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COMMON FIXED POINTS OF ALMOST GENERALIZED $(\alpha, \psi, \varphi, F)$-CONTRACTION TYPE MAPPING IN b-METRIC SPACES

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Abstract. In this manuscript, we introduce almost generalized $(\alpha, \psi, \varphi, F)$-contraction type mapping and prove the existence and uniqueness of common fixed point in the setting of b-metric spaces of these maps. The result presented in this paper extend/generalize some of the earlier results in the existing literature. Further, we provide an example to illustrate our main result.

1. Introduction and Preliminaries

In the development of nonlinear functional analysis fixed point theory plays a vital role in many aspects. It has been used in various branches of engineering and sciences. The Banach contraction principle was introduced by Banach [**7**] is one of the most important result in fixed point theory. Over the years, many authors extended and generalized the Banach contraction principle in many directions and spaces, we refer [**11, 15, 16, 14, 21, 25, 28, 29, 30**]. The main idea of b-metric was initiated from the works of Bourbaki [**8**] and Bakhtin [**6**]. The concept of b-metric space or metric type space was introduced by Czerwik [**12**] as a generalization of metric space. Since then several authors proved fixed point results of single valued and multivalued operators in b-metric spaces, we refer [**1, 5, 9, 10, 13, 19, 20, 22, 23, 26, 27**].

In 2014, Ansari [**2**] introduced the concept of C-class functions and established fixed point theorems for certain contractive mappings with respect to the C-class function, we refer [**4, 18, 24**].

Definition 1.1. ([**12**]) Let X be a non-empty set. A function $d : X \times X \to [0, \infty)$ is said to be a b-metric if the following conditions are satisfied:

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(i) $0 \leqslant d(x,y)$ for all $x,y \in X$ and $d(x,y) = 0$ if and only if $x = y$,

(ii) $d(x,y) = d(y,x)$ for all $x,y \in X$, and

(iii) there exists $s \geqslant 1$ such that $d(x,z) \leqslant s\big[d(x,y)+d(y,z)\big]$ for all $x,y,z \in X$.
In this case, the pair (X,d) is called a b-metric space with coefficient s. Here, we observe that every metric space is a b-metric space with $s = 1$.

Definition 1.2. ([**10**]) Let (X,d) be a b-metric space and $\{x_n\}$ a sequence in X.

(i) A sequence $\{x_n\}$ in X is called b-convergent if there exists $x \in X$ such that $d(x_n,x) \to 0$ as $n \to \infty$. In this case, we write $\lim_{n\to\infty} x_n = x$.

(ii) A sequence $\{x_n\}$ in X is called b-Cauchy if $d(x_n,x_m) \to 0$ as $n,m \to \infty$.

(iii) A b-metric space (X,d) is said to be a complete b-metric space if every b-Cauchy sequence in X is b-convergent.

(iv) A set $B \subset X$ is said to be b-closed if for any sequence $\{x_n\}$ in B such that $\{x_n\}$ is b-convergent to $z \in X$ then $z \in B$.

In general, a b-metric is not necessarily continuous.

Example 1.1. ([**19**]) Let $X = \mathbb{N}\cup\{\infty\}$. We define a mapping $d : X\times X \to \mathbb{R}^+$ as follows:

$$d(m,n) = \begin{cases} 0 & \text{if } m = n, \\ |\frac{1}{m} - \frac{1}{n}| & \text{if one of } m,n \text{ is even and the other is even or } \infty\ , \\ 5 & \text{if one of } m,n \text{ is odd and the other is odd or } \infty, \\ 2 & \text{otherwise.} \end{cases}$$

Then (X,d) is a b-metric space with coefficient $s = \frac{5}{2}$.

Definition 1.3. ([**10**]) Let (X,d_X) and (Y,d_Y) be two b-metric spaces. A function $f : X \to Y$ is b-continuous at a point $x \in X$, if it is b-sequentially continuous at x *i.e.*, whenever $\{x_n\}$ is b-convergent to x, $\{fx_n\}$ is b-convergent to fx.

In 2014, Ansari [**2**] introduced the concept of C-class functions as follows.

Definition 1.4. ([**2**]) A continuous mapping $F : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ is called a C-class function if it satisfies the following conditions:

(a) $F(s,t) \leqslant s$ for all $s,t \in \mathbb{R}^+$, and

(b) $F(s,t) = s$ implies that either $s = 0$ or $t = 0$.

We denote the family of all C-class functions by $\mathcal{C}$.

Example 1.2. ([**2**]) The following functions $F : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ are elements of $\mathcal{C}$, for all $s,t \in [0,\infty)$:

(i) $F(s,t) = s - t$, $F(s,t) = s$ implies that $t = 0$;

(ii) $F(s,t) = ks$, where $0 < k < 1$, $F(s,t) = s$ implies that $s = 0$;

(iii) $F(s,t) = \frac{s}{(1+t)^r}$, where $r > 0$;

(iv) $F(s,t) = s\beta(s)$, $\beta : \mathbb{R}^+ \to [0,1)$ and is continuous, $F(s,t) = s$ implies that $s = 0$;

(v) $F(s,t) = s - \varphi(s)$, $F(s,t) = s$ implies that $s = 0$, where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a continuous function such that $\varphi(t) = 0$ if and only if $t = 0$;

(vi) $F(s,t) = sh(s,t)$, $F(s,t) = s$ implies that $s = 0$,where $h : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is a continuous function such that $h(t,s) < 1$ for all $t, s > 0$. In particular $F(s,t) = \frac{st}{1+t}$;

(vii) $F(s,t) = s - \frac{t}{k+t}$, $F(s,t) = s$ implies that $t = 0$;

(viii) $F(s,t) = ln\frac{1+e^s}{2}$.

We now define the F-class functions as follows:

Definition 1.5. A continuous mapping $F : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ is said to be F-class function if

(a') $F(s,t) < s$ for all $s > 0$ and $t > 0$.
We denote the family of F-class functions by $\mathcal{F}$

Here we observe that the classes $\mathcal{C}$ and $\mathcal{F}$ both are same.

Let $F \in \mathcal{C}$. Let $s, t > 0$. From (b), $F(s,t) \neq s$. From (a), $F(s,t) \leqslant s$ so that $F(s,t) < s$. Therefore $F \in \mathcal{F}$. Hence $\mathcal{C} \subseteq \mathcal{F}$.

Let $F \in \mathcal{F}$. Now we show that the condition (b). Suppose that it fails to hold, so that there exist $s, t > 0$ such that $F(s,t) = s$. But from (a') $s = F(s,t) < s$, a contradiction. Now we show that condition (a) from the following cases.

Case (i): Let $s, t > 0$. By condition (a'), we have $F(s,t) < s$, follows that $F(s,t) \leqslant s$.

Case (ii): We show that $F(s,0) \leqslant s$ for all $s \geqslant 0, t = 0$. Let $s \geqslant 0, t = 0$. Then there exists a sequence $\{(s_n,t_n)\}$ in $(0.\infty) \times (0,\infty)$ such that $\lim\limits_{n\to\infty}(s_n,t_n) = (s,0)$. Since F is continuous at $(s,0)$, it follows that $\lim\limits_{n\to\infty} F(s_n,t_n) = F(s,0)$. From (a'), we have $F(s_n,t_n) < s_n$ for all n. On taking limits as $n \to \infty$, we have $F(s,0) = \lim\limits_{n\to\infty} F(s_n,t_n) \leqslant \lim\limits_{n\to\infty} s_n = s$.

Case (iii). $t \geqslant 0, s = 0$. In this case, we follow similar to Case (ii) to show $F(0,t) \leqslant 0$. Therefore $F \in \mathcal{C}$, so that $\mathcal{F} \subseteq \mathcal{C}$. Hence $\mathcal{F} = \mathcal{C}$.

Therefore, we identify $\mathcal{C}$ by $\mathcal{F}$.

From the following examples, we observe that for any $F \in \mathcal{C}$, $F(0,0)$ may not be equal to zero.

Example 1. $F(s,t) = s - \frac{1}{1+t} \in \mathcal{C}$ and $F(0,0) = -1 \neq 0$.

2. $F(s,t) = s - e^{s+t} \in \mathcal{C}$ and $F(0,0) = -1 \neq 0$.

If we restrict the codomain of F to $\mathbb{R}^+$ then $F(0,t) = 0$ for all $t \geqslant 0$, by Case (iii).

Here onwards, in this paper, we use F-class functions to prove our results.

In 2017, Huang, Deng and Radenović [**18**] proved the following result in b-metric spaces for a single selfmap by using a C-class function.

THEOREM 1.1. *Let (X,d) be a complete b-metric space with coefficient $s>1$ and $f:X\to X$ be a mapping such that*

$$\psi(s^{\alpha}d(fx,fy)) \leqslant F(\psi(M_i(x,y)),\varphi(M_i(x,y))) + LN(x,y) \tag{1.1}$$

for all $x,y\in X$, where $\alpha>0, L\geqslant 0$ are constants, $\psi,\varphi:\mathbb{R}^+\to\mathbb{R}^+$ are altering distance functions, $F:\mathbb{R}^+\times\mathbb{R}^+\to\mathbb{R}$ is a C-class function, and $M_i(x,y)(i=0,1,2)$ and N(x,y) are defined as follows:
$M_1(x,y)=\max\{d(x,y),\frac{d(x,fx)d(y,fy)}{1+d(x,y)},\frac{d(x,fx)d(y,fy)}{1+d(fx,fy)}\}$,
$M_2(x,y)=\max\{d(x,y),\frac{d(x,fx)d(x,fy)+d(y,fy)d(y,)fx}{1+s[d(x,fx)+d(y,fy)]},\frac{d(x,fx)d(x,fy)+d(y,fy)d(y,fx)}{1+s[d(x,fy)+d(y,fx)}\}$,
$M_3(x,y)=\max\{d(x,y),\frac{d(x,fx)d(y,fy)}{1+s[d(x,y)+d(x,fy)+d(y,fx)},\frac{d(x,fy)d(x,y)}{1+sd(x,fx)+s^3[d(y,fx)+d(y,fy)]}\}$ *and*
$N(x,y)=\min\{d(x,fx),d(y,fy),d(x,fy),d(y,fx)\}$.
Then for each $i\in\{1,2,3\}$, f has a unique fixed point in X. Moreover, for any $x\in X$, the iterative sequence $\{f^n(x)\}(n\in\mathbb{N})$ b-converges to the fixed point.

In 2017, He, Sun and Zhao [**17**] proved the following theorem in complete metric spaces for a pair of selfmaps.

THEOREM 1.2. *Let (X,d) be a complete metric space and $f,g:X\to X$ such that for every $x,y\in X$*

$$\frac{1}{2}\min\{d(x,fx),d(y,gy)\}\leqslant d(x,y)$$

implies that

$$\psi(d(fx,gy))\leqslant\psi(M(x,y))-\varphi(M(x,y))) \tag{1.2}$$

where

(i) *$\psi:\mathbb{R}^+\to\mathbb{R}^+$ is a continuous and monotone nondecreasing function with $\psi(t)=0$ if and only if $t=0$;*
(ii) *$\varphi:\mathbb{R}^+\to\mathbb{R}^+$ is a lower semi continuous function with $\varphi(t)=0$ if and only if $t=0$ and*

$$M(x,y)=\max\{d(x,y),d(x,fx),d(y,gy),\frac{d(x,gy)+d(y,fx)}{2}\}.$$

Then f and g have a unique common fixed point in X.

LEMMA 1.1 (Huang, Deng, Radenovic, [**18**]). *Let (X,d) be a b-metric space with coefficient $s\geqslant 1$. Suppose that $\{x_n\}$ is a sequence in X such that*

$$d(x_n,x_{n+1})\leqslant kd(x_{n-1},x_n) \text{ for all } n\in\mathbb{N}, \text{ where } k\in[0,1) \text{ is a constant.} \tag{1.3}$$

Then $\{x_n\}$ is a b-Cauchy sequence in X.

We denote Ψ to be the set of all continuous functions $\psi:\mathbb{R}^+\to\mathbb{R}^+$ satisfying the following conditions:

(i) ψ is monotonically increasing function;
(ii) $\psi(t)=0$ if and only if $t=0$.

We call an element $\psi \in \Psi$, an altering distance function [**21**].

We denote Φ to be the set of all continuous functions $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ such that $\varphi(t) = 0$ if and only if $t = 0$.

Inspired and motivated by the results of He, Sun and Zhao [**17**] and Huang, Deng and Radenović [**18**], we introduce a pair of almost (α,ψ,φ,F)-contraction type maps in b-metric spaces through F-class functions and prove the existence of common fixed points .

DEFINITION 1.6. Let (X,d) be a b-metric space with coefficient $s \geqslant 1$ and $f,g : X \to X$ be two selfmaps of X. If there exist $\alpha > 0, L \geqslant 0, \psi \in \Psi$, $\varphi \in \Phi$ and $F \in \mathcal{F}$ such that for every $x,y \in X$

$$\frac{1}{2s}\min\{d(x,fx),d(y,gy)\} \leqslant d(x,y)$$

implies that

$$\psi(s^\alpha d(fx,gy)) \leqslant F(\psi(M(x,y)),\varphi(M(x,y))) + LN(x,y) \tag{1.4}$$

where

$$M(x,y) = \max\{d(x,y),d(x,fx),d(y,gy),\frac{d(x,gy)+d(y,fx)}{2s}\}$$

and

$$N(x,y) = \min\{d(x,fx),d(y,gy),d(x,gy),d(y,fx)\}$$

then we say that the pair (f,g) is an almost generalized (α,ψ,φ,F)- contraction type maps.

2. Main results

PROPOSITION 2.1. *Let (X,d) be a b-metric space with coefficient $s \geqslant 1$ and $f,g : X \to X$ be two selfmaps. Assume that the pair (f,g) is an almost generalized (α,ψ,φ,F)-contraction type maps. Then z is a fixed point of f if and only if z is a fixed point of g. In that case, z is a common fixed point of f and g, and z is unique.*

PROOF. Let z be a fixed point of f. i.e., $fz = z$. Now, we prove that z is a fixed point of g. Suppose that $z \neq gz$. i.e., $d(z,gz) > 0$. We have $\frac{1}{2s}\min\{d(z,fz),d(z,gz)\} = 0 \leqslant d(z,z)$. Hence from (1.4), we have

$$\psi(s^\alpha d(fz,gz)) \leqslant F(\psi(M(z,z)),\varphi(M(z,z))) + LN(z,z) \tag{2.1}$$

where

$$M(z,z) = \max\{d(z,z),d(z,fz),d(z,gz),\frac{d(z,gz)+d(z,fz)}{2s}\} = d(z,gz)$$

and

$$N(x,y) = \min\{d(z,fz),d(z,gz),d(z,gz),d(z,fz)\} = 0.$$

Therefore, using the values of $M(z,z)$ and $N(z,z)$ in (2.1) and using the property (a') of F-class function, we get

$$\psi(s^\alpha d(z,gz)) \leqslant F(\psi(d(z,gz)),\varphi(d(z,gz))) + L.0 < \psi(d(z,gz)). \tag{2.2}$$

Since ψ is monotonically increasing, we have $s^\alpha d(z,gz) \leqslant d(z,gz)$. Since $s^\alpha - 1 \geqslant 0$, we have $d(z,gz) \leqslant 0$, what is a contradiction. Therefore $z = gz$. Hence z is a fixed point of g.

Similarly, it is easy to see that if z is a fixed point of g then z is a fixed point of f also.

Now, we prove that z is a unique common fixed point of $f$and g. Let w be another common fixed point of $f$and g. i.e., $fw = gw = w$. Since

$$\frac{1}{2s}\min\{d(z,fz), d(w,gw)\} = 0 \leqslant d(z,w),$$

from (1.4), we have

$$\psi(s^\alpha d(fz,gw)) \leqslant F(\psi(M(z,w)), \varphi(M(z,w))) + LN(z,w) \tag{2.3}$$

where

$$M(z,w) = \max\{d(z,w), d(z,fz), d(w,gw), \frac{d(z,gw) + d(w,fz)}{2s}\} = d(z,w)$$

and

$$N(x,y) = \min\{d(z,fz), d(w,gw), d(z,gw), d(w,fz)\} = 0.$$

Therefore, using the values of $M(z,z)$ and $N(z,z)$ in (2.3) and by property (a') of F-class function, we get

$$\psi(s^\alpha d(z,w)) \leqslant F(\psi(d(z,w)), \varphi(d(z,w))) + L.0 < \psi(d(z,w)). \tag{2.4}$$

Since ψ is increasing, we have $s^\alpha d(z,w) \leqslant d(z,w)$. Since $s^\alpha - 1 \geqslant 0$, we have $d(z,w) \leqslant 0$, which is a contradiction. Therefore $z = w$. Hence z is a unique common fixed point of f and g. □

THEOREM 2.1. *Let (X,d) be a complete b-metric space with coefficient $s \geqslant 1$ and $f, g : X \to X$ be two self maps. Suppose that the pair (f,g) is an almost generalized (α,ψ,φ,F)-contraction type maps. Then f, g have a unique common fixed point in X, provided either f or g is b-continuous.*

PROOF. Let $x_0 \in X$ be an arbitrary point. We define a sequence $\{x_n\} \subset X$ by

$$x_{2n+1} = fx_{2n} \text{ and } x_{2n+2} = gx_{2n+1} \text{ for } n = 0,1,2,\ldots.$$

We note that

$$\frac{1}{2s}\min\{d(x,fx), d(y,gy)\} \leqslant d(x,y)$$

if and only if either

$$\tfrac{1}{2s}d(x,fx) \leqslant d(x,y) \text{ or } \tfrac{1}{2s}d(y,gy) \leqslant d(x,y).$$

Suppose that $x_{2n} = x_{2n+1}$ for some n, then $x_{2n} = fx_{2n}$ so that x_{2n} is a fixed point of f. Hence by Proposition 2.1, we have x_{2n} is a fixed point of g also so that x_{2n} is a common fixed point of f and g.

Similarly if $x_{2n+1}=x_{2n+2}$ then x_{2n+1} is a fixed point of g so that by using Proposition 2.1, we have x_{2n+1} is a common fixed point of f and g. Hence, with out loss of generality, we may assume that $x_n \neq x_{n+1}$ for $n=0,1,2,\ldots$. Since

$$\frac{1}{2s}\min\{d(x_{2n},fx_{2n}),d(x_{2n+1},gx_{2n+1})\}\leqslant d(x_{2n},x_{2n+1}),$$

from (1.4), we have

$$\begin{aligned}\psi(s^{\alpha}d(x_{2n+1},x_{2n+2})&=\psi(s^{\alpha}d(fx_{2n},gx_{2n+1})\\&\leqslant F(\psi(M(x_{2n},x_{2n+1}),\varphi(M(x_{2n},x_{2n+1})))\\&\quad+LN(x_{2n},x_{2n+1})\end{aligned}\tag{2.5}$$

where

$$\begin{aligned}M(x_{2n},x_{2n+1})&=\max\{d(x_{2n},x_{2n+1}),d(x_{2n},fx_{2n}),d(x_{2n+1},gx_{2n+1}),\\&\qquad\frac{1}{2s}(d(x_{2n},gx_{2n+1})+d(x_{2n+1},fx_{2n})\}\\&=\max\{d(x_{2n},x_{2n+1}),d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2}),\\&\qquad\frac{1}{2s}(d(x_{2n},x_{2n+2})+d(x_{2n+1},x_{2n+1})\}\\&=\max\{d(x_{2n},x_{2n+1}),d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2}),\\&\qquad\frac{1}{2s}(d(x_{2n},x_{2n+2})\}\\&\leqslant\max\{d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2})\}\leqslant M(x_{2n},x_{2n+1}),\end{aligned}$$

so that $M(x_{2n},x_{2n+1})=\max\{d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2})\}$
and

$$\begin{aligned}N(x_{2n},x_{2n+1})&=\min\{d(x_{2n},fx_{2n}),d(x_{2n+1},gx_{2n+1}),d(x_{2n},gx_{2n+1}),\\&\qquad d(x_{2n+1},fx_{2n})\}\\&=\min\{d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2}),d(x_{2n},x_{2n+2}),\\&\qquad d(x_{2n+1},x_{2n+1})\}\\&=\min\{d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2}),d(x_{2n},x_{2n+2}),0\}\\&=0.\end{aligned}$$

Now using the values of $M(x_{2n},x_{2n+1})$, $N(x_{2n},x_{2n+1})$ and property (a') of F-class function in (2.5), we get

$$\begin{aligned}\psi(s^{\alpha}d(x_{2n+1},x_{2n+2}))&=\psi(s^{\alpha}d(fx_{2n},gx_{2n+1}))\\&\leqslant F(\psi(M(x_{2n},x_{2n+1})),\varphi(M(x_{2n},x_{2n+1})))+L.0.\\&<\psi(M(x_{2n},x_{2n+1})).\end{aligned}\tag{2.6}$$

Suppose that $d(x_{2n+1},x_{2n+2})>d(x_{2n+1},x_{2n+2})$ for some n then from (2.6), we have

$$\psi(s^{\alpha}d(x_{2n+1},x_{2n+2})\leqslant F(\psi(d(x_{2n+1},x_{2n+2})),\varphi(d(x_{2n+1},x_{2n+2})))$$

$$< \psi(d(x_{2n+1}, x_{2n+2})). \tag{2.7}$$

Since ψ is monotonically increasing and from (2.7), we have

$$s^{\alpha} d(x_{2n+1}, x_{2n+2}) \leqslant d(x_{2n+1}, x_{2n+2}). \tag{2.8}$$

Since $s^{\alpha} - 1 \geqslant 0$, we have $d(x_{2n+1}, x_{2n+2}) \leqslant 0$, which is a contradiction. Hence $M(x_{2n}, x_{2n+1}) = d(x_{2n}, x_{2n+1})$. Therefore from (2.6), we have

$$\psi(s^{\alpha} d(x_{2n+1}, x_{2n+2})) \leqslant F(\psi(d(x_{2n}, x_{2n+1})), \varphi(d(x_{2n}, x_{2n+1}))) < \psi(d(x_{2n}, x_{2n+1}). \tag{2.9}$$

Similarly, we can prove that

$$\psi(s^{\alpha} d(x_{2n+2}, x_{2n+3})) \leqslant F(\psi(d(x_{2n+1}, x_{2n+2})), \varphi(d(x_{2n+1}, x_{2n+2})))$$

$$< \psi(d(x_{2n+1}, x_{2n+2})). \tag{2.10}$$

Therefore from (2.9) and (2.10), we have

$$\psi(s^{\alpha} d(x_{n+1}, x_{n+2})) \leqslant F(\psi(d(x_n, x_{n+1})), \varphi(d(x_n, x_{n+1})))$$

$$< \psi(d(x_n, x_{n+1})) \text{ for } n = 0, 1, 2, \dots. \tag{2.11}$$

Since ψ is monotonically increasing, we have

$$s^{\alpha} d(x_{n+1}, x_{n+2}) \leqslant d(x_n, x_{n+1}) \text{ for } n = 0, 1, 2, \dots. \tag{2.12}$$

which implies that

$$d(x_{n+1}, x_{n+2}) \leqslant \frac{1}{s^{\alpha}} d(x_n, x_{n+1}) \text{ for } n = 0, 1, 2, \dots. \tag{2.13}$$

We now study the following two cases.

Case (i): $s = 1$. From (2.13), we have

$$d(x_{n+1}, x_{n+2}) \leqslant d(x_n, x_{n+1}) \quad \text{for } n = 0, 1, 2 \dots. \tag{2.14}$$

and hence $\{d(x_n, x_{n+1})\}$ is a decreasing sequence which is bounded below by zero. Therefore there exists $r \geqslant 0$ such that $\lim_{n\to\infty} d(x_n, x_{n+1}) = r$. Suppose that $r > 0$. From (2.11) with $s = 1$, we have

$$\psi(d(x_{n+1}, x_{n+2})) \leqslant F(\psi(d(x_n, x_{n+1})), \varphi(d(x_n, x_{n+1}))). \tag{2.15}$$

Now by taking limits as $n \to \infty$ on (2.15) and using property (a') of F-class function, we get

$$\psi(r) \leqslant F(\psi(r), \varphi(r)) < \psi(r), \tag{2.16}$$

which is a contradiction. Therefore

$$r = \lim_{n\to\infty} d(x_n, x_{n+1}) = 0. \tag{2.17}$$

Also, by the triangular inequality, we have

$$d(x_n, x_{n+2}) \leqslant d(x_n, x_{n++1}) + d(x_{n+1}, x_{n+2}) \tag{2.18}$$

Now taking limits as $n \to \infty$ on (2.18) and using (2.17), we get

$$\lim_{n\to\infty} d(x_n, x_{n+2}) = 0. \tag{2.19}$$

We now prove the following:

"For any $\epsilon > 0$ there exists $N \in \mathbb{N}$ such that $d(x_m, x_n) < \epsilon$ for all $m, n \in \mathbb{N}$ with

$$m > n > N \text{ and } m - n \equiv 1 \ (mod\ 2)." \tag{2.20}$$

On the contrary, we assume that there exists $\epsilon > 0$ such that for any $N \in \mathbb{N}$ there exist $m, n \in \mathbb{N}$ with $m > n > N$, $m - n \equiv 1$ $(mod\ 2)$ and $d(x_m, x_n) \geqslant \epsilon$. From (2.17) and (2.19), there exists N_0 such that for every $n > N_0$

$$d(x_n, x_{n+1}) < \epsilon \ \text{ and } \ d(x_n, x_{n+2}) < \epsilon. \tag{2.21}$$

We now construct two subsequences $\{x_{m_k}\}$ and $\{x_{n_k}\}$ of the sequence $\{x_n\}$ such that

$$d(x_{m_k}, x_{n_k}) \geqslant \epsilon, d(x_{m_k-2}, x_{n_k}) < \epsilon \text{ and } m_k - n_k \equiv 1 \ (mod\ 2) \tag{2.22}$$

as follows.

From our assumption there exists $l_1 > n_1 > N_0$ such that $l_1 - n_1 \equiv 1$ $(mod\ 2)$ and $d(x_{l_1}, x_{n_1}) \geqslant \epsilon$. From (2.21), we choose the smallest integer $m_1 \in \{n_1 + 3, n_1 + 5, n_1 + 7, \ldots, l_1\}$ such that $d(x_{m_1}, x_{n_1}) \geqslant \epsilon$. Therefore

$$d(x_{m_1}, x_{n_1}) \geqslant \epsilon, d(x_{m_1-2}, x_{n_1}) < \epsilon \text{ and } m_1 - n_1 \equiv 1 \ (mod\ 2).$$

Again, from our assumption there exist $l_2 > n_2 > m_1$ such that $l_2 - n_2 \equiv 1$ $(mod\ 2)$ and $d(x_{l_2}, x_{n_2}) \geqslant \epsilon$. From (2.21), we choose the smallest integer $m_2 \in \{n_2 + 3, n_2 + 5, n_2 + 7, \ldots, l_2\}$ such that $d(x_{m_2}, x_{n_2}) \geqslant \epsilon$. Therefore

$$d(x_{m_2}, x_{n_2}) \geqslant \epsilon, d(x_{m_2-2}, x_{n_2}) < \epsilon \text{ and } m_2 - n_2 \equiv 1 \ (mod\ 2).$$

On continuing this process, we can get two subsequences $\{x_{m_k}\}$ and $\{x_{n_k}\}$ of the sequence $\{x_n\}$ satisfying (2.22). Also, we can easily see that the following hold:

(i) $\lim_{k\to\infty} d(x_{m_k}, x_{n_k}) = \epsilon$

(ii) $\lim_{k\to\infty} d(x_{m_k+1}, x_{n_k}) = \epsilon$

(iii) $\lim_{k\to\infty} d(x_{m_k}, x_{n_k+1}) = \epsilon$ and

(iv) $\lim_{k\to\infty} d(x_{m_k+1}, x_{n_k+1}) = \epsilon$.

From the condition $m_k - n_k \equiv 1$ $(mod\ 2)$, the following two subcases arise.

Subcase (i): $m_k = 2p_k$ and $n_k = 2q_k - 1$ for some $p_k, q_k \in \mathbb{N}$.
From (2.21) and (2.22), we have

$$\frac{1}{2}\min\{d(x_{m_k}, fx_{m_k}), d(x_{n_k}, gx_{n_k})\} \leqslant \frac{1}{2}d(x_{m_k}, x_{m_k+1}) < \frac{\epsilon}{2} < \epsilon \leqslant d(x_{m_k}, x_{n_k}).$$

From (1.4) with $s = 1$, we have

$$\begin{aligned}\psi(d(x_{m_k+1}, x_{n_k+1})) = \psi(d(fx_{m_k}, gx_{n_k})) &\leqslant F(\psi(M(x_{m_k}, x_{n_k})), \varphi(M(x_{m_k}, x_{n_k}))) \\ &\quad + LN(x_{m_k}, x_{n_k})\end{aligned} \tag{2.23}$$

where

$$\begin{aligned} M(x_{m_k}, x_{n_k}) &= \max\{d(x_{m_k}, x_{n_k}), d(x_{m_k}, fx_{m_k}), d(x_{n_k}, gx_{n_k}), \\ &\qquad \frac{1}{2}(d(x_{m_k}, gx_{n_k}) + d(fx_{m_k}, x_{n_k})\} \\ &= \max\{d(x_{m_k}, x_{_k}), d(x_{m_k}, x_{m_k+1}), d(x_{n_k}, x_{n_k+1}), \\ &\qquad \frac{1}{2}(d(x_{m_k}, x_{n_k+1}) + d(x_{m_k+1}, x_{n_k})\}. \end{aligned}$$

and

$$\begin{aligned} N(x_{m_k}, x_{n_k}) &= \min\{d(x_{m_k}, fx_{m_k}), d(x_{n_k}, gx_{n_k}), d(x_{m_k}, gx_{n_k}), d(fx_{m_k}, x_{n_k})\} \\ &= \min\{d(x_{m_k}, x_{m_k+1}), d(x_{n_k}, x_{n_k+1}), d(x_{m_k}, x_{n_k+1}), d(x_{m_k+1}, x_{n_k})\}. \end{aligned}$$

On taking limits as $k \to \infty$ on $M(x_{m_k}, x_{n_k})$ and $N(x_{m_k}, x_{n_k})$, we get

$$\lim_{k\to\infty} M(x_{m_k}, x_{n_k}) = \epsilon \text{ and } \lim_{k\to\infty} N(x_{m_k}, x_{n_k}) = 0. \tag{2.24}$$

Now on taking limits as $k \to \infty$ on (2.23), using (2.24) and property (a') of F-class function, we get

$$\psi(\epsilon) \leqslant F(\psi(\epsilon), \varphi(\epsilon)) + L.0 < \psi(\epsilon), \tag{2.25}$$

a contradiction.

Subcase (ii): $m_k = 2p_k - 1$ and $n_k = 2q_k$ for some $p_k, q_k \in \mathbb{N}$.
From (2.21) and (2.22), we have

$$\frac{1}{2}\min\{d(x_{n_k}, fx_{n_k}), d(x_{m_k}, gx_{m_k})\} \leqslant \frac{1}{2}d(x_{m_k}, x_{m_k+1}) < \frac{\epsilon}{2} < \epsilon \leqslant d(x_{n_k}, x_{m_k}).$$

From (1.4) with $s = 1$, we have

$$\begin{aligned} \psi(d(x_{m_k+1}, x_{n_k+1})) = \psi(d(fx_{n_k}, gx_{m_k})) &\leqslant F(\psi(M(x_{n_k}, x_{m_k})), \varphi(M(x_{n_k}, x_{m_k}))) \\ &\quad + LN(x_{n_k}, x_{m_k}). \end{aligned} \tag{2.26}$$

Now as in Subcase (i), we get a contradiction in this case also. Therefore, (2.20) holds.

Now, we prove that the sequence $\{x_n\}$ is a Cauchy sequence in X. Let $\epsilon > 0$. From the claim, we find $N_1 \in \mathbb{N}$ such that if $m > n > N_1$ with $m - n \equiv 1$ $(mod\ 2)$ then

$$d(x_n, x_m) < \frac{\epsilon}{2}.$$

From (2.17), there exists N_2 such that for any $n > N_2$

$$d(x_n, x_{n+1}) < \frac{\epsilon}{2}.$$

Let $N = \max\{N_1, N_2\}$ with $m > n > N$. Here we have the following two cases.

Case (a): $m - n \equiv 1$ $(mod\ 2)$. In this case,

$$d(x_m, x_n) < \frac{\epsilon}{2} < \epsilon,$$

so that $\{x_n\}$ is a b-Cauchy sequence in X.

Case (b): $m - n \equiv 0$ $(mod\ 2)$. Here

$$d(x_m, x_n) \leqslant d(x_m, x_{n+1}) + d(x_{n+1}, x_n) < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon.$$

Hence sequence $\{x_n\}$ is a Cauchy sequence when $s = 1$.

Case (ii): $s > 1$. In this case, from (2.13), we have

$$d(x_{n+1}, x_{n+2}) \leqslant \frac{1}{s^\alpha} d(x_n, x_{n+1}) \text{ for } n = 0, 1, 2, \ldots. \tag{2.27}$$

Now from Lemma 1.1, we have, the sequence $\{x_n\}$ is a b-Cauchy sequence in X. Since X is a complete b-metric space, we have $\{x_n\}$ is b-convergent to some point z(say) in X. Therefore

$$z = \lim_{n\to\infty} x_{2n+1} = \lim_{n\to\infty} fx_{2n} \text{ and } z = \lim_{n\to\infty} x_{2n+2} = \lim_{n\to\infty} gx_{2n+1}$$

so that

$$\lim_{n\to\infty} fx_{2n} = z = \lim_{n\to\infty} gx_{2n+1}.$$

We, assume that f is b-continuous. Since $x_{2n} \to z$ as $n \to \infty$, we have $fx_{2n} \to fz$ as $n \to \infty$. Hence

$$0 \leqslant d(z, fz) \leqslant s(d(z, fx_{2n}) + d(fx_{2n}, fz)) \to 0 \text{ as } n \to \infty$$

so that $d(z, fz) = 0$. Hence z is a fixed point of f.

Now by proposition 2.1, we have z is a unique common fixed point of f and g.

Similarly, we can prove that z is a unique common fixed point of f and g whenever g is b-continuous.

Hence the theorem follows. □

3. Corollaries and examples

By choosing $f = g$ and $F(s, t) = s - t$ in Theorem 2.1, we get the following corollary.

Corollary 3.1. *Let (X, d) be a complete b-metric space and $f : X \to X$ be a selfmapping such that for every $x, y \in X$, $\frac{1}{2s} d(x, fx) \leqslant d(x, y)$ implies*

$$\psi(s^\alpha d(fx, fy)) \leqslant \psi(M(x, y)) - \varphi(M(x, y))) + LN(x, y)$$

where ψ, φ, $M(x, y)$ and $N(x, y)$ as in Theorem 2.1. Then f has a unique fixed point in X, provided f is b-continuous.

By choosing $F(s, t) = s\beta(s)$ in Theorem 2.1, where $\beta : [0, \infty) \to [0, 1)$ is continuous, we get the following corollary.

Corollary 3.2. *Let (X, d) be a complete b-metric space and $f : X \to X$ be a selfmapping such that for every $x, y \in X$, $\frac{1}{2s} d(x, fx) \leqslant d(x, y)$ implies*

$$\psi(s^\alpha d(fx, fy)) \leqslant \psi(M(x, y))\beta(\psi(M(x, y))) + LN(x, y)$$

where ψ, φ, $M(x, y)$ and $N(x, y)$ as in Theorem 2.1. Then f has a unique fixed point in X, provided f is b-continuous.

By choosing $L = 0$ in Theorem 2.1, we obtain the following result.

COROLLARY 3.3. *Let (X, d) be a complete b-metric space and $f, g : X \to X$ be such that for every $x, y \in X$,*

$$\frac{1}{2s}\{d(x, fx), d(y, gy)\} \leqslant d(x, y)$$

implies that

$$\psi(s^\alpha d(fx, gy)) \leqslant F(\psi(M(x, y)), \varphi(M(x, y)))$$

where ψ, φ and $M(x, y)$ as in Theorem 2.1. Then f and g have a unique common fixed point in X, provided either f or g is b-continuous.

By choosing $f = g$ in Theorem 2.1, we get the following corollary.

COROLLARY 3.4. *Let (X, d) be a complete b-metric space and $f, g : X \to X$ be such that for every $x, y \in X$, $\frac{1}{2s}d(x, fx) \leqslant d(x, y)$ implies*

$$\psi(s^\alpha d(fx, fy)) \leqslant F(\psi(M(x, y)), \varphi(M(x, y))) + LN(x, y)$$

where ψ, φ, $M(x, y)$ and $N(x, y)$ as in Theorem 2.1. Then f has a unique fixed point in X, provided f is b-continuous.

REMARK 3.1. Theorem 2.1 extends Theorem 1.2 for a pair (f, g) of almost generalized $(\alpha, \psi, \varphi, F)$-contraction type maps for the case of continuous function φ in b-metric spaces.

The following corollaries are consequences of Theorem 2.1 in b-metric spaces with $s = 1$.

COROLLARY 3.5. *Let (X, d) be a complete metric space and $f : X \to X$. Assume that there exist $\varphi \in \Phi$ and $\psi \in \Psi$ such that $\frac{1}{2}d(x, fx) \leqslant d(x, y)$ implies*

$$\psi(d(fx, fy)) \leqslant \psi(M(x, y)) - \varphi(M(x, y)),$$

for all $x, y \in X$, and $M(x, y)$ as in Theorem 2.1. Then f has a unique fixed point in X.

PROOF. Follows by choosing $f = g$, $F(s, t) = s - t$, $s, t \geqslant 0$ and $L = 0$ in Theorem 2.1 □

COROLLARY 3.6. *Let (X, d) be a complete metric space and $f, g : X \to X$. Assume that there exist $\varphi \in \Phi$ and $\psi \in \Psi$ such that*

$$\psi(d(fx, gy)) \leqslant \psi(M(x, y)) - \varphi(M(x, y)),$$

for all $x, y \in X$, and $M(x, y)$ as in Theorem 2.1. Then f and g have a unique common fixed point in X.

PROOF. Follows by choosing $F(s, t) = s - t$, $s, t \geqslant 0$ and $L = 0$ in Theorem 2.1. □

The following is an example in support of Theorem 2.1.

EXAMPLE 3.1. Let $X = \{0, 1, 2, 4, 5, 7, 9\}$. Define the b-metric by $d(x, y) = (x - y)^2$ with coefficient $s = 2$. Define two self maps $f$and g such that

$$f = \begin{pmatrix} 0 & 1 & 2 & 4 & 5 & 7 & 9 \\ 7 & 7 & 7 & 9 & 7 & 7 & 7 \end{pmatrix}, \ g = \begin{pmatrix} 0 & 1 & 2 & 4 & 5 & 7 & 9 \\ 7 & 5 & 9 & 9 & 9 & 7 & 7 \end{pmatrix}.$$

Define $\psi, \varphi : [0, \infty) \to [0, \infty)$ by $\psi(t) = t^2$, $\varphi(t) = \frac{t}{1+t}$ and $F : [0, \infty) \times [0, \infty) \to \mathbb{R}$ by $F(s, t) = \frac{st}{1+t}$.Then clearly F is a F-class function. By choosing $\alpha = \frac{1}{7}$, we get $s^\alpha = 2^{\frac{1}{7}}$. In this example 49 cases arise to verify the inequality (1.4). Among these we consider the non trivial cases for the verification of the inequality (1.4) which are mentioned in the following tabular form.

For simplicity, we write $\psi(s^\alpha d(fx, gy)) = A$, $F(\psi(M(x, y))), \varphi(M(x, y))) = B$ and $N(x, y) = C$.

(x,y)	$s^\alpha d(fx,gy)$	A	$M(x,y)$	B	C	$B + L.C$
(0,1)	$\frac{44}{10}$	$\frac{1936}{100}$	49	$\frac{117649}{99}$	16	$\frac{117649}{99} + L.16$
(0,4)	$\frac{44}{10}$	$\frac{1936}{100}$	49	$\frac{117649}{99}$	9	$\frac{117649}{99} + L.9$
(2,4)	$\frac{44}{10}$	$\frac{1936}{100}$	25	$\frac{15625}{51}$	9	$\frac{15625}{51}+L.9$
(2,5)	$\frac{44}{10}$	$\frac{1936}{100}$	25	$\frac{15625}{51}$	4	$\frac{15625}{51} + L.4$
(4,1)	$\frac{176}{10}$	$\frac{30976}{100}$	25	$\frac{15625}{51}$	1	$\frac{15625}{51} + L.1$
(4,7)	$\frac{44}{10}$	$\frac{1936}{100}$	25	$\frac{15625}{51}$	0	$\frac{15625}{51} + L.0$
(5,1)	$\frac{44}{10}$	$\frac{1936}{100}$	16	$\frac{4096}{33}$	0	$\frac{4096}{33} + L.0$
(5,2)	$\frac{44}{10}$	$\frac{1936}{100}$	25	$\frac{15625}{51}$	4	$\frac{15625}{51} + L.4$
(5,4)	$\frac{44}{10}$	$\frac{1936}{100}$	25	$\frac{15625}{51}$	4	$\frac{15625}{51} + L.4$
(7,4)	$\frac{44}{10}$	$\frac{1936}{100}$	25	$\frac{15625}{51}$	0	$\frac{15625}{51} + L.0$
(9,1)	$\frac{44}{10}$	$\frac{1936}{100}$	25	$\frac{262144}{129}$	4	$\frac{262144}{129} + L.4$
(9,5)	$\frac{44}{10}$	$\frac{1936}{100}$	16	$\frac{4096}{33}$	0	$\frac{4096}{33} + L.0$

In the following, we mention that the importance of L in the inequality (1.4) of Theorem 2.1.

When $(x, y) = (4, 1)$, we have $d(f4, g1) = d(9, 5) = 16$, $M(4, 1) = 25$, $N(x, y) = 1$ and

$$\begin{aligned} \frac{1}{2s} \min\{d(x, fx), d(y, gy)\} &= \frac{1}{4} \min\{d(4, f4), d(1, g1)\} \\ &= \frac{1}{4} \min\{d(4, 9), d(1, 5) \\ &= 4 \leqslant d(4, 1) = 9 = d(x, y)\} \end{aligned}.$$

Therefore from (1.4), we have

$$\begin{aligned}\psi(s^{\alpha}d(fx,gy)) = \psi(2^{\frac{1}{7}}d(f4,g1)) &= \psi(\frac{176}{10}) = \frac{30976}{100} \\ &\leqslant \frac{15625}{51} + \frac{7}{2}.1 \\ &= F(\psi(25),\varphi(25)) + \frac{7}{2}.1 \\ &= F(\psi(M(4,1)),\varphi(M(4,1)) + \frac{7}{2}.N(4,1) \\ &= F(\psi(M(x,y)),\varphi(M(x,y)) + L.N(x,y). \qquad (3.1)\end{aligned}$$

Therefore the inequality (1.4) holds for any $L \geqslant \frac{7}{2}$. Further, it easy to see from the inequality (3.1) that it fails to hold when $L = 0$ which in turn the inequality (1.4) fails to hold. Further 7 is the unique common fixed point of f and g in X.

Also we observe that, in the above example, if we choose $f = g$ then g does not satisfy the condition (1.1) of Theorem 1.1 at $x = 5$ and $y = 7$ for $M_i(x,y)$ $(i = 1,2,3)$ and $N(x,y)$ as in Theorem 1.1, even though g satisfies all conditions of Theorem 2.1. Hence Theorem 1.1 is not applicable.

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