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TEACHING MATHEMATICS THROUGH ACTION LEARNING DURING THE COVID-19 PANDEMIC

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Abstract. The Covid-19 pandemic has suspended traditional face-to-face classes across the K-20 educational spectrum. Instead, the course delivery was facilitated using various meeting utilities and learning management platforms. The paper suggests that, during the crisis, teaching of mathematics continues being more effective if it incorporates the action learning framework which includes students' individual work on various types of projects requiring application of mathematics and supervised by more knowledgeable others. It has been observed that, despite teaching being seemingly in dire straits, the notion of individual projects as the means of action learning of mathematics has grown in its educational significance. This growth is in part due to the increase in parental assistance available to students due to a greater interfamily connectivity in the time of the pandemic.

Key words: mathematics education, online teaching, interdisciplinary projects, pandemic, parental assistance, technology, instrumental act

1. Introduction

This paper reexamines the educational model of *Teaching Mathematics through Action Learning* [9] in the context of the Covid-19 pandemic currently affecting much of our planet. Action learning stems from the notion that it is pragmatic to do what works. With this in mind, in the 1940s, Reginald Revans began promoting the applied side of education to business by describing the concept of action learning as an educational pedagogy for business development and problem solving [33, 34]. In general, action learning is characterized by taking an action and reflecting on the results. The pioneering ideas about action learning have come to describe a variety of activities and contexts. For example, in the context of achieving high quality teaching at the tertiary level Biggs [14, p. 7] contends that "the target of action learning is the teaching of

the individual teacher". Similarly, in the context of improving professional performance, Dilworth [18, p. 36] states that action learning starts with an inquiry into a real problem so that regardless whether the problem is "tactical or strategic… [the process of] learning is strategic". A real problem, in the words of Dixon [20, p. 45], deals with "an issue that has salience for the participants, that is, ... if participants doubt that a solution they construct would actually be implemented, less learning occurs".

Action learning in mathematics education for future engineers and schoolteachers, the main focus of our earlier work [9] can be defined as learning through student individual work on a real-life problem supervised by a more knowledgeable other and followed by reflection on this work. This approach to learning mathematics can operate anywhere within the K-20 educational spectrum. It implies students' personal engagement in the studies motivated by the posed real-life problems. It is an alternative to the formal teaching of mathematics with its "rigid autocratic controls ... not consistent with action learning" [19, p. 5]. A problem posing for action learning may be due to instructors (i.e., anyone teaching at K-20 levels), subject area advisors of different kinds [8, 9], parents and other members of a nuclear family. The role of the last two groups in one's mathematics action learning has come to be especially worthwhile during the time of pandemic as discussed below in section 5. Action learning in the context of K-12 mathematics where various contributing roles of teacher candidates including self-studies, parental assistance, and professional advisement on mini projects of young children is discussed in section 3. An increasing role of the action learning framework in teaching calculus to non-mathematics majors during the pandemic and a brief review of student interdisciplinary projects dealing with engineering and health issues are the focus of section 4. As noted in section 4, there were several application projects which prudently included aspects of the deadly Covid-19 pandemic. Pragmatic students used the context of the pandemic in their problem topics.

As always, looking at the past by educators "must only be a means of understanding more clearly what and who they are so that they can more wisely build the future" [22, p. 84]. With this in mind, a historical account of teaching during the time of pandemic is presented in section 2. Finally, the current efforts of teaching and learning online are actualized in section 6using a psychological concept of the instrumental act [38].

In this paper, the authors provide several fact-based observations and instructional responses to Covid-19 disaster. Included are new ideas concerning the parental assistance benefits for students who study mathematics through action learning. This is a truly relevant concept since the pandemic safety requirements cause many families, while being globally isolated, to become locally integrated. Through this integration, parents of primary and secondary students are more available to assist with their children's mathematical homework. There are families where parents are teacher candidates who can immediately test teaching ideas learned in an online mathematics education course with their children. College students often involve parents with their mathematics application coursework when it happens to connect with their parents' professional experiences. The effectiveness of online teaching is facilitated by student maturity. The loss of direct oversight in a classroom setting is less detrimental to the learning experiences of most college students than it is to grade-school students. Understandably, it is more challenging to engage younger students remotely, so that parental guidance is valuable. The continued parental assistance into college is worth noting, however we've found that it is rather difficult to accurately measure relative parental assistance (see section 4).

2. Historical comments: pandemics in the past

History recounts that pandemics have afflicted the human population many times throughout our occupation of planet Earth . Pandemics and other natural catastrophes may have even prompted the invention of writing. Early humans were probably motived to record events originally conveyed only through speech. For a long time the best defense against contagion was believed to be isolation. That was Newton's thinking as the Great Plague ravaged Europe in 1665. Burton [15, p. 349] quotes Newton as saying, "All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention and minded Mathematics and Philosophy [physics] more than at any time since." Indeed, Newton was quite productive during the plague years. The year 1666 came to be considered an *Annus Mirabilis* for all of the new ideas of the time, epitomized by the profusion of work by Sir Isaac Newton. Newton was himself a student at the time of the Great Plague, so college students might well keep Newton's achievements in mind. We do not wish to be isolated, but while we are, we should make the best of things. We *will* solve this problem; and we will do it by improving our understanding of biology, cytology, mathematics, and physics. Therefore, we should continue our studies, just as Newton did. Not everyone needs to concern themselves with the details of virology; however, some students will find it rewarding to do just that. It may be difficult to imagine the year 2021 as being a similarly profuse *Annus Mirabilis* to that of 1666 , but there are like to be many people focusing now on their work and being productive. With a positive overall attitude toward study, students can still learn and grow intellectually. The positive attitude required is often supplied by action learning. In the case of Newton, he was undoubtedly drawing from his formal education, however his discoveries during the Great Plague were largely "informal." The point is that students also learn using self-direction. Instructors, of course, are interested in providing formal direction to their students.

It seems appropriate to notice that the development of the New World is shaped by diseases. Native Americans consequently became infected by diseases for which they had no immunities. As early as in 1516, epidemics began effecting the Americas and they continued into the 20th century [29]. The transmission of disease is part of the Western "legacy." From what we have seen in the summer of 2020, disease causes societal xenophobia. While there is no need to fear any particular people, it is reasonable to give ourselves physical distance. Franklin Roosevelt, who was a polio victim, asserted that phobophobia was our only fear. While the consolation of Roosevelt may not do much to aleviate our concern over the outcome of the present pandemic, it does allude to our tenacious hope which keeps us going.

About a century ago, when the tools of distance education were not available, the United States was struggling with another pandemic. As a result, most urban schools were closed; in some cases, for as long as seven months. At the same time, New York City and Chicago kept schools open because health officials in these two cities believed that, all things considered, keeping students in schools under supervision possessed less risk than allowing them to be outside without any supervision [36]. Los Angeles, on the contrary, established mail-in correspondence courses for secondary students and special courses for teachers were developed similar to how nowadays, in some universities (e.g., State University of New York), professors who applied to teach fully online have to take a special course on the use of tools of distance education.

Together with the use of correspondence technology for remote instruction of K-12 students in 1918-19, hundreds of instructors were themselves given content enhancement courses at that time [36]. Today, teachers can use online teaching to continue instruction while prudently

averting face-to-face meetings during Covid-19. Because the technology of the twenty-first century now includes real-time image transmission, classes can still largely meet, if all parties have adequate computer equipment. While it is not an invisible medium, it is tenable. Even elementary students can attend virtual meetings and benefit from virtual lessons. Online teaching appears to be more personal than mail correspondence. Retention of the personal touch is what instructors desire. While any students in K-16 might be hesitant to ask questions while online [2], others may be emboldened by the medium. Evidently, as online teaching is used more, the participants will naturally become more comfortable with one-to-one (while not physically face-to-face) interaction.

3. Action Learning for Teacher Candidates During Covid -19

3.1 On the benefits of learning from home

Teaching mathematics remotely has both pros and cons to future elementary schoolteachers. Our examples and ideas are taken from SUNY Potsdam instruction experience [2]. In the face-to-face setting, the use of concrete materials (commonly known as manipulatives) including two-sided and multi-colored counters, pattern and base-ten blocks, square tiles is critical as teacher candidates are not familiar with the use of these tools in the teaching of mathematics. A teacher candidate at SUNY Potsdam reflecting on her observation of mathematics lesson in grade two admits: "*I was lost, as I had no clue what she was talking about HTO [place value charts for three-digit numbers], base-ten blocks, counters, and many other things were not something we ever did when I was a child.*" Unfortunately, whereas the above-mentioned tools can be demonstrated on a computer screen, they cannot be touched physically. At the same time as another teacher candidate admits: "*despite our long classes, we were engaged and interested in the content due to the frequent use of virtual manipulatives. I think that although it is not the same as hands on, for example, it can be hard when sorting cookies on a plate by using circles, it is a good adaptation to being able to still use visualization to teach certain skills to students*". As the teacher candidate mentions cookies, one has to keep in mind, it is due to teaching remotely that physical manipulatives can be easily substituted for even more concrete things having an authentic real-life flavor. Indeed, cookies, candies, cakes, pizzas, apples, oranges, juice, water, etc., are commonly available at home and, in the context of mathematical problem solving through action learning, can be used in place of standard physical manipulatives. Consider the following problem: *How many ways can one put eight candies on two plates without regard to the order of the plates, no plate having three candies and with at least one candy on each plate?* Such a problem is typically discussed in the traditional face-to-face classroom of elementary teacher candidates by drawing pictures. However, the problem can be explored in a home setting using real plates and candies rather than drawing them on a white board. In that case, candies might not be identical unlike their circular images on a white board and one may even wonder whether candies are assumed to be identical or not. The emergence of such problem-solving curiosity can be considered through the lens of a supervised action learning as one takes action of putting *real* candies on *real* plates when in the presence of family members the reflection on the results is more likely to occur than in the context of a white board. Through such reflection mediated by the concreteness of a problem-solving setting an elementary teacher may encounter a dual nature of implicit/explicit assumptions when teaching to solve word problems. Indeed, a simple question about putting candies on two plates in the absence of the assumption of candies being identical may require a mathematically complicated resolution. Such discussion when initiated within a nuclear family would naturally demonstrate

an intricate relationship between context from which posing a word problem stems and mathematics which frames the ensuing problem solving. In the real classroom when the assumption about candies being identical is implicitly presumed it may be challenged by a curious student and, likewise when it is made explicit another curious student might ask a teacher to explain the need for this seemingly self-evident declaration. In both cases to address students' curiosity an elementary teacher needs to have what is commonly referred to as deep understanding or flexible knowledge of mathematics (e.g., [10, 12, 17]). The discussion around this issue is an example of why elementary teachers are expected to possess such knowledge. In section 6 below this kind of extended exchange of ideas within a nuclear family is interpreted in terms of an instrumental act when sharing knowledge is considered as a psychological tool that mediates learning.

3.2 On the interplay between listening and taking notes

Another issue in an online mathematics course for prospective teachers (or in any mathematics course for that matter) deals with listening and taking notes vs. just listening. When teaching online, a mathematics instructor writes and displays everything using multiple, commonly available, applications (e.g., Word, Excel, and Power Point). Very often, especially within teacher education courses, an instructor draws pictures and diagrams on a white board, something that teacher candidates are not familiar with, e.g., images of candies and plates, not to mention more abstract drawings. These are new forms of representation of mathematical ideas advocated in the United States by Common Core State Standards [16] and Standards for Preparing Teachers of Mathematics [10]. That is why, taking a note of a picture is something new for a student and, while being time consuming, may significantly hinder conceptual understanding. Furthermore, students take notes selectively for a number of reasons and once a student chooses not to draw a picture because of its perceived complexity the picture is gone before you know it. Finally, students take notes with different pace and waiting for the entire class to complete taking notes may be an unnecessary waste of class time. During an online teaching a picture drawn by an instructor is saved and students may concentrate their attention on comprehending the meaning of a picture knowing that it will be saved for them to keep. Even when drawing a picture online takes more time than on a white board, the time difference is not significant and the slowness by an instructor gives students more time to understand and, if necessary, to ask questions. Furthermore, all classes have been recorded. These issues of recording and saving class notes may be considered as pros of teaching online. As another teacher candidate puts it: "*I also really appreciate the lessons being recorded for me to review after so as to gain a deeper understanding of concepts that we reviewed in class. We discussed many technology resources that I noted for use in my future classroom that we likely would not have used if we were learning in-person*". Nonetheless, for some teacher candidates recording belongs to the cons of online classes, as "*student participation lacks in online courses, being on video can be very intimidating, explaining what parts of a problem or concept that are not understood can be a difficult task, more than one would think*".

3.3 On the social interaction and family participation

Unfortunately, among other cons of online classes many teacher candidates mention the lack of social interaction, something that face-to-face teaching and learning typically provides. According to a teacher candidate's opinion "*Learning happens best in the social context of the classroom ... if I were learning in person, I would definitely work through mathematics problems*

with the help of fellow students. In person, professors can more easily read the cues of confusion or understanding on the students' faces. The lack of social aspect is definitely hard". In addition, some teacher candidates are concerned about the lack of privacy, an aspect "*that can be difficult during remote learning, sometimes it's nice to be able to ask your teacher or professor a question privately, as opposed to having the entire class hear your question*", as one teacher candidate believes. Often after a question being asked privately a professor can discuss this question with the entire class without revealing its source. There is no such opportunity in a virtual setting.

Notwithstanding, the lack of social interaction among students and professors in online classes can be made up by increased interaction among family members. During 2020/21 many teacher candidates with children of school age had an opportunity to test teaching ideas taught online at home, especially, mathematical problems involving food. Three such problems are worth mentioning. One problem with candies and plates was discussed above. Another problem with an applied flavor that has recently been explored in a nuclear family of four in a home setting is as follows: *What is the smallest number of cuts one can make when fairly dividing three snickers among four people?* Figure 1 shows three solutions to the problem of dividing three snickers or any three food items that can be easily cut into pieces among four people – with nine cuts on the left, five cuts in the center and three cuts on the right. When two members of a nuclear family are professionals, such a problem can contribute to mutually shared knowledge through action learning. In particular, in a family of four where mother is a teacher candidate and father works in a restaurant industry, the younger child, an elementary school student familiar with fractions as an extension of integer arithmetic, takes an action by dividing each of the three snickers into four equal parts. Through this action, which requires nine cuts, three snickers are divided into twelve equal pieces allowing for a fair division among four family members. Obviously, the child's action is based on the most facile assumption about fair division that all pieces have to be identical. This assumption nonetheless ignores an applied element of the problem about minimizing the number of cuts, something that may be important in real life when the number of food items, the dividend, and the number of servings, the divisor, are large numbers. In turn, mother reflects on the child's action by saying that fewer than nine cuts or twelve pieces can be made. In this, she does pay attention to an applied character of the problem dealing with the issue of minimization[1]. Being familiar with Egyptian fractions from her studies of mathematics in a teacher education program, she suggests dividing two snickers in half and the third snicker in fourths thus representing 3/4 through the sum of two unit fractions, 1/2 and 1/4. This division results in five cuts or eight pieces which are not all identical but match each of the four serving by shape. In turn, father with experience in food industry claims that he can do fair division with fewer cuts and pieces although not everybody would get same number of pieces. He suggests cutting one fourth from each of the three snickers to keep the three small pieces for himself so that the remaining three family members would have a single piece, each measuring 3/4 of a snicker. This division results in three cuts or six pieces. Finally, the older child, a freshman in high school familiar with algebra, generalizes his father's solution as follows: in order to divide m snickers among n people, $n > m$, one can cut them in such a way that $(n - m)$ individuals can be given m pieces, each piece measuring $1/n$ of a snicker, and the remaining m individuals can get a single piece measuring m/n of a snicker. This example illustrates the essence of action learning in a home setting when mutually shared knowledge

[1] According to Euler, "nothing happens in this world in which some reason of maximum or minimum would not come to light" (cited in [30, p. 121]).

becomes inserted between parents and children leading to some significant epistemic outcomes[2]. In section 6 below this episode is discussed further under the umbrella of an instrumental act. Moreover, the above problem once again demonstrates how different assumptions about a real-life problem when not made explicit yield different solutions prompted by either educational background or professional experience.

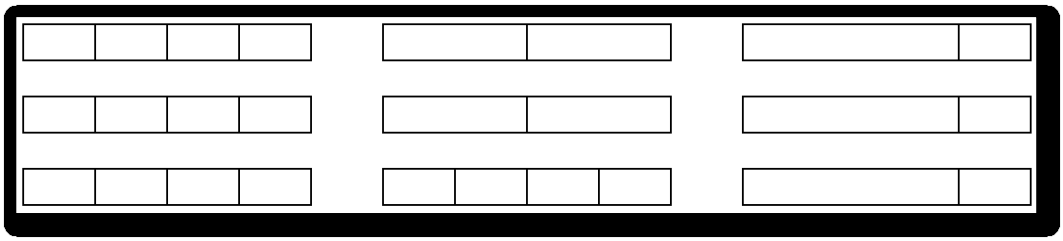

Figure 1. From nine cuts (left) to five cuts (center) to three cuts (right).

Finally, the third problem deals with cutting a circular pizza delivered from a pizzeria to a family of four hosting another such family. As usual, pizza is placed in a square-shaped box. As it is reported by one of the primary teacher candidates reflecting on their experience of learning mathematics online during the pandemic, the concept of symmetry is implicitly applied when pizza is divided in eight equal pieces. Using a pizza wheel, it is cut twice from corner to corner of the box thus creating four equal pieces. In the process, the center of pizza is created, and the wheel is then driven from the mid points of the sides of the box to go through the center. The teacher candidate notes that in order to recognize that this action learning of mathematics results in the number of pizza pieces enough to serve eight people, a pre-school child does not even need to know how to count as such recognition can result from using the principle of one-to-one correspondence. Just as the problem with snickers, the pizza problem is a wonderful mini project for schoolchildren's learning mathematical ideas at home during the pandemic while being advised by more knowledgeable others.

4. Collegiate Action Learning During Covid-19

4.1 On the importance of projects in the time of pandemic

Covid-19 quarantine not only changed the traditional mathematics teaching from face-to-face to online. Also, it led to an important development of action learning approach in collegiate mathematics education. Many STEM faculty and students do their research being motivated by Covid-19 related problems. In the case of mathematics action learning at the tertiary level there is double supervision of a mathematics student's work from both the course instructor and a subject area advisor. The latter is a human resource which is drawn upon for their expertise in the topic of concern for a given project. The subject area advisor serves the role of an "expert" on matters of which the mathematics instructor may not have sufficient background. It is considered a "good practice" to add more professional minds to a project, something that makes the ultimate success of action learning more likely and the outcome of this action potentially more significant.

Students involved in undergraduate mathematics courses might ask "How is face-to-face instruction different from teaching online?" Although there is a change to the medium of course

[2] Epistemically speaking, a (non-trivial) question still remains whether the three methods of cutting m pizzas among n people fairly, suggested through knowledge sharing among the family members, always go from the largest to the smallest number of pieces resulting from each method for all m and n, $n > m$.

delivery, the content remains unchanged. Nowadays the use of technology in education is nearly guaranteed. Requiring the use of a computer for education should not cause anxiety to any modern-day student. Students studying mathematics often have an online coursework component. Additionally, students may find that there are resources available online that are helpful for understanding some of their course material. If the participants are willing and able to communicate virtually, the online presentation of course materials is effective in mathematics education.

There are numerous university action learning examples listed in [8, 9] that come to light during face-to-face interaction with students. However, none of these examples require face-to-face setting in order to be discussed. A relative success of undergraduate students in mathematics courses with an action learning coursework compared to those mathematics students in courses without action learning (see data for several years in [8, 9, 21]) is not diminished, but rather is increased by the response to resort to online delivery of instruction for the duration of the Covid-19 pandemic. This observation is based on teaching calculus at the University of South Florida in 2020/21.

In composing the individual action learning there are face-to-face discussions between the subject area advisors and mathematics instructors [8, 9] which could easily be replaced by a remote meeting technique. These remote meetings might even be facilitated by telephone. It is observed that telephonic communication has largely merged with computing devices with today's smart phones. It is important that we have the Internet to help get us through this pandemic by allowing for convenient remote interaction. University libraries are still open remotely, so we can "do our time" with Newtonian resilience. We all can read, unlike the cases of pandemic in the past as described above in section 2, and can even pragmatically do action learning while learning remotely.

We should try to understand the entire situation surrounding this epidemic. The fuller understanding will take the complete spectrum of academia with medicine and mathematics at the forefront. There is much to be learned by studying the pandemic situation in detail. We should know more about the virus' geometry, for instance. These considerations are just the kind of ideas that might be fashioned into action learning. There are unarguably many levels to our virus problem. The usual problems are still here, and we will have to get back to them. Either way, teaching with action learning can be done outside the classroom. Face-to-face meetings can be discouraged for all participants while the virus is still a threat. Many have learned that a freely available utility such as Microsoft's Teams or Zoom, can offer reasonable student-teacher interaction. Including action learning in the teaching of calculus requires a short submission process which is accessible online.

4.2 Action learning improves success rate in undergraduate mathematics

There are likely tens of thousands of instructors in K-16 schools who have recently learned of meeting utilities, and then immediately began using them for online instruction at the onset of the pandemic. Teams and Zoom function with little or no technological savvy. All that is needed is electric power and a modern computer device. At the college level, assuming students and instructors are dedicated to their tasks, a meeting utility and email appear to be all the equipment needed to conduct a course structured by action learning. Students who can log onto an online course can equally well submit an action learning project. The basic telephonic ability exists as well. These electronic utilities are the best ways to educate right now. A personal interaction between students and instructors in classrooms has pedagogical and social

significance which should be resumed once the pandemic is over. Instructors are challenged to remotely replicate as best as they can the personal contact of actual classrooms. With dedication, instructors can approach this simulated interaction; however, it is virtually impossible since the medium is not invisible. Accepting online teaching is probably more difficult for younger students who may be less educationally self-motivated than are college students. Nevertheless, young students are notably resilient and adaptable. Dedicated parents are likely to assist them with their studies, and now with their new learning paraphernalia. With adequate direction, many students at all grade levels can continue their education through the Covid-19 period in this instructional context.

Several papers concerning action learning and based on numerous studies conducted at USF and SUNY have been published [3, 5, 7, 8, 21, 23, 24, 26]. The bottom line for instructors and students is that the success rate in certain undergraduate mathematics courses is higher when an action learning option is available in their courses (see, for example, [8, 9. 21, 28]).

There is a good cause to fully utilize action learning during the current crisis. As stated, students availing themselves of online teaching already have everything they need to virtually submit action learning projects. It is of historical note that collegiate action learning predates the online submission process which has been successfully capturing these data at the University of South Florida for the past twenty years. Since 2000 online submission has been seen as a technological convenience and a means of effectively promoting action learning. During the remote instruction this became a necessity.

4.3 Undergraduate research during the pandemic

That action learning can be effectively produced with or without face-to-face interaction of participants as it is exemplified by the publications in the *Undergraduate Journal of Mathematical Modeling: One + Two* (UJMM; https://scholarcommons.usf.edu/ujmm/) sponsored by the University of South Florida. Most of the instructor-student interaction that goes into completing this work is done through electronic mail. This is typical of individual action learning, without the threat of contagion. Meetings are fine, but when they are discouraged, they can be adequately replaced by an online meeting utility. The student conferences can be carried out remotely, as well.

A number of projects presented by USF undergraduates in 2020/21 are directly motivated by the Covid-19 pandemic, several students used in the context of the virus for all or a portion of their project work. For instance, projects such as "Year of Covid-19 Population Growth Rate," "Investigating Covid-19 Epidemic," "Using the Gompertz function to predict the Progression of Covid-19 Pandemic in Puerto Rico," "Use of Personal Protective Equipment during Covid-19 Pandemic," "Determining the Theoretical Rate of HDAC3 Inhibition Using PDA106 in Humans," and "Right Amount of Dosage" have been submitted. Some of the projects which draw upon the Covid-19 pandemic have already been published in UJMM. This further validates the educational importance of interdisciplinary projects. One published work [35], entitled "Calculating AMBU Bag Dimensions for Use in Portable Ventilators", considers various physical aspects of the AMBU bag of an air respirator. A second publication produced at that time is titled "The Relationship between Suicide Rates and Mental Health Provider Ratio" [11]. The project may have us ponder if the added psychological strain imposed on everyone by the Covid-19 pandemic exerts itself more harshly on those of us who might have already had some mental strain. Another paper [37] "The Famous Coin Change Problem and its Possible New Applications" describes some possible beneficial solutions to the Coin Change Problem that are

worth paying attention. Now its application in finance and fiscal policy is of particular importance because of Covid-19 pandemic. A paper entitled "Possible Contamination from Rainwater in Community Pool" [27] describes an approximate number of organic contaminants present in the pool after an amount of time in days. It shows that the pool – during its Covid-19 closure – has been cleaned often enough. The author of the paper "Using Calculus to Plan an Open-Air Concert" [25] notices that huge overcrowded indoor events are still a hazardous place to be. In light of this, outside, where the air is naturally circulating and it is easier for airborne contaminants to disperse, is a good solution for holding a big gathering.

It is a worthy observation that the disruption to classroom availability does not slow down the submission of action learning projects in 2020/21 at USF. Action learning proved to thrive in certain courses. In the sections of a calculus course offering an action learning option the overall success rate is as high as, or even higher than, that reported in [21]. It is reasonable to expect that several students look forward to producing some creative work through action learning while practicing physical distancing. Those students may wish to do action learning regardless of the online imposition, but the crisis may now add further impetus to action learning work. A way such project work can be performed by curious high school students at collegiate level is discussed in [8]. There is always an action taken and a reflection on the results occurs. Action learning at the secondary level would not be unlike that experienced in the primary school. Projects at the college level are a natural extension of those carried out at the secondary level. This reaffirms our assertion that action learning can and does encompass the full K-16 levels of formal mathematics education.

Note that parental assistance in problem selection, data suggesting, and subject area advising increase in different fields**.** During 2020/21 at USF about 15% of the projects had knowingly involved parental assistance. This compares to roughly 5% of projects suggested by parents that were submitted in 2019.

5. Parental Assistance in Locally Integrated Nuclear Families

As mentioned earlier the parental assistance for students of all levels is broadened by the response to the Covid-19 pandemic. Parents of K-12 students may feel obliged to smooth out the educational bumps where they might occur. Parents are often a support feature for students in their early mathematics schoolwork. Those students who have more educated parents have an educational advantage during the pandemic. Parents with foresight might urge their children to embark on projects, since projects and related experiences are important to students.

There is generally less parental assistance offered as students move on to secondary and tertiary levels, since not many parents are comfortable with mathematics taught beyond the primary level. Still, because support from parents can be helpful for students, mathematics instructors seek to elicit that support whenever possible. Parental support of students' mathematical learning can take different forms. As mentioned above, some college students' parents assist them with their projects. Parents are known to suggest real-life problems and serve as subject area advisors of the projects stemming from the problems. It is important to recognize that parents are involved in projects, and the situation now draws more from them. In fact, parents and, occasionally, close relatives (or even neighbors) may become subject area advisors in the area of their non-mathematical expertise. This positive side of learning mathematics via projects has been highlighted in the time of the pandemic. At the college level, at least, it is evident that encouraging an interdisciplinary project with parents or "significant others" as subject area advisors is one of the best responses to the Covid-19 imposition of online teaching.

The graph of Figure 2 represents the following observation by the authors averaged across various demographics. At the preschool level, parental assistance with mini, game-like, projects having hidden mathematical flavor such as using a pizza wheel aimed at intuitive understanding of symmetry is more significant than with traditional mathematics because the latter is almost non-existent at that level. Then at the primary level parental assistance takes over the assistance with projects because parents are good with integer arithmetic involving addition and subtraction and thus they do not see projects as helpful tools of learning mathematics. But then, as traditional mathematics becomes more and more involved, parental help with mathematics considerably reduces while that of with interdisciplinary projects increases and reaches its maximum at the tertiary level.

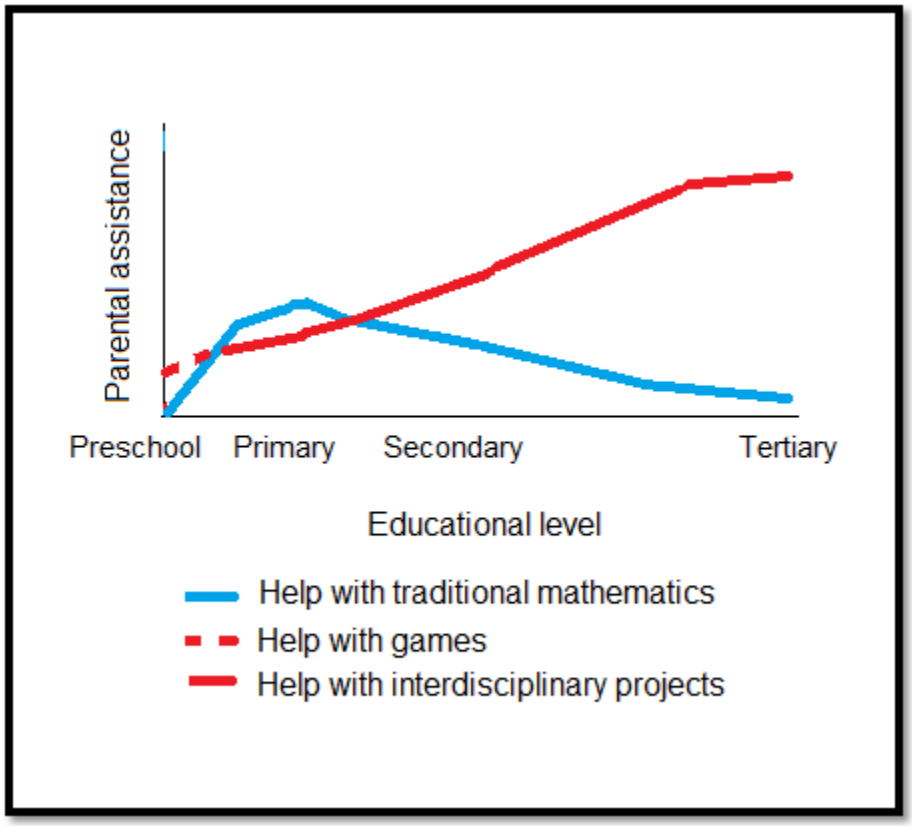

Figure 2. Parental assistance: traditional mathematics vs. interdisciplinary projects.

At the tertiary level, action learning has been termed projects, and these require the inclusion of a subject area advisor. Although the collegiate projects have a layer of formality, they universally involve action learning. As mentioned in [8], in a non-tertiary setting, there are mini projects supervised by teacher candidates during their field experience. Specific examples of school-based mini projects are given in [3, 4, 6]. That way, there is an analogy between tertiary level projects described in section 4 and the primary school mini projects in which teacher candidates play the role of subject area advisors.

Given the plots in Figure 2, parents have a heightened potential to assist in projects at later grade levels. This can be quite significant with college courses, where parents often have little ability to assist with higher mathematics but may more readily assist with a project work. While it may seem obvious that educated parents are more able to assist with traditional

mathematics, Figure 2 indicates that parental assistance in collegiate mathematics courses can best be enacted when projects are involved. To some extent, this is true even at the primary level but appears to become more significant at the secondary and tertiary levels. In the time of the pandemic, parents are free from driving long distance to work and thus are quite able to assist in projects. Regardless of the parental level of education, the situation with the pandemic creates a bridge which takes parents into mathematical work of children even with insufficient understanding of the details of mathematics involved. Parents have diverse professional skills and background knowledge which are important components to projects. As noted above, a parent might even propose a problem or serve as a subject area advisor for their student. It can be beneficial to have parents with personal and professional interests involved with student projects.

6. Online Teaching as an Instrumental Act

The online teaching, in general, and specific activities within nuclear families described above can be conceptualized through the lens of psychology thus filling mathematics action learning with a theoretical flavor. Obviously, technology is the crux of monumental efforts worldwide to provide learners with online education in the time of pandemic. Understanding psychological aspects of those efforts using the notion of instrumental act [38] is important for all parties involved – students, instructors, parents, and professionals of all educationally-relevant stripes.

The advent of technology in education gave birth to the theory of instrumental genesis [31], rooted in the seminal ideas of Vygotsky [38]. Nowadays it is used to study and develop the pedagogical ideas dealing with the appropriation of an artifact as a material object and its elaboration to become an instrument as a psychological concept. After an artifact is turned into an instrument, e.g., Zoom and its specific features have been appropriated for an online delivery of a mathematics course, the latter being inserted between the instructor and the student can act bi-directionally. In the context of using Zoom this popular instrument of online education and social interaction affects both the course delivery and the intellectual development of instructors and students. Such role of an instrument constitutes the essence of the instrumental method, whatever the context. According to Vygotsky [38], "the instrumental method distinguishes a twofold relation between behavior and an external phenomenon [which] … can play the role of the object toward which the act of behavior is directed … [and] of a means by means of which we direct and realize the psychological operations (memorizing, comparing, selecting, etc.) necessary for the solution of the problem." By acting towards the task, in our case, a technological alternative to the face-to-face agency, an instrument evinces pragmatic mediation and by acting towards the user, in our case, an instructor and their students as agents of the selected alternative, it supports epistemic mediation [26].

Furthermore, in the context of an integrated family confined to home due to the pandemic, the concept of the instrumental act can be extended to include the exchange of information among two groups of actors, e.g., children and parents who are positioned by the opposite sides of mutually shared knowledge. In that way, knowledge as a psychological instrument plays the role of the middle term with a bi-directional functionality towards the actors. This duality of the role of an instrument both material and psychological as an agent of education allows one not only to see online teaching through the lens of the concept of the instrumental act but it enables the emergence of new ideas conducive to the improvement of a course delivery. An online teaching system (OTS), which may include such tools as Zoom, a learning management platform, a virtual lab and the Internet (see Figure 3), enables alternative

ways of a course delivery for students. By the same token, when preparing a course an instructor, by transforming an artifact into an instrument, grows naturally in their epistemic advancement. Furthermore, the exchange of knowledge within a nuclear family, homebound due to the pandemic, has great potential for epistemic advancement of both parties involved. In particular, a parent as an ever-present vicarious teacher has multiple opportunities to ask a child information type questions who, in turn, is more likely to ask the parent questions requesting explanation. At the very simple level a parent may ask a child how many ice cream cones are left in the fridge and a child may be curious to know why ice cream melts outside the fridge. Obviously, the latter question is more intelligent than the former one. The diagram of Figure 4 reflects the following observation by the authors. Even in a pre-pandemic time, the younger the students as members of a nuclear family are, the larger is the size of the mutually shared knowledge within the family. During the pandemic a buildup of the mutually shared knowledge takes place and, once again, the younger are the students as members of a nuclear family, the larger is the buildup of the mutually shared knowledge within the family. This general subject matter observation is consistent with the specific graphs of Figure 2 indicating that the buildup of mutually shared traditional mathematical knowledge between parents and students decreases with the increase of the students' level of studies. At the same time, especially, during the pandemic on the average across multiple demographics parental involvement with mathematical action learning as subject area advisors of their children increases and reaches its saturation at the tertiary level.

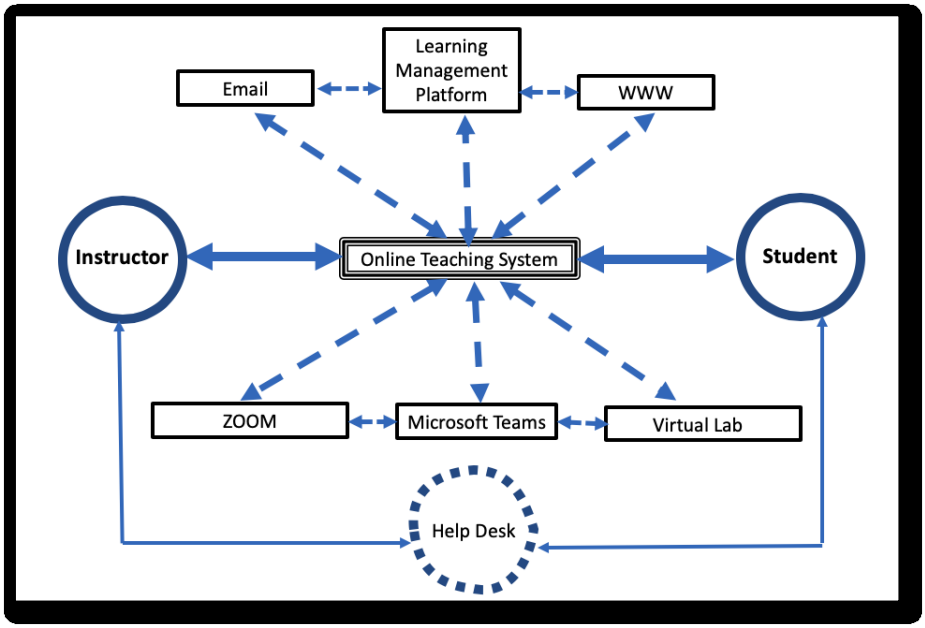

Figure 3. Online teaching as an instrumental act.

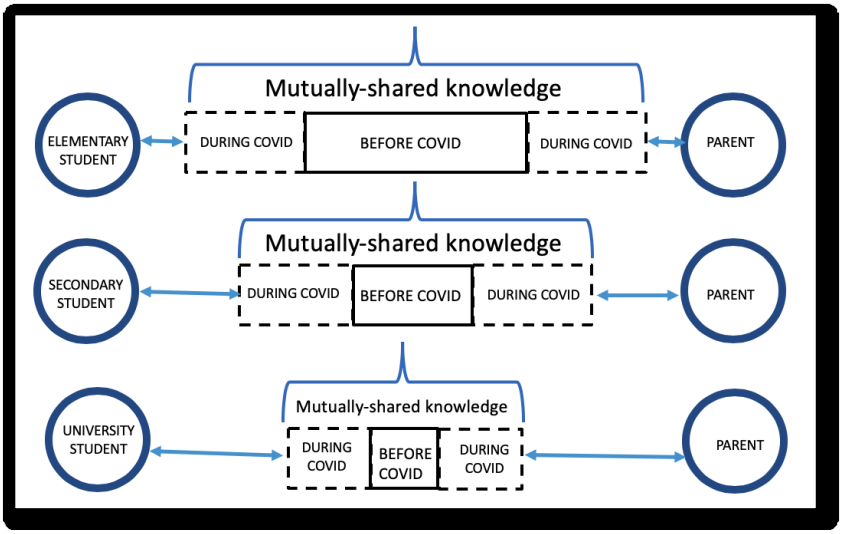

Figure 4. Mutually shared knowledge and its various buildups as a function of educational level.

The use of an OTS (online teaching system) and its adaptation is not straightforward which means that the effective appropriation of the system motivates and often gives birth to specific intellectual efforts on the part of the instructor. In other words, by acting towards epistemic advancement of an instructor an OTS concurrently acts towards a student by amplifying the efficiency of a course delivery due to the instructor's contribution. In terms of the theory of instrumental genesis, the work of an instructor on preparing the system for effective online delivery of a course may be construed as "*instrumentation*" – a process through which he or she develops intellectually; the application of the system to online teaching may be construed as "*instrumentalization*" – a process through which an artifact broadens the realm of utilization. As Lonchamp [26, p. 216] puts it, "In the instrumentation process, the subject develops, while in the instrumentalization process, the artifact evolves". At the same time the artifacts included into an OTS "have potentials that the user may or may not develop ... [becoming evident only after] the developments will be tried out, validated or rejected and artifacts will take on functions that will be temporary or permanent, following the diversity of the situations and projects the users set for themselves" [13, p. 186]. Therefore, 2020/21 years are very important for revealing the potentials of Zoom and like tools of distance education for the teaching of mathematics. Just as mathematical activities can differ in complexity on the spectrum from basic to advanced one may distinguish between basic and advanced instrumentations as a subject turns an artifact into an instrument [1]. In the context of mathematics an important part of an OTS is a virtual computer lab, something that allows students to access campus computers from home (or dorm) and takes on functions to support the diversity of computational experiments necessary for the teaching and learning of mathematics.

To conclude this section, note that any instrument is a combination of an artifact (or a group of artifacts) and recipes for its (their) utilization. These recipes may vary in terms of complexity even within a single artifact and often stem from various subject matter domains – education, psychology, mathematics, and computer science – each recipe enabling specific mediation by and highlighting a particular orientation of an artifact within an instrumental act. Rabardel and Bourmaud [32] studied three orientations of an instrument mediated activity – the orientation towards the object, e.g., regarding a student learning mathematics remotely, the orientation towards the subject, e.g., regarding an instructor teaching a mathematics course using

an OTS, and the orientation towards other subjects, e.g., regarding the remote teaching support personnel. These three major orientations of an instrumental act can be extended to include an orientation regarding specific elements of an OTS. The diagram of Figure 3 shows multiple orientations stemming from bi-directional relations among the elements that the OTS includes. For example, within the use of Zoom, one can demonstrate to the students how a virtual computer lab can be used or what has been uploaded to a learning management platform (LMP). Also, within the Moodle, a specific type of LMP, Wiki activity that involves the use of the WWW can be originated. Another example associated with a LMP is the submission portfolios by students and portfolio grading by an instructor, something that is supported by internally generated email exchange between the two parties. In other words, through mastering multiple recipes for utilizing an OTS as an instrument comprised of multiple artifacts and their mutual orientation, one becomes a coherent participant of the online teaching and learning in the time of pandemic.

7. Conclusion

The paper was written to reflect on the authors' experience of teaching mathematics online while the Covid-19 pandemic affected both teachers and students worldwide. The focus of the paper is on mathematics teaching through the action learning framework defined in [9] as a supervised work of students on real-life projects varied on the spectrum from mini projects involving curious young children to interdisciplinary industrial projects requiring application of undergraduate mathematics. The authors found that this framework is not only preserved in the time of the pandemic but, better still, it prospers due to specific circumstances of this merciless juncture. More specifically, mathematics action learning grows in its academic value and educational significance due to the increase in parental assistance provided within a homebound nuclear family with students of different age.

The paper provided a brief history of education during the time of pandemics spanning from the 16^{th} century to the 20^{th} century and reveals some political and organizational similarities between the past and the present. The paper discussed different pros and cons associated with online teaching and learning. In particular, one of the advantages for students learning mathematics is a possibility of paying more attention to listening rather than concentrating on taking notes as everything written by an instructor is not only displayed on the computer screen but is saved and uploaded to a learning management platform (Figure 3). The paper reflected on the online mathematical preparation of teacher candidates and it provided anecdotal evidence of various contributions to the mutually shared knowledge (Figure 4) by four members of a nuclear family – a teacher-candidate, an expert in food industry, an elementary pupil and a secondary student – through the discussion of a seemingly simple real-life problem involving fractions as mathematical tools of fair sharing. Several examples of mini projects appropriate for the primary school level are provided. It is noted that the graph describing the measure of parental assistance with traditional mathematics and interdisciplinary projects as the function of their students' educational level intersects twice which means that there is a period when the former assistance takes over the latter one. In the case of teaching calculus to non-mathematics majors, the paper notes that many face-to-face interactions among a student, a mathematics instructor, and a subject area advisor suspended due the pandemic can be replaced without any significant abatement by an online interaction. The paper references several undergraduate mathematics projects dealing with the issue of Covid-19, indicating the increase of undergraduate students' scholarly initiatives despite the unprecedented circumstances.

Finally, the online teaching in general and specific activities within a nuclear family are conceptualized through the lens of psychology using the classic notion of the instrumental act. An online teaching system and mutually shared knowledge among family members are described as physical and psychological instruments inserted, respectively, between the instructor and the students as well as between the children and their parents. It is shown how these two instruments enable and motivate both pragmatic and epistemic mediation of all parties involved in the teaching and learning process during the time of pandemic. Different dynamics of the mutually shared knowledge buildup during the pandemic across families with students of different educational levels is observed. Knowing multiple orientations among the artifacts included into an OTS (Figure 3) and mastering skills in improving recipes for their utilization by all parties involved is noted as the pivotal condition of coherent participation in the online teaching and learning during the pandemic.

To conclude, the authors note that for the moment there is little or no good news concerning Covid-19. It seems that our situation has not improved since the pandemic was declared early in 2020. We would like the pandemic to end and any writings concerning the virus to be made retrospective. Instead, we are still in the thick of this crisis, and we are dealing with matters of immediacy. We hope things to improve and we say that the action learning approach to the teaching of mathematics helps to get us through this. Educational writings, such as this paper, may not be of immediate importance when the virus subsides; however, they may be useful should a new strain emerge, or if this virus reappears. There are the warnings that a new wave may wash over us just as the previous one ebbs, so we have to remain vigilant.

Performing requisite work toward a collegiate project involves several important steps, including communications with a subject area advisor. These communications as well as those with the mathematics instructor can be safely and effectively done electronically.

In requiring a write-up, collegiate interdisciplinary projects emphasize some mathematical content with a student's exposition of prose. This is a solid, active way of expressing knowledge. Projects likely serve as a good start for those of us who enjoy communicating about our crafts. When instruction is administered online, adding projects to remote instruction is a natural process. Normal course anxiety is reduced by including project work; making projects even more desirable in these extraordinary times.

We must do good work and improve our overall condition. Project instruction is part of the good work that helps us solve problems. Parental assistance for students should be recognized in order to further encourage collaboration from more parents. Our efforts can remain productive and students can gain much from their project experience. It is more important than ever for students and instructors to value and participate in pre-college and collegiate interdisciplinary projects.

References

[1] Abramovich, S. (2020). Connecting technology immune – technology enabled mathematics pedagogy to the theory of instrumental genesis. In G. Metivier (Ed.), *Computers in Education: Trends, Applications and Challenges* (pp. 27-55). New York, NY: Nova Science Publishers.

[2] Abramovich, S. (2021a). Teaching mathematics in the era of COVID 19: pros and cons of online education through the voices of elementary teacher candidates. In I. Mattsson (Ed.), *Teacher Education: Opportunities, Challenges and Perspectives* (pp. 149-180). New York, NY: Nova Science Publishers.

[3] Abramovich, S. (2021b). Using Wolfram Alpha with elementary teacher candidates: From more than one correct answer to more than one correct solution. *MDPI Mathematics* (special issue Research on Teaching and Learning Mathematics in Early Years and Teacher Training), **9**, 2121, 18 pages. Available on-line at: https://doi.org/10.3390/math9172112.

[4] Abramovich, S., Stanton, M., and Baer, E. (2002). What are Billy's chances? Computer spreadsheet as a learning tool for younger children and their teachers alike. *Journal of Computers in Mathematics and Science Teaching*, **21**(2):127-145.

[5] Abramovich, S. and Grinshpan, A. Z. (2008). Teaching mathematics to non-mathematics majors through applications. *PRIMUS*, **18**(5): 411–428.

[6] Abramovich, S., Easton, J., and Hayes, V. O. (2012). Parallel structures of computer-assisted signature pedagogy: The case of integrated spreadsheets. *Computers in the Schools*, **29**(1-2): 174-190.

[7] Abramovich, S. and Grinshpan, A. Z. (2012). Bridging K-12 and university mathematics: building the staircase from the top. *Open Mathematical Education Notes*, **2**: 1–21.

[8] Abramovich, S., Burns, J., Campbell, S., and Grinshpan, A. Z. (2016). STEM education: action learning in primary, secondary, and post-secondary mathematics. *Open Mathematical Education Notes*, **6**(2): 65–106.

[9] Abramovich, S., Grinshpan, A. Z, and Milligan, D. (2019). Teaching mathematics through concept motivation and action learning, *Education Research International*, Vol. 2019: ID 3745406.

[10] Association of Mathematics Teacher Educators, (2017). *Standards for Preparing Teachers of Mathematics*. Available on-line at: https://amte.net/standards.

[11] Bates, Ch. (2020). The Relationship between Suicide Rates and Mental Health Provider Ratio, *Undergraduate Journal of Mathematical Modeling: One + Two*, **11**(1): Article 1. https://scholarcommons.usf.edu/ujmm/vol11/iss1/7/

[12] Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., and Tsai, Y.-M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, **47**(1): 133–180.

[13] Béguin, P. and Rabardel, P. (2000). Designing instrument-mediated activity. *Scandinavian journal of information systems*, **12**(1): 173-190.

[14] Biggs, J. (2003). Teaching for Quality Learning at University: What the Student Does. Philadelphia, PA: The Society for Research into Higher Education & Open University Press.

[15] Burton, D. (1991) *The History of Mathematics: An Introduction* (2nd ed). Dubuque, IA: William C. Brown.

[16] Common Core State Standards. (2010). *Common core standards initiative: Preparing America's students for college and career.* Available on-line at: http://www.corestandards.org/Math/.

[17] Conference Board of the Mathematical Sciences. (2012). *The Mathematical education of teachers II*. Washington, DC: The Mathematical Association of America.

[18] Dilworth, R. (1998a). Action learning in a nutshell. *Performance Improvement Quarterly*, **11**(1): 28–43.

[19] Dilworth, R. (1998b). Setting the stage. *Performance Improvement Quarterly*, **11**(1): 5–8.

[20] Dixon, N. M. (1998a). Action learning: More than just a task force. *Performance Improvement Quarterly*, **11**(1): 44–58.

[21] Fox, G., Campbell, S., Grinshpan, A. Z., et al. (2017). Implementing projects in calculus on a large scale at the university of South Florida. *Journal of STEM Education*, **18**(3): 30–38.

[22] Freire, P. (2003). *Pedagogy of the Oppressed* (with an introduction by D. Macedo). New York, NY: Continuum.

[23] Grinshpan, A. Z. (2005). The Mathematics Umbrella: modeling and education. Mathematics in Service to the Community: Concepts and Models for Service-Learning in the Mathematical Sciences, *MAA Notes* **66**: 59–68. Washington, DC: The Mathematical Association of America.

[24] Grinshpan, A. Z. (2014). Interdisciplinary mathematics in STEM education: undergraduate retention and research. In NSF STEP Grantees Meeting: Identifying Best Practices, Washington, DC.

[25] Kalandarov, T. (2021). Using calculus to plan and open-air concert. *Undergraduate Journal of Mathematical Modeling: One + Two,* **12**(1): Article 4.
[26] Lonchamp, J. (2012). An instrumental perspective on CSCL systems. *International Journal of Computer-Supported Collaborative Learning*, **7**(2): 211-237.
[27] McGregor, M. (2021). Possible contamination from rainwater in community pool. *Undergraduate Journal of Mathematical Modeling: One + Two*, **12**(1): Article 3.
[28] Milligan, D. (2019). The Orbital Torus of Arrokoth -- A Sample Project. *Open Mathematical Education Notes*, **9**(2): 87-102.
[29] Parvis, S. (Ed.) (2003). *The New York Public Library American History Desk Reference*. New York, NY: Hyperion.
[30] Pólya, G. (1954). *Mathematics and Plausible Reasoning* (volume 1: *Induction and Analogy in Mathematics*. Princeton, NJ: Princeton University Press.
[31] Rabardel, P. (1995). *Les Hommes et les Technologies, Une Approche Cognitive des Instruments Contemporains* [*Men and Technologies, A cognitive Approach to Contemporary Instruments*]. Paris, France: Armand Colin.
[32] Rabardel, P. and Bourmaud, G. (2003). From computer to instrument system: a developmental perspective. *Interacting with Computers*, **15**(5): 665-691.
[33] Revans, R. (1980). *Action Learning: New Techniques for Management*. London, England: Blond & Briggs.
[34] Revans, R. (1982). *The Origin and Growth of Action Learning*. Brickley, England: Chartwell-Bratt.
[35] Smith, C. (2020). UJMM "Calculating AMBU Bag Dimensions for Use in Portable Ventilators," *Undergraduate Journal of Mathematical Modeling: One + Two*, **11**(1): Article 4. Available online at: https://scholarcommons.usf.edu/ujmm/vol11/iss1/4/
[36] Stern, A. M., Cetron, M. S., and Markel, H. (2009). Closing the schools: Lessons from the 1918-19 U.S. influenza pandemic. *Health Affairs*, **28** (1). Available on-line at: https://doi.org/10.1377/hlthaff.28.6.w1066.
[37] Vu, Q. (2020). The Famous Coin Change Problem and its Possible New Applications, *Undergraduate Journal of Mathematical Modeling: One + Two*, **11**(1): Article 5. Available on-line at: https://digitalcommons.usf.edu/ujmm/vol11/iss1/5/.
[38] Vygotsky, L. S. (1930). The instrumental method in psychology (talk given in 1930 at the Krupskaya Academy of Communist Education). *Lev Vygotsky Archive*. [On-line materials]. Available at: https://www.marxists.org/archive/vygotsky/works/1930/instrumental.htm.