σ
Number of candidates	38	2	0	0	2	0	0	2	0	0	1	**45**
%	84.44	4.44	0	0	4.44	0	0	4.44	0	0	2.22	**100.0**

Reflection: The three questions in Task 5 should be used as indicators of the establishing of conceptual understanding.[4] Furthermore, these three questions allow the researcher to observe whether the student's scoring function as an *object* or as a *process* or as an *object-process.* This research and our previous research experiences in the establishing student's reflection on issues related to the functions of the various surrounding suggest that candidates prefer to look at the function as an object instead as a *process* or *action.* Hence, they look at the mapping $f : x \rightarrow 2x - 3$ as an object $2x - 3$ in the field ***R*** of real

[3] Of course, in this paper we do not deal with the confirmation or denial of this hypothesis.
[4] On the development of conceptual understanding, see [11].

numbers. This justifies our hypothesis about their capability to recognize functional relation $(x,2x\text{-}3)\in f$, between the variable x and the term $2x - 3$ in a variety of environments.

Task 6 (5 points) (Establishing geometric thinking levels)
Determine the accuracy / inaccuracy (i.e, correctness / incorrectness) of the following statements:

6.1. *A polygon is a closed two-dimensional shape formed by straight lines*.
6.2. *Quadrilateral is a polygon with only four sides.*
6.3. *The rectangle is a quadrilateral.*
6.4. *The square is a rectangle*.
6.5. *The rectangle is never a square*.

Task goals: The goal of this task is, according to Bloom's taxonomy [2], to develop knowledge of mathematical concepts and objects of polygons, quadrilaterals and understand the existence of specifications within quadrilateral subcategories.

Comment: The task is within 'level 0' (visualization) and within 'level 1' (description) in accordance with the classification of van Hiele's levels of understanding of geometry [8]. Candidates were supposed to identify two basic geometric concepts (Question 6.1. and Question 6.2) and to know how to precisely determine them. Questions 6.3, 6.4, and 6.5 in this task are the statements about the concept of quadrilateral. Thus, they are on 'level 1' and inside 'level 2'.

Successful distribution:

Number of points	∅	0	1	2	3	4	5	**Σ**
Number of candidates	1	2	0	5	16	12	7	**45**
%	2.22	4.44	0.0	11.11	35.56	26.67	15.56	**100.0**

Reflection: The task was designed so that the candidates recognize the descriptions of polygons. It is obvious that conceptual knowledge (level 0) is present in the corresponding cognitive plane of the tested population since 77.78% of the population offered acceptable answers. To the last two questions, for which acceptable answers should expose geometrical knowledge within 'level 1, offered only 19 (42.22%) candidates. Unfortunately, it was observed the presence of a candidate who did not recognize the geometric figure of a polygon.

Task 7 (10 points) (Establishing advanced geometric thinking)
Describe the classification of quadrilaterals and for each kind give an example.

Task goals: The goal of this task is the classification of quadrilateral subcategories according to one or more predicates.

Successful distribution:

Number of points	∅	0	1	2	3	4	5	6	7	8	**9**	10	**Σ**
Number of candidates	12	24	6	0	2	0	0	2	0	1	0	0	**45**
%	26.67	53.33	13.33	0	4.44	0	0	4.44	0	2.22	0	0	**100.0**

Reflection: Object classification problem of a category, in this case the category of quadrilaterals, the subcategories according to the essential predicates (one or more) is a sure indicator for the registration of more advanced levels of geometric thinking among students. Very small number (6.67%) of the tested candidates exposed possessing higher mathematical thinking elements than elements described by 'level 0' (in the van Hiele's classification).

Task 8 (4 points) (Establishing of the Set-Relation thinking)

For the sets A = {2, 4, 6, 8, ...} and B = {4, 8, 12, 16, ...} *determine*:

8.1. $A\cup B$; 8.2. $A\cap B$; 8.3. $A \setminus B$; 8.4. $B \setminus A$.

Task goals: The main objective of this task is to establish the Set-Relation thinking by finding answers to the questions whether the candidates: (A) Understand the use of brackets. (B) Understand of the symbol "..." (ellipsis). (C) Recognize the predicates which are determined subsets A and B of the set **N** of natural numbers. (G) Knowing the concepts of union, intersection and difference between seta (sets-theoretic symbols).

Successful distribution:

Number of points	∅	0	1	2	3	4	**Σ**
Number of candidates	6	15	5	7	4	8	**45**
%	13.33	33.33	11.11	15.56	8.89	17.78	**100.0**

Reflection: A significant number of candidates did not offer any answers (13.33%) and completely unacceptable responses (33.33%) of the questions in this assignment. Most candidates (73.33%) were not skilled in dealing with the difference of sets.

6. Conclusion

Problems related to the design of mathematical tasks have long been identified as an important, quite complex and extremely subtle activity within the community of mathematics educators and mathematics education researchers. Questions that are naturally asked:

1. *Do such selected tasks correspond to the goals of testing*?
2. *Are tasks properly designed to unambiguously establish significant existence or absence of the selected indicators*?
3. *How to influence the process of teaching students through the implementation of mathematical courses at the Faculty of Mechanical Engineering to increase mathematical literacy* ?

These and similar questions were the subject of discussion at 2013 meeting of ICMI (Oxford, 2-22 July 2013). Then the mathematics education community of researchers accepted the attitude that designing mathematical tasks is the core of quality teaching mathematics at all levels of education. Designing mathematical tasks as an independent problem ([1], [3] - [6]) or within the paradigm of 'methodological knowledge necessary for teaching mathematics' ([16], [17], [19], [20], [25], [26]) are the sporadic subjects of the community of mathematics education researchers in last twenty years.

Designing tasks is essential for teacher/researcher's activity in assessing candidates' mathematical literacy as well as the establishment of their mathematical ability and mathematical skills. In this paper, we pointed out both sides of designing tasks for the candidates' mathematics classification test: Expecting a solution that candidates should offer and the evaluation of the candidate's successfulness. Therefore, analyzing the test is not intended to determine the level of candidates' mathematical literacy and professional competence of their teachers. Primarily, it is to gain experience in understanding how and why the applied teaching methods of tested students raises the possibility of registration of student's incomplete mathematical knowledge, insufficient algorithmic skills, and low ability to cope in solving problems.

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References

[1] Ainley, J., and Pratt, D. (2005). The significance of task design in mathematics education: Examples from propositional reasoning. In Chick, H. L. and Vincent, J. L. (Eds), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, (Vol. 1, pp. 93-122)

[2] Anderson, L.W., and Krathwohl, D. R. (Eds). (2001). *A Taxonomy for Learning, Teaching, and Assessing: A Revision of Bloom's Taxonomy of Educational Objectives*. New York: Longman.

[3] Berg, C. V. (2012). From designing to implementing mathematical tasks: Investigating the changes in the nature of the T-shirt task, *The Mathematics Enthusiast*, **9**(3): 347-358

[4] Breen, S. and O'Shea, A. (2010). Mathematical Thinking and Task Design. *Irish Math. Soc. Bulletin* **66:** 39-49

[5] Breen, S., and O'Shea, A. (2011). The Design and Implementation of Mathematical Tasks to Promote Advanced Mathematical Thinking, *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education*, University of Rzeszów, Poland, 9th -13th February 2011.

[6] Breen, S., and O'Shea, A. (2012) *Designing tasks to aid understanding of mathematical functions*, In the 4th Biennial Threshold Concepts Conference and 6th NAIRTL Annual Conference, 27-29th June 2012. Trinity College: Dublin.

[7] Brousseau, G. (1997). *Theory of Didactic Situation in Mathematics*, Dordrecht, The Netherlands: Kluwer.

[8] Burger, W. F., and Shaughnessy, J. M. (1986). Characterizing of van Hiele levels of development in geometry, *Journal for Research in Mathematics Education*, **17**(1): 31-48.

[9] Doig, B. (2013), *Mathematical Tasks and Learning Goals: Examples from Japanese Lesson Study*, In V. Steinle, L. Ball and C. Bardini (Eds), *Mathematics education: Yesterday, today and tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne, VIC: MERGA. (pp. 715-718)

[10] Doig, B., Groves, S., and Fujii, T. (2011). The critical role of task development in lesson study, In L.C.Hart, S.C. Alston, and A.Murata (Eds), *Lesson study research and practice in mathematics education:* 181-199, Dordrecht, The Netherlands: Springer.

[11] Fujii, T., and Stephens, M. (2001), Fostering an understanding of algebraic generalization through numerical expressions: The role of quasi-variables. In H. Chick, K. Stacey, J. Vincent, and J. Vincent (Eds), *Proceedings of the 12th ICMI Study Conference: The Future of the Teaching and Learning of Algebra* (pp. 258-264). Melbourne: University of Melbourne.

[12] Fujii, T. and Stephens, M. (2008), Using Number Sentences to Introduce the Idea of Variable, Algebra and Algebraic Thinking in School Mathematics. National Council of Teachers of Mathematics, Seventeenth Yearbook, pp.127-140

[13] Galbraith, P. (2006). *Real World Problems: Developing Principles of Design*, Conference Proceedings MERGA 29, 229-236

[14] Groves, S., and Doig, B. (2002), Developing conceptual understanding: The role of the task in communities of mathematical enquiry. In: *Proceedings of the 26th conference of the International Group for the Psychology of Mathematics Education*, Norwich, UK: University of East Anglia.

[15] Kilpatrick, J., Swafford, J., and Findel, B. (Eds) (2001). *Adding It Up: Helping Children Learn Mathematics*, Washington, DC: The National Academies Press,.

[16] Lewis, C. (2002). *Lesson Study: A Handbook of Teacher-Led Instructional Change*. Research for Better Schools, Inc. Philadelphia, PA.

[17] Lewis, C., and Hurd, J. (2011). *Lesson Study step by step: How teacher learning communities improve instruction.* Portsmouth, NH: Heinneman

[18] Pointon, A., and Sangwin, C. (2003). An analysis of undergraduate core material in the light of hand-held computer algebra systems, *International Journal of Mathematical Education in Science and Technology*, **34**(5): 671-686.

[19] Radović, V.Ž., Đokić, O., and Trmčić, M. (2013). Didactic-teaching methodological function of a question in mathematics teaching at beginner's level, *Pedagoška stvarnost*, **59**(3): 474-487

[20] Rezat, S., and Strässer, R. (2012). *From the didactical triangle to the socio-didactical tetrahedron: artifacts as fundamental constituents of the didactical situation*. ZDM, **44**(5): 641-651

[21] Sangwin, C. (2003). New opportunities for encouraging higher level mathematical learning by creative use of emerging computer assessment, *International Journal of Mathematical Education in Science and Technology*, **34**(6): 813-829.

[22] Stigler, J., and Hiebert, J. (1999). *The Teaching Gap: Best ideas from the world's teachers for improving education in the classroom.* New York: The Free Press.

[23] Vygodsky, L.S. (1978). *Mind in Society.* Cambridge, MA: MIT Press.

[24] Vinner, S. (1983). Concept definition, concept image and the notion of function. *International Journal of Mathematical Education in Science and Technology*, **14** (3): 293-305.

[25] Watanabe, T., Takahashi, A., and Yoshida, M. (2008). A critical step for conducting effective lesson study and beyond. In: F. Arbaugh and P. M. Taylor (Eds), *Inquiry into Mathematics Teacher Education.* Association of Mathematics Teacher Educators Monograph Series, Volume 5.

[26] Yoshida, M. (1999). *Lesson study: A case study of a Japanese approach to improving instruction through school-based teacher development,* Unpublished doctoral dissertation, University of Chicago, Department of Education.

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