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GENERALIATION OF FIXED POINT RESULTS FOR (α^*, η^*, ψ)- CONTRACTIVE MAPPINGS IN FUZZY METRIC SPACES

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ABSTRACT. In this paper, we extend the results of Supak, Cho, Kumam [**12**] to a more general, (α^*, η^*) admissible condition and prove a new fixed point theorem in generalized modified (α^*, η^*, ψ)- contractive mapping. Moreover, we present some examples to show the necessity of the obtained results.

1. Introduction

Fixed point theorems in fuzzy metric spaces satisfying some contractive condition is a central area of research now a days. The concept of fuzzy sets was introduced by Zadeh [**17**] in 1965. The theory of fuzzy metric space has been studied by many mathematicians. The first mathematicians, who introduced fuzzy metric spaces, in 1975 are Kramosil and Michálek [**8**]. In 1994, George and Veeramani [**2**] modified the concept of fuzzy metric space introduced by Kramosil and Michálek [**8**]. In 2002, Gregori and Sapene [**3**] initiated fuzzy contraction mappings and proved an important fixed point theorem for this class of mappings. In 2008, Miheţ [**9**] introduced ψ contractive mappings in non-Archimedean fuzzy metric spaces. For the last 41 years, the concept of fuzzy metric space and fixed point theorems were studied, generalized and proved by different mathematicians (see [**5**] - [**13**]). In 2012, Samet, Vitero and Vetro [**14**] introduced the concept of admissible mapping for single valued map, and in the same year Asl et al. [**1**] extended the concept of admissible for single valued mappings to multi valued mappings. Soon after, Hussain, Salimi and Latif [**7**] proved fixed point theorem for single and set

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valued (α, η, ψ) contractive mappings. Very recently Supak, Cho, Kumam [**12**] introduced a new contractive condition and proved fixed point theorems for modified (α^*, η^*, ψ)- contractive mappings in fuzzy metric space.

2. Preliminaries

We begin with some basic definitions and results which will be used in main part of our paper.

Definition 2.1. ([**16**]) A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t-norm if it satisfies the following conditions :

(T1): $*$ is associative and commutative,
(T2): $*$ is continuous,
(T3): $a * 1 = a$ for all $a \in [0,1]$,
(T4): $a * b \leqslant c * d$ whenever $a \leqslant c$ and $b \leqslant d$ for all $a, b, c, d \in [0,1]$.

Remark 2.1. A t-norm $*$ is called positive, if $a * b > 0$ for all $a, b \in (0,1)$. $\Diamond$

Examples of continuous t-norms are Lukasievicz t-norm, i.e,

$$a *_L b = \max\{a + b - 1, 0\},$$

product t-norm, i.e, $a * b = ab$ and minimum t-norm, *i.e.,* $a *_M b = \min\{a, b\}$, for $a, b \in [0,1]$.

The concept of fuzzy metric space is defined by George and Veeramani [**2**] as follows.

Definition 2.2. ([**2**]) Let X be a nonempty set, $*$ be a continuous t-norm. Assume that a fuzzy set $M : X \times X \times (0,\infty) \to [0,1]$ satisfies the following conditions; for each $x, y, z \in X$ and $t, s > 0$,

(M1): $M(x, y, t) > 0$,
(M2): $M(x, y, t) = 1$ if and only if $x = y$,
(M3): $M(x, y, t) = M(y, x, t)$,
(M4): $M(x, y, t) * M(y, z, s) \leqslant M(x, z, t + s)$,
(M5): $M(x, y, \cdot) : (0,\infty) \to [0,1]$ is continuous.

Then we call M a fuzzy metric on X, and we call the 3-tuple $(X, M, *)$ a fuzzy metric space.

Lemma 2.1 ([**4**]). *Let $(X, M, *)$ be a fuzzy metric space. For all $x, y \in X$, $M(x, y, \cdot)$ is a non-decreasing function.*

Remark 2.2. We observe that $0 < M(x, y, t) < 1$ provided $x \neq y$, for all $t > 0$ (see [**10**]).

Let $(X, M, *)$ be a fuzzy metric space. For $t > 0$, the open ball $B(x, r, t)$ with a center $x \in X$ and radius $0 < r < 1$ is defined by $B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}$. A subset $A \subset X$ is called open if for each $x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Let τ denote the family of all open subsets of X. Then τ is a topology on X, called the topology induced by the fuzzy metric M. This topology is metrizable (see [**6**]). $\Diamond$

EXAMPLE 2.1. ([**2**]). Let (X, d) be a metric space. We define $a * b = ab$ (or $a * b = \min\{a, b\}$) for all $a, b \in [0, 1]$, and $M : X \times X \times (0, \infty) \to [0, 1]$ as

$$M(x, y, t) = \frac{t}{t+d(x,y)} \text{ for all } \quad x, y \in X \text{ and } t > 0.$$

Then $(X, M, *)$ is a fuzzy metric space. We call this fuzzy metric M as the fuzzy metric induced by the metric d, and this M is known as the standard fuzzy metric. $\diamondsuit$

Now we give some examples of fuzzy metric spaces due to Gregori, Morillas and Sapena ([**5**]).

EXAMPLE 2.2. [**5**] Let (X, d) be a metric space and $g : R^+ \to [0, \infty)$, $R^+ = [0, \infty)$ be an increasing continuous function. Define $M : X \times X \times (0, \infty) \to [0, 1]$ as $M(x, y, t) = e^{(\frac{-d(x,y)}{g(t)})}$ for all $x, y \in X$ and $t > 0$. Then $(X, M, *)$ is a fuzzy metric space on X where $*$ is the product t-norm. $\diamondsuit$

EXAMPLE 2.3. ([**5**]). Let (X, d) be a bounded metric space with $d(x, y) < k$ for all $x, y \in X$, where k is fixed constant in $(0, \infty)$ and $g : R^+ \to (k, \infty)$, $R^+ = [0, \infty)$ be an increasing continuous function. Define a function $M : X \times X \times (0, \infty) \to [0, 1]$ as $M(x, y, t) = 1 - \frac{d(x,y)}{g(t)}$ for all $x, y \in X$ and $t > 0$. Then $(X, M, *)$ is a fuzzy metric space, where $*$ is a Lukasievicz t-norm. $\diamondsuit$

DEFINITION 2.3. [**2**] Let $(X, M, *)$ be a fuzzy metric space.

(1) A sequence$\{x_n\}$ in X is said to be convergent to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all $t > 0$.
(2) A sequence$\{x_n\}$ in X is called a Cauchy sequence if, for each $0 < \epsilon < 1$ and $t > 0$, there exits $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for each $n, m \geqslant n_0$.
(3) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.
(4) A fuzzy metric space in which every sequence has a convergent subsequence is said to be compact.

REMARK 2.3. In a fuzzy metric space the limit of a convergent sequence is unique. $\diamondsuit$

DEFINITION 2.4. ([**12**]) Let $(X, M, *)$ be a fuzzy metric space. Then the mapping M is said to be continuous on $X \times X \times (0, \infty)$ if

$$\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t),$$

when $\{(x_n, y_n, t_n)\}$ is a sequence in $X \times X \times (0, \infty)$ which converges to a point $(x, y, t) \in X \times X \times (0, \infty)$, *i.e.,*

$$\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1 \text{ and } \lim_{n\to\infty} M(x, y, t_n) = M(x, y, t).$$

LEMMA 2.2 ([**11**]). *If $(X, M, *)$ is a fuzzy metric space, then M is a continuous function on $X \times X \times (0, \infty)$.*

The concept of α-admissible mapping was introduced by Samet,Vetro and Vetro [**14**] as follows.

Definition 2.5. ([**14**]) Let X be a nonempty set, $T : X \to X$ and $\alpha : X \times X \to [0, \infty)$ be maps. We say that T is an α-admissible mapping if for all $x, y \in X$, we have $\alpha(x, y) \geqslant 1 \Rightarrow \alpha(Tx, Ty) \geqslant 1$.

In 2013, Salimi, Latif and Hussain ([**13**]) modified the concept of $\alpha-$ admissible mapping as follows.

Definition 2.6. ([**13**]) Let X be a nonempty set, $T : X \to X$ and $\alpha, \eta : X \times X \to [0, \infty)$. We say that T is an α-admissible mapping with respect to η if for all $x, y \in X$, we have $\alpha(x, y) \geqslant \eta(x, y) \Rightarrow \alpha(Tx, Ty) \geqslant \eta(Tx, Ty)$.

If we take $\eta(x, y) = 1$ for all $x, y \in X$, then T is an α-admissible mapping. If we take $\alpha(x, y) = 1$, then we say that T is an η-subadmissible mapping.

In 2016, Supak, Cho and Kumam ([**12**]) introduced the α^*- admissible mappings in fuzzy metric spaces.

Definition 2.7. ([**12**]) Let $(X, M, *)$ be a fuzzy metric space. A mapping $T : X \to X$ and let $\alpha^* : X \times X \times (0, \infty) \to [0, \infty)$ be a function. We say that T is an α^*-admissible mapping if, for all $x, y \in X$ and $t > 0$, $\alpha^*(x, y, t) \geqslant 1 \Rightarrow \alpha^*(Tx, Ty, t) \geqslant 1$.

In 2016, Supak, Cho and Kumam ([**12**]) introduced the (α^*, η^*)- admissible mappings in fuzzy metric spaces.

Definition 2.8. ([**12**]) Let$(X, M, *)$ be a fuzzy metric space. A mapping $T : X \to X$and let $\alpha^*, \eta^* : X \times X \times (0, \infty) \to [0, \infty)$ be two functions. We say that T is an (α^*, η^*)- admissible mapping if, for all $x, y \in X$ and $t > 0$, $\alpha^*(x, y, t) \geqslant \eta^*(x, y, t) \Rightarrow \alpha^*(Tx, Ty, t) \geqslant \eta^*(Tx, Ty, t)$.

Note that, if $\eta^*(x, y, t) = 1$ then it is clear that T is an α^* admissible mapping. if we take $\alpha^*(x, y, t) = 1$, then we say that T is an η^*-subadmissible mapping.

Now we sate a proposition which is useful to prove our main result.

Proposition 2.1. ([**15**]) *Suppose $(X, M, *)$ is fuzzy metric space. Let $\{x_n\}$ be a sequence in X such that $M(x_n, x_{n+1}, t) \to 1$ as $n \to \infty$, for all $t > 0$. If $\{x_n\}$ is not a Cauchy sequence then there exist $0 < \epsilon < 1$, $t_0 > 0$ and sequences of positive integers $\{m(k)\}$ and $\{n(k)\}$ with $m(k) > n(k) \geqslant k$ for each $k \in \mathbb{N}$ such that $M(x_{m(k)}, x_n(k), t_0) \leqslant 1 - \epsilon$ and*

(i) $\lim_{k\to\infty} M(x_{m(k)}, x_{n(k)}, t_0) = 1 - \epsilon,$
(ii) $\lim_{k\to\infty} M(x_{m(k)}, x_{n(k)}, \frac{t_0}{2}) = 1 - \epsilon,$
(iii) $\lim_{k\to\infty} M(x_{m(k)}, x_{n(k)}, \frac{t_0}{4}) = 1 - \epsilon,$
(iv) $\lim_{k\to\infty} M(x_{m(k)-1}, x_{n(k)+1}, \frac{t_0}{2}) = 1 - \epsilon,$
(v) $\lim_{k\to\infty} M(x_{m(k)-1}, x_{n(k)}, \frac{t_0}{2}) = 1 - \epsilon,$
(vi) $\lim_{k\to\infty} M(x_{m(k)+1}, x_{n(k)+1}, \frac{t_0}{2}) = 1 - \epsilon.$

Throughout the paper we denote Ψ be a class of all mappings $\psi : [0, 1] \to [0, 1]$ which are satisfying the following conditions:

(a): ψ is continuous
(b): ψ is non-decreasing

(c): $\psi(a) > a, \forall a \in (0,1)$

REMARK 2.4. For $\psi \in \Psi$, $\psi(a) = a$ if and only if $a = 0$ or $a = 1$. $\Diamond$

In 2016, Supak, Cho and Kumam Introduced modified (α^*, η^*, ψ) contractive mapping in fuzzy metric space and prove a fixed point theorem.

DEFINITION 2.9. ([**12**]) Let $(X, M, *)$ be a fuzzy metric space. Let $T : X \to X$ and let $\alpha^*, \eta^* : X \times X \times (0, \infty) \to [0, \infty)$ be two functions. We say that T is a modified (α^*, η^*, ψ)-contractive mapping if there exists a function $\psi \in \Psi$ such that,

$$\alpha^*(x, Tx, t)\alpha^*(y, Ty, t) \geqslant \eta^*(x, Tx, t)\eta^*(y, Ty, t) \Rightarrow M(Tx, Ty, t) \geqslant \psi(N(x, y, t)),$$

where

$$N(x, y, t) = \min\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\}.$$

THEOREM 2.1 ([**12**]). *Let $(X, M, *)$ be a complete fuzzy metric space. Let $T : X \to X$ be a modified (α^*, η^*, ψ)- contractive mapping. Suppose that the following assertions hold:*

(a): *T is (α^*, η^*) admissible mapping;*

(b): *there exists $x_0 \in X$ such that $\alpha^*(x_0, Tx_0, t) \geqslant \eta^*(x_0, Tx_0, t)$ for all $t > 0$;*

(c): *for any sequence$\{x_n\} \subset X$ such that $\alpha^*(x_n, x_{n+1}, t) \geqslant \eta * (x_n, x_{n+1}, t)$, for all $n \in N, t > 0$ and $\{x_n\} \to x$ as $n \to \infty$, then $\alpha^*(x, Tx, t) \geqslant \eta * (x, Tx, t)$ for all $t > 0$.*

Then T has a fixed point.

Now, we define generalized modified (α^*, η^*, ψ)- contractive mapping in fuzzy metric spaces.

DEFINITION 2.10. Let $(X, M, *)$ be a fuzzy metric space. Let $T : X \to X$ be a self map and let $\alpha^*, \eta^* : X \times X \times (0, \infty) \to [0, \infty)$ be two functions. We say that T is a generalized modified (α^*, η^*, ψ)- contractive mapping if there exists a function $\psi \in \Psi$ such that,

$$\begin{aligned} \alpha^*(x, Tx, t)\alpha^*(y, Ty, t) \geqslant \eta^*(x, Tx, t)\eta^*(y, Ty, t) \implies \\ M(Tx, Ty, t) \geqslant \psi(N(x, y, t))K(x, y, t), \end{aligned} \tag{2.1}$$

where

$$N(x, y, t) = \min\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\}$$

and

$$K(x, y, t) = \max\{M(x, y, t), M(x, Tx, t), M(y, Ty, t), M(x, Ty, t), M(y, Tx, t)\}.$$

Now we give an example of a generalized (α^*, η^*, ψ) modified contractive mapping.

EXAMPLE 2.4. let $X = [0, \infty)$ and $M(x, y, t) = \left(\frac{t}{t+1}\right)^{d(x,y)}$, where $d(x, y) = |x - y|$ for $x, y \in X$, $*$ is product continuous t-norm. Here $(X, M, *)$ is complete fuzzy metric space. Let $T : X \to X$ be a map defined by

$$Tx = \begin{cases} \frac{x^2}{4} & \text{if } x \in [0, \frac{1}{2}] \\ 0, & \text{if} x \in (\frac{1}{2}, \infty) \end{cases}$$

Let $\alpha^*, \eta^* : X \times X \times (0, \infty) \to [0, \infty)$ defined by

$$\alpha^*(x, y, t) = \begin{cases} 3 & \text{if } x, y \in [0, 1], \\ 0, & \text{otherwise,} \end{cases} \quad \text{and } \eta^*(x, y, t) = \begin{cases} 1, & \text{if } x, y \in [0, 1], \\ 4 & \text{otherwise.} \end{cases}$$

$\Diamond$

Suppose $\alpha^*(x, Tx, t)\alpha^*(y, Ty, t) \geqslant \eta^*(x, Tx, t)\eta^*(y, Ty, t)$. Then we have $x, y \in [0, 1]$.

Case i: If $x, y \in [0, \frac{1}{2}]$ and $\psi(t) = t^{\frac{1}{2}}$. This implies

$$\begin{aligned} M(Tx, Ty, t) = \left(\frac{t}{t+1}\right)^{d(Tx,Ty)} = \left(\frac{t}{t+1}\right)^{|Tx-Ty|} &= \left(\frac{t}{t+1}\right)^{|\frac{x^2}{4} - \frac{y^2}{4}|} \\ &= \left(\frac{t}{t+1}\right)^{\frac{1}{4}|x-y||x+y|} \\ &\geqslant \left(\frac{t}{t+1}\right)^{\frac{1}{2}|x-y|}) \\ &= \psi(M(x, y, t)). \end{aligned} \tag{2.2}$$

Since ψ is non decreasing, from (2.2) we have

$$M(Tx, Ty, t) \geqslant \psi(M(x, y, t)) \geqslant \psi(N(x, y, t)) \geqslant \psi(N(x, y, t))K(x, y, t).$$

Thus, the theorem follows in this case.

Case ii: If $x \in [0, \frac{1}{2}]$ and $y \in (\frac{1}{2}, 1]$
$M(Tx, Ty, t) = \left(\frac{t}{t+1}\right)^{|\frac{x^2}{4} - 0|} = \left(\frac{t}{t+1}\right)^{\frac{x^2}{4}}$, $M(x, y, t) = \left(\frac{t}{t+1}\right)^{y-x}$,
$M(x, Tx, t) = \left(\frac{t}{t+1}\right)^{x - \frac{x^2}{4}}$, $M(y, Ty, t) = \left(\frac{t}{t+1}\right)^{y}$, $M(x, Ty, t) = \left(\frac{t}{t+1}\right)^{x}$,
$M(y, Tx, t) = \left(\frac{t}{t+1}\right)^{(y - \frac{x^2}{4})}$.

Here we have

$$N(x,y,t) = \min\{M(x,y,t), \max\{M(x,Tx,t), M(y,Ty,t)\}\}$$
$$= \min\left\{(\frac{t}{t+1})^{y-x}, \max\{(\frac{t}{t+1})^{x-\frac{x^2}{4}}, (\frac{t}{t+1})^{y}\}\right\}$$
$$(2.3) \qquad = \begin{cases} \left(\frac{t}{t+1}\right)^{y-x}, \text{if } y \geqslant 2x - \frac{x^2}{4} \\ \left(\frac{t}{t+1}\right)^{x-\frac{x^2}{4}}, \text{if } y \leqslant 2x - \frac{x^2}{4} \end{cases}$$

Sub case i: If $y \geqslant 2x - \frac{x^2}{4}$.

$$K(x,y,t) = \max\{M(x,y,t), M(x,Tx,t), M(y,Ty,t), M(x,Ty,t), M(y,Tx,t)\}$$
$$= \max\left\{\left(\frac{t}{t+1}\right)^{y-x}, \left(\frac{t}{t+1}\right)^{x-\frac{x^2}{4}}, \left(\frac{t}{t+1}\right)^{y}, \left(\frac{t}{t+1}\right)^{x}, \left(\frac{t}{t+1}\right)^{(y-\frac{x^2}{4})}\right\}$$
$$= \left(\frac{t}{t+1}\right)^{(x-\frac{x^2}{4})}.$$

Since

$$\frac{x^2}{4} \leqslant \frac{y-x}{2} + x - \frac{x^2}{4},$$

we get that

$$\left(\frac{t}{t+1}\right)^{\frac{x^2}{4}} \geqslant \left(\frac{t}{t+1}\right)^{\frac{y-x}{2}} \left(\frac{t}{t+1}\right)^{(x-\frac{x^2}{4})}.$$

This implies

$$M(Tx,Ty,t) \geqslant \psi(N(x,y,t))K(x,y,t).$$

Sub case ii: If $y \leqslant 2x - \frac{x^2}{4}$ then $N(x,y,t) = \left(\frac{t}{t+1}\right)^{(x-\frac{x^2}{4})}$.

Here

$$K(x,y,t)$$
$$= \max\left\{\left(\frac{t}{t+1}\right)^{y-x}, \left(\frac{t}{t+1}\right)^{x-\frac{x^2}{4}}, \left(\frac{t}{t+1}\right)^{y}, \left(\frac{t}{t+1}\right)^{x}, \left(\frac{t}{t+1}\right)^{(y-\frac{x^2}{4})}\right\}$$
$$= \max\left\{\left(\frac{t}{t+1}\right)^{y-x} \left(\frac{t}{t+1}\right)^{x}\right\}.$$

If $K(x,y,t) = (\frac{t}{t+1})^{y-x}$. Then we can easily observe that

$$\frac{x^2}{4} \leqslant y - \frac{x^2}{8} - \frac{x}{2}.$$

Thus

$$\begin{aligned}\left(\frac{t}{t+1}\right)^{\frac{x^2}{4}} = M(Tx,Ty,t) &\geqslant \left(\frac{t}{t+1}\right)^{\frac{x}{2}-\frac{x^2}{4}}\left(\frac{t}{t+1}\right)^{y-x}\\ &= \psi(N(x,y,t))K(x,y,t).\end{aligned}$$

On the other hand if $K(x,y,t) = \left(\frac{t}{t+1}\right)^{x}$.
From the fact that

$$\frac{x^2}{4} \leqslant \frac{x}{2} - \frac{x^2}{8} + x.$$

we have

$$\begin{aligned}\left(\frac{t}{t+1}\right)^{\frac{x^2}{4}} = M(Tx,Ty,t) &\geqslant \left(\frac{t}{t+1}\right)^{\frac{x}{2}-\frac{x^2}{8}}\left(\frac{t}{t+1}\right)^{x}\\ &= \psi(N(x,y,t))K(x,y,t).\end{aligned}$$

It is easy to see that $M(Tx,Ty,t) \geqslant \psi(N(x,y,t))K(x,y,t)$ for $x \in (\frac{1}{2},\infty)$, $y \in (\frac{1}{2},\infty)$.

Hence T is a generalized modified (α^*,η^*,ψ)- contractive mapping.

In Section 3, we prove the existence of fixed points for generalized modified (α^*,η^*,ψ)-contractive mappings in a complete fuzzy metric spaces. We provide an example to show the validity of our results. Our results generalize the results of ([**12**]).

3. Main results

The following proposition is needed to establish the main result.

PROPOSITION 3.1. *Let $(X,M,*)$ be a fuzzy metric space. Let $T : X \to X$ be a generalized modified (α^*,η^*,ψ)- contractive mapping. Fix $x_0 \in X$ and define a sequence $\{x_n\}$ by $x_{n+1} = Tx_n$ for all $n = 0,1,2,\cdots$. If $\alpha^*(x_n,x_{n+1},t) \geqslant \eta^*(x_n,x_{n+1},t)$ for all n and $\lim_{n\to\infty} M(x_n,x_{n+1},t) = 1$ then $\{x_n\}$ is Cauchy sequence in X.*

PROOF. Suppose on the contrary that $\{x_n\}$ is not a Cauchy sequence. By Proposition 2.1, there exist $0 < \epsilon < 1$, $t_0 > 0$ and a positive integers $\{m(k)\}, \{n(k)\}$ with $m(k) > n(k) \geqslant k$ for any $k \in \mathbb{N}$ such that

$$\begin{aligned}\lim_{k\to\infty} M(x_{n(k)},x_{m(k)},t_0) = 1-\epsilon, \lim_{k\to\infty} M(x_{n(k)},x_{m(k)},\frac{t_0}{2}) = 1-\epsilon \text{ and}\\ \lim_{k\to\infty} M(x_{n(k)+1},x_{m(k)+1},t_0) = 1-\epsilon.\end{aligned} \tag{3.1}$$

Here we have

$$\begin{aligned}&\alpha^*(x_{n(k)},Tx_{n(k)},\tfrac{t_0}{2})\alpha^*(x_{m(k)},Tx_{m(k)},\tfrac{t_0}{2})\\ &\geqslant \eta^*(x_{n(k)},Tx_{n(k)},\tfrac{t_0}{2})\eta^*(x_{m(k)},Tx_{m(k)},\tfrac{t_0}{2}).\end{aligned}$$

Hence, from (2.1), we have

$$M(Tx_{n(k)}, Tx_{m(k)}, \frac{t_0}{2}) \geqslant \psi(N(x_{n(k)}, x_{m(k)}, \frac{t_0}{2}))K(x_{n(k)}, x_{m(k)}, \frac{t_0}{2}). \tag{3.2}$$

where

$$K(x_{n(k)}, x_{m(k)}, \frac{t_0}{2}) = \max\{M(x_{n(k)}, x_{m(k)}, \frac{t_0}{2}), M(x_{n(k)}, x_{m(k)+1}, \frac{t_0}{2}),$$
$$M(x_{m(k)}, x_{n(k)+1}, \frac{t_0}{2}), M(x_{n(k)}, x_{n(k)+1}, \frac{t_0}{2}), M(x_{m(k)}, x_{m(k)+1}, \frac{t_0}{2})\}$$

and

$$N(x_{n(k)}, x_{m(k)}, \frac{t_0}{2}) = \min\{M(x_{n(k)}, x_{m(k)}, \frac{t_0}{2}), \max\{M(x_{n(k)}, x_{n(k)+1}, \frac{t_0}{2}),$$
$$M(x_{m(k)}, x_{m(k)+1}, \frac{t_0}{2})\}\}.$$

Hence we have

$$\lim_{k\to\infty} K(x_{n(k)}, x_{m(k)}, \frac{t_0}{2}) = 1 \text{ and } \lim_{k\to\infty} N(x_{n(k)}, x_{m(k)}, \frac{t_0}{2}) = 1 - \epsilon. \tag{3.3}$$

On taking limit as $k \to \infty$ in (3.2), and by using (3.1) and (3.3) it follows that

$$1 - \epsilon \geqslant \psi(1 - \epsilon) > 1 - \epsilon,$$

which is a contradiction. Therefore,$\{x_n\}$ is a Cauchy sequence. □

THEOREM 3.1. *Let $(X, M, *)$ be a complete fuzzy metric space. Let $T : X \to X$ be a generalized modified (α^*, η^*, ψ)- contractive mapping. Suppose that the following assertions hold:*

- **(a):** *T is (α^*, η^*) admissible mapping;*
- **(b):** *there exists $x_0 \in X$ such that $\alpha^*(x_0, Tx_0, t) \geqslant \eta^*(x_0, Tx_0, t)$ for all $t > 0$;*
- **(c):** *for any sequence$\{x_n\} \subset X$ such that $\alpha^*(x_n, x_{n+1}, t) \geqslant \eta * (x_n, x_{n+1}, t)$, for all $n \in N, t > 0$ and $\{x_n\} \to x$ as $n \to \infty$, then $\alpha^*(x, Tx, t) \geqslant \eta * (x, Tx, t)$ for all $t > 0$.*

Then T has a fixed point.

PROOF. Let $x_0 \in X$ be such that $\alpha^*(x_0, Tx_0, t) \geqslant \eta^*(x_0, Tx_0, t)$ for all $t > 0$. Define a sequence $\{x_n\}$ in X such that $x_n = T^n x_0 = Tx_{n-1}$ for all $n \in N$. If $x_n = x_{n+1}$ for some $n \in N$, then $x_n = Tx_n$ and hence x_n is a fixed point of T and we are done. Assume that $x_n \neq x_{n+1}$ for all $n \in N$. Since T is (α^*, η^*) admissible mapping and since $\alpha^*(x_0, Tx_0, t) \geqslant \eta^*(x_0, Tx_0, t)$ it follows that

$$\alpha^*(x_1, x_2, t) = \alpha^*(Tx_0, Tx_1, t) \geqslant \eta^*(Tx_0, Tx_1, t) = \eta^*(x_1, x_2, t),$$

so that

$$\alpha^*(x_0, Tx_0, t)\alpha^*(x_1, Tx_1, t) \geqslant \eta^*(x_0, Tx_0, t)\eta^*(x_1, Tx_1, t).$$

On continuing this process, we have $\alpha^*(x_n, Tx_n, t) \geqslant \eta^*(x_n, Tx_n, t)$, for all $n \geqslant 1$ and so we have

$\alpha^*(x_{n-1}, Tx_{n-1}, t)\alpha^*(x_n, Tx_n, t) \geqslant \eta^*(x_{n-1}, Tx_{n-1}, t)\eta^*(x_n, Tx_n, t)$ for all $n \in N$ and $t > 0$. Now, from the inequality in (2.1), we have

$$\begin{aligned} M(x_n, x_{n+1}, t) &= M(Tx_{n-1}, Tx_n, t) \\ &\geqslant \psi(N(x_{n-1}, x_n, t))K(x_{n-1}, x_n, t), \end{aligned} \tag{3.4}$$

where

$$\begin{aligned} N(x_{n-1}, x_n, t) &= \min\{M(x_{n-1}, x_n, t), \max\{M(x_{n-1}, Tx_{n-1}, t), M(x_n, Tx_n, t)\}\} \\ &= \min\{M(x_{n-1}, x_n, t), \max\{M(x_{n-1}, x_n, t), M(x_n, x_{n+1}, t)\}\}. \end{aligned} \tag{3.5}$$

and

$$\begin{aligned} K(x_{n-1}, x_n, t) &= \max\{M(x_{n-1}, x_n, t), M(x_{n-1}, Tx_n, t), M(x_n, Tx_{n-1}, t), \\ &\quad M(x_n, Tx_n, t), M(x_{n-1}, Tx_{n-1}, t)\}. \end{aligned}$$

Since $M(x_n, Tx_{n-1}, t) = 1$ for all $n \in \mathbb{N}$ and $t > 0$, we have

$$K(x_{n-1}, x_n, t) = 1. \tag{3.6}$$

Moreover, since $\min\{a, \max\{a, b\}\} = a$, we have

$$N(x_{n-1}, x_n, t) = M(x_{n-1}, x_n, t) \text{ for each } n \in \mathbb{N} \text{ and } t > 0. \tag{3.7}$$

From (3.4), (3.6) and (3.7) we have

$$M(x_n, x_{n+1}, t) \geqslant \psi(M(x_{n-1}, x_n, t)) > M(x_{n-1}, x_n, t) \quad \forall n \in \mathbb{N} \quad \text{and} \quad t > 0. \tag{3.8}$$

It follows that the sequence$\{M(x_n, x_{n+1}, t)\}$ is an increasing sequence in $(0, 1]$. Thus, there exists $l_t \in (0, 1]$ such that

$$\lim_{n\to\infty} M(x_n, x_{n+1}, t) = l_t \quad \text{for all} \quad t > 0.$$

We now prove that $l_t = 1$ for each $t > 0$. Let $t > 0$. Now from (3.8), we have

$$M(x_n, x_{n+1}, t) \geqslant \psi(M(x_{n-1}, x_n, t)) > M(x_{n-1}, x_n, t).$$

Since ψ is continuous, if we take limit as $n \to \infty$ in the above inequality, we get that

$$l_t \geqslant \psi(l_t) \geqslant l_t.$$

This implies $\psi(l_t) = l_t$. By remark 2.4 and the sequence $\{M(x_n, x_{n+1}, t)\}$ is increasing follows $l_t = 1$. Thus by Proposition 3.1, $\{x_n\}$ is Cauchy Sequence.

Since $(X, M, *)$ is a complete fuzzy metric space and $\{x_n\}$ is a Cauchy sequence in X, there exist $x^* \in X$ such that $x_n \to x^*$ as $n \to \infty$. i.e $M(x_n, x^*, t) \to 1$ as $n \to \infty$ for each $t > 0$.

Moreover, Since

$$\alpha^*(x_n, Tx_n, t) \geqslant \eta^*(x_n, Tx_n, t) \text{ for all } n \in \mathbb{N} \cup \{0\}, t > 0,$$

by condition (c) in the Theorem 3.1 we have $\alpha^*(x^*, Tx^*, t) \geqslant \eta^*(x^*, Tx^*, t)$.

Hence we get that $\alpha^*(x_n, Tx_n, t)\alpha^*(x^*, Tx^*, t) \geqslant \eta^*(x_n, Tx_n, t)\eta^*(x^*, Tx^*, t)$, for all $n \in \mathbb{N} \cup \{0\}$ and $t > 0$.

By the hypothesis of the theorem

$$\begin{aligned} M(Tx^*, Tx_n, t) = M(Tx^*, x_{n+1}, t) &\geqslant \psi(N(x^*, x_n, t))K(x^*, x_n, t) \\ &\geqslant N(x^*, x_n, t)K(x^*, x_n, t), \end{aligned} \tag{3.9}$$

where

$$N(x^*, x_n, t) = \min\{M(x^*, x_n, t), \max\{M(x^*, Tx^*, t), M(x_n, Tx_n, t)\}\}.$$

and

$$K(x^*, x_n, t) = \max\{M(x^*, x_n, t), M(x^*, x_{n+1}, t), M(x_{n+1}, Tx^*, t)M(x^*, Tx^*, t), \\ M(x_n, Tx_n, t)\}.$$

As $n \to \infty$,

$$\lim_{n\to\infty} N(x^*, x_n, t) = 1 \text{ and } \lim_{n\to\infty} K(x^*, x_n, t) = 1.$$

This implies that

$$1 \geqslant \lim_{n\to\infty} M(Tx^*, x_{n+1}, t) \geqslant 1.$$

Therefore, $\lim_{n\to\infty} M(Tx^*, x_{n+1}, t) = 1$. Hence, the sequence $\{x_n\}$ converges to Tx^*. Since the limit of a convergent sequence in a fuzzy metric space is unique, it follows that $Tx^* = x^*$.

Therefore, x^* is a fixed point of T. □

THEOREM 3.2. *In addition to the hypotheses of Theorem 3.1, we assume the following.*

condition (H): $\alpha^*(x, y, t) = \eta^*(x, y, t)$ *if and only if* $x = y$.

Then T *has a unique fixed point in* X.

PROOF. Suppose x and y are fixed points of T. Thus, $Tx = x$ and $Ty = y$ which implies that

$$\alpha^*(x, Tx, t) = \alpha^*(x, x, t) \text{ and } \alpha^*(y, Ty, t) = \alpha^*(y, y, t).$$

By condition (H), we have

$$\alpha^*(x, x, t) = \eta^*(x, x, t) \text{ and } \alpha^*(y, y, t) = \eta^*(y, y, t)$$

for all $t > 0$. This implies that

$$\alpha^*(x, Tx, t)\alpha^*(y, Ty, t) \geqslant \eta^*(x, Tx, t)\eta^*(y, Ty, t).$$

Since T is a generalized modified (α^*, η^*, ψ) contractive mapping, we have that

$$M(x, y, t) = M(Tx, Ty, t) \geqslant \psi(N(x, y, t))K(x, y, t),$$

where

$$N(x, y, t) = \min\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\} = M(x, y, t)$$

and

$$K(x, y, t) = \max\{M(x, y, t), M(x, Tx, t), M(y, Ty, t), M(x, Ty, t), M(y, Tx, t)\} = 1.$$

This implies that

$$M(x,y,t) \quad \geqslant \quad \psi(M(x,y,t)).$$

By the property of ψ we have $M(x,y,t) = 1$. This implies that $x = y$. Therefore T has a unique fixed point. □

By taking $\eta^*(x,y,t) = 1$ in Theorem 3.1 we obtain the following corollary.

COROLLARY 3.1. *Let $(X, M, *)$ be a complete fuzzy metric space and $T : X \to X$ be α^*-admissible map. Assume that there exists a function $\psi \in \Psi$ such that*

$$\alpha^*(x,Tx,t)\alpha^*(y,Ty,t) \geqslant 1 \Rightarrow M(Tx,Ty,t) \geqslant \psi(N(x,y,t))K(x,y,t),$$

where

$$N(x,y,t) = \min\{M(x,y,t), \max\{M(x,Tx,t), M(y,Ty,t)\}\},$$

and

$$K(x,y,t) = \max\{M(x,y,t), M(x,Tx,t), M(y,Ty,t), M(x,Ty,t), M(y,Tx,t)\}$$

for all $x, y \in X$ and $t > 0$. Suppose that the following assertions hold

- **(a):** *there exists $x_0 \in X$ such that $\alpha^*(x_0, Tx_0, t) \geqslant 1$ for all $t > 0$,*
- **(b):** *for any sequence $\{x_n\} \subset X$ such that $\alpha^*(x_n, x_{n+1}, t) \geqslant 1$, for all $n \in \mathbb{N}, t > 0$ and $x_n \to x$ as $n \to \infty$, then $\alpha^*(x, Tx, t) \geqslant 1$ for all $t > 0$.*

Then T has a fixed point.

COROLLARY 3.2. *Let $(X, M, *)$ be a complete fuzzy metric space and $T : X \to X$ be α^*-admissible map. Assume that there exists a function $\psi \in \Psi$ such that*

$$\frac{1}{\alpha^*(x,Tx,t)\alpha^*(y,Ty,t)} M(Tx,Ty,t) \geqslant \psi(N(x,y,t)))K(x,y,t),$$

where

$$N(x,y,t) = \min\{M(x,y,t), \max\{M(x,Tx,t), M(y,Ty,t)\}\}$$

and

$$K(x,y,t) = \max\{M(x,y,t), M(x,Tx,t), M(y,Ty,t), M(x,Ty,t), M(y,Tx,t)\},$$

for all $x, y \in X$ and $t > 0$. Suppose that the following assertions hold

- **(a):** *there exists $x_0 \in X$ such that $\alpha^*(x_0, Tx_0, t) \geqslant 1$ for all $t > 0$,*
- **(b):** *for any sequence $\{x_n\} \subset X$ such that $\alpha^*(x_n, x_{n+1}, t) \geqslant 1$, for all $n \in \mathbb{N}, t > 0$ and $x_n \to x$ as $n \to \infty$, then $\alpha^*(x, Tx, t) \geqslant 1$ for all $t > 0$.*

Then T has a fixed point.

PROOF. Suppose $\alpha^*(x,Tx,t)\alpha^*(y,Ty,t) \geqslant 1$. it follows that

$$\frac{1}{\alpha^*(x,Tx,t)\alpha^*(y,Ty,t)} M(Tx,Ty,t) \leqslant M(Tx,Ty,t).$$

Which implies that $M(Tx,Ty,t) \geqslant \psi(N(x,y,t))K(x,y,t)$. By Corollary 3.1 T has a fixed point. □

By taking $\alpha^*(x,y,t) = 1$ in Theorem 3.1, we draw the following corollary.

Corollary 3.3. *Let $(X, M, *)$ be a complete fuzzy metric space. A mapping $T : X \to X$ be η^*-sub admissible map. Assume that there exists a function $\psi \in \Psi$ such that*

$$\eta^*(x, Tx, t)\eta^*(y, Ty, t) \leqslant 1 \Rightarrow M(Tx, Ty, t) \geqslant \psi(N(x, y, t))K(x, y, t),$$

where

$$N(x, y, t) = \min\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\}$$

and

$$K(x, y, t) = \max\{M(x, y, t), M(x, Tx, t), M(y, Ty, t), M(x, Ty, t), M(y, Tx, t)\},$$

for all $x, y \in X$ and $t > 0$. Suppose that the following assertions hold

(a): *there exists $x_0 \in X$ such that $\eta^*(x_0, Tx_0, t) \leqslant 1$ for all $t > 0$,*

(b): *for any sequence $\{x_n\} \subset X$ such that $\eta^*(x_n, x_{n+1}, t) \leqslant 1$, for all $n \in \mathbb{N}, t > 0$ and $x_n \to x$ as $n \to \infty$, then $\alpha^*(x, Tx, t) \leqslant 1$ for all $t > 0$.*

Then T has a fixed point.

Corollary 3.4. *Let $(X, M, *)$ be a complete fuzzy metric space. $T : X \to X$ be η^*-sub admissible map. Assume that there exists a function $\psi \in \Psi$ such that*

$$M(Tx, Ty, t) \geqslant \frac{1}{\eta^*(x, Tx, t)\eta^*(y, Ty, t)}\psi(M(x, y, t))K(x, y, t),$$

where

$$N(x, y, t) = \min\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\}$$

and

$$K(x, y, t) = \max\{M(x, y, t), M(x, Tx, t), M(y, Ty, t), M(x, Ty, t), M(y, Tx, t)\},$$

for all $x, y \in X$ and $t > 0$. Suppose that the following assertions hold

(a): *there exists $x_0 \in X$ such that $\eta^*(x_0, Tx_0, t) \leqslant 1$ for all $t > 0$,*

(b): *for any sequence $\{x_n\} \subset X$ such that $\eta^*(x_n, x_{n+1}, t) \leqslant 1$, for all $n \in \mathbb{N}, t > 0$ and $x_n \to x$ as $n \to \infty$, then $\alpha^*(x, Tx, t) \leqslant 1$ for all $t > 0$.*

Then T has a fixed point.

If we take $\alpha^*(x, y, t) = 1$ in Corollary3.2 or $\eta^*(x, y, t) = 1$ in Corollary 3.4, we draw the following Corollary

Corollary 3.5. *Let $(X, M, *)$ be a complete fuzzy metric space. Let $T : X \to X$ be a self map. Assume that there exists a function $\psi \in \Psi$ such that*

$$M(Tx, Ty, t) \geqslant \psi(N(x, y, t))K(x, y, t)$$

where

$$N(x, y, t) = \min\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\}$$

and

$$K(x, y, t) = \max\{M(x, y, t), M(x, Tx, t), M(y, Ty, t), M(x, Ty, t), M(y, Tx, t)\},$$

for all $x, y \in X$ and $t > 0$. Then T has a fixed point.

Remark 3.1. Theorem 2.1 follows as a corollary to Theorem 3.1 since $K(x, y, t) \leqslant 1$ for all $x, y \in X$ and $t > 0$. $\Diamond$

Now we give an example in which to Theorem 2.1 fail to apply but Theorem 3.1 can be applied to prove the existence of fixed point. From this it follows that Theorem 3.1 is a generalization of Theorem 2.1.

EXAMPLE 3.1. let $X = [0, \frac{2}{3}] \cup [1, \infty)$ and $M(x, y, t) = \left(\frac{t}{t+1}\right)^{d(x,y)}$, where $d(x, y) = |x - y|$, $*$ is product continuous t-norm. Here $(X, M, *)$ is complete fuzzy metric space.

Let $T : X \to X$ be a map defined by

$$Tx = \begin{cases} \frac{2}{3}x, & \text{if } \quad x \in [0, \frac{2}{3}], \\ 0, & \text{if } \quad x \in [1, \infty). \end{cases}$$

Let $\alpha^*, \eta^* : X \times X \times (0, \infty) \to [0, \infty)$ defined by

$$\alpha^*(x, y, t) = \begin{cases} 3, & \text{if } x, y \in [0, \frac{2}{3}] \cup \{1\} \\ 0, & \text{otherwise} \end{cases} ,$$

$$\eta^*(x, y, t) = \begin{cases} 1, & \text{if } x, y \in [0, \frac{2}{3}] \cup \{1\} \\ 2, & \text{otherwise.} \end{cases}$$

Claim T is (α^*, η^*) admissible map.

Suppose $\alpha^*(x, y, t) \geqslant \eta^*(x, y, t)$, then $x, y \in [0, \frac{2}{3}] \cup \{1\}$. On the other hand , for all $x, y \in [0, \frac{2}{3}] \cup \{1\}$, we have $Tx \in [0, \frac{2}{3}]$ and $Ty \in [0, \frac{2}{3}]$. This implies

$$\alpha^*(Tx, Ty, t) \geqslant \eta^*(Tx, Ty, t).$$

Hence T is (α^*, η^*) admissible mapping.

Moreover, $\alpha^*(\frac{1}{4}, T\frac{1}{4}, t) \geqslant \eta^*(\frac{1}{4}, T\frac{1}{4}, t)$.

Let $\{x_n\}$ be a sequence in X such that $\alpha^*(x_n, x_{n+1}, t) \geqslant \eta^*(x_n, x_{n+1}, t)$ for all $n \in \mathbb{N} \cup \{1\}$ and $x_n \to x$ as $n \to \infty$, then $\{x_n\} \subset [0, \frac{2}{3}] \cup \{1\}$, and hence $x \in [0, \frac{2}{3}] \cup \{1\}$. This implies that $\alpha^*(x, Tx, t) \geqslant \eta^*(x, Tx, t)$ for all $n \in \mathbb{N}$ and $t > 0$.

Suppose

$$\alpha^*(x, Tx, t)\alpha^*(y, Ty, t) \geqslant \eta^*(x, Tx, t)\eta^*(y, Ty, t),$$

then $x, y \in [0, \frac{2}{3}] \cup \{1\}$.

Case i: If $x, y \in [0, \frac{2}{3}]$ and $\psi(t) = t^{\frac{2}{3}}$

Let $a_t = \frac{t}{t+1} < 1$. Then $M(Tx, Ty, t) = {a_t}^{\frac{2}{3}|x-y|}$, $M(x, y, t) = {a_t}^{|x-y|}$, $M(x, Tx, t) = {a_t}^{\frac{x}{3}}$, $M(y, Ty, t) = {a_t}^{\frac{y}{3}}$. We wish to show that

$$M(Tx, Ty, t) \geqslant \psi(N(x, y, t))K(x, y, t),$$

where

$$N(x, y, t) = \min\{M(x, y, t), \max\{M(x, Tx, t), M(y, Ty, t)\}\}.$$

$$K(x, y, t) = \max\{M(x, y, t), M(x, Tx, t), M(y, Ty, t), M(x, Ty, t), M(y, Tx, t)\}.$$

Since ψ is non-decreasing and $K(x, y, t) \leqslant 1$, we get that

$${a_t}^{\frac{2}{3}|x-y|} = \psi(M(x, y, t)) \geqslant \psi(N(x, y, t)).$$

Thus, $M(Tx, Ty, t) \geqslant \psi(N(x, y, t))K(x, y, t)$. So, the hypothesis of Theorem 3.1 is satisfied in this case.

Case ii: Let $x \in [0, \frac{2}{3}]$ and $y = 1$.

$$M(Tx, Ty, t) = a_t{}^{\frac{2}{3}x}, M(x, y, t) = a_t{}^{1-x},\ M(x, Tx, t) = a_t{}^{\frac{x}{3}}, M(y, Ty, t) = a_t,$$
$$M(x, Ty, t) = a_t{}^{x}, M(y, Tx, t) = a_t{}^{1-\frac{2}{3}x}.$$

Now, we may see that $N(x, y, t) = a_t{}^{1-x}$ and $K(x, y, t) = a_t{}^{\frac{x}{3}}$. Since

$$\frac{2}{3}x \leqslant \frac{2}{3}(1-x) + \frac{x}{3}, \forall x \leqslant \frac{2}{3},$$

we have

$$M(Tx, Ty, t) \geqslant \psi(N(x, y, t))K(x, y, t).$$

Case iii: If $x = 1,\ y = 1$ then $M(Tx, Ty, t) = 1$, $\psi(N(x, y, t)) \leqslant 1$ and $K(x, y, t) \leqslant 1$. This implies that $M(Tx, Ty, t) \geqslant \psi(N(x, y, t))K(x, y, t)$, So the result follows in this case. Therefore, from case i-case iii all conditions in Theorem 3.1 are satisfied. Therefore, T has a fixed point. In fact, 0 is a fixed point of T.

Here we can't apply Theorem 2.1 to this example to prove the existence of fixed point, because if we take $x = \frac{3}{5}$ and $y = 1$, we have

$$M(Tx, Ty, t) = a_t{}^{\frac{2}{5}}, M(x, y, t) = a_t{}^{\frac{2}{5}}, M(x, Tx, t) = a_t{}^{\frac{1}{5}}, M(y, Ty, t) = a_t.$$

Now, $N(x, y, t) = a^{\frac{2}{5}}$. If there exist $\psi \in \Psi$ such that $M(Tx, Ty, t) \geqslant \psi(N(x, y, t))$ then $a_t{}^{\frac{2}{5}} \geqslant \psi(a^{\frac{2}{5}}) > a_t{}^{\frac{2}{5}}$, which is a contradiction. $\Diamond$

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