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SOME PARAMETERS OF THE V_n-ARITHMETIC GRAPH

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Abstract. In this paper, we continue the study of V_n-Arithmetic graph by determining domination parameters of V_n. Bounds for some V_n graph parameters are established.

1. Introduction

Let $G = (V, E)$ be a graph. As usual $|V|$ and $|E|$ denote the number of vertices and edges of a graph G, respectively. In general, we use $\langle X \rangle$ to denote the subgraph induced by the set of vertices X. The open and closed neighborhoods of v are defined by $N(v) = \{u \in V(G) : uv \in E\}$ and $N[v] = N(v) \cup \{v\}$, respectively. The degree of a vertex v in a graph G denoted by $deg(v)$ is defined to be the number of edges incident with v. In simple graphs, $deg(v) = |N(v)|$. The maximum and minimum degrees in a graph G are denoted respectively by Δ and δ. The complement $\overline{G}$ of a graph G has $V(G)$ as its vertex set but two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. A set of vertices which cover all the edges of a graph G is called a cover for G. The minimum number of vertices in any cover of G is called its covering number and is denoted by α. A set of edges which cover all the vertices of a graph G is called an edge cover for G. The minimum number of edges in any edge cover of G is called edge covering number of G and is denoted by α'. A subset S of V is said to be independent if no two vertices in S are adjacent in G. The maximum number of vertices in an independent set is called the independence number of G and is denoted by β. A subset M of edges in a graph G is said to be edge independent set or a matching if no two edges in M are adjacent. The maximum cardinality of an edge independent set of G is called the

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edge independence number or matching number of G and is denoted by β', and for any graph G, the equations $\alpha(G) + \beta(G) = |V|$, $\alpha'(G) + \beta'(G) = |V|$ are satisfied. All the definitions in this paper available in [**5**].

The concept of domination in graph theory was formalized by Berge [**3**] and Ore [**11**] and is strengthened by Haynes, Hedetniemi, Slater in two books [**6, 7**]. Domination in graphs has been studied extensively and at present it is an emerging area of research in graph theory. A subset D of V is said to be a dominating set of G if every vertex in $V-D$ is adjacent to a vertex in D. The minimum cardinality of a dominating set is called the domination number of G and is denoted by $\gamma(G)$. A dominating set D of a graph G is a global dominating set if D is also a dominating set of $\overline{G}$. The global domination number $\gamma_g(G)$ is the minimum cardinality of a global dominating set of G. This concept had introduced independently by Brigham and Dutton [**4**] and Sampathkumar [**13**]. For a minimum dominating set D in a graph G, if $V - D$ contains a dominating set D' of G, then D' is called an inverse dominating set with respect to D. The inverse domination number $\gamma^{-1}(G)$ is the cardinality of a smallest inverse dominating set. Kulli and Sigarkanti had introduced this concept in [**8**].

The common neighborhood graph (congraph) of G, denoted by $con(G)$, is the graph with the vertex set $V(G)$, in which two vertices are adjacent if and only if they have at least one common neighbor in the graph G. The common neighborhood (CN-neighborhood) of a vertex $v \in V(G)$ denoted by $N_{cn}(v)$ is defined as $N_{cn}(v) = \{u \in V(G) : uv \in E(G) \text{ and } |\Gamma(u,v)| \geqslant 1\}$, where $|\Gamma(u,v)|$ is the number of common neighborhood between the vertices u and v, [**1**]. A subset D of $V(G)$ is called an injective dominating set (Inj-dominating set) if for every vertex $v \in V-D$ there exists a vertex $u \in D$ such that $|\Gamma(u,v)| \geqslant 1$. The minimum cardinality of such dominating set denoted by $\gamma_{in}(G)$ and is called injective domination number (Inj-domination number) of G. Anwar Alwardi, R. Rangarajan and Akram Alqesmah had introduced this concept in [**2**].

Proposition 1.1 ([**2**]). *Let G be a nontrivial connected graph. Then $\gamma_{in}(G) = 1$ if and only if there exists a vertex $v \in V(G)$ such that $N(v) = N_{cn}(v)$ and $e(v) \leqslant 2$.*

Vasumathi and Vangipuram have introduced the concept of V_n-Arithmetic graphs and studied some of its properties [**15**]. Let n be a positive integer with $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots {p_k}^{\alpha_k}$ as its prime factorization. Then the V_n-Arithmetic graph is defined as the graph whose vertex set consists of the divisors of n and two vertices u, v are adjacent in V_n graph if and only if $\gcd(u,v) = p_i$ for some prime divisor p_i of n. In this graph vertex 1 becomes an isolated vertex. Hence, we consider V_n-Arithmetic graph without vertex 1 as the contribution of this isolated vertex is nothing when the properties of these graphs and enumeration of some domination parameters are studied. Clearly, V_n graph is a connected graph. In the trivial case when n is a prime, the V_n graph consists of a single vertex. Hence it is a connected graph. In other cases, by the definition of adjacency in V_n there exist edges between prime number vertices and their prime power vertices and also to their prime product and prime power product vertices. Also, any two vertices p_i and p_j,

$(i \neq j)$ are both adjacent to $p_i p_j$ hence they lie in a path of length 2. Therefore each vertex of V_n is connected to some vertex in V_n.
Some properties like number of vertices, number of edges, maximum degree, minimum degree, diameter, radius, Hamiltonian and Eulerian are presented in [**12**].

In [**14**], the authors have determined the domination number γ of the V_n-Arithmetic graph, where $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k{}^{\alpha_k}$ as in the following theorem.

THEOREM 1.1 ([**14**]). *For any V_n-Arithmetic graph G with $n = p_1^{\alpha_1} \dots p_k{}^{\alpha_k}$ as the prime factorization of n, the domination number of G is given by*

$$\gamma(G) = \begin{cases} k-1, & \text{if } \alpha_i = 1 \text{ for more than one } i; \\ k, & \text{otherwise.} \end{cases}$$

where k is the core of n.

Note that, the core of a positive integer $n \geqslant 2$ is the number of its distinct prime factors. Actually, the study of various domination parameters shows that the domination parameters of the V_n graphs are functions of k, where k is the core of n (see [**14, 10, 9**]).

One of the main goals to constrict and study the different relations between the different branches of mathematics is to solve and analysis the mathematical problems by using different trends. Some difficult problems in algebra can be solve easily geometrically or by using algebraic topology and visa versa. The arithmetic graph is one of the methods to represent the numerical sets. Every V_n-Arithmetic graph represent some numerical subset of positive integers, so some properties and problems in the numerical subset of integers can be study and resolve by using graph theory and vise versa, this motivated us to continue the study of V_n-arithmetic graph by studying some different graph parameters of this graph.

In this paper, some parameters of V_n-Arithmetic graph like vertex covering number, edge covering number for some special cases, domination number of $\overline{V_n}$, global domination number, inverse domination number and injective domination number are studied and some bounds are established.

2. Domination Parameters of V_n-Arithmetic Graph

DEFINITION 2.1. *Let G be a V_n-Arithmetic graph with $n = p_1^{\alpha_1} \dots p_k{}^{\alpha_k}$, where p_i are primes and α_i are integers greater than or equal one, $i \in \{1, 2, \dots, k\}$. Then $\overline{V_n}$ is an Arithmetic graph with the same vertices of V_n such that any two vertices $u, v \in V$ are adjacent if and only if the greatest common divisor of u and v doesn't equal to p_i for some $i \in \{1, 2, \dots, k\}$.*

THEOREM 2.1. *Let G be a V_n-Arithmetic graph and $\overline{G}$ is its complement. Then*

(1) $\gamma(\overline{G}) = 2$.

(2) $\gamma(G) + \gamma(\overline{G}) = \begin{cases} k+1, & \text{if } \alpha_i = 1 \text{ for more than one } i; \\ k+2, & \text{otherwise.} \end{cases}$

(3) $\gamma(G) \cdot \gamma(\overline{G}) = \begin{cases} 2(k-1), & \text{if } \alpha_i = 1 \text{ for more than one } i; \\ 2k, & \text{otherwise.} \end{cases}$

Proof. Let G be a V_n-Arithmetic graph. By Definition 2.1, it is clear that, the vertex $u = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k{}^{\alpha_k}$ is adjacent to all the vertices of $\overline{G}$ except the primes p_i, $i \in \{1, 2, \dots, k\}$ because $\gcd(u, p_i) = p_i$, $i \in \{1, 2, \dots, k\}$ and any prime vertex p_j is adjacent to all the other primes because $\gcd(p_j, p_i) = 1$, $j, i \in \{1, 2, \dots, k\}$ then $\gamma(\overline{G}) \leqslant 2$, and there is no vertex of full degree. Hence, $\gamma(\overline{G}) = 2$

The proofs of (ii) and (iii) are straight forward and may be obtained by applying Theorem 1.1 and Part (i) of Theorem 2.1. □

Theorem 2.2. *For any V_n-Arithmetic graph G, the global domination number of G is equal to k, where k is the core of n.*

Proof. We characterize two cases.

Case 1. Suppose $\alpha_i = 1$ for at most one i. In this case $\gamma(G) = k$. The subset $D = \{p_1, p_1^2 p_2, \dots, p_1^2 p_k\}$ of order k is a dominating set of both G and $\overline{G}$. Since $\gamma_g(G) \geqslant \gamma(G)$, then $\gamma_g(G) = k$.

Case 2. Suppose $\alpha_i = 1$ for more than one i. In this case $\gamma(G) = k - 1$. The subset $D_g = \{p_1, p_2, \dots, p_{k-1}p_k, n\}$ of order k is a dominating set of both G and $\overline{G}$, so $k - 1 \leqslant \gamma_g(G) \leqslant k$. Since any minimum dominating set D ($|D| = k - 1$) of G must contain a vertex of the form $p_i p_j$, where $\alpha_i = \alpha_j = 1$, then for any minimum dominating set D of G we can find at least a vertex $v \in V - D$ which is adjacent to all the vertices of D in G, so v is not dominated in $\overline{G}$. Hence, $\gamma_g(G) = k$. □

Theorem 2.3. *Let G be a V_n-Arithmetic graph, where $n = \prod_{i=1}^{j} p_i \prod_{i=j+1}^{k} p_i^{\alpha_i}$, j is a nonnegative integer, $\alpha_i > 1$, $\forall i \in \{j+1, \dots, k\}$, and $n \neq p^\alpha$. Then the inverse domination number of G is given by*

$$\gamma^{-1}(G) = \begin{cases} k, & \text{if } j \leqslant 2; \\ k - 1, & \text{if } 3 \leqslant j \leqslant k. \end{cases}$$

where k is the core of n.

Proof. We have two cases.

Case I. Let $j \leqslant 2$. Then

Subcase 1. Suppose $j = 0$ or $j = 1$ this means $n = \prod_{i=1}^{k} p_i^{\alpha_i}$, all $\alpha_i > 1$, for all $i \in \{1, 2, \dots, k\}$ or $n = p_1 \prod_{i=2}^{k} p_i^{\alpha_i}$, $\alpha_i > 1$, $\forall i \in \{2, \dots, k\}$, respectively. In this case $\gamma(G) = k$, so the subset $D = \{p_1, p_1p_2, p_1p_3, \dots, p_1p_k\}$ which has order k is a minimum dominating set of G and hence the subset $D' \subset V - D$ where $D' = \{p_2, p_3, \dots, p_k, p_1p_2^2\}$ which has order k is also a minimum dominating set of G because the vertices p_i, $i = 2, 3, \dots, k$ dominate all the vertices of G except p_1 (and its powers in case $j = 0$) so we have to choose one more vertex to dominate p_1 and its powers. Since $\gamma(G) \leqslant \gamma^{-1}(G)$ for any graph G has no isolated vertices, then $\gamma^{-1}(G) = |D'| = k$.

Subcase 2. Suppose $j = 2$ this implies that $n = p_1p_2\prod_{i=3}^{k} p_i^{\alpha_i}$, $\alpha_i > 1$, for all $i \in \{3, 4, \ldots, k\}$. In this case $\gamma(G) = k - 1$, where $D = \{p_1p_2, p_3, p_4, \ldots, p_k\}$ is a minimum dominating set of G (note that in this case any dominating set of order $k-1$ must contain the vertex p_1p_2), so the dominating set $D' \subset V - D$, where $D' = \{p_1, p_2, p_1p_3, p_1p_4, \ldots, p_1p_k\}$ is a minimal dominating set of G of order $|D'| = k$. Since $\gamma(G) \leqslant \gamma^{-1}(G)$, then $\gamma^{-1}(G) = |D'| = k$.

Case II. Let $3 \leqslant j \leqslant k$. In this case $\gamma(G) = k - 1$, thus we have the following subcases.

Subcase 1. Suppose $j = k$. Then $n = \prod_{i=1}^{k} p_i$. Let $D = \{p_1, p_2, \ldots, p_{k-1}p_k\}$ is a minimum dominating set of G. Then we have two possibilities.

(1) If k is even, then the subset

$D' = \{p_{k-1}, p_k, p_1p_2, p_3p_4, \ldots, p_{k-3}p_{k-2}, p_1p_k, p_3p_k, \ldots, p_{k-5}p_k\}$

which has order

$$|D'| = 2 + \frac{k-2}{2} + \frac{k-4}{2} = k - 1$$

is a minimum inverse dominating set of G with respect to D. Then $\gamma^{-1}(G) = |D'| = k - 1$.

(2) If k is odd, we have two situations,

(a) $k = 3$, then $D = \{p_1, p_2p_3\}$ and $D' = \{p_3, p_1p_2\}$.

(b) $k \geqslant 5$, then the subset

$D' = \{p_{k-1}, p_k, p_1p_2, p_3p_4, \ldots, p_{k-2}p_{k-1}, p_1p_k, p_3p_k, \ldots, p_{k-6}p_k\}$

which has order

$$|D'| = 2 + \frac{k-1}{2} + \frac{k-5}{2} = k - 1$$

is a minimum inverse dominating set of G with respect to D. Then $\gamma^{-1}(G) = |D'| = k - 1$.

Subcase 2. Suppose $3 \leqslant j < k$. Let $D = \{p_1, p_2, \ldots, p_{j-1}p_j, p_{j+1}, \ldots, p_k\}$ is a minimum dominating set of G. Consider $n = n_1.n_2$, where $n_1 = \prod_{i=1}^{j} p_i$ and $n_2 = \prod_{i=j+1}^{k} p_i^{\alpha_i}$. Then from Case II (Subcase 1), the vertices of the factorization n_1 of n has a minimum inverse dominating set D'_1 of order $j - 1$. Also from Case I (Subcase 1), the vertices of the factorization n_2 of n has a minimum inverse dominating set D'_2 of order $k - j$, where $D'_2 = \{p_jp_{j+1}, p_jp_{j+2}, \ldots, p_jp_k\}$, and any vertex consists of a product of some divisors of n_1 and n_2 is dominated by at least one vertex of D'_1 or D'_2, so the dominating set $D' = D'_1 \bigcup D'_2$ of order $|D'| = j - 1 + k - j = k - 1$ is a minimum inverse dominating set of G. Hence, $\gamma^{-1}(G) = |D'| = k - 1$. □

THEOREM 2.4. *Let G be a non-trivial V_n-Arithmetic graph, where*

$$n = p_1^{\alpha_1} p_2^{\alpha_2} \dots {p_k}^{\alpha_k} \text{ such that } \alpha_1 \geqslant \alpha_2 \geqslant \dots \geqslant \alpha_k.$$

Then the injective domination number of G is given by

$$\gamma_{in}(G) = \begin{cases} 2, & \text{if } n = p_1p_2 \text{ or } n = p^{\alpha}; \\ 1, & \text{otherwise,} \end{cases}$$

PROOF. We have three cases.

Case 1. Suppose $n = p_1p_2$ or $n = p^{\alpha}$. Then G is a star. So any end vertex $v \in V$ inj-dominates all the other vertices except the center vertex. Hence, $\gamma_{in}(G) = 2$.

Case 2. Suppose $\alpha_1 \neq 1$, $k \geqslant 2$. The vertex $v = p_1^2$ satisfies $N(v) = N_{cn}(v)$ and since $diam(G) \leqslant 2$, then by Proposition 1.1, $\gamma_{in}(G) = 1$.

Case 3. Suppose $\alpha_1 = 1$ and $n \neq p_1p_2$. The vertex $v = p_1p_2$ satisfies $N(v) = N_{cn}(v)$ and since $diam(G) \leqslant 2$, then by Proposition 1.1, $\gamma_{in}(G) = 1$. □

Now, it is our turn to obtain some bounds on the domination number of V_n-Arithmetic graph.

PROPOSITION 2.1. *For any V_n-Arithmetic graph G, $\delta(G) \geqslant \gamma(G)$. Furthermore, $\delta(G) = \gamma(G)$ if and only if $\alpha_i = 1$ for at most one i or $n = p_1p_2$.*

PROOF. Let G be a V_n-Arithmetic graph. Then from Theorem 1.1, and since

$$\delta(G) = \begin{cases} 1, & \text{if } n = p_1p_2; \\ k, & \text{otherwise,} \end{cases}$$

then $\delta(G) \geqslant \gamma(G)$. □

THEOREM 2.5. *For any non-trivial V_n-Arithmetic graph G, where $\alpha_1 \geqslant \alpha_2 \geqslant \dots \geqslant \alpha_k$. Then*

$$\gamma(G) \leqslant \left\lceil \frac{\Delta(G) - \delta(G)}{2} \right\rceil + 1.$$

Furthermore, the equality is attained if and only if $n = p^2$ or $p_1^2p_2$ or $p_1p_2p_3$ or $p_1p_2p_3p_4$.

PROOF. We have the following cases.

Case 1. Suppose $k = 1$ ($n = p^{\alpha}$, $\alpha > 1$). Then $\Delta(G) = \alpha - 1$, $\delta(G) = 1$ and $\gamma(G) = 1$. Hence,

$$\left\lceil \frac{\Delta(G) - \delta(G)}{2} \right\rceil + 1 = \left\lceil \frac{\alpha}{2} \right\rceil \geqslant 1.$$

Case 2. Suppose $k \geqslant 2$ and at most $\alpha_k = 1$. In this case the smallest choice of G is when $n = p_1^2p_2$, so $\Delta(G) = 4$, $\delta(G) = 2$ and $\gamma(G) = 2$. Hence,

$$\left\lceil \frac{\Delta(G) - \delta(G)}{2} \right\rceil + 1 = 2 \geqslant 2.$$

Case 3. Suppose $\alpha_i = 1$ for more than one i and at least $\alpha_1 \neq 1$ (here $k \geqslant 3$). Then $\Delta(G) = \alpha_1 \prod_{i=2}^{k}(\alpha_i + 1) - 1$, $\delta(G) = k$ and $\gamma(G) = k - 1$, thus

$$\left\lceil \frac{\Delta(G) - \delta(G)}{2} \right\rceil + 1 = \left\lceil \frac{\alpha_1 \prod_{i=2}^{k}(\alpha_i + 1) - 1 - k}{2} \right\rceil + 1$$
$$\geqslant \left\lceil \frac{\alpha_1 2^{k-1} - 1 - k}{2} \right\rceil + 1 \geqslant \alpha_1 2^{k-2} - \left\lceil \frac{k}{2} \right\rceil \geqslant k - 1.$$

Case 4. Suppose $\alpha_1 = 1$ (here $k \geqslant 2$). Then $\Delta(G) = 2^{k-1}$, $\delta(G) = k$ and $\gamma(G) = k - 1$. Hence,

$$\left\lceil \frac{\Delta(G) - \delta(G)}{2} \right\rceil + 1 = \left\lceil 2^{k-2} - \frac{k}{2} \right\rceil + 1 \geqslant k - 1.$$

For the equality, if $n = p^2$ or $p_1^2 p_2$ or $p_1p_2p_3$ or $p_1p_2p_3p_4$, then from Case 1, Case 2 and Case 4, respectively, we easily can see that,

$$\gamma(G) = \left\lceil \frac{\Delta(G) - \delta(G)}{2} \right\rceil + 1.$$

□

PROPOSITION 2.2. *For any V_n-Arithmetic graph G, where $\alpha_1 \geqslant \alpha_2 \geqslant \ldots \geqslant \alpha_k$. Then*

$$\gamma(G) \geqslant \begin{cases} \left\lceil \dfrac{\alpha_1 + 1}{\alpha_1} - \dfrac{1}{\Delta(G) + 1} \right\rceil, & \text{if } \alpha_1 \neq 1; \\ \left\lceil 2 - \dfrac{3}{2^{k-1} + 1} \right\rceil, & \text{if } \alpha_i = 1, \forall i \in \{1, 2, \ldots, k\}. \end{cases}$$

PROOF. This bound comes from [**16**], for any graph G, then

$$\gamma(G) \geqslant \left\lceil \frac{|V|}{\Delta(G) + 1} \right\rceil.$$

□

3. Vertex (Edge) Covering of V_n-Arithmetic Graph

THEOREM 3.1. *Let G be a V_n-Arithmetic graph, where $n = p_1p_2 \ldots p_k$ (all $\alpha_i = 1$, $i \in \{1, 2, \ldots, k\}$). Then the vertex covering number of G is given by*

$$\alpha(G) = \begin{cases} 2^k - k - 1, & \text{if } k \leqslant 3; \\ 2^{k-1} + \dfrac{1}{2}\dbinom{k}{\frac{k}{2}} - 1, & \text{if } k \text{ is even, } k > 2; \\ 2^{k-1} + \dbinom{k-1}{\lfloor \frac{k}{2} \rfloor} - 1, & \text{if } k \text{ is odd, } k > 3. \end{cases}$$

Proof. In this case $|V| = 2^k - 1$. We have three cases.

Case 1. Assume that, k is even and $k > 2$. Let $S_1 \subset V$ be the subset of those vertices of V such that each vertex consists of at least a product of $\frac{k}{2} + 1$ primes i.e. $S_1 = \left\{ u = \prod_{i \in B_1} p_i : B_1 \subset \{1, 2, \ldots, k\}, |B_1| \geqslant \frac{k}{2} + 1 \right\}$. Then S_1 has no two adjacent vertices because any two vertices of S_1 have at least two common primes, so S_1 is an independent set of G.

It is clear that the cardinality of S_1 is

$$|S_1| = 2^k - 1 - \left[\binom{k}{1} + \binom{k}{2} + \cdots + \binom{k}{\frac{k}{2}} \right] = 2^{k-1} - \frac{1}{2}\binom{k}{\frac{k}{2}}.$$

We show that S_1 is a maximal independent set of G. First, suppose S_1 is not maximal. Then there exists an independent set S_2 of G such that $S_1 \subset S_2$ and $|S_2| \geqslant |S_1| + 1$. Let $v \in S_2$, $v = \prod_{i \in B_2} p_i$, where $B_2 \subset \{1, 2, \ldots, k\}$, $|B_2| \leqslant \frac{k}{2}$. Then $v \notin S_1$. So there exists at least a vertex $u \in S_1$, such that $B_1 \cap B_2 = \{j\}$ for some $j \in \{1, 2 \ldots, k\}$, so u and v are adjacent, a contradiction. Hence, S_1 is maximal. On the other hand, a maximal independent set of G can be obtained by two other manners. First of them is the set of the prime vertices which clearly has cardinality k, less than the cardinality of S_1 when $k > 3$, and the second is the set of all the vertices of G which has more than one common prime divisor. Let S' be the set of vertices of G which consists all the vertices of two common prime divisors. Obviously, S' has the greatest cardinality of such maximal independent sets of G, where $|S'| = 2^{k-2}$. But $|S_1| \geqslant |S'|$ for all $k > 2$. Then, S_1 is a maximum independent set of G and

$$|S_1| = \beta(G) = 2^{k-1} - \frac{1}{2}\binom{k}{\frac{k}{2}}.$$

Hence, $\alpha(G) = |V| - \beta(G) = 2^{k-1} + \frac{1}{2}\binom{k}{\frac{k}{2}} - 1.$

Case 2. Assume that, k is odd and $k > 3$. As in (Case 1) let $S_1 \subset V$ be the subset of those vertices of V such that each vertex consists of at least a product of $\left\lceil \frac{k}{2} \right\rceil + 1$ primes. Then S_1 is an independent set of G and

$$|S_1| = \binom{k}{\lceil \frac{k}{2} \rceil + 1} + \binom{k}{\lceil \frac{k}{2} \rceil + 2} + \cdots + \binom{k}{k} = 2^{k-1} - \binom{k}{\lceil \frac{k}{2} \rceil}.$$

Let $X_1 = \left\{ u = \prod_{i \in B_1} p_i : B_1 \subset \{1, 2, \ldots, k\}, |B_1| = \left\lceil \frac{k}{2} \right\rceil \right\}$ and

$X_2 = \left\{ u = \prod_{i \in B_2} p_i : B_2 \subset \{1, 2, \ldots, k\}, |B_2| < \left\lceil \frac{k}{2} \right\rceil \right\}$. Then the vertices of X_1 are not adjacent to any vertex of S_1 and any vertex of X_2 is adjacent to at least one vertex of S_1. Then S_1 is not maximal independent set of G. So to make a maximal independent set of G containing S_1 we have to add a maximum independent set of vertices of X_1 to S_1. Observably, $|X_1| = \binom{k}{\lceil \frac{k}{2} \rceil}$. So, $\binom{k-1}{\lceil \frac{k}{2} \rceil}$ gives a number of vertices of X_1 which are not adjacent one to each others. Hence, the subset S_2 of V, where

$$|S_2| = |S_1| + \binom{k-1}{\lceil \frac{k}{2} \rceil} = 2^{k-1} - \binom{k-1}{\lfloor \frac{k}{2} \rfloor},$$

is a maximal independent set of G. Also, it is easy to see that $|S_2| > k$ and $|S_2| \geqslant |S'|$ for all $k > 3$ (S' is that independent set in the proof of Case 1). Thus, S_2 is a maximum independent set of G. Hence, $\beta(G) = |S_2| = 2^{k-1} - \binom{k-1}{\lfloor \frac{k}{2} \rfloor}$ and

$$\alpha(G) = 2^k - 1 - \beta(G) = 2^{k-1} + \binom{k-1}{\lfloor \frac{k}{2} \rfloor} - 1.$$

Case 3. Assume that, $k \leqslant 3$. In this case one can simply observe that the maximum independent set of G is $S = \{p_i : i = 1, 2, \dots, k\}$ which has cardinality k. Then $\alpha(G) = 2^k - 1 - k$. □

THEOREM 3.2. *If G is a V_n-Arithmetic graph, where $\alpha_1 \geqslant \dots \geqslant \alpha_k$, k is even and $\alpha_1 \neq 1$, then the covering number of G is given by*

$$\alpha(G) = \begin{cases} \displaystyle\sum_{i=1}^{k} \alpha_i + \sum_{i=1}^{k-1} \alpha_i \sum_{j=i+1}^{k} \alpha_j + \dots + \\ \displaystyle\quad + \sum_{i=1}^{k-\frac{k}{2}+1} \alpha_i \sum_{j=i+1}^{k-\frac{k}{2}+2} \alpha_j \dots \sum_{z=y+1}^{k} \alpha_z, & \text{if } |S_1| \geqslant (\alpha_1 - 1) \displaystyle\prod_{i=2}^{k} (\alpha_i + 1); \\ \displaystyle 2 \prod_{i=2}^{k} (\alpha_i + 1) - 1, & \text{if } |S_1| \leqslant (\alpha_1 - 1) \displaystyle\prod_{i=2}^{k} (\alpha_i + 1), \end{cases}$$

where $|S_1| = \sum_{i=1}^{k-\frac{k}{2}} \alpha_i \sum_{j=i+1}^{k-\frac{k}{2}+1} \alpha_j \dots \sum_{z=y+1}^{k} \alpha_z + \dots + \prod_{i=1}^{k} \alpha_i$ and y is the index of the before last summation.

PROOF. In this case $|V| = \prod_{i=1}^{k} (\alpha_i + 1) - 1$. Let $S_1 \subset V$ be the subset of those vertices of V such that each vertex consists of at least a product of $\frac{k}{2} + 1$ primes or their powers i.e. $S_1 = \left\{ u = \prod_{i \in B_1} p_i^{a_i} : 1 \leqslant a_i \leqslant \alpha_i, \forall i \in B_1 \right\}$, where $B_1 \subset \{1, 2, \dots, k\}$, $|B_1| \geqslant \frac{k}{2} + 1$. Then S_1 has no two adjacent vertices because any two vertices of S_1 have at least two common primes or their powers, so S_1 is a maximal independent set of G (by the same argument in the proof of Theorem 3.1, Case 1). It is clear that the cardinality of S_1 is

$$|S_1| = \sum_{i=1}^{k-\frac{k}{2}} \alpha_i \sum_{j=i+1}^{k-\frac{k}{2}+1} \alpha_j \dots \sum_{z=y+1}^{k} \alpha_z + \dots + \prod_{i=1}^{k} \alpha_i.$$

Let $S_2 \in V$ be the subset of V which consists of all the powers of p_1 (except p_1 itself) and their product to the other divisors of n i.e. $S_2 = \big\{u = p_1^a \prod_{i \in B_2} p_i^{b_i} : 1 < a \leqslant \alpha_1, 0 \leqslant b_i \leqslant \alpha_i, \forall i \in B_2\big\}$, where $B_2 \subset \{2, \dots, k\}$ (the powers of p_1 are contained in S_2 when $b_i = 0$, $\forall i \in B_2$). From the definitions of V_n-Arithmetic graph and the divisors of n all the vertices of S_2 are independent one to each others and for each vertex $v \in V - S_2$ there exist at least one vertex $u \in S_2$ which has a common prime divisor with v. Hence, S_2 is a maximal independent set of G of order $|S_2| = \big(\alpha_1 - 1\big) \prod_{i=2}^{k} (\alpha_i + 1)$.

In fact, there are three other manners to get a maximal independent set of G which are as follows.

- The set of all the vertices of G which has more than one common divisor. Since $\alpha_1 \geqslant \alpha_2 \geqslant \dots \geqslant \alpha_k$, then the set S' of all the vertices of G which has $p_1 p_2$ as a common divisor has the greatest cardinality among such sets, where $|S'| = \alpha_1 \alpha_2 \prod_{i=3}^{k} (\alpha_i + 1)$. But $|S'|$ less than or equal $|S_1|$ or $|S_2|$.
- The set S'' of all the power vertices (except the primes) and their products, which has cardinality $|S''| = -1 + \prod_{i=1}^{k} \alpha_i$. But again $|S''|$ less than or equal $|S_1|$ or $|S_2|$.
- The set of prime divisors of n which has cardinality k less than or equal the cardinalities of $|S_1|$ and $|S_2|$.

Since α_1 is the greatest exponent of a prime divisor of n, then either S_1 or S_2 is a maximum independent set of G depending on the powers α_i's, $i \in \{1, 2, \dots, k\}$. Therefore,

(1) If $|S_1| \geqslant |S_2|$, then

$$\alpha(G) = |V| - |S_1| = \sum_{i=1}^{k} \alpha_i + \sum_{i=1}^{k-1} \alpha_i \sum_{j=i+1}^{k} \alpha_j + \dots + \sum_{i=1}^{k-\frac{k}{2}+1} \alpha_i \sum_{j=i+1}^{k-\frac{k}{2}+2} \alpha_j \dots \sum_{z=y+1}^{k} \alpha_z.$$

(2) If $|S_1| \leqslant |S_2|$, then

$$\alpha(G) = |V| - |S_2| = \prod_{i=1}^{k}(\alpha_i + 1) - 1 - (\alpha_1 - 1)\prod_{i=2}^{k}(\alpha_i + 1)$$
$$= 2\prod_{i=2}^{k}(\alpha_i + 1) - 1.$$

□

COROLLARY 3.1. *For any V_n-Arithmetic graph G, where $n = p_1^{\alpha_1} p_2^{\alpha_2}$, $\alpha_1 \geqslant \alpha_2$ and $\alpha_1 \neq 1$, the vertex covering number of G is*

$$\alpha(G) = \begin{cases} \alpha_1 + \alpha_2, & \text{if } \alpha_1 \leqslant \alpha_2 + 1; \\ 2\alpha_2 + 1, & \text{otherwise.} \end{cases}$$

THEOREM 3.3. *If G is a V_n graph, where $\alpha_1 \geqslant \alpha_2 \geqslant \ldots \geqslant \alpha_k$, k is odd and $\alpha_1 \neq 1$, then the covering number of G is given by*

$$\alpha(G) = \begin{cases} \sum_{i=1}^{k}\alpha_i + \sum_{i=1}^{k-1}\alpha_i \sum_{j=i+1}^{k}\alpha_j + \cdots + \alpha_k \sum_{i=1}^{k-\lceil\frac{k}{2}\rceil+1}\alpha_i \sum_{j=i+1}^{k-\lceil\frac{k}{2}\rceil+2}\alpha_j \ldots \sum_{z=y+1}^{k-1}\alpha_z, \\ \text{if } |S_1| \geqslant (\alpha_1 - 1)\prod_{i=2}^{k}(\alpha_i + 1); \\ 2\prod_{i=2}^{k}(\alpha_i + 1) - 1, \quad \text{if } |S_1| \leqslant (\alpha_1 - 1)\prod_{i=2}^{k}(\alpha_i + 1), \end{cases}$$

where

$$|S_1| = \sum_{i=1}^{k-\lceil\frac{k}{2}\rceil}\alpha_i \sum_{j=i+1}^{k-\lceil\frac{k}{2}\rceil+1}\alpha_j \ldots \sum_{z=y+1}^{k}\alpha_z + \cdots + \prod_{i=1}^{k}\alpha_i + \sum_{i=1}^{k-\lceil\frac{k}{2}\rceil}\alpha_i \sum_{j=i+1}^{k-\lceil\frac{k}{2}\rceil+1}\alpha_j \ldots \sum_{z=y+1}^{k-1}\alpha_z,$$

and y is the index of the before last summation.

PROOF. Consider $S = \{u = \prod_{i\in B} p_i^{a_i} : 1 \leqslant a_i \leqslant \alpha_i, \forall i \in B\}$, where $B \subset \{1, 2, \ldots, k\}$ and $|B| \geqslant \left\lceil\frac{k}{2}\right\rceil + 1$. Then S is an independent set of G, but it is not maximal because all the vertices of $X^* = \{u = \prod_{i\in B^*} p_i^{a_i} : 1 \leqslant a_i \leqslant \alpha_i, \forall i \in B^*\}$, where $B^* \subset \{1, 2, \ldots, k\}$, $|B^*| = \left\lceil\frac{k}{2}\right\rceil$ are not adjacent to any vertex of S and any vertex of $X^{**} = \{u = \prod_{i\in B^{**}} p_i^{a_i} : 1 \leqslant a_i \leqslant \alpha_i, \forall i \in B^{**}\}$, where $B^{**} \subset \{1, 2, \ldots, k\}$, $|B^{**}| < \left\lceil\frac{k}{2}\right\rceil$ is adjacent to at least one vertex of S. Therefore, to get a maximal independent set S_1 of G containing S, we have to determine a maximum independent set of X^* and add it to S.

From the prove of Theorem 3.2, if k is even the subset $V_0 \subset V$ such that $V_0 = \{u = \prod_{i \in B_0} p_i^{a_i} : 1 \leqslant a_i \leqslant \alpha_i, \forall i \in B_0\}$, where $B_0 \subset \{1, 2, \ldots, k\}$, $|B_0| = \frac{k}{2}+1$ is an independent set which has cardinality $|V_0| = \sum_{i=1}^{k-\frac{k}{2}} \alpha_i \sum_{j=i+1}^{k-\frac{k}{2}+1} \alpha_j \ldots \sum_{z=y+1}^{k} \alpha_z$, where y is the index of the before last summation. Since k is odd, then $k-1$ is even and the subset $V_0^* \subset X^*$ which has cardinality $|V_0^*| = \sum_{i=1}^{k-\lceil \frac{k}{2} \rceil} \alpha_i \sum_{j=i+1}^{k-\lceil \frac{k}{2} \rceil+1} \alpha_j \ldots \sum_{z=y+1}^{k-1} \alpha_z$, is a maximum independent set of X^*. Thus,

$$|S_1| = \sum_{i=1}^{k-\lceil \frac{k}{2} \rceil} \alpha_i \sum_{j=i+1}^{k-\lceil \frac{k}{2} \rceil+1} \alpha_j \ldots \sum_{z=y+1}^{k} \alpha_z + \cdots + \prod_{i=2}^{k} \alpha_i + \sum_{i=1}^{k-\lceil \frac{k}{2} \rceil} \alpha_i \sum_{j=i+1}^{k-\lceil \frac{k}{2} \rceil+1} \alpha_j \ldots \sum_{z=y+1}^{k-1} \alpha_z,$$

where y is the index of the before last summation.

Now, let $S_2 \subset V$ be the subset of V which consists of all the powers of p_1 (except p_1 itself) and their product to the other divisors of n i.e. $S_2 = \{u = p_1^a \prod_{i \in B_2} p_i^{b_i} : 1 < a \leqslant \alpha_1, 0 \leqslant b_i \leqslant \alpha_i, \forall i \in B_2\}$, where $B_2 \subset \{2, \ldots, k\}$. Then S_2 is a maximal independent set of G of order $|S_2| = (\alpha_1 - 1) \prod_{i=2}^{k} (\alpha_i + 1)$.

By the same argument in the proof of Theorem 3.2, either S_1 or S_2 is a maximum independent set of G depending on the powers α_i's, $i \in \{1, 2, \ldots, k\}$. Therefore,

(1) If $|S_1| \geqslant |S_2|$, then

$$\begin{aligned}\alpha(G) = |V| - |S_1| = \prod_{i=1}^{k} (\alpha_i + 1) - 1 - \sum_{i=1}^{k-\lceil \frac{k}{2} \rceil} \alpha_i \sum_{j=i+1}^{k-\lceil \frac{k}{2} \rceil+1} \alpha_j \ldots \sum_{z=y+1}^{k} \alpha_z \\ - \cdots - \prod_{i=1}^{k} \alpha_i - \sum_{i=1}^{k-\lceil \frac{k}{2} \rceil} \alpha_i \sum_{j=i+1}^{k-\lceil \frac{k}{2} \rceil+1} \alpha_j \ldots \sum_{z=y+1}^{k-1} \alpha_z.\end{aligned}$$

Since,

$$\begin{aligned}\sum_{i=1}^{k-\lceil \frac{k}{2} \rceil+1} \alpha_i \sum_{j=i+1}^{k-\lceil \frac{k}{2} \rceil+2} \alpha_j \ldots \sum_{z=y+1}^{k} \alpha_z = \sum_{i=1}^{k-\lceil \frac{k}{2} \rceil} \alpha_i \sum_{j=i+1}^{k-\lceil \frac{k}{2} \rceil+1} \alpha_j \ldots \sum_{z=y+1}^{k-1} \alpha_z \\ + \alpha_k \sum_{i=1}^{k-\lceil \frac{k}{2} \rceil+1} \alpha_i \sum_{j=i+1}^{k-\lceil \frac{k}{2} \rceil+2} \alpha_j \ldots \sum_{z=y+1}^{k-1} \alpha_z\end{aligned}$$

then

$$\alpha(G) = \sum_{i=1}^{k} \alpha_i + \sum_{i=1}^{k-1} \alpha_i \sum_{j=i+1}^{k} \alpha_j + \cdots + \alpha_k \sum_{i=1}^{k-\lceil \frac{k}{2} \rceil+1} \alpha_i \sum_{j=i+1}^{k-\lceil \frac{k}{2} \rceil+2} \alpha_j \ldots \sum_{z=y+1}^{k-1} \alpha_z.$$

(2) If $|S_1| \leqslant |S_2|$, then

$$\alpha(G) = |V| - |S_2| = \prod_{i=1}^{k}(\alpha_i+1) - 1 - (\alpha_1 - 1)\prod_{i=2}^{k}(\alpha_i+1)$$
$$= 2\prod_{i=2}^{k}(\alpha_i+1) - 1.$$

□

COROLLARY 3.2. *Let G be a V_n-Arithmetic graph, where $n = p_1^{\alpha_1}p_2^{\alpha_2}p_3^{\alpha_3}$, $\alpha_1 \geqslant \alpha_2 \geqslant \alpha_3$ and $\alpha_1 \neq 1$. Then the vertex covering number of G is*

$$\alpha(G) = \begin{cases} \sum_{i=1}^{3}\alpha_i + \alpha_3(\alpha_1+\alpha_2), & \text{if } \alpha_1 \leqslant \alpha_2 + 1; \\ 2(\alpha_2+1)(\alpha_3+1) - 1, & \text{otherwise.} \end{cases}$$

THEOREM 3.4. *For any V_n-Arithmetic graph G, where $n = p_1p_2\dots p_k$ (all $\alpha_i = 1$, $i \in \{1,2,\dots,k\}$), the edge covering number of G is given by $\alpha'(G) = 2^{k-1}$.*

PROOF. Let G be a V_n-Arithmetic graph, where $n = p_1p_2\dots p_k$. Then $|V| = 2^k - 1$. Since $\binom{k}{k-1} = \binom{k}{1} = k$, then we can make a matching of size k join the prime vertices with the vertices which consist of a product of $k-1$ primes. Also, since $\binom{k}{k-2} = \binom{k}{2}$, then we can make a matching of size $\binom{k}{2}$ join the vertices p_ip_j, $i,j \in \{1,2,\dots,k\}$ and the vertices which consist of a product of $k-2$ primes, and so on to $\binom{k}{\lceil \frac{k}{2} \rceil} = \binom{k}{\lfloor \frac{k}{2} \rfloor}$. Therefore,

(1) If k is odd, then $\beta'(G) = \left\lfloor \frac{2^k-1}{2} \right\rfloor$ and hence,

$$\alpha'(G) = \left\lceil \frac{2^k-1}{2} \right\rceil = 2^{k-1}.$$

(2) If k is even, then $\binom{k}{\lceil \frac{k}{2} \rceil}$ and $\binom{k}{\lfloor \frac{k}{2} \rfloor}$ are same which are $\binom{k}{\frac{k}{2}}$. Therefore, we can make a matching of size $\frac{1}{2}\binom{k}{\frac{k}{2}}$ and hence, $\beta'(G) = \left\lfloor \frac{2^k-1}{2} \right\rfloor$ and $\alpha'(G) = 2^{k-1}$.

□

PROPOSITION 3.1. *If G is a V_n-Arithmetic graph, where $n = p_1^{\alpha}p_2$, $\alpha > 1$, then the edge covering number of G is*

$$\alpha'(G) = \begin{cases} 3, & \text{if } \alpha = 2; \\ 2(\alpha-1), & \text{if } \alpha \geqslant 3. \end{cases}$$

PROOF. Let $n = p_1^{\alpha} p_2$, $\alpha > 1$. Then $|V| = 2\alpha + 1$. In this case all the vertices of the form $p_1^a p_2$, $1 \leqslant a \leqslant \alpha$ form an independent set in G. Therefore,

(1) If $\alpha = 2 \Rightarrow n = p_1^2 p_2$. The subset $M \subset E$, where $M = \{\{p_1, p_1^2 p_2\}, \{p_2, p_1 p_2\}\}$ is a maximum matching set in G. Then $\beta'(G) = 2$ and hence, $\alpha'(G) = 3$.

(2) If $\alpha \geqslant 3$. Since the number of vertices which are adjacent only to the primes p_1, p_2 is $\alpha - 1 \geqslant 2$, then we have exactly a matching of size two join p_1 and p_2 with only two vertices from these vertices. Also, we have exactly one edge joins one vertex from the power vertices of p_1 with the vertex $p_1 p_2$. Then the maximum matching set M of G has cardinality $|M| = 3$. Thus, $\beta'(G) = 3$ and hence, $\alpha'(G) = 2(\alpha - 1)$.

□

PROPOSITION 3.2. *If G is a V_n-Arithmetic graph, where $n = p_1^{\alpha_1} p_2^{\alpha_2}$, $\alpha_1 \geqslant \alpha_2 > 1$, then the edge covering number of G is*

$$\alpha'(G) = \begin{cases} \alpha_1 \alpha_2, & \text{if } \alpha_1 \leqslant \alpha_2 + 1; \\ (\alpha_1 - 1)(\alpha_2 + 1), & \text{otherwise.} \end{cases}$$

PROOF. Let $n = p_1^{\alpha_1} p_2^{\alpha_2}$, $\alpha_1 \geqslant \alpha_2 > 1$. Then $|V| = (\alpha_1 + 1)(\alpha_2 + 1) - 1$. In this case the number of vertices which are adjacent only to the primes p_1, p_2 is $(\alpha_1 - 1)(\alpha_2 - 1)$. Therefore, we have a matching of size two joins the primes p_1 and p_2 with only two vertices from these vertices. Since the power vertices of p_1 and p_2 are adjacent to the vertices of the forms $\{p_1 p_2^b : 1 \leqslant b \leqslant \alpha_2\}$ and $\{p_1^a p_2 : 1 \leqslant a \leqslant \alpha_1\}$ which have orders α_2 and α_1, respectively, then

(1) If $\alpha_1 = \alpha_2$ or $\alpha_1 = \alpha_2 + 1$, then we have a matching of size $\alpha_1 - 1$ joins the powers of p_1 with $\alpha_1 - 1$ or all the vertices of $\{p_1 p_2^b : 1 \leqslant b \leqslant \alpha_2\}$ respectively. Also, we have a matching of size $\alpha_2 - 1$ joins the powers of p_2 with $\alpha_2 - 1$ vertex of $\{p_1^a p_2 : 1 \leqslant a \leqslant \alpha_1\}$. Thus, $\beta'(G) = 2 + \alpha_1 - 1 + \alpha_2 - 1 = \alpha_1 + \alpha_2$ and hence, $\alpha'(G) = \alpha_1 \alpha_2$.

(2) If $\alpha_1 > \alpha_2 + 1$, then we have a matching of size $\alpha_2 - 1$ joins the powers of p_2 with $\alpha_2 - 1$ vertex of $\{p_1^a p_2 : 1 < a \leqslant \alpha_1\}$ and a matching of size α_2 joins exactly α_2 vertex of the power of p_1 with all the vertices of $\{p_1 p_2^b : 1 \leqslant b \leqslant \alpha_2\}$. Thus,

$$\beta'(G) = 2 + \alpha_2 - 1 + \alpha_2 = 2\alpha_2 + 1,$$

and hence, $\alpha'(G) = (\alpha_1 - 1)(\alpha_2 + 1)$.

□

4. Conclusion

As remarked earlier, the authors in [**12**] have studied some properties of the V_n-Arithmetic graph. In this research work, we continue the study of V_n-Arithmetic graph by determining domination parameters of V_n. Bounds for some V_n parameters are established. The authors recommend many problems for future studies like domatic number, coloring problems and the eigenvalues of V_n-Arithmetic graph.

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