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SOME RESULTS ON EDGE IRREGULAR TOTAL LABELING

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Abstract. An edge irregular total k-labeling $f : V \cup E \to \{1, 2, 3, \dots, k\}$ of a graph $G = (V, E)$ is a labeling of vertices and edges of G in such a way that for any two different edges uv and $u'v'$ their weights $f(u) + f(uv) + f(v)$ and $f(u') + f(u'v') + f(v')$ are distinct. The total edge irregularity strength $tes(G)$ is defined as the minimum k for which the graph G has an edge irregular total k-labeling. In this paper, we study the total edge irregularity strength for shadow graph of cycle and path, total graph of cycle and path, lotus inside a circle, double wheel graph.

1. Introduction

The graphs in this paper are simple, finite and undirected. In [**11**] Bača et al. defined the notion of edge irregular total k-labeling of a graph G as a function $\phi : V \cup E \to \{1, 2, \dots, k\}$ such that the edge weights $wt_\phi(uv) = \phi(u) + \phi(uv) + \phi(v)$ are distinct for all the edges. That is $wt_\phi(uv) \neq wt_\phi(u'v')$ for every pair of edges $uv, u'v' \in E$. The minimum k for which the graph G has an edge irregular total k-labeling is called the *total edge irregularity strength* of G, $tes(G)$. They found a lower bound for the total edge irregularity strength of any graph as

$$tes(G) \geqslant max\left\{ \left\lceil \frac{(|E(G)| + 2)}{3} \right\rceil, \left\lceil \frac{(\Delta(G) + 1)}{2} \right\rceil \right\} \tag{1.1}$$

where $\Delta(G)$ is the maximum degree of G. Ivančo and Jendroľ [**13**] posed the following conjecture.

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Key words and phrases. irregularity strength; total edge irregularity strength; edge irregular total labeling; shadow graph; total graph; lotus inside a circle; doublewheel graph.

CONJECTURE 1.1. [13] Let G be an arbitrary graph different from K_5. Then

$$tes(G) = max\left\{ \left\lceil \frac{(|E(G)| + 2)}{3} \right\rceil, \left\lceil \frac{(\Delta(G) + 1)}{2} \right\rceil \right\} \tag{1.2}$$

Conjecture (1.1) has been verified for the categorical product of a cycle [1], strong product [2], categorical product of two paths $P_n \times P_m$ [3], categorical product of two cycles[4], strong product of cycles and paths [5], disjoint union of prisms and cycles [6], zigzag graphs[7], hexagonal grid graphs [8], generalized prism [9], toroidal fullerene [10], large dense graphs with $\frac{(|E(G)|+2)}{3} \leqslant \frac{(\Delta(G)+1)}{2}$ [12], trees [13], complete graphs [14], complete bipartite graphs [15], cartesian product of two paths $P_n \times P_m$[20], corona product of a path with certain graphs [21] categorical product of cycle and path [22], and for subdivision of star [23] . We found [16, 17, 18, 19] the total edge irregularity strength of closed helm graph CH_n and flower graph Fl_n, the disjoint union of wheel graphs, double wheel graphs, armed crown graph, splitting graph, tadpole graph. In this paper, we determine exact values of the total edge irregularity strength for shadow graph of cycle and path, total graph of cycle and path, lotus inside a circle, double wheel graph.

DEFINITION 1.1. The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G''. Join each vertex u'in G' to the neighbours of the corresponding vertex u'' in G''.

DEFINITION 1.2. The total graph $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and the two vertices are adjacent in $T(G)$whenever they are either adjacent or incident in G.

DEFINITION 1.3. The lotus inside a circle LC_n is a graph obtained from the cycle $C_n : u_1, u_2, \dots, u_n, u_1$ and the star $K_{1,n}$ with central vertex v and the end vertices $v_1, v_2, v_3, \dots, v_n$ by joining each u_i to v_i and $v_{i+1}(mod n)$.

DEFINITION 1.4. A double-wheel graph DW_n of size n can be composed of $2C_n + K_1$, that is it consists of two cycles of size n, where all the vertices of the two cycles are connected to a common hub.

2. Main Results

THEOREM 2.1. $tes(D_2(C_n)) = \left\lceil \frac{4n+2}{3} \right\rceil, n \geqslant 3$.

PROOF. Let $V(D_2(C_n)) = \{u_i, v_i : 1 \leqslant i \leqslant n\}$ and $E(D_2(C_n)) = \{u_i v_{i+1}, v_i u_{i+1}, v_i v_{i+1}, u_i u_{i+1} : 1 \leqslant i \leqslant n\}$ with indices taken modulo n. Let $k = \left\lceil \frac{4n+2}{3} \right\rceil$, then from (1.1) it follows that, $tes(D_2(C_n)) \geqslant \left\lceil \frac{4n+2}{3} \right\rceil = k$. That is $tes(D_2(C_n)) \geqslant k$. To prove the reverse inequality we define a function f from $V \cup E$ to $\{1, 2, 3, \dots, k\}$ by considering the following two cases.

Case(i): n is odd

$n = 3, n = 5, n = 7$, we consider the following labeling.

when $n = 3$

$f(u_1) = 1, f(u_2) = 3, f(u_3) = 2, f(v_1) = f(v_3) = 5, f(v_2) = 1, f(v_1v_2) = f(v_3v_1) = f(u_3u_1) = 4, f(v_1u_2) = f(u_3v_1) = f(v_2v_3) = 5, f(u_1u_2) = f(v_2u_3) = f(u_1v_2) = f(u_2v_3) = f(u_2u_3) = 1, f(v_3u_1) = 2.$

when $n = 5$

$f(u_1) = 1, f(u_2) = 5, f(u_3) = 2, f(v_1) = f(v_3) = f(v_5) = 8, f(v_2) = 1, f(v_4) = 2, f(u_4) = 6, f(u_5) = 3, f(v_1v_2) = 5, f(v_2v_3) = f(v_3v_4) = 6, f(v_4v_5) = 7, f(v_5v_1) = 2, f(u_1u_2) = f(u_2u_3) = f(u_3u_4) = f(u_4u_5) = 1, f(u_5u_1) = 8, f(u_1v_2) = f(u_3v_4) = f(v_2u_3) = f(v_4u_5) = 1, f(u_2v_3) = f(v_3u_4) = 7, f(u_5v_1) = f(v_5u_1) = 2, f(v_1u_2) = 6, f(u_4v_5) = 8.$

when $n = 7$

$f(u_1) = 1, f(u_2) = 7, f(u_3) = 2, f(u_4) = 8, f(u_5) = 3, f(u_6) = 9, f(u_7) = 4, f(v_1) = f(v_3) = f(v_5) = f(v_7) = 10, f(v_2) = 1, f(v_4) = 2, f(v_6) = 3, f(v_1v_2) = 7, f(v_2v_3) = f(v_3v_4) = 8, f(v_4v_5) = f(v_5v_6) = 9, f(v_6v_7) = f(v_7v_1) = 10, f(u_1u_2) = f(u_2u_3) = f(u_3u_4) = f(u_4u_5) = f(u_5u_6) = f(u_6u_7) = 1, f(u_7u_1) = 10, f(u_1v_2) = f(u_3v_4) = f(u_5v_6) = f(v_4u_5) = f(v_6u_7) = 1, f(u_7v_1) = 2, f(u_2v_3) = f(v_3u_4) = 9, f(u_4v_5) = f(v_5u_6) = f(u_6v_7) = 5, f(v_1u_2) = 8, f(v_2u_3) = 1, f(v_7u_1) = 6.$

Now we define a labeling for $n \geqslant 9$

$$f(v_i) = \begin{cases} k, & if\ i\ is\ odd \\ \frac{i}{2}, & if\ i\ is\ even; \end{cases}$$

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n \\ n - 1 + \frac{i}{2}, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ k, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n-1; \end{cases}$$

$$f(u_iv_{i+1}) = \begin{cases} 1, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-2 \\ 2n + 4 - \frac{i}{2} - k + i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ 3n + 3 - 2k + i, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n-1 \\ 2, & if\ i = n; \end{cases}$$

$$f(v_iu_{i+1}) = \begin{cases} 2n + 4 - k - \frac{i+1}{2} + i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n)-1 \\ 3n + 3 - 2k + i, & if\ i\ is\ odd,\ 2(k-n)+1 \leqslant i \leqslant n-2 \\ 2n - k + 2, & if\ i = n \\ 1, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n-1; \end{cases}$$

$$f(u_iu_{i+1}) = \begin{cases} 1, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n)-1 \\ n+1-\frac{i+1}{2}-k+i, & if\ i\ is\ odd,\ 2(k-n)+1 \leqslant i \leqslant k+\lceil\frac{n}{2}\rceil-n-1 \\ n+3-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ k+\lceil\frac{n}{2}\rceil\ -n \leqslant i \leqslant n-1 \\ k, & if\ i=n \\ 1, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ n+1-k-\frac{i+2}{2}+i, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant k+\lceil\frac{n}{2}\rceil-n-1 \\ n+3-k-\frac{i+2}{2}+i, & if\ i\ is\ even,\ k+\lceil\frac{n}{2}\rceil-n \leqslant i \leqslant n-1; \end{cases}$$

$$f(v_iv_{i+1}) = \begin{cases} 2n+3-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-2 \\ 3n+3-2k, & if\ i=n \\ 2n+3-k-\frac{i}{2}+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n-1. \end{cases}$$

We observe that,

$$wt(u_iv_{i+1}) = \begin{cases} 2+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-2 \\ k+\frac{n+1}{2}+2 & if\ i=n \\ 3n+3+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n-1; \end{cases}$$

$$wt(v_iu_{i+1}) = \begin{cases} 3n+3+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-2 \\ 2n+3 & if\ i=n \\ 2+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n-1; \end{cases}$$

$$wt(u_iu_{i+1}) = \begin{cases} n+1+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant k+\lceil\frac{n}{2}\rceil-n-1 \\ n+3+i, & if\ i\ is\ odd,\ k+\lceil\frac{n}{2}\rceil-n \leqslant i \leqslant n-1 \\ n+1+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant k+\lceil\frac{n}{2}\rceil-n-1 \\ n+3+i, & if\ i\ is\ even,\ k+\lceil\frac{n}{2}\rceil-n \leqslant i \leqslant n-1 \\ k+\frac{n+1}{2}+1, & if\ i=n; \end{cases}$$

$$wt(v_iv_{i+1}) = \begin{cases} 2n+3+i, & if\ \ 1 \leqslant i \leqslant n-1 \\ 3n+3, & if\ \ i=n. \end{cases}$$

Case(ii): n is even

when $n = 4$, we consider the following labeling.

$f(u_1) = 1, f(u_2) = 4, f(u_3) = 2, f(u_4) = 6, f(v_1) = f(v_3) = 6, f(v_4) = 2, f(v_2) = 1, f(v_1v_2) = 4, f(v_2v_3) = f(v_3u_4) = f(v_3v_4) = f(v_1u_2) = 5, f(v_4v_1) = 6, f(u_1u_2) = f(u_2u_3) = 2, f(u_3u_4) = f(u_3v_4) = f(u_1v_2) = f(v_2u_3) = 1, f(u_2v_3) = 6, f(u_4v_1) = 6, f(v_4u_1) = f(u_4u_1) = 3.$

Now we define a labeling for $n \geqslant 6$.

$$f(v_i) = \begin{cases} k, & if\ i\ is\ odd \\ \frac{i}{2}, & if\ i\ is\ even; \end{cases}$$

$$f(u_i)=\begin{cases}\frac{i+1}{2}, & if\ i\ is\ odd,\ 1\leqslant i\leqslant n-1\\ n-1+\frac{i}{2}, & if\ i\ is\ even,\ 2\leqslant i\leqslant 2(k-n)\\ k, & if\ i\ is\ even,\ 2(k-n)+2\leqslant i\leqslant n;\end{cases}$$

$$f(u_iv_{i+1})=\begin{cases}1, & if\ i\ is\ odd,\ 1\leqslant i\leqslant n-1\\ 2n+3-\frac{i}{2}-k+i, & if\ i\ is\ even,\ 2\leqslant i\leqslant 2(k-n)\\ 3n+2-2k+i, & if\ i\ is\ even,\ 2(k-n)+2\leqslant i\leqslant n;\end{cases}$$

$$f(v_iu_{i+1})=\begin{cases}2n+3-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1\leqslant i\leqslant 2(k-n)-1\\ 3n+2-2k+i, & if\ i\ is\ odd,\ 2(k-n)+1\leqslant i\leqslant n-1\\ 1, & if\ i\ is\ even,\ 2\leqslant i\leqslant n;\end{cases}$$

$$f(u_iu_{i+1})=\begin{cases}2, & if\ i\ is\ odd,\ 1\leqslant i\leqslant 2(k-n)-1\\ n+2-\frac{i+1}{2}-k+i, & if\ i\ is\ odd,\ 2(k-n)+1\leqslant i\leqslant n-1\\ 2, & if\ i\ is\ even,\ 2\leqslant i\leqslant 2(k-n)\\ n+2-\frac{i+2}{2}-k+i, & if\ i\ is\ even,\ 2(k-n)+2\leqslant i\leqslant n;\end{cases}$$

$$f(v_iv_{i+1})=\begin{cases}2n+2-k-\frac{i+1}{2}+i, & if\ i\ is\ odd\\ 2n+2-k-\frac{i}{2}+i, & if\ i\ is\ even.\end{cases}$$

We observe that,

$$wt(u_iv_{i+1})=\begin{cases}2+i, & if\ i\ is\ odd,\ 1\leqslant i\leqslant n-1\\ 3n+2+i, & if\ i\ is\ even,\ 2\leqslant i\leqslant n;\end{cases}$$

$$wt(v_iu_{i+1})=\begin{cases}3n+2+i, & if\ i\ is\ odd,\ 1\leqslant i\leqslant n-1\\ 2+i, & if\ i\ is\ even,\ 2\leqslant i\leqslant n;\end{cases}$$

$$wt(u_iu_{i+1})=n+2+i, 1\leqslant i\leqslant n;$$

$$wt(v_iv_{i+1})=2n+2+i, 1\leqslant i\leqslant n.$$

From the above two cases the weights of the edges of $D_2(C_n)$ under the labeling f constitute the set $\{3,4,5,\ldots,4n+2\}$ and the function f is a mapping from $V(D_2(C_n))\cup E(D_2(C_n))$ into $\{1,2,3,\ldots,k\}$. The total labeling f has the required properties of an edge irregular total labeling, then we have $tes(D_2(C_n))\leqslant k$. This completes the proof. An edge irregular total labeling of $D_2(C_8)$ is given in Figure 1. □

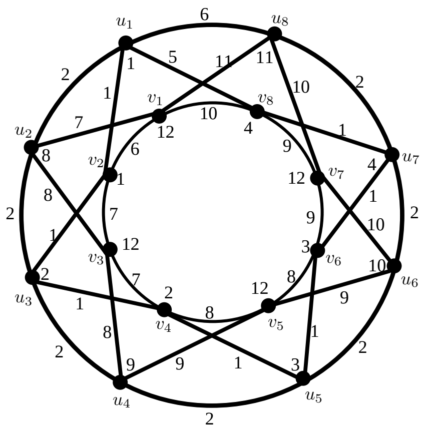

FIGURE 1. $tes(D_2(C_8)) = 12$.

THEOREM 2.2. $tes(D_2(P_n)) = \left\lceil \frac{4n-2}{3} \right\rceil, n \geqslant 2$.

PROOF. Let $k = \left\lceil \frac{4n-2}{3} \right\rceil$, then from (1.1) it follows that, $tes(D_2(P_n)) \geqslant \left\lceil \frac{4n-2}{3} \right\rceil = k$. That is $tes(D_2(P_n)) \geqslant k$. To prove the reverse inequality we define a function f from $V \cup E$ to {1,2,3,. . . ,k}.

$$f(v_i) = \begin{cases} 1, & if\ i\ is\ odd \\ n-1+\frac{i}{2}, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ k, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n; \end{cases}$$

$$f(u_i) = \begin{cases} k, & if\ i\ is\ odd \\ \frac{i}{2}, & if\ i\ is\ even; \end{cases}$$

$$f(v_i u_{i+1}) = \begin{cases} 1, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-1 \\ 2n + \left\lceil \frac{n-1}{2} \right\rceil - k, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ 3n-2k-1+\left\lceil \frac{n-1}{2} \right\rceil + \frac{i}{2}, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n-1; \end{cases}$$

$$f(u_i v_{i+1}) = \begin{cases} 2n-k, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n)-1 \\ 3n-2k-1+\frac{i+1}{2}, & if\ i\ is\ odd,\ 2(k-n)+1 \leqslant i \leqslant n-1 \\ 1+\left\lceil \frac{n-1}{2} \right\rceil, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n-1; \end{cases}$$

$$f(v_iv_{i+1}) = \begin{cases} 2 + \lceil \frac{n-2}{2} \rceil + \lceil \frac{n-1}{2} \rceil - n - \frac{i+1}{2} + i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n)-1 \\ 1 + \lceil \frac{n-2}{2} \rceil + \lceil \frac{n-1}{2} \rceil - k + i, & if\ i\ is\ odd,\ 2(k-n)+1 \leqslant i \leqslant n-1 \\ 2 + \lceil \frac{n-2}{2} \rceil + \lceil \frac{n-1}{2} \rceil - n - \frac{i}{2} + i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ 1 + \lceil \frac{n-2}{2} \rceil + \lceil \frac{n-1}{2} \rceil - k + i, & if\ i\ is\ even, 2(k-n)+2 \leqslant i \leqslant n-1; \end{cases}$$

$$f(u_iu_{i+1}) = \begin{cases} 2n - k - \frac{i+1}{2} + i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-2 \\ 2n - k - \frac{i}{2} + i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n-1. \end{cases}$$

We observe that,

$$wt(v_iu_{i+1}) = \begin{cases} 2 + \frac{i+1}{2}, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-1 \\ 3n - 1 + \lceil \frac{n-1}{2} \rceil + \frac{i}{2}, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n-1; \end{cases}$$

$$wt(u_iv_{i+1}) = \begin{cases} 3n - 1 + \frac{i+1}{2}, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-1 \\ 2 + \lceil \frac{n-1}{2} \rceil + \frac{i}{2}, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n-1; \end{cases}$$

$$wt(v_iv_{i+1}) = 2 + \left\lceil \frac{n-2}{2} \right\rceil + \left\lceil \frac{n-1}{2} \right\rceil + i, 1 \leqslant i \leqslant n-1;$$

$$wt(u_iu_{i+1}) = 2n + i, 1 \leqslant i \leqslant n-1.$$

The weights of the edges of $D_2(P_n)$ under the labeling f constitute the set $\{3, 4, 5, \ldots, 4n-2\}$ and the function f is a mapping from $V(D_2(P_n)) \cup E(D_2(P_n))$ into $\{1, 2, 3, \ldots, k\}$. The total labeling f has the required properties of an edge irregular total labeling, then we have $tes(D_2(P_n)) \leqslant k$. This completes the proof. An edge irregular total labeling of $D_2(P_9)$ is given in Figure 2. □

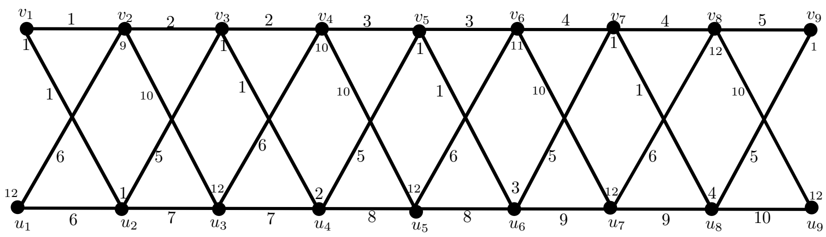

FIGURE 2. $tes(D_2(P_9)) = 12$.

THEOREM 2.3. $tes(T(C_n)) = \left\lceil \frac{4n+2}{3} \right\rceil, n \geqslant 3.$

PROOF. Let $V(T(C_n)) = \{u_i, v_i : 1 \leqslant i \leqslant n\}$ and $E(T(C_n)) = \{u_iv_i, u_iv_{i+1}, u_i u_{i+1}, v_iv_{i+1} : 1 \leqslant i \leqslant n\}$ with indices taken modulo n. Let $k = \left\lceil \frac{4n+2}{3} \right\rceil$, then from (1.1) it follows that, $tes(T(C_n)) \geqslant \left\lceil \frac{|E(T(C_n))|+2}{3} \right\rceil = \left\lceil \frac{4n+2}{3} \right\rceil = k$. That is $tes(T(C_n)) \geqslant k$. To prove the reverse inequality we define a function f from $V \cup E$ to $\{1, 2, 3, \ldots, k\}$ by considering the following two cases.

Case(i): n is odd

$n = 3, 5$, we consider the following labeling.

when $n = 3$

$f(u_1) = f(u_3) = 5, f(u_2) = 3, f(v_1) = f(v_2) = 1, f(v_3) = 2, f(v_1v_2) = f(v_2v_3) = 1, f(v_3v_1) = f(v_2u_2) = 2, f(v_1u_1) = 3, f(u_1v_2) = f(u_1u_2) = f(u_3u_1) = f(v_3u_3) = 4, f(u_2u_3) = 5, f(u_2v_3) = f(u_3v_1) = 2.$

when $n = 5$

$f(u_1) = f(u_3) = f(u_5) = 8, f(u_2) = 5, f(u_4) = 6, f(v_1) = f(v_2) = 1, f(v_3) = f(v_4) = 2, f(v_5) = 3, f(v_1v_2) = f(v_2v_3) = f(v_3v_4) = f(v_4v_5) = 1, f(v_5v_1) = 3, f(u_1v_1) = 4, f(u_2v_2) = f(u_4v_4) = 2, f(u_3v_3) = f(u_1v_2) = f(u_1u_2) = 5,$
$f(u_5v_5) = 6, f(u_2v_3) = f(u_4v_5) = 2, f(u_3v_4) = f(u_2u_3) = f(u_3u_4) = f(u_5u_1) = 6, f(u_5v_1) = 3, f(u_4u_5) = 7.$

Now we define a labeling for $n \geqslant 7$.

$$f(v_i) = \left\lceil \frac{i}{2} \right\rceil, 1 \leqslant i \leqslant n;$$

$$f(u_i) = \begin{cases} k, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n \\ n - 1 + \frac{i}{2}, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ k, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n-1; \end{cases}$$

$$f(v_iv_{i+1}) = 1, 1 \leqslant i \leqslant n;$$

$$f(u_iv_i) = \begin{cases} 2n + 2 - k - \frac{i+1}{2} + i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n \\ 2, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ n + 1 - k - \frac{i}{2} + i, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n-1; \end{cases}$$

$$f(u_iv_{i+1}) = \begin{cases} 2n+3-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-2 \\ 2n+1-k, & if\ i=n \\ 2, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ n+2-k-\frac{i+2}{2}+i, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n-1; \end{cases}$$

$$f(u_iu_{i+1}) = \begin{cases} 2n+3-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n)-1 \\ 3n+2-2k+i, & if\ i\ is\ odd,\ 2(k-n)+1 \leqslant i \leqslant n \\ 2n+3-k-\frac{i}{2}+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ 3n+2-2k+i, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n-1. \end{cases}$$

We observe that,

$$wt(v_iv_{i+1}) = 2+i, 1 \leqslant i \leqslant n;$$

$$wt(u_iv_i) = \begin{cases} 2n+2+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n \\ n+1+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n-1; \end{cases}$$

$$wt(u_iv_{i+1}) = \begin{cases} 2n+3+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-2 \\ n+2+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n-1 \\ 2n+2, & if\ i=n; \end{cases}$$

$$wt(u_iu_{i+1}) = 3n+2+i, 1 \leqslant i \leqslant n.$$

Case(ii): n is even

when $n = 4$ we consider the following labeling.

$f(u_1) = f(u_3) = 6, f(u_2) = 4, f(u_4) = 6, f(v_1) = f(v_2) = 1, f(v_3) = f(v_4) = 2, f(u_1v_1) = 4, f(u_1v_2) = 5, f(u_2v_2) = 2, f(u_2v_3) = 2, f(u_3v_4) = 6, f(u_3v_3) = 5, f(u_4v_4) = 1, f(v_1v_2) = f(v_2v_3) = f(v_3v_4) = 1, f(u_4v_1) = 3, f(u_1u_2) = 5, f(u_2u_3) = f(u_4u_1) = 6, f(u_3u_4) = 5, f(v_4v_1) = 3.$

Now we define a labeling for $n \geqslant 6$.

$$f(v_i) = \left\lceil \frac{i}{2} \right\rceil, 1 \leqslant i \leqslant n;$$

$$f(u_i) = \begin{cases} k, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-1; \\ n-1+\frac{i}{2}, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ k, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n; \end{cases}$$

$$f(v_iv_{i+1}) = 1, 1 \leqslant i \leqslant n;$$

$$f(u_iv_i)=\begin{cases}2n+2-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1\leqslant i\leqslant n-1\\ 2, & if\ i\ is\ even,\ 2\leqslant i\leqslant 2(k-n)\\ n+1-k-\frac{i}{2}+i, & if\ i\ is\ even,\ 2(k-n)+2\leqslant i\leqslant n;\end{cases}$$

$$f(u_iv_{i+1})=\begin{cases}2n+3-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1\leqslant i\leqslant n-1\\ 2, & if\ i\ is\ even,\ 2\leqslant i\leqslant 2(k-n)\\ n+2-k-\frac{i+2}{2}+i, & if\ i\ is\ even,\ 2(k-n)+2\leqslant i\leqslant n;\end{cases}$$

$$f(u_iu_{i+1})=\begin{cases}2n+3-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1\leqslant i\leqslant 2(k-n)-1\\ 3n+2-2k+i, & if\ i\ is\ odd,\ 2(k-n)+1\leqslant i\leqslant n-1\\ 2n+3-k-\frac{i}{2}+i, & if\ i\ is\ even,\ 2\leqslant i\leqslant 2(k-n);\\ 3n+2-2k+i, & if\ i\ is\ even,\ 2(k-n)+2\leqslant i\leqslant n.\end{cases}$$

We observe that,

$$wt(v_iv_{i+1})=2+i, 1\leqslant i\leqslant n;$$

$$wt(u_iv_i)=\begin{cases}2n+2+i, & if\ i\ is\ odd\\ n+1+i, & if\ i\ is\ even;\end{cases}$$

$$wt(u_iv_{i+1})=\begin{cases}2n+3+i, & if\ i\ is\ odd\\ n+2+i, & if\ i\ is\ even;\end{cases}$$

$$wt(u_iu_{i+1})=3n+2+i, 1\leqslant i\leqslant n.$$

From the above two cases the weights of the edges of $T(C_n)$ under the labeling f constitute the set $\{3,4,5,\ldots,4n+2\}$ and the function f is a mapping from $V(T(C_n))\cup E(T(C_n))$ into $\{1,2,3,\ldots,k\}$. The total labeling f has the required properties of an edge irregular total labeling, then we have $tes(T(C_n))\leqslant k$. This completes the proof. An edge irregular total labeling of $T(C_8)$ is given in Figure 3. □

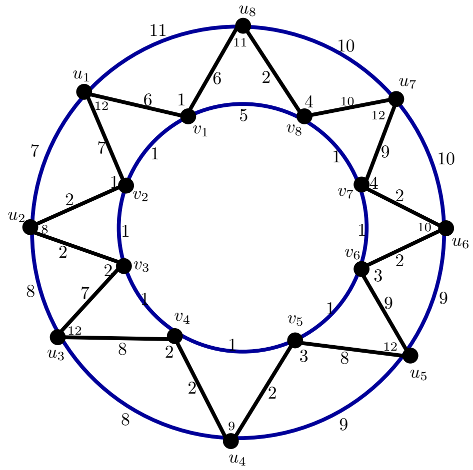

FIGURE 3. $tes(T(C_8)) = 12$.

THEOREM 2.4. $tes(T(P_n)) = \left\lceil \frac{4n-3}{3} \right\rceil, n \geqslant 2$.

PROOF. Let $k = \left\lceil \frac{4n-3}{3} \right\rceil$, then from (1.1) it follows that, $tes(T(P_n)) \geqslant \left\lceil \frac{|E(T(P_n))|+2}{3} \right\rceil = \left\lceil \frac{4n-3}{3} \right\rceil = k$. That is $tes(T(P_n)) \geqslant k$. To prove the reverse inequality we define a function f from $V \cup E$ to $\{1, 2, 3, \ldots, k\}$ by considering the following two cases.

Case(i): n is odd

$$f(v_i) = \left\lceil \frac{i}{2} \right\rceil, 1 \leqslant i \leqslant n;$$

$$f(u_i) = \begin{cases} n - 1 + \frac{i+1}{2}, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n) - 1 \\ k, & if\ i\ is\ odd,\ 2(k-n) + 1 \leqslant i \leqslant n - 2 \\ k, & if\ i\ is\ even; \end{cases}$$

$$f(v_i v_{i+1}) = 1, 1 \leqslant i \leqslant n - 1;$$

$$f(u_i v_i) = \begin{cases} 1, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n) - 1 \\ n + 1 - k - \frac{i+1}{2} + i, & if\ i\ is\ odd,\ 2(k-n) + 1 \leqslant i \leqslant n - 2 \\ 2n - 1 - k - \frac{i}{2} + i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n - 1; \end{cases}$$

$$f(u_i v_{i+1}) = \begin{cases} 2, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n) - 1 \\ n + 2 - k - \frac{i+1}{2} + i, & if\ i\ is\ odd,\ 2(k-n) + 1 \leqslant i \leqslant n - 2 \\ 2n - k - \frac{i+2}{2} + i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n - 1; \end{cases}$$

$$f(u_iu_{i+1}) = \begin{cases} 2n-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n)-1 \\ 3n-1-2k+i, & if\ i\ is\ odd,\ 2(k-n)+1 \leqslant i \leqslant n-2 \\ 2n-k-\frac{i}{2}+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n)-2 \\ 3n-1-2k+i, & if\ i\ is\ even,\ 2(k-n) \leqslant i \leqslant n-3. \end{cases}$$

We observe that,

$$wt(v_iv_{i+1}) = 2+i, 1 \leqslant i \leqslant n-1;$$

$$wt(u_iv_i) = \begin{cases} n+1+i, & if\ i\ is\ odd \\ 2n-1+i, & if\ i\ is\ even; \end{cases}$$

$$wt(u_iv_{i+1}) = \begin{cases} n+2+i, & if\ i\ is\ odd \\ 2n+i, & if\ i\ is\ even; \end{cases}$$

$$wt(u_iu_{i+1}) = 3n-1+i, 1 \leqslant i \leqslant n-2.$$

Case(ii): n is even

$$f(v_i) = \left\lceil \frac{i}{2} \right\rceil, 1 \leqslant i \leqslant n;$$

$$f(u_i) = \begin{cases} n-1+\frac{i+1}{2}, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n)-1 \\ k, & if\ i\ is\ odd,\ 2(k-n)+1 \leqslant i \leqslant n-1 \\ k, & if\ i\ is\ even; \end{cases}$$

$$f(v_iv_{i+1}) = 1, 1 \leqslant i \leqslant n-1;$$

$$f(u_iv_i) = \begin{cases} 1, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n)-1 \\ n+1-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 2(k-n)+1 \leqslant i \leqslant n-1 \\ 2n-k-\frac{i}{2}+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n-2; \end{cases}$$

$$f(u_iv_{i+1}) = \begin{cases} 2, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n)-1 \\ n+2-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 2(k-n)+1 \leqslant i \leqslant n-1 \\ 2n+1-k-\frac{i+2}{2}+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant n-2; \end{cases}$$

$$f(u_iu_{i+1}) = \begin{cases} 2n-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n)-1 \\ 3n-1-2k+i, & if\ i\ is\ odd,\ 2(k-n)+1 \leqslant i \leqslant n-3 \\ 2n-k-\frac{i}{2}+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n)-2 \\ 3n-1-2k+i, & if\ i\ is\ even,\ 2(k-n) \leqslant i \leqslant n-2. \end{cases}$$

We observe that,

$$wt(v_iv_{i+1}) = 2 + i, 1 \leqslant i \leqslant n - 1;$$

$$wt(u_iv_i) = \begin{cases} n+1+i, & if\ i\ is\ odd \\ 2n+i, & if\ i\ is\ even; \end{cases}$$

$$wt(u_iv_{i+1}) = \begin{cases} n+2+i, & if\ i\ is\ odd \\ 2n+1+i, & if\ i\ is\ even; \end{cases}$$

$$wt(u_iu_{i+1}) = 3n - 1 + i, 1 \leqslant i \leqslant n - 2.$$

From the above two cases, the weights of the edges of $T(P_n)$ under the labeling f constitute the set $\{3, 4, 5, \ldots, 4n + 2\}$ and the function f is a mapping from $V(T(P_n)) \cup E(T(Pn))$ into $\{1, 2, 3, \ldots, k\}$. The total labeling f has the required properties of an edge irregular total labeling, then we have $tes(T(P_n)) \leqslant k$. This completes the proof. An edge irregular total labeling of $T(P_9)$ is given in Figure 4. □

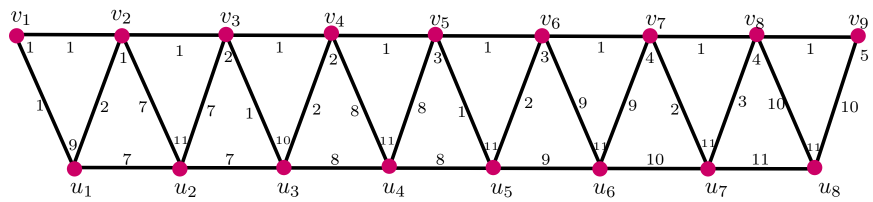

FIGURE 4. $tes(T(P_9)) = 11$.

THEOREM 2.5. $tes(LC_n) = \left\lceil \frac{4n+2}{3} \right\rceil, n \geqslant 3$.

PROOF. Let $V(LC_n) = \{v, u_i, v_i : 1 \leqslant i \leqslant n\}$ and $E(LC_n) = \{vv_i, u_iv_i, u_iv_{i+1}, u_iu_{i+1} : 1 \leqslant i \leqslant n\}$ with indices taken modulo n. Let $k = \left\lceil \frac{4n+2}{3} \right\rceil$, then from (1.1) it follows that, $tes(LC_n) \geqslant \left\lceil \frac{|E(LC_n)|+2}{3} \right\rceil = \left\lceil \frac{4n+2}{3} \right\rceil = k$. That is $tes(LC_n) \geqslant k$. To prove the reverse inequality we define a function f from $V \cup E$ to $\{1, 2, 3, \ldots, k\}$ by considering the following two cases.

Case(i): n is odd

$n = 3, 5$ we consider the following labeling.

when $n = 3$

$f(u_1)=5, f(u_2)=3, f(u_3)=5, f(v)=f(v_1)=f(v_2)=1, f(v_3)=2,$
$f(vv_1)=1, f(vv_2)=f(vv_3)=2, f(u_1v_1)=3, f(u_2v_2)=2, f(u_3v_3)=4,$
$f(u_1v_2)=4, f(u_2v_3)=2, f(u_1u_2)=4, f(u_2u_3)=5, f(u_3u_1)=4, f(u_3v_1)=2.$

when $n=5$

$f(u_1)=f(u_3)=f(u_5)=8, f(u_2)=5, f(u_4)=6, f(v)=f(v_1)=f(v_2)=1, f(v_3)=f(v_4)=2, f(v_5)=3, f(vv_1)=1, f(vv_2)=f(vv_3)=2, f(vv_4)=f(vv_5)=3, f(u_1v_1)=4, f(u_2v_2)=f(u_4v_4)=2, f(u_3v_3)=f(u_1v_2)=f(u_1u_2)=5, f(u_5v_5)=6, f(u_2v_3)=f(u_4v_5)=2, f(u_3v_4)=f(u_2u_3)=f(u_3u_4)=f(u_5u_1)=6, f(u_5v_1)=3, f(u_4u_5)=7.$

Now we define a labeling for $n\geqslant 7$

$$f(v)=1;$$
$$f(v_i)=\left\lceil\frac{i}{2}\right\rceil, 1\leqslant i\leqslant n;$$
$$f(u_i)=\begin{cases}k, & if\ i\ is\ odd,\ 1\leqslant i\leqslant n\\ n-1+\frac{i}{2}, & if\ i\ is\ even,\ 2\leqslant i\leqslant 2(k-n)\\ k, & if\ i\ is\ even,\ 2(k-n)+2\leqslant i\leqslant n-1;\end{cases}$$
$$f(vv_i)=1-\left\lceil\frac{i}{2}\right\rceil+i, 1\leqslant i\leqslant n;$$

$$f(u_iv_i)=\begin{cases}2n+2-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1\leqslant i\leqslant n\\ 2, & if\ i\ is\ even,\ 2\leqslant i\leqslant 2(k-n)\\ n+1-k-\frac{i}{2}+i, & if\ i\ is\ even,\ 2(k-n)+2\leqslant i\leqslant n-1;\end{cases}$$

$$f(u_iv_{i+1})=\begin{cases}2n+3-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1\leqslant i\leqslant n-2\\ 2n+1-k, & if\ i=n\\ 2, & if\ i\ is\ even,\ 2\leqslant i\leqslant 2(k-n)\\ n+2-k-\frac{i+2}{2}+i, & if\ i\ is\ even,\ 2(k-n)+2\leqslant i\leqslant n-1;\end{cases}$$

$$f(u_iu_{i+1})=\begin{cases}2n+3-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1\leqslant i\leqslant 2(k-n)-1\\ 3n+2-2k+i, & if\ i\ is\ odd,\ 2(k-n)+1\leqslant i\leqslant n\\ 2n+3-k-\frac{i}{2}+i, & if\ i\ is\ even,\ 2\leqslant i\leqslant 2(k-n)\\ 3n+2-2k+i, & if\ i\ is\ even,\ 2(k-n)+2\leqslant i\leqslant n-1.\end{cases}$$

We observe that,

$$wt(vv_i)=2+i, 1\leqslant i\leqslant n;$$
$$wt(u_iu_{i+1})=3n+2+i, 1\leqslant i\leqslant n;$$

$$wt(u_iv_i) = \begin{cases} 2n+2+i, & if\ i\ is\ odd \\ n+1+i, & if\ i\ is\ even; \end{cases}$$

$$wt(u_iv_{i+1}) = \begin{cases} 2n+3+i, & if\ i\ is\ odd \\ 2n+2 & if\ i=n \\ n+2+i, & if\ i\ is\ even. \end{cases}$$

Case(ii): n is even

when $n=4$, we consider the following labeling.

$f(u_1) = f(u_3) = f(u_4) = 6, f(u_2) = 4, f(v) = f(v_1) = f(v_2) = 1, f(v_3) = f(v_4) = 2, f(vv_1) = 1, f(vv_2) = f(vv_3) = 2, f(vv_4) = 3, f(u_1v_2) = 5, f(u_2v_3) = 2, f(u_3v_4) = 6, f(u_4v_1) = 3, f(u_1u_2) = 5, f(u_2u_3) = 6, f(u_3u_4) = 5, f(u_4u_1) = 6, f(u_1v_1) = 4, f(u_2v_2) = 2, f(u_3v_3) = 5, f(u_4v_4) = 1.$

Now we define a labeling for $n \geqslant 6$

$$f(v) = 1;$$

$$f(v_i) = \left\lceil \frac{i}{2} \right\rceil, 1 \leqslant i \leqslant n;$$

$$f(u_i) = \begin{cases} k, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-1 \\ n-1+\frac{i}{2}, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ k, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n; \end{cases}$$

$$f(vv_i) = 1 - \left\lceil \frac{i}{2} \right\rceil + i, 1 \leqslant i \leqslant n;$$

$$f(u_iv_i) = \begin{cases} 2n+2-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-1 \\ 2, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ n+1-k-\frac{i}{2}+i, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n; \end{cases}$$

$$f(u_iv_{i+1}) = \begin{cases} 2n+3-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant n-1 \\ 3-\frac{i}{2}-\frac{i+2}{2}+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ n+2-k-\frac{i+2}{2}+i, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n; \end{cases}$$

$$f(u_iu_{i+1}) = \begin{cases} 2n+3-k-\frac{i+1}{2}+i, & if\ i\ is\ odd,\ 1 \leqslant i \leqslant 2(k-n)-1 \\ 3n+2-2k+i, & if\ i\ is\ odd,\ 2(k-n)+1 \leqslant i \leqslant n-1 \\ 2n+3-k-\frac{i}{2}+i, & if\ i\ is\ even,\ 2 \leqslant i \leqslant 2(k-n) \\ 3n+2-2k+i, & if\ i\ is\ even,\ 2(k-n)+2 \leqslant i \leqslant n. \end{cases}$$

We observe that,

$$wt(vv_i) = 2+i, 1 \leqslant i \leqslant n;$$

$$wt(u_iu_{i+1}) = 3n+2+i, 1 \leqslant i \leqslant n;$$

$$wt(u_iv_i) = \begin{cases} 2n+2+i, & if\ i\ is\ odd, \\ n+1+i, & if\ i\ is\ even; \end{cases}$$

$$wt(u_iv_{i+1}) = \begin{cases} 2n+3+i, & if\ i\ is\ odd \\ n+2+i, & if\ i\ is\ even. \end{cases}$$

From the above two cases the weights of the edges of LC_n under the labeling f constitute the set $\{3, 4, 5, \ldots, 4n+2\}$ and the function f is a mapping from $V(LC_n) \cup E(LC_n)$ into $\{1, 2, 3, \ldots, k\}$. The total labeling f has the required properties of an edge irregular total labeling, then we have $tes(LC_n) \leqslant k$. This completes the proof. An edge irregular total labeling of LC_8 is given in Figure 5. □

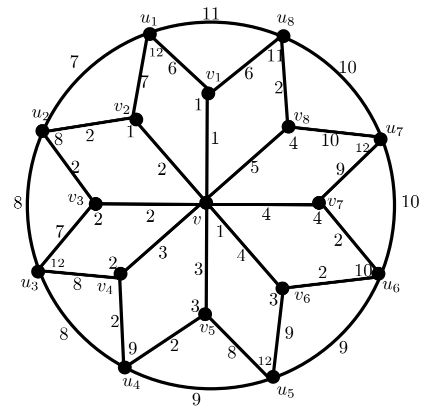

FIGURE 5. $tes(LC_8) = 12$.

THEOREM 2.6. $tes(DW_n) = \left\lceil \frac{4n+2}{3} \right\rceil, n \geqslant 3$.

PROOF. Let $V(DW_n) = \{v, u_i, v_i : 1 \leqslant i \leqslant n\}$ and $E(DW_n) = \{vu_i, vv_i, v_iv_{i+1}, u_iu_{i+1} : 1 \leqslant i \leqslant n\}$ with indices taken modulo n. Let $k = \left\lceil \frac{4n+2}{3} \right\rceil$, then from(1.1) it follows that $tes(DW_n) \geqslant \left\lceil \frac{|E(DW_n)|+2}{3} \right\rceil = \left\lceil \frac{4n+2}{3} \right\rceil = k$. We define a total labeling f as follows.

$$f(v) = \left\lceil \frac{n}{2} \right\rceil, 1 \leqslant i \leqslant n;$$
$$f(v_i) = k, 1 \leqslant i \leqslant n;$$
$$f(u_i) = \left\lceil \frac{i}{2} \right\rceil, 1 \leqslant i \leqslant n;$$
$$f(u_iu_{i+1}) = 1, 1 \leqslant i \leqslant n-1;$$

$$f(u_n u_1) = \left\lceil \frac{n+1}{2} \right\rceil;$$

$$f(vv_i) = 2n + 2 - \left\lceil \frac{n}{2} \right\rceil - k + i, 1 \leqslant i \leqslant n;$$
$$f(vu_i) = n + 2 - \left\lceil \frac{n}{2} \right\rceil - \left\lceil \frac{i}{2} \right\rceil + i, 1 \leqslant i \leqslant n;$$

$$f(v_i v_{i+1}) = 3n + 2 - 2k + i, 1 \leqslant i \leqslant n.$$

We observe that,

$$w(vv_i) = 2n + 2 + i, 1 \leqslant i \leqslant n;$$
$$wt(u_i u_{i+1}) = 2 + i, 1 \leqslant i \leqslant n;$$
$$wt(v_i v_{i+1}) = 3n + 2 + i, 1 \leqslant i \leqslant n;$$
$$wt(vu_i) = n + 2 + i, 1 \leqslant i \leqslant n.$$

The weights of the edges of DW_n under the labeling f constitute the set $\{3, 4, 5, \ldots, 4n+2\}$ and the function f is a mapping from $V(DW_n) \cup E(DW_n)$ into $\{1, 2, 3, \ldots, k\}$. The total labeling f has the required properties of an edge irregular total labeling, then we have $tes(DW_n) \leqslant k$. This completes the proof. An edge irregular total labeling of DW_6 is given in Figure 6.

□

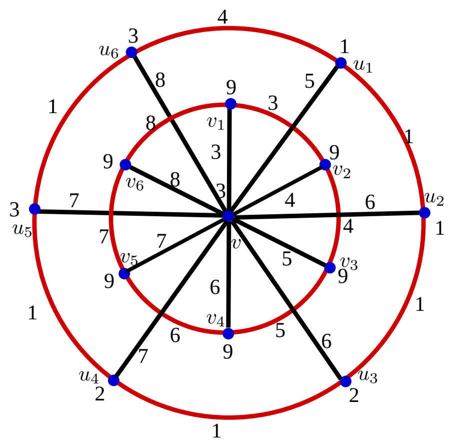

FIGURE 6. $tes(DW_6) = 9$.

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