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S-IDEALS IN ALMOST SEMILATTICES

G. Nanaji Rao, Ch. Swapna and Terefe Getachew Beyene

Abstract. The concepts of strong ideal (S-ideal) and prime S-ideal in an Almost Semilattice (ASL) are introduced and given certain examples of S-ideal in an ASL. Proved several basic properties of S-ideals in an ASL. Established the set $PSI(L)$, of all principal S-ideals in an ASL L form a semilattice. We proved an isomorphism of the semilattice $SI(L)$ of all S-ideals in an ASL L onto the semilattice of all ideals of a semilattice $PSI(L)$, moreover, this isomorphism gives one-to-one correspondence between the prime S-ideals of L and those of $PSI(L)$. Proved that every amicable set in an ASL L is embedded in a semilattice $PSI(L)$. Derived the semilattices $PSI(L)$ and $PF(L)$ are isomorphic. Obtain a set of equivalent conditions for an intersection of any family of filters is again a filter in terms of principal S-ideals. Finally, we proved that the filter lattice $F(L)$ of all filters in an ASL L and the filter lattice $F(PSI(L))$ of all filters in ASL $PSI(L)$ are isomorphic.

1. Introduction

Ideals were first studied by Dedekind, who defined the concept for rings of algebraic integers. Later the concept of ideal was extended to rings in general. M.H.Stone investigated ideals in Boolean rings, which are lattice of special kind. There is already a well-developed theory of ideals in lattice. We wish to show that it is useful to extend the notion of ideal to the more general systems called Almost Semilattice. There are only one reasonable way of defining what is to be meant by an ideal in a lattice. Recall that Dedekind's definition of an ideal in a ring R is that it is a collection J of elements of R which (1) contains the difference $a-b$, and hence the sum $a+b$, of any two of its elements a and b for all $a, b \in J$, and (2) contains all multiples such as ax or ya of any of $x, y \in R$ and $a \in J$, by analogy, a collection J of elements of a lattice L is called an ideal if (1) it contains the lattice

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sum $a \cup b$ of any two of its elements a and b, and (2) it contains all multiplies $a \cap x$ of any $x \in L$ *and* $a \in J$. The analogy is that the greatest lower bound, or lattice meet $a \cap b$ corresponds to product in a ring, and the least upper bound, or lattice join $a \cup b$ corresponds to the sum of two elements in a ring. In [2], Nanaji Rao, G. and Terefe G.B. introduced the concept of an ideal in an ASL L and proved several properties of ideals, principle ideals, prime ideals, filters and principal filters in an ASL.

In this paper, we introduced the concept of strong ideal(S-ideal) in an ASL L and gave certain examples to this concept. We proved that if L is an ASL with minimal element, then the set of all minimal elements in L form an S-ideal and also proved the set $PSI(L)$, of all principal S-ideals of an ASL L is a semilattice. We established an isomorphism of the semilattice $SI(L)$ of all S-ideals in an ASL L onto the semilattice of all ideals of a semilattice $PSI(L)$, moreover, this isomorphism gives one-to-one correspondence between the prime S-ideals of L and those of $PSI(L)$. We proved that every amicable set in L is embedded in the semilattice $PSI(L)$. Also, we proved the semilattices $PSI(L)$ and $PF(L)$ are isomorphic. Derived a set of equivalent conditions for an intersection of any arbitrary family of filters is again a filter in an ASL L. Finally, we proved that the lattices $F(L)$ of all filters in an ASL L and all filters in an ASL $PSI(L)$ are isomorphic.

2. Preliminaries

In this section we collect a few important definitions and results which are already known and which will be used more frequently in the paper.

Definition 2.1. Let A and B be nonempty sets. Then a relation R from A to B is a subset of $A \times B$. Relations from A to A are called relation on A.

A relation R on a nonempty set A may have the following properties:

(1) R is reflexive if for all a in A we have $(a, a) \in R$.
(2) R is symmetric if for all a and b are in A: $(a, b) \in R$ imply $(b, a) \in R$.
(3) R is antisymmetric if for all a and b are in A: $(a, b) \in R$ and $(b.a) \in R$ imply $a = b$.
(4) R is transitive if for all a, b, c are in A: $(a, b) \in R$ and $(a, c) \in R$ imply $(a, c) \in R$.

Definition 2.2. A relation R on a nonempty set A is called an equivalence relation if R is reflexive, symmetric and transitive.

Definition 2.3. A relation R on a set A is called a partial order relation if R is reflexive, antisymmetric and transitive. In this case (A, R) is called partially ordered set or poset.

Definition 2.4. A partial order $\leqslant$ on a set P is called a total order, if for any $a, b \in R$, either $a \leqslant b$ or $b \leqslant a$ holds. In this case the poset $(P, \leqslant)$ is called a totally ordered set or a chain.

Definition 2.5. Let $(P, \leqslant)$ be a poset. An element a in P is called greatest (least) element of P if for all $x \in P,\ x \leqslant a\ (a \leqslant x)$.

Definition 2.6. Let $(P, \leqslant)$ be a poset. An element a in P is called a maximal (minimal) element of P if $a \leqslant x$ $(x \leqslant a)$ implies $a = x$ for all $x \in P$.

It can be easily seen that every poset has almost one greatest (least) element. However, there may be none, one or several maximal (minimal) elements. Also, seen that greatest (least) element is maximal (minimal), but not converse.

Definition 2.7. Let $(P, \leqslant)$ be a poset and $S \subseteq P$. Let $a \in P$. Then

(1) a is called a lower bound of S if $a \leqslant x$ for all $x \in S$.
(2) a is called an upper bound of S if $x \leqslant a$ for all $x \in S$.
(3) a is called the greatest lower bound (g.l.b or infimum) of S if a is greatest among lower bounds of S in P.
(4) a ia called the least upper bound (l.u.b or supremum) of S if a is least among upper bounds of S in P.

Definition 2.8. (Zorn's Lemma) If $(P, \leqslant)$ is a poset such that every chain of elements in P has an upper bound in P, then P has at least one maximal element.

Definition 2.9. A semilattice is an algebra $(S, \star)$, where S is a nonempty set and $\star$ is a binary operation on S satisfying:

(1) $x \star (y \star z) = (x \star y) \star z$
(2) $x \star y = y \star x$
(3) $x \star x = x$

Definition 2.10. An Almost Semilattice (ASL) is an algebra $(L, \circ)$, where L is a nonempty set and $\circ$ is a binary operation on L, satisfies:

(1) $(x \circ y) \circ z = x \circ (y \circ z)$
(2) $(x \circ y) \circ z = (y \circ x) \circ z$
(3) $x \circ x = x$

Definition 2.11. An ASL with 0 is an algebra $(L, \circ, 0)$ of type (2, 0) satisfies the following conditions:

(1) $(x \circ y) \circ z = x \circ (y \circ z)$
(2) $(x \circ y) \circ z = (y \circ x) \circ z$
(3) $x \circ x = x$
(4) $0 \circ x = 0,\ for\ all\ x, y, z \in L.$

Definition 2.12. Let L be a nonempty set. Define a binary operation $\circ$ on L by $x \circ y = y$ for all $x, y \in L$. Then clearly $(L, \circ)$ is an ASL and called discrete ASL.

Definition 2.13. Let L be an ASL. Then for any $a, b \in L$ where L is an ASL, we say that a is less or equal to b and write $a \leqslant b$ if and only if $a \circ b = a$.

Definition 2.14. Let L be an ASL. Then for any $a, b \in L$, we say that a is *compatible* with b and write $a \sim b$ if $a \circ b = b \circ a$. A subset S of L is said to be *compatible set* if $a \sim b$ for all $a, b \in S$.

Definition 2.15. Let L be an ASL. Then a maximal compatible set of L is called a *maximal set*.

Definition 2.16. Let M be a maximal set in an ASL L. Then an element $x \in L$ is said to be $M-$*amicable* if there exists $a \in M$ such that $a \circ x = x$.

Lemma 2.1. *Let L be an ASL. Then for any* $a, b \in L,\ a \circ b = b \circ a$ *whenever* $a \leqslant b$.

Theorem 2.1. *Let M be a maximal set in an ASL L and* $a \in M$. *Then for any* $x \in L,\ x \circ a \in M$.

Corollary 2.1. *If M is a maximal set and x is in an ASL L is M-amicable, then there is a smallest element* $a \in M$ *with the property* $a \circ x = x$. *We denote this element a of L by* x^M.

Corollary 2.2. *Let M is a maximal set and x is an ASL L is M-amicable. Then* x^M *is the unique element of M such that* $x^M \circ x = x$ *and* $x \circ x^M = x^M$.

Definition 2.17. If M is a maximal set in an ASL L, then we denote the set of all M-amicable elements of L by $A_M(L)$.

Theorem 2.2. *Let M be a maximal set in an ASL L. Then* $(A_M(L), \circ)$ *in an ASL. Moreover, for any* $x, y \in A_M(L)$, *we have* $(x \circ y)^M = x^M \circ y^M$.

Definition 2.18. A maximal set M in an ASL L is said to be *amicable* if $A_M(L) = L$. That is, every element in L is M-amicable.

Definition 2.19. An element m in an ASL L is said to be *unimaximal* if $m \circ x = x$ for all $x \in L$.

Definition 2.20. A nonempty subset F of an ASL L is said to be a *filter* if F satisfies the following conditions:

(1) $x,\ y \in F$ implies $x \circ y \in F$
(2) If $x \in F$ *and* $a \in L$ *such that* $a \circ x = x$, *then* $a \in F$

Definition 2.21. Let L be an ASL and $a \in L$. Then $[a) = \{x \in L : x \circ a = a\}$ is a filter of L and is called principal filter generated by a.

Corollary 2.3. *Let L be an ASL. Then for any* $a, b \in L,\ a \in [b)$ *if and only if* $a \circ b = b$.

Theorem 2.3. *Let L be an ASL with unimaximal element. Then the set* $F(L)$, *of all filters in L form a lattice with respect to set inclusion, where for any* $F, G \in F(L)$, $F \wedge G = F \cap G$ *and* $F \vee G = \{t \in L : t \circ (a \circ b) = a \circ b$ *for some* $a \in F, b \in G\}$.

Theorem 2.4. *Let L be an ASL. Then the set PF(L), of all principal filters of L is a semilattice with respect to a binary operation* $\vee$, *defined by* $[a) \vee [b) = [a \circ b)$.

Theorem 2.5. *Let* $(P, \leqslant)$ *be a poset which is bounded above. If every nonempty subset of P has glb, then every nonempty subset of P has lub and hence P is a complete lattice.*

3. S-ideals

In this section we introduce the concept of strong ideal (S-ideal) in an ASL L and give few examples to this concept. We prove certain basic properties of S-ideals. If L is an ASL with minimal element, then we prove that the set of all minimal elements in L form an S-ideal. We prove that the set $PSI(L)$, of all principal S-ideals form a semilattice with respect to set intersection.
Here onwards, by L, we mean an ASL unless otherwise mentioned.

DEFINITION 3.1. Let L be an ASL. A nonempty subset I of L is said to be an S-ideal if I satisfies the following conditions:

(1) If $x \in I$ *and* $a \in L$, then $x \circ a \in I$.
(2) Let $x, y \in I$. Then there exists $d \in I$ such that $d \circ x = x, d \circ y = y$.

In the following we give some examples of S-ideals.

EXAMPLE 3.1. Let L ={ a, b, c, 0} and define a binary operation $\circ$ on L as follows:

$\circ$	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

Then clearly $(L, \circ)$ is an ASL with 0. In this ASL, if we take $I = \{0, a\}$ then clearly I is an S-ideal of L.

EXAMPLE 3.2. Let L ={ a, b, c, 0} and define a binary operation $\circ$ on L as follows:

$\circ$	0	a	b	c
0	0	0	0	0
a	0	a	a	a
b	0	a	b	c
c	0	a	b	c

Then clearly $(L, \circ)$ is an ASL with 0. In this ASL, if we take $I = \{0, a\}$ then clearly I is an S-ideal of L.

EXAMPLE 3.3. Let L ={ a, b, c, 0} and define a binary operation $\circ$ on L as follows:

$\circ$	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	a	b	c
c	0	c	c	c

Then clearly $(L, \circ)$ is an ASL with 0. In this ASL, if we take $I = \{0, c\}$ then clearly I is an S-ideal of L.

EXAMPLE 3.4. Let L ={ a, b, c, 0} and define a binary operation $\circ$ on L as follows:

$\circ$	0	a	b	c
0	0	0	0	0
a	0	a	a	0
b	0	a	b	c
c	0	0	c	c

Then clearly $(L, \circ)$ is an ASL with 0. In this ASL, if we take $I = \{0, a\}$ then clearly I is an S-ideal of L.

Next, we prove certain basic properties of S-ideals in an ASL L. First, we begin with the following.

LEMMA 3.1. *Let I be an S-ideal of an ASL L. Then for any $a, b \in L, a \circ b \in I$ if and only if $b \circ a \in I$.*

PROOF. Suppose I is an S-ideal of L and suppose $a \circ b \in I$. Then we have $(a \circ b) \circ a \in I$. It follows that $b \circ a = b \circ (a \circ a) = (b \circ a) \circ a = (a \circ b) \circ a \in I$. Similarly, we can prove that if $b \circ a \in I$ then $a \circ b \in I$. $\square$

Recall that for any $a, b \in L,$ *with* $a \leqslant b$, we have $a \circ b = b \circ a$. Now, we have the following.

LEMMA 3.2. *Let L be an ASL and let I be an S-ideal of L. Then I is an initial segment of L.*

PROOF. Suppose I is an S-ideal of L and suppose $x \in I$ *and* $a \in L$ *such that* $a \leqslant x$. Then $a = a \circ x = x \circ a$. It follows that $a \in I$. $\square$

But, converse of the above lemma 3.2 is not true. For in example 3.1, we have $I = \{0, a, b\}$ is an initial segment, but I is not an S-ideal, since there is no $d \in I$ such that $d \circ a = a$ *and* $d \circ b = b$. Also, in example 3.1, if $I_1 = \{0, a\}$ *and* $I_2 = \{0, b\}$, then clearly, I_1 *and* I_2 are S-ideals. But, $I_1 \cup I_2 = \{0, a, b\}$ is not an S-ideal. However, we prove the following.

THEOREM 3.1. *Let I and J be S-ideals of an ASL L. Then $I \cap J$ is an S-ideal.*

PROOF. Suppose I and J are S-ideals of L. Then clearly I and J are nonempty subsets of L. Hence we can choose $x \in I$ *and* $y \in J$. Then we have $x \circ y \in I$ and hence $y \circ x \in I$, since by lemma 3.1. Also, we have $y \circ x \in J$. Therefore $y \circ x \in I \cap J$. Hence $I \cap J$ is nonempty. Let $x \in I \cap J$ *and* $t \in L$. Then $x \in I$ *and* $x \in J$. Thus $x \circ t \in I$ *and* $x \circ t \in J$. Therefore $x \circ t \in I \cap J$. Suppose $x, y \in I \cap J$. Then $x, y \in I$ *and* $x, y \in J$. Thus there exists $c_1 \in I$ *and* $c_2 \in J$ such that $c_1 \circ x = x, c_1 \circ y = y$ *and* $c_2 \circ x = x, c_2 \circ y = y$. Since $c_1 \in I$ *and* $c_2 \in J$, it follows that $c_1 \circ c_2 \in I \cap J$. Now, consider $(c_1 \circ c_2) \circ x = c_1 \circ (c_2 \circ x) = c_1 \circ x = x$ and $(c_1 \circ c_2) \circ y = c_1 \circ (c_2 \circ y) = c_1 \circ y = y$. Therefore $I \cap J$ is an S-ideal. $\square$

It can be easily seen that the set $SI(L)$, of all S-ideals in an ASL L is a semilattice with respect to set intersection.

In the following, we define principal S-ideal generated by an element in an ASL L.

THEOREM 3.2. *Let L be an ASL and $a \in L$. Then $(a] = \{a \circ x : x \in L\}$ is an S-ideal of L.*

PROOF. Suppose $a \in L$. Then we have $a \circ a \in (a]$ and hence $a \in (a]$. Therefore (a] is nonempty subset of L. Let $x \in (a]$ *and* $t \in L$. Then $x = a \circ y$ *for some* $y \in L$ *and* $t \in L$. Now, consider $x \circ t = (a \circ y) \circ t = a \circ (y \circ t)$ *and* $y \circ t \in L$. Therefore $x \circ t \in (a]$. Again, let $x, y \in (a]$. Then $x = a \circ t_1, y = a \circ t_2$ *for some* $t_1, t_2 \in L$. Now, we have $a \in (a]$ and $a \circ x = a \circ (a \circ t_1) = (a \circ a) \circ t_1 = a \circ t_1 = x$. Similarly, $a \circ y = y$. Hence (a] is an S-ideal of L. □

DEFINITION 3.2. Let L be an ASL. Then for any $a \in L$, (a] is called principal S-ideal generated by a.

Next, we prove that the set $PSI(L)$, of all principal S-ideal in an ASL L is a semilattice with respect to set intersection and hence $PSI(L)$ become a subsemilattice of $SI(L)$. For this first, we need the following.

LEMMA 3.3. *Let L be an ASL and $a \in L$. Then $x \in (a]$ if and only if $x = a \circ x$.*

LEMMA 3.4. *Let L be an ASL. Then for any $a, b \in L$, $b \in (a]$ if and only if $(b] \subseteq (a]$.*

PROOF. Suppose $b \in (a]$. Then $b = a \circ b$. Now, let $t \in (b]$. Then $t = b \circ t = (a \circ b) \circ t = a \circ (b \circ t) \in (a]$. Thus $(b] \subseteq (a]$. Converse is trivial, since $b \in (b]$. □

LEMMA 3.5. *Let L be an ASL. Then for any $a, b \in L, (a \circ b] = (b \circ a]$.*

PROOF. We have $(a \circ b) \circ t = (b \circ a) \circ t$, *for all* $t \in L$. It follows that $(a \circ b] = (b \circ a]$. □

LEMMA 3.6. *Let L be an ASL. Then for any $a, b \in L$, $(a \circ b] = (a] \cap (b]$.*

PROOF. Suppose $t \in (a] \cap (b]$. Then $t \in (a]$ *and* $t \in (b]$. Thus we have $t = a \circ t$ *and* $t = b \circ t$. Now, $t = a \circ t = a \circ (b \circ t) = (a \circ b) \circ t \in (a \circ b]$. Hence $(a] \cap (b] \subseteq (a \circ b]$. Conversely, let $t \in (a \circ b]$. Then $t = (a \circ b) \circ t$. Now, $t = (a \circ b) \circ t = a \circ (b \circ t)$. Hence $t \in (a]$. Again, we have $t = (a \circ b) \circ t = (b \circ a) \circ t = b \circ (a \circ t)$. Therefore $t \in (b]$. Hence $t \in (a] \cap (b]$. Thus $(a \circ b] \subseteq (a] \cap (b]$. Therefore $(a \circ b] = (a] \cap (b]$. □

THEOREM 3.3. *Let L be an ASL. Then the set $PSI(L)$, of all principal S-ideals of L is a semilattice with respect to set intersection.*

In the following we prove some properties of S-ideals in an ASL L.

THEOREM 3.4. *Let L be an ASL with a minimal element. Then the set of all minimal elements of L forms an S-ideal.*

PROOF. Suppose L has a minimal element say a. Now, put

$$I = \{m : m \; is \; a \; mini - -mal \; element \; in \; L\}.$$

Then clearly I is nonempty, since $a \in I$. Let $x \in I \; and \; t \in L$. Then $s \circ x = x \; for \; all \; s \in L$. Now, $s \circ (x \circ t) = (s \circ x) \circ t = x \circ t \; for \; all \; s \in L$. Thus $x \circ t$ is a minimal element of L. Hence $x \circ t \in I$. Let $x, y \in I$. Then x, y are minimal. Now, let $t \in L$. Then $t \circ x \circ y = (t \circ x) \circ y = x \circ y$. Therefore $x \circ y$ is minimal. Hence $x \circ y \in I$, by the definition of minimal element and $(x \circ y) \circ x = x \; and \; (x \circ y) \circ y = y$. Therefore I is an S-ideal. $\square$

THEOREM 3.5. *Let L be an ASL. Then the following are equivalent:*

(1) *The semilattice $PSI(L)$ has least element.*
(2) *L has minimal element.*

PROOF. (1) $\Rightarrow$ (2): Suppose $PSI(L)$ has least element say $(t]$. Then we have $(t] \subseteq (x] \; for \; all \; (x] \in PSI(L)$. Thus $(t] \cap (x] = (t]$. Therefore $(t \circ x] = (t]$. Now, $t \in (t] = (t \circ x] = (x \circ t]$. Thus $t = (x \circ t) \circ t = x \circ t$. Hence $x \circ t = t, \; for \; all \; x \in L$. Therefore t is minimal. Hence L has minimal element.

(2) $\Rightarrow$ (1): Suppose L has minimal element say a. Then $(a] \in PSI(L)$. Since a is minimal, $x \circ a = a$ for all $x \in L$. This implies $(x \circ a] = (a]$. Hence $(x] \cap (a] = (a]$. Therefore $(a] \subseteq (x] \; for \; all \; x \in L$. Thus $(a]$ is the least element in the semilattice $PSI(L)$. $\square$

4. The Semilattice $PSI(L)$

In this section we introduce the concept of prime S-ideal and give necessary and sufficient condition for an S-ideal to become prime S-ideal. Also, prove that the mapping $I \mapsto I^e$ is an isomorphism of the semilattice $SI(L)$ of all S-ideals in an ASL L onto the semilattice of all ideals of a semilattice $PSI(L)$, moreover, this isomorphism gives one-to-one correspondence between the prime S-ideals of L and those of $PSI(L)$. We prove that if M_0 is the least element in the semilattice $SI(L)$, then M_0 consisting precisely the set of all minimal element in an ASL. Finally, in this section we prove that if M is a amicable set in an ASL, then M is isomorphic to the semilattice $PSI(L)$. First we begin with the following.

DEFINITION 4.1. A proper S-ideal P of an ASL L is said to be a prime S-ideal if for any $x, y \in L, x \circ y \in P$ implies that $x \in P \; or \; y \in P$.

In the following we give necessary and sufficient for an S-ideal to become prime S-ideal.

THEOREM 4.1. *Let L be an ASL and P be a proper S-ideal of L. Then P is prime S-ideal if and only if for any S-ideals I and J of L, $I \cap J \subseteq P$ imply $I \subseteq P$ or $J \subseteq P$.*

PROOF. Suppose I and J are S-ideals of L such that $I \cap J \subseteq P$ and suppose $I \nsubseteq P$. Then there exists $x \in I$ such that $x \notin P$. Let $y \in J$. Then $y \circ x \in J$ and hence $x \circ y \in J$. Also, $x \circ y \in I$. Hence $x \circ y \in I \cap J \subseteq P$. Since P is prime S-ideal and $x \notin P, y \in P$. Thus $J \subseteq P$. Conversely, assume the condition. Now, we shall prove that P is a prime S-ideal. Let $x, y \in L$ such that $x \circ y \in P$. Then

$(x] \cap (y] = (x \circ y] \subseteq P$. Therefore either $(x] \subseteq P$ *or* $(y] \subseteq P$. It follows that either $x \in P$ *or* $y \in P$. Therefore P is prime S-ideal. □

The following, we prove that the semilattices $SI(L)$ and $PSI(L)$ are isomorphic. For, this we prove the following.

LEMMA 4.1. *Let L be an ASL. Then we have the following:*

(1) *For any S-ideal I of L,* $I^e := \{(a] : a \in I\}$ *is an ideal of* $PSI(L)$. *Moreover, I is prime if and only if* I^e *is prime.*
(2) *For any ideal K of the semilattice* $PSI(L)$, $K^c := \{a \in L : (a] \in K\}$ *is an S-ideal of L. Further, K is prime if and only if so is* K^c.
(3) *For any S-ideals* I_1 *and* I_2 *of L,* $I_1 \subseteq I_2$ *if and only if* ${I_1}^e \subseteq {I_2}^e$.
(4) *For any ideals* K_1 *and* K_2 *of the semilattice* $PSI(L)$, $K_1 \subseteq K_2$ *if and only if* ${K_1}^c \subseteq {K_2}^c$.
(5) $I^{e^c} = I$ *for all S-ideals I of L.*
(6) $K^{c^e} = K$ *for all ideals K of the semilattice* $PSI(L)$.

PROOF. (1) Suppose I is an S-ideal of L. Now, we shall prove that $I^e := \{(a] : a \in I\}$ is an ideal of the semilattice $PSI(L)$. Since I is nonempty, it follows that I^e is nonempty. Let $(a] \in I^e$ *and* $(t] \in PSI(L)$ *such that* $(t] \subseteq (a]$. Then $(t] = (t] \cap (a] = (t \circ a] = (a \circ t] \in I^e$ since $a \in I, t \in L$ and I is an ideal of L. Therefore $(t] \in I^e$. Hence I^e is a down set. Let $(a], (b] \in I^e$. Then $a, b \in I$. Therefore, there exists $d \in I$ *such that* $d \circ a = a$ *and* $d \circ b = b$. This implies $(d \circ a] = (a]$ *and* $(d \circ b] = (b]$. Therefore $(d] \cap (a] = (a]$ *and*$(d] \cap (b] = (b]$. It follows that $(a] \subseteq (d], (b] \subseteq (d]$ *and* $(d] \in I^e$. Thus I^e is an ideal of the semilattice $PSI(L)$. Suppose I is a prime S-ideal of L. Now, we shall prove that I^e is a prime ideal of the semilattice $PSI(L)$. Since I is proper, I^e is proper. Let $(x], (y] \in PSI(L)$ such that $(x] \cap (y] \in I^e$. Then $(x \circ y] \in I^e$. Therefore $(x \circ y] = (a]$ *for some* $a \in I$. Since $x \circ y \in (x \circ y] = (a], x \circ y = a \circ (x \circ y)$. Therefore $x \circ y \in I$. Since I is prime, either $x \in I$ *or* $y \in I$. It follows that either $(x] \in I^e$ *or* $(y] \in I^e$. Thus I^e is a prime ideal of $PSI(L)$. Conversely, suppose I^e is a prime ideal of the semilattice $PSI(L)$. Since I^e is proper, I is proper. Let $x, y \in L$ *such that* $x \circ y \in I$. Then $(x] \cap (y] = (x \circ y] \in I^e$. Therefore either $(x] \in I^e$ *or* $(y] \in I^e$, since I^e is a prime ideal of the semilattice $PSI(L)$. It follows that either $x \in I$ *or* $y \in I$. Thus I is a prime S-ideal of L.

(2) Suppose K is an ideal of the semilattice $PSI(L)$. Now, we shall prove that $K^c := \{a \in L : (a] \in K\}$ is an S-ideal of L. Since K is nonempty, it follows that K^c is nonempty. Let $x \in K^c$ *and* $t \in L$. Then $(x] \in K$ *and* $(t] \in PSI(L)$. Therefore there exists $(d] \in K$ *such that* $(x] \subseteq (d]$ *and* $(y] \subseteq (d]$, since K is an ideal of the semilattice PSI(L). This implies $(d] \cap (x] = (d \circ x] = (x]$ *and* $(d] \cap (y] = (d \circ y] = (y]$. Since $x \in (x] = (d \circ x],\ x = (d \circ x) \circ x = d \circ (x \circ x) = d \circ x$. Similarly, $d \circ y = y$. Thus $d \circ x = x,\ d \circ y = y$ *and* $d \in K^c$. Hence K^c is an S-ideal of L. Suppose K is a prime ideal of the semilattice $PSI(L)$. Now, we shall prove that K^c is a S-ideal of L. Since K is proper, K^c is proper. Let $x, y \in L$ *such that* $x \circ y \in K^c$. Then $(x) \cap (y] = (x \circ y] \in K$. Therefore either $(x] \in K$ *or* $(y] \in K$, since K is prime ideal in $PSI(L)$. It follows that either $x \in K^c$ *or* $y \in K^c$. Thus K^c is a

prime S-ideal of L. Conversely, suppose K^c is a prime S-ideal of L. Now, we shall prove that K is a prime ideal of the semilattice $PSI(L)$. Let $(x], (y] \in PSI(L)$ such that $(x] \cap (y] = (x \circ y] \in K$. Then $x \circ y \in K^c$. Since K^c is prime, either $x \in K^c$ *or* $y \in K^c$. It follows that either $(x] \in K$ *or* $(y] \in K$. Thus K is a prime ideal of the semilattice $PSI(L)$.

(3) Suppose I_1 *and* I_2 are S-ideals of L such that $I_1 \subseteq I_2$. Let $(a] \in {I_1}^e$. Then $a \in I_1$ and hence $a \in I_2$. Therefore $(a] \in {I_2}^e$. Thus ${I_1}^e \subseteq {I_2}^e$. Conversely, suppose ${I_1}^e \subseteq {I_2}^e$. Let $a \in I_1$. Then $(a] \in {I_1}^e \subseteq {I_2}^e$. Therefore $(a] = (t]$ *for some* $t \in I_2$. Hence $a = t \circ a \in I_2$. Thus $I_1 \subseteq I_2$.

(4) Suppose K_1 *and* K_2 are ideals of the semilattice $PSI(L)$ such that $K_1 \subseteq K_2$. Let $a \in {K_1}^c$. Then $(a] \in K_1$. Thus $(a] \in K_2$. Therefore $(a] = (t]$ *for some* $(t] \in K_2$. Hence $a = t \circ a \in {K_2}^c$. Thus ${K_1}^c \subseteq {K_2}^c$. Conversely, suppose ${K_1}^c \subseteq {K_2}^c$. Let $(a] \in K_1$. Then $a \in {K_1}^c \subseteq {K_2}^c$. Therefore $a \in {K_2}^c$. Hence $(a] \in K_2$. Thus $K_1 \subseteq K_2$.

(5) Suppose I is an S-ideal of L. Now, let $a \in I^{e^c}$. Then $(a] \in I^e$. Therefore $(a] = (t]$ *for some* $t \in I$. Hence $a = t \circ a \in I$. Therefore $I^{e^c} \subseteq I$. Clearly, $I \subseteq I^{e^c}$. Thus $I = I^{e^c}$.

(6) Suppose K is an ideal of the semilattice $PSI(L)$. Now, let $(a] \in K^{c^e}$. Then $a \in K^c$. Therefore $(a] \in K$. Thus $K^{c^e} \subseteq K$. Conversely, suppose $(a] \in K$. Then $a \in K^c$ and hence $(a] \in K^{c^e}$. Therefore $K \subseteq K^{c^e}$. Thus $K = K^{c^e}$. □

LEMMA 4.2. *Let I and J be an S-ideals of an ASL L. Then* $(I \cap J)^e = I^e \cap J^e$.

PROOF. Suppose I and J are S-ideals of L. Then $I \cap J \subseteq I, J$. Therefore by lemma 4.1, we have $(I \cap J)^e \subseteq I^e, J^e$. Hence $(I \cap J)^e \subseteq I^e \cap J^e$. Conversely, suppose $(a] \in I^e \cap J^e$. Then $(a] \in I^e$ *and* $(a] \in J^e$. Hence $(a] = (t]$ *for some* $t \in I$ *and* $(a] = (s]$ *for some* $s \in J$. Therefore $a \in (a] = (t]$ and hence $a = t \circ a \in I$. Similarly, we get $a = s \circ a \in J$. Hence $a \in I \cap J$. It follows that $(a] \in (I \cap J)^e$. Therefore $I^e \cap J^e \subseteq (I \cap J)^e$ and hence $(I \cap J)^e = I^e \cap J^e$. □

Thus we have the following theorem, whose proof follows by lemma 4.1 and lemma 4.2.

THEOREM 4.2. *The mapping* $I \mapsto I^e$ *is an isomorphism of the semilattice* $SI(L)$ *of all S-ideals in an ASL L onto the semilattice of all ideals of a semilattice* $PSI(L)$. *Moreover, this isomorphism gives one-to-one correspondence between the prime S-ideals of L and those of* $PSI(L)$.

Next, we prove that if M_0 is the least element in the semilattice $SI(L)$ then M_0 contains precisely of all minimal elements in an ASL L. For, this first we need the following lemma.

LEMMA 4.3. *Let L be an ASL. Then for any* $a, b \in L$, $(a] \subseteq (b]$ *whenever* $a \leqslant b$.

PROOF. Sppose $a \leqslant b$. Then $a = a \circ b$. Now, let $t \in (a]$. Then $t = a \circ t$. Now, $t = a \circ t = (a \circ b) \circ t = (b \circ a) \circ t = b \circ (a \circ t) \in (b]$. Therefore $(a] \subseteq (b]$. □

THEOREM 4.3. *Let L be an ASL with minimal element and let* M_0 *denote the least element of* $SI(L)$. *Then* M_0 *contains precisely the all minimal elements of L.*

Proof. Suppose $x \in M_0$. Now, we shall prove that x is minimal. Suppose $a \in L$ *such that* $a \leqslant x$. Then by lemma 4.3, $(a] \subseteq (x]$ *and* $(x] \subseteq M_0$. On the other hand, M_0 is the least element of SI(L), $M_0 \subseteq (a] \subseteq (x]$. It follows that $M_0 \subseteq (a] \subseteq (x] \subseteq M_0$. Hence $M_0 = (a] = (x]$. Now, $x \in (x] = (a]$ and hence $x = a \circ x = a$, since $a \leqslant x$. Thus x is minimal. Now, suppose $x \in L$ such that x is minimal. Since M_0 is S-ideal, we can choose $a \in M_0$. Therefore $x = a \circ x$, since x is minimal. Thus $x \in M_0$, since $a \in M_0$. Hence M_0 contains precisely all minimal elements in L. □

Corollary 4.1. *Let L be an ASL with a minimal element. Then for any* $x, y \in L$, $x \circ y$ *is minimal if and only if* $y \circ x$ *is minimal.*

Proof. We have for any S-ideal I of L, $x \circ y \in I$ if and only if $y \circ x \in I$. It follows that $x \circ y$ is minimal if and only if $y \circ x$ is minimal. □

It can be easily seen that if an S-ideal I of an ASL L contains a unimaximal element then I = L. For, suppose I is an S-ideal of L and suppose a is a unimaximal element in L such that $a \in I$. Then for any $x \in L$, we have $x = a \circ x \in I$. It follows that I = L. Recall that if M is a maximal set and $x \in L$ is M-amicable, then there exists unique element x^M in M with the property $x^M \circ x = x$ *and* $x \circ x^M = x^M$. In the following, we prove that every amicable set in an ASL L is isomorphic to the semilattice $PSI(L)$. First we need the following.

Lemma 4.4. *Let M be an amicable set in an ASL L. Then for any* $x \in L$, $(x] = (x^M]$.

Lemma 4.5. *let M be an amicable set in an ASL L. Then for any* $x, y \in L$, *the following are equivalent:*

(1) $(x] = (y]$
(2) $(x^M] = (y^M]$
(3) $x^M = y^M$

Proof.

(1) ⇒ (2): Suppose M is an amicable set in L. Then we have $A_M(L) = L$. Now, let $x, y \in L = A_M(L)$. Then there exists unique element $x^M, y^M \in M$ such that $x^M \circ x = x$ *and* $x \circ x^M = x^M$ and also $y^M \circ y = y$ *and* $y \circ y^M = y^M$. Therefore by lemma 4.4, we get $(x] = (x^M]$ *and* $(y] = (y^M]$. Thus $(x^M] = (y^M]$.

(2) ⇒ (3): Assume (2). Then we have $x^M \in (x^M] = (y^M]$. Thus $x^M = y^M \circ x^M = x^M \circ y^M$ *since* x^M, $y^M \in M$. Hence $x^M \leqslant y^M$. Similarly, we get $y^M \leqslant x^M$. Therefore $x^M = y^M$.

(3) ⇒ (1): Assume (3). We need to show that (x] = (y]. Let $t \in (x]$. Then $t = x \circ t = (x^M \circ x) \circ t = (x \circ x^M) \circ t = x^M \circ t = y^M \circ t = (y \circ y^M) = (y^M \circ y) \circ t = y \circ t \in (y]$. Hence $(x] \subseteq (y]$. Similarly we can prove that $(y] \subseteq (x]$. Therefore (x] = (y]. □

Lemma 4.6. *Let M be a maximal set in an ASL L. Then for any* $x, y \in M$, *the following are equivalent:*

(1) $x = y$

(2) *(x] = (y]*

Proof.
(1) $\Rightarrow$ (2): Assume (1). Since $x \in (x] = (y], x \in (y]$. Hence $(x] \subseteq (y]$. Similarly, $(y] \subseteq (x]$. Thus $(x] = (y]$.
(2) $\Rightarrow$ (1): Assume (2). Now, $x \in (x] = (y]$. It follows that $x = y \circ x = x \circ y$. Therefore $x \leqslant y$. Similarly, we can prove that $y \leqslant x$. Thus x = y. □

Now, we prove the following.

Theorem 4.4. *Let M be an amicable set in an ASL L. Then the mapping $x \mapsto (x]$ is an isomorphism of M onto the semilattice $PSI(L)$.*

Proof. Define $f : M \to PSI(L)$ by f(x) = (x] for all $x \in M$. Then clearly, f is both well defined and one-one. Now, let $(x] \in PSI(L)$. Then $x \in L = A_M(L)$. Therefore there exists $a \in M$ such that $a \circ x = x$. Now, for any $t \in M, (x \circ a) \circ t = x \circ (a \circ t) = x \circ (t \circ a) = (x \circ t) \circ a = (t \circ x) \circ a = t \circ (x \circ a)$. Therefore $x \circ a \in M$. Now, $f(x \circ a) = (x \circ a] = (a \circ x] = (x]$. Hence f is onto. Now, it remains to show that f is a homomorphism. Suppose $x, y \in M$. Then $f(x \circ y) = (x \circ y] = (x] \cap (y] = f(x) \cap f(y)$. Thus f is an isomorphism. □

5. Properties of S-ideals and Filters in ASLs

In this section we prove that the semilattices $PSI(L)$ and $PF(L)$ are isomorphic. Also, we give a set of equivalent conditions for the intersection of any family of filters in ASL L is again a filter. We establish the filter lattice $F(L)$ of an ASL L and the filter lattice $F(PSI(L))$ of an ASL $PSI(L)$ are isomorphic. First, we prove the following.

Lemma 5.1. *Let L be an ASL and $a, b \in L$. Then the following are equivalent:*
(1) $(a] \subseteq (b]$
(2) $b \circ a = a$
(3) $[b) \subseteq [a)$

Proof. (1) $\Rightarrow$ (2): Assume (1). Then we have $a \in (a] \subseteq (b]$. It follows that $b \circ a = a$.
(2) $\Rightarrow$ (3): Assume (2). Let $t \in [b)$. Then $t \circ b = b$. But, by (2), $a = b \circ a$. Hence $t \circ a = t \circ (b \circ a) = (t \circ b) \circ a = b \circ a = a$. Therefore $t \in [a)$. Thus $[b) \subseteq [a)$.
(3) $\Rightarrow$ (1): Assuume (3). Then $b \in [b) \subseteq [a)$. Hence $b \circ a = a$. Therefore $a \in (b]$. Thus $(a] \subseteq (b]$. □

Lemma 5.2. *Let L be an ASL L. Then for any*

$$a, b \in L, (a] = (b] \textit{ if and only if } [a) = [b).$$

In the following we prove that the semilattices $PSI(L)$ and PF(L) are isomorphic.

Theorem 5.1. *Let L be an ASL. Then a mapping $\psi : PSI(L) \to PF(L)$ defined by $\psi((a]) = [a)$ for all $(a] \in PSI(L)$ is an isomorphism.*

Proof. Clearly ψ is well defined and one-one, since by lemma 5.2. Also, clearly ψ is onto. Hence ψ is a bijection. Let $(a], (b] \in PSI(L)$. Then $\psi((a \circ b]) = [a \circ b) = [a) \vee [b) = \psi((a]) \vee \psi((b])$. Therefore ψ is a homomorphism. Thus ψ is an isomorphism. □

Theorem 5.2. *Let L be an ASL. Then the following are equivalent:*

(1) *The semilattice $PSI(L)$ has least element.*
(2) *The semilattice PF(L) has greatest element.*
(3) *L has minimal element.*

Proof. (1) ⇒ (2): Suppose $PSI(L)$ has least element say $(a]$. Then $(a] \subseteq (x]$ for all $x \in L$. Therefore by Lemma 5.1, we have $[x) \subseteq [a)\ for\ all\ x \in L$. Thus $[a)$ is the greatest element of PF(L).

(2) ⇒ (3): Suppose PF(L) has greatest element say $[a)$. Then $[x) \subseteq [a)$ for all $x \in L$. Hence by Lemma 5.1, $x \circ a = a\ for\ all\ x \in L$. Thus a is a minimal element of L. Therefore L has minimal element.

(3) ⇒ (1): Suppose L has minimal element say a. Then we have $x \circ a = a$ for all $x \in L$. Therefore $a \in (x]$. It follows that $(a] \subseteq (x]\ for\ all\ x \in L$. Hence $(a]$ is the least element in $PSI(L)$. Thus $PSI(L)$ has least element. □

Next, we give a set of equivalent conditions for the intersection of any family of filters is again a filter in terms of principal S-ideal in an ASL L.

Theorem 5.3. *Let L be an ASL. Then the following are equivalent:*

(1) *The intersections of any family of filters is nonempty.*
(2) *The intersections of any family of filters is again a filter.*
(3) *The lattice F(L) has least element.*
(4) *The lattice F(L) is complete.*
(5) *The semilattice PF(L) has least element.*
(6) *The semilattice $PSI(L)$ has greatest element.*
(7) *L has unimaximal element.*

Proof. Proof of (1) ⇒ (2) and (2) ⇒ (3) are clear.

(3) ⇒ (4): Suppose the lattice F(L) has least element. Now, we shall prove that F(L) is complete. Let $\{F_\alpha\}_{\alpha \in \Delta}$ be a nonempty subset of F(L). Define $F = \{x \in L : x \circ (a_1 \circ a_2 \circ ... \circ a_n) = a_1 \circ a_2 \circ ... \circ a_n$; where $a_i \in F_{\alpha_i}$ for all $\alpha_i \in \Delta$ and n is a positive integer $\}$. Then clearly, F is a filter of L and $F_\alpha \subseteq F$ for all $\alpha \in \Delta$. Therefore F is an upper bound of $\{F_\alpha\}_{\alpha \in \Delta}$. Now, let H be a filter of L such that $F_\alpha \subseteq H$ for all $\alpha \in \Delta$. Now, let $x \in F$. Then $x \circ (a_1 \circ a_2 \circ ... \circ a_n) = a_1 \circ a_2 \circ ... \circ a_n$ and $a_i \in F_{\alpha_i} \subseteq H$ for all i. It follows that $x \in H$. Hence $F \subseteq H$. Thus F is a least upper bound of F(L). Hence the lattice F(L) is complete.

(4) ⇒ (5): Suppose F(L) is complete. Since $PF(L) \subseteq F(L)$, $PF(L)$ has greatest lower bound say F. We shall prove that F is a principal filter. Let $a \in F$. Then $[a) \subseteq F$. Now, let $b \in F$. Then $[b) \subseteq F$. Therefore $[b)$ is a lower bound of $PF(L)$ and hence $[b) \subseteq [a)$. It follows that $b \in [a)$. Hence $F \subseteq [a)$. Therefore $F = [a) \in PF(L)$. Thus $PF(L)$ has least element.

(5) $\Rightarrow$ (6): Suppose $PF(L)$ has a least element say $[a)$. Then $[a) \subseteq [x)$ for all $x \in L$. Then we have $(x] \subseteq (a]$ for all $x \in L$. It follows that $(a]$ is a greatest element of $PSI(L)$.

(6) $\Rightarrow$ (7): Suppose $PSI(L)$ has a greatest element say $(a]$. Then $(x] \subseteq (a]$,for all $x \in L$. This implies $x \in (a]$ for all $x \in L$. Therefore $a \circ x = x$ for all $x \in L$. Thus a is a unimaximal element of L.

(7) $\Rightarrow$ (1): Suppose L has a unimaximal element. Since every filters in L contains a unimaximal element, it follows that the intersection of all filters in L is nonempty. □

In the following, we prove that filter lattices $F(L)$ of all filters in an ASL L, $F(PSI(L))$ of all filters in an ASL L $PSI(L)$ are isomorphic. First, we need the following.

Lemma 5.3. *If F is a filters of an ASL L then* $\{(a] : a \in F\}$ *is filter of the semilattice* $PSI(L)$.

Proof. Let F be a filter of an ASL. Now, we shall prove that $\{(a] : a \in F\}$ is a filter of the semilattice $PSI(L)$. Put $\bar{F} = \{(a] : a \in F\}$. Clearly $\bar{F}$ is nonempty, since F is nonempty. Let $(a], (b] \in \bar{F}$. Then $a, b \in F$ and hence $a \circ b \in F$, since F is a filter of L. Therefore $(a] \cap (b] = (a \circ b] \in \bar{F}$. Also, let $(a] \in \bar{F}$ *and* $(t] \in PSI(L)$ such that $(a] \subseteq (t]$. Then by lemma 5.1, $t \circ a = a$. Since $t \circ a = a,\ a \in F$. Therefore $t \in F$, since F is filter of L. Hence $(t] \in \bar{F}$. Thus $\bar{F}$ is a filter of the semilattice $PSI(L)$. □

Theorem 5.4. *The mapping* $F \mapsto \{(a] : a \in F\}$ *is an isomorphism of the filter lattice* $F(L)$ *of* L *onto the filter lattice of an ASL* $PSI(L)$.

Proof. Define $\psi : F(L) \to F(PSI(L))$ by $\psi(F) = \{(a] : a \in F\}$. Then clearly, ψ is well defined. Suppose $F_1, F_2 \in F(L)$ such that $\psi(F_1) = \psi(F_2)$. Now, we shall prove that $F_1 = F_2$. Let $x \in F_1$. Then $(x] \in \psi(F_1) = \psi(F_2)$. Therefore $(x] = (y]$, for some $y \in F_2$. Hence by lemma 5.2, $[x) = [y)$. Now, $y \in [y) = [x)$. This implies $y \circ x = x$. Since $y \in F_2,\ x \in F_2$. Hence $F_1 \subseteq F_2$. Similarly, we can prove that $F_2 \subseteq F_1$. Therefore $F_1 = F_2$. Thus ψ is one-one. Let $\bar{F} \in F(PSI(L))$. Define $F = \{a \in L : (a] \in \bar{F}\}$. First we shall prove that F is a filter of L. Let $a, b \in F$. Then $(a], (b] \in \bar{F}$. Therefore $(a \circ b] = (a] \cap (b] \in \bar{F}$, since $\bar{F}$ is a filter of $PSI(L)$. Thus $a \circ b \in F$. Let $a \in F$ and $t \in L$ such that $t \circ a = a$. Then $(t \circ a] = (a]$. Therefore $(t] \cap (a] = (t \circ a] = (a]$ and $(a] \in \bar{F}$. Hence $(t] \in \bar{F}$, since $\bar{F}$ is a filter of an $ASL\ PSI(L)$. Thus $t \in F$ and hence F is a filter of L. Therefore $F \in F(L)$ and clearly $\psi(F) = \bar{F}$. Thus ψ is onto. Now, it remains to show that ψ is a homomorphism. Now, for any $(a] \in PSI(L)$, we have

$$\begin{aligned}
(a] \in \psi(F_1 \cap F_2) &\Leftrightarrow a \in F_1 \cap F_2 \\
&\Leftrightarrow a \in F_1 \ \textit{and} \ a \in F_2 \\
&\Leftrightarrow (a] \in \psi(F_1) \ \textit{and} \ (a] \in \psi(F_1) \\
&\Leftrightarrow (a] \in \psi(F_1) \cap \psi(F_1)
\end{aligned}$$

and

$$
\begin{aligned}
(a] \in \psi(F_1 \vee F_2) \Leftrightarrow\ & a \in F_1 \vee F_2. \\
& \Leftrightarrow a \circ (x \circ y) = x \circ for\ some\ x \in F_1\ and\ y \in F_2 \\
& \Leftrightarrow (a \circ (x \circ y)] = (x \circ y] \\
& \Leftrightarrow (a] \cap ((x] \cap (y]) = (x] \cap (y] \text{ and } (x] \in \psi(F_1)
\end{aligned}
$$

and

$$(x] \in \psi(F_2) \Leftrightarrow (a] \in \psi(F_1) \vee \psi(F_2).$$

Thus ψ is a homomorphism and hence it is an isomorphism. $\square$

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G. Nanaji Rao: Department of Mathematics, Andhra University, Visakhapatnam-530003, India.

E-mail address: nani6us@yahoo.com, drgnanajirao.math@auvsp.edu.in,

Ch. Swapna: Department of Mathematics, Andhra University, Visakhapatnam-530003, India,

E-mail address: swapnachettupilli@yahoo.com

Terefe Getachew Beyene: Addis Ababa Science and Technology University, Addis Ababa, Ethiopia

E-mail address: gterefe4@gmail.com