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ON GENERALIZED RIGHT DERIVATIONS OF Γ - INCLINE

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Abstract. In this paper, we introduce the concept of generalized right derivation of $\Gamma-$incline and we study some of the properties of generalized right derivation of $\Gamma-$incline.

Semiring, the best algebraic structure, which is a common generalization of rings and distributive lattices, was first introduced by American mathematician Vandiver [**21**] in 1934 but non trivial examples of semirings had appeared in the studies on the theory of commutative ideals of rings by German mathematician Richard Dedekind in 19th century. Semiring is an universal algebra with two binary operations called addition and multiplication, where one of them distributive over the other,Bounded distributive lattices are commutative semirings, which are both additively idempotent and multiplicatively idempotent. A natural example of semiring is the set of all natural numbers under usual addition and multiplication of numbers. In particular, if I is the unit interval on the real line then $(I, \max, \min)$ in which 0 is the additive identity and 1 is the mutilative identity. The theory of rings and the theory of semigroups have considerable impact on the development of the theory of semirings. In structure, semirings lie between semigroups and rings. The study of rings shows that multiplicative structure of ring is independent of additive structure whereas in semiring multiplicative structure of semiring is not independent of additive stricture of semiring. Additive and multiplicative structures of a semiring play an important role in determining the structure of a semiring. Semiring, as the basic algebraic structure was used in the areas of theoretical computer science as well as in the solutions of graph theory and optimization theory and in particular for studying automata, coding theory and formal languages. Semiring theory has many applications in other branches.

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The notion of $\Gamma-$ring was introduced by Nobusawa [**18**] as a generalization of ring in 1964. Sen [**20**] introduced the notion of $\Gamma-$ semigroup in 1981. The notion of ternary algebraic system was introduced by Lehmer [**9**] in 1932, Lister [**10**] introduced ternary ring. Dutta & Kar [**6**] introduced the notion of ternary semiring which is a generalization of ternary ring and semiring. In 1995, Murali Krishna Rao [**15**] introduced the notion of Γ- semiring which is a generalization of Γ- ring, ternary semiring and semiring. After the paper [**15**] was published, many mathematicians obtained interesting results on Γ-semirings.

The concept of incline was first introduced by Cao et al. [**5**] in 1984. Inclines are additively idempotent semiring in which products are less than or equal to either factor. Products reduce the values of quantities and make them go down which is why the structures were named inclines. Idempotent semirings and Kleene algebras have recently been established as fundamental structures in computer sciences. An incline is a generalization of Boolean algebra, fuzzy algebra and distributive lattice and incline is a special type of semiring. An incline has both semiring structure and the poset structure. Every distributive lattice and every boolean algebra is an incline but an incline need not be a distributive lattice. Set of all idempotent elements in an incline is a distributive lattice. Von Neumann [**17**] introduced the concept of regular elements in a ring. Meenakshi and Anbalagan [**13**] studied regular elements in an incline and proved that regular commutative incline is a distributive lattice. An incline is a more general algebraic structure than a distributive lattice. Ahn et al. [**1**] have proved that " In an incline every prime ideal of incline M is equivalent to irreducible ideal of M''. In an incline every ideal is a $k-$ideal. Meenakshi and Jayabalan [**14**] have proved that every prime ideal in incline M is an irreducible ideal of M and every maximal ideal in incline of M is an irreducible ideal. Cao et al. [**5**] studied the incline and its applications. Kim and Rowsh [**7, 8**] have studied matrices over an incline. Many research scholars have been researched the theory of incline matrices. Few research scholars studied the algebraic structure of incline. Inclines and Matrices over inclines are useful tools in diverse areas such as automata theory, design of switching circuits, graph theory, information systems, modeling, decision making, dynamical programming, control theory, classical and non classical path finding problems in graphs, fuzzy set theory, data analysis, medical diagnosis, nervous system, probable reasoning, physical measurement and so on. Murali Krishna Rao and Venkteswarlu [**16**] studied $\Gamma-$incline and field $\Gamma-$semiring.

Over the last few decades several authors have investigated the relationship between the commutativity of ring R and the existence of certain specified derivations of R. The first result in this direction is due to Posner [**19**] in 1957. In the year 1990, Bresar and Vukman [**4**] established that a prime ring must be a commutative if it admits a non-zero left derivation. Ayar et al. [**3**] studied generalized derivations of incline algebras. The notion of derivation of ring is useful for characterization of rings. The notion of generalized right derivation is a generalization of right derivation. In this paper, we introduce the concept of generalized right derivation of $\Gamma-$incline and we study some of the properties of generalized right derivation of $\Gamma-$incline.

1. Preliminaries

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

DEFINITION 1.1. [2] A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called a semiring provided

(i) addition is a commutative operation.
(ii) multiplication distributes over addition both from the left and from the right.
(iii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$.

DEFINITION 1.2. [5] A commutative incline M with additive identity 0 and multiplicative identity 1 is a non-empty set M with operations addition $(+)$ and multiplication $(.)$ defined on $M \times M \to M$ such that satisfying the following conditions for all $x, y, z \in M$

(i) $x + y = y + x$ (ii) $x + x = x$ (iii) $x + xy = x$
(iv) $y + xy = y$ (v) $x + (y + z) = (x + y) + z$
(vi) $x(yz) = x(yz)$ (vii) $x(y + z) = xy + xz$
(viii) $(x + y)z = xz + yz$ (ix) $x1 = 1x = x$
(x) $x + 0 = 0 + x = x$ (xi) $xy = yx$

DEFINITION 1.3. [15] Let M and Γ be additive abelian groups. If there exists a mapping $M \times \Gamma \times M \to M$ (images to be denoted by $x\alpha y, x, y \in M, \alpha \in \Gamma$) satisfying the following conditions for all $x, y, z \in M, \alpha, \beta \in \Gamma$

(i) $x\alpha(y\beta z) = (x\alpha y)\beta z$ (ii) $x\alpha(y + z) = x\alpha y + x\alpha z$
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$ (iv) $(x + y)\alpha z = x\alpha z + y\alpha z$.

Then M is called a $\Gamma-$ ring.

DEFINITION 1.4. [15] Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M as a $\Gamma-$semiring, if there exists a mapping $M \times \Gamma \times M \to M$ is written (x, α, y) as $x\alpha y$ such that it satisfies the following axioms for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$
(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$
(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

Every semiring R is a $\Gamma-$semiring with $\Gamma = R$ and ternary operation $x\gamma y$ as the usual semiring multiplication.

We illustrate the definition of $\Gamma-$semiring by the following example

EXAMPLE 1.1. [13] Let S be a semiring and $M_{p,q}(S)$ denote the additive abelian semigroup of all $p \times q$ matrices with identity element whose entries are from S. Then $M_{p,q}(S)$ is a $\Gamma-$semiring with $\Gamma = M_{p,q}(S)$ ternary operation is defined by $x\alpha z = x(\alpha^t)z$ as the usual matrix multiplication, where α^t denote the transpose of the matrix α; for all x, y and $\alpha \in M_{p,q}(S)$.

Definition 1.5. [15] A $\Gamma-$semiring M is said to have zero element if there exists an element $0 \in M$ such that $0+x = x = x+0$ and $0\alpha x = x\alpha 0 = 0,$ for all $x \in M, \alpha \in \Gamma$.

Definition 1.6. [15] A $\Gamma-$semiring M is said to be commutative $\Gamma-$semiring if $x\alpha y = y\alpha x$, for all $x, y \in M$ and $\alpha \in \Gamma$.

Definition 1.7. [15] Let M be a $\Gamma-$semiring. An element $a \in M$ is said to be n idempotent of M if there exists $\alpha \in \Gamma$ such that $a = a\alpha a$ and a is also said to be α idempotent.

Definition 1.8. [15] Let M be a $\Gamma-$semiring. Every element of M, is an idempotent of M then M is said to be idempotent $\Gamma-$semiring M.

Definition 1.9. [15] Let M be a $\Gamma-$semiring. An element $a \in M$ is said to be regular element of M if there exists $x \in M, \alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$.

Definition 1.10. [15] Let M be a $\Gamma-$semiring. Every element of M is a regular element of M then M is said to be regular $\Gamma-$semiring M.

Definition 1.11. [15] A non-empty subset A of $\Gamma-$semiring M is called a $\Gamma-$subsemiring M if $(A, +)$ is a subsemigroup of $(M, +)$ and $a\alpha b \in A$ for all $a, b \in A$ and $\alpha \in \Gamma$.

Definition 1.12. [15] An additive subsemigroup I of $\Gamma-$semiring M is said to be left (right) ideal of M if $M\Gamma I \subseteq I$ $(I\Gamma M \subseteq I)$. If I is both left and right ideal then I is called an ideal of $\Gamma-$semiring M.

Definition 1.13. [16] Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. If there exists a mapping $M \times \Gamma \times M \rightarrow\ M\big((x, \alpha, y) \Rightarrow\ x\alpha y\big)$ such that it satisfies the following axioms for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$ (ii) $(x + y)\alpha z = x\alpha z + y\alpha z$
(iii) $x(\alpha + \beta)y = x\alpha y + x\beta y$ (iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$
(v) $x + x = x$ (vi) $x + x\alpha y = x$ (vii) $y + x\alpha y = y$.

Then M is called a $\Gamma-$incline.

Every incline M is a $\Gamma-$incline with $\Gamma = M$ and ternary operation $x\gamma y$ as the usual incline multiplication. In an $\Gamma-$incline define the order relation such that for all $x, y \in M, y \leqslant x$ if and only if $y + x = x$. Obviously, $\leqslant$ is a partially order relation over M.

Definition 1.14. [16] A $\Gamma-$incline M is said to have zero element if there exists an element $0 \in M$ such that $0 + x = x = x + 0$ and $0\alpha x = x\alpha 0 = 0$ for all $x \in M$.

Definition 1.15. [16] A $\Gamma-$incline M is said to be commutative $\Gamma-$incline if $x\alpha y = y\alpha x$ for all $x, y \in M$.

Example 1.2. [16] Let $M = [0,\ 1]$ and $\Gamma = N$. $+$ is defined as $x + y = \max\{x, y\}$ and ternary operation is defined as $xry = \min\{x, r, y\}$ for all $x, y \in M, r \in \Gamma$. Then M is a $\Gamma-$ incline.

Definition 1.16. [16] A $\Gamma-$subincline I of a $\Gamma-$incline M is a non-empty subset of M which is closed under the $\Gamma-$incline operations addition and ternary operation.

Definition 1.17. [16] Let M be a $\Gamma-$incline. An element $a \in M$ is said to be idempotent of M if there exists $\alpha \in \Gamma$ such that $a = a\alpha a$ and a is also said to be α idempotent.

Definition 1.18. [16] Let M be a $\Gamma-$incline. If every element of M is an idempotent of M then M is said to be idempotent $\Gamma-$incline M.

Definition 1.19. [16] Let M be a $\Gamma-$incline. An element $a \in M$ is said to be regular element of M if there exist $x \in M, \alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$. Then x is called generalized inverse of a. Let $a\{1\}$ denotes the set of all $g-$inverses of a.

Definition 1.20. [16] Let M be a $\Gamma-$ incline M. If every element of M is a regular element of M then M is said to be regular $\Gamma-$incline M.

Definition 1.21. [16] Let M be a $\Gamma-$incline. If $x \leqslant y$ for all $y \in M$ then x is called the least element of M and denoted as 0. If $x \geqslant y$ for all $y \in M$ then x is called the greatest element of M and denoted as 1.

Definition 1.22. [16] Let M be a $\Gamma-$incline. An element $a \in M$ is called an anti regular, if there exists an element $x \in M, \alpha, \beta \in \Gamma$ such that $x\alpha a\beta x = x$ then x is called $2-$inverse of a.

$a[2]$ denotes the set of all 2-inverses of a.

Definition 1.23. [16] Let M be a $\Gamma-$incline, $a, x \in M$ and $\alpha, \beta, \delta, \gamma \in \Gamma$. If $a = a\alpha x\beta a$ and $x = x\delta a\gamma x$ then x is called $1, 2-$ inverse of a.

$a[1, 2]$ denotes the set of all $1, 2$ inverse of a.

Definition 1.24. [16] A $\Gamma-$incline M is said to be linearly ordered, if $x, y \in M$ then either $x \leqslant y$ or $y \leqslant x$, where $\leqslant$ is an incline order relation.

Definition 1.25. [16] Let M be a $\Gamma-$incline. An element $1 \in M$ is said to be unity if for each $x \in M$ there exists $\alpha \in \Gamma$ such that $x\alpha 1 = 1\alpha x = x$.

Definition 1.26. [16] In a $\Gamma-$incline M with unity 1, an element $a \in M$ is said to be invertible if there exist $b \in M, \alpha \in \Gamma$ such that $a\alpha b = b\alpha a = 1$.

Definition 1.27. [16] A non-zero element a in a $\Gamma-$incline M is said to be zero divisor if there exists a non-zero element $b \in M, \alpha \in \Gamma$ such that $a\alpha b = b\alpha a = 0$.

Definition 1.28. [16] A $\Gamma-$incline M is said to be field $\Gamma-$incline if M is a commutative $\Gamma-$incline with unity 1 and every non-zero element of M is an invertible.

Definition 1.29. [16] A $\Gamma-$incline M with unity 1 and zero element 0 is called Integral $\Gamma-$incline if it has no zero divisors.

Definition 1.30. [16] A $\Gamma-$incline M with zero element 0 is said to be hold cancelation laws if $a \neq 0, a\alpha b = a\alpha c, b\alpha a = c\alpha a,$ where $a, b, c \in M, \alpha \in \Gamma$ then $b = c$.

Definition 1.31. [16] A $\Gamma-$incline M with unity 1 and zero element 0 is called pre -integral $\Gamma-$ incline if M holds cancelation laws.

Example 1.3. [16] If $M = [0, 1], \Gamma = \{0, 1\}$, binary operation $+$ is maximum, ternary operation $a\alpha b$ is the usual multiplication for all $a, b \in M, \alpha \in \Gamma$ then M is $\Gamma-$incline with unity 1.

2. Right derivation and generalized right derivation

In this section we introduce the notion of right derivation and generalized right derivation of $\Gamma-$incline. We study the properties of generalized right derivation of $\Gamma-$incline.

Definition 2.1. Let M be a $\Gamma-$incline. A function $d : M \to M$ is called a right derivation on M if

(i) $d(x + y) = d(x) + d(y)$
(ii) $d(x\alpha y) = d(x)\alpha y + d(y)\alpha x$, for all $x, y \in M, \alpha \in \Gamma$

Definition 2.2. Let M be a $\Gamma-$incline. A function $D : M \to M$ is called a generalized right derivation on M if there exists a right derivation $d : M \to M$ such that

(i) $D(x + y) = D(x) + D(y)$
(ii) $D(x\alpha y) = D(x)\alpha y + d(y)\alpha x$, for all $x, y \in M, \alpha \in \Gamma$

Example 2.1. Let $M = \{a, b, c\}$ and $\Gamma = \{a\}$ be sets. Define $+$ and ternary operation by

+	a	b	c
a	a	b	a
b	b	b	b
c	a	b	c

and

a	a	b	c
a	a	a	c
b	a	b	c
c	c	c	c

.

Then M is a $\Gamma-$incline. Define a map $d : M \to M$ by $d(x) = \begin{cases} a & \text{if } \ x = ab \\ c & \text{if } \ x = c \end{cases}$ and a map $D : M \to M$ by $D(x) = x$, for all $x \in M$. Then D is a generalized right derivation associated with right derivation d of M.

Lemma 2.1. *Let D be a generalized right derivation of $\Gamma-$incline. Then $D(0) = 0$.*

Proof. Let D be a generalized right derivation of $\Gamma-$incline. Then we have $D(0) = D(0\alpha 0) = D(0)\alpha 0 + d(0)\alpha 0 = 0 + 0 = 0.$ □

Corollary 2.1. *Let d be right derivation of $\Gamma-$incline. Then $d(0) = 0$.*

Theorem 2.1. *Let M be an idempotent $\Gamma-$incline and D be a generalized right derivation of M associated with right derivation d of M. Then*

(i) $d(x) \leqslant D(x) \leqslant x$
(ii) $D(x\alpha y) \leqslant D(x) + D(y)$
(iii) *If* $x \leqslant y$ *then* $D(x\alpha y) \leqslant y$, *for all* $x, y \in M, \alpha \in \Gamma$.

Proof. Let M be an idempotent $\Gamma-$incline and D be a generalized right derivation of M associated with right derivation d of M.

(i)

$$\begin{aligned} D(x) &= D(x\alpha x) \\ &= D(x)\alpha x + d(x)\alpha x \\ &\geqslant d(x)\alpha x \\ &\geqslant d(x)\alpha d(x), \text{ since } d(x) \leqslant x \\ &= d(x) \\ \text{Thus } d(x) &\leqslant D(x), \text{ for all } x \in M, \alpha \in \Gamma. \\ D(x) &= D(x)\alpha x + d(x)\alpha x \\ D(x) + x &= D(x)\alpha x + x + d(x)\alpha x + x \\ &= x + x = x \\ \text{Therefore } D(x) &\leqslant x. \end{aligned}$$

Hence $d(x) \leqslant D(x) \leqslant x$.

(ii)

$$\begin{aligned} D(x\alpha y) &= D(x)\alpha y + d(y)\alpha x \\ &\leqslant D(x) + d(y) \\ &\leqslant D(x) + D(y). \end{aligned}$$

(iii) If $x, y \in M, \alpha \in \Gamma$ and $x \leqslant y$ then $d(y)\alpha x \leqslant d(y)\alpha y \leqslant y$ and $D(x)\alpha y \leqslant y$.

$$\begin{aligned} D(x\alpha y) &= D(x)\alpha y + d(y)\alpha x \\ &\leqslant y + y \\ &= y. \end{aligned}$$

Hence the theorem holds.

□

Corollary 2.2. *Let* M *be an idempotent* $\Gamma-$*incline and* d *be a right derivation of* M. *Then the following.*

(i) $d(x) \leqslant x$
(ii) $d(x\alpha y) \leqslant d(x + y)$
(iii) *If* $x \leqslant y$ *then* $d(x\alpha y) \leqslant y$, *for all* $x, y \in M, \alpha \in \Gamma$.

Theorem 2.2. *Let* M *be an idempotent* $\Gamma-$*incline and* D *be a generalized right derivation of* M, *associated with right derivation* d *of* M. *Then*

(i) $D(x\alpha y) = D(x) + d(y)$
(ii) $D(x\alpha y) \leqslant D(x)$
(iii) $D(x\alpha y) \leqslant D(y)$

(iii) *If* $x \leqslant y$ *then* $D(x) \leqslant D(y)$, *for all* $x, y \in M, \alpha \in \Gamma$.

Proof. Let M be an idempotent $\Gamma-$incline and D be a generalized right derivation of M, associated with right derivation d of M. and $x, y \in M, \alpha \in \Gamma$.

(i) $D(x\alpha y) = D(x)\alpha y + d(y)\alpha x \leqslant D(x) + d(y)$.
(ii) $D(x) = D(x + x\alpha y) = D(x) + D(x\alpha y)$. Therefore $D(x\alpha y) \leqslant D(x)$.
(iii) $D(y) = D(y + x\alpha y) = D(y) + D(x\alpha y)$. Therefore $D(x\alpha y) \leqslant D(y)$.
(iii) Suppose $x \leqslant y, x, y \in M$. Then $x + y = y$.
$D(y) = D(x + y) = D(x) + D(y)$. Therefore $D(x) \leqslant D(y)$.
Hence D is an isotone.

□

Theorem 2.3. *Let* M *be an idempotent* $\Gamma-$*incline and* D *be a generalized right derivation of* M, *associated with right derivation* d *of* M. *Then*

(i) *If* $d(D(x)) = D(x)$ *then* $D(x\alpha D(x)) = D(x)$, *for all* $x \in M, \alpha \in \Gamma$.
(ii) *If* I *is an ideal of* M *then* $D(I) \subseteq I$.

Proof. Let M be an idempotent $\Gamma-$incline and D be a generalized right derivation of M, associated with right derivation d of M.

(i) $D(x\alpha D(x)) = D(x)\alpha D(x) + d(D(x))\alpha x = D(x) + D(x)\alpha x = D(x)$, for all $x \in M, \alpha \in \Gamma$.
(ii) Let I be an ideal of M and $x \in I$. We have $D(x) \leqslant x$. Therefore $D(x) \in I$, since I is an ideal. Hence $D(I) \subseteq I$.

□

Theorem 2.4. *Let* M *be an idempotent of* $\Gamma-$*incline with unity* 1 *and* D *be generalized right derivation of* $\Gamma-$*incline* M *associated with right derivation* d *of* M. *Then*

(i) $D(x) = D(1)\alpha x + d(x)\alpha 1$, *for some* $x \in M, \alpha \in \Gamma$
(ii) *If* $x \geqslant D(1)$ *then* $D(x) \geqslant D(1)$.
(iii) *If* $x \leqslant D(1)$ *then* $D(x) = x$
(iv) $D(1) = 1$ *if and only if* $D(x) = 1$.

Proof. Let M be an idempotent of $\Gamma-$incline with unity 1 and D be generalized right derivation of $\Gamma-$incline M associated with right derivation d of M.

(i) Since M is an idempotent of $\Gamma-$incline with unity 1, then for each $x \in M$ there exists $\alpha \in \Gamma$ such that $x\alpha 1 = x = 1\alpha x$.
$D(x) = D(1\alpha x) = D(1)\alpha x + d(x)\alpha 1$, for $x \in M, \alpha \in \Gamma$.
(ii) Suppose $x \geqslant D(1)$, $x \in M$. Then there exists $\alpha \in \Gamma$ such that $x = 1\alpha x$.

$$\begin{aligned} D(x) &= D(1)\alpha x + d(x)\alpha 1 \\ &\geqslant D(1)\alpha x \\ &\geqslant D(1)\alpha D(1) \\ &= D(1). \end{aligned}$$

(iii) Suppose $x \leqslant D(1)$, $x \in M$. Then there exists $\alpha \in \Gamma$ such that $x = 1\alpha x$.

$$\begin{aligned} D(x) &= D(1)\alpha x + d(x)\alpha 1 \\ &\geqslant D(1)\alpha x \\ &\geqslant x\alpha x \\ &= x. \end{aligned}$$

We have, by Theorem 2.1, $D(x) \leqslant x$. Therefore $D(x) = x$.

(iv) Suppose $D(1) = 1$ and $x \in M$. Then there exists $\alpha \in \Gamma$ such that $x = 1\alpha x$.

$$\begin{aligned} D(x) &= D(1)\alpha x + d(x)\alpha 1 \\ &= 1\alpha x + d(x)\alpha 1 \\ &\geqslant 1\alpha x \\ &= x. \end{aligned}$$

We have $D(x) \leqslant x$. Therefore $D(x) = x$. Converse is obvious.

□

THEOREM 2.5. *Let M be an idempotent $\Gamma-$incline with unity 1 and D be generalized right derivation of $\Gamma-$incline M associated with right derivation d of M. Then, for $x \in M, \alpha \in \Gamma$, the following*

(i) $d(1)\alpha x \leqslant D(x)$

(ii) *If $d(1) = 1$ then $x \leqslant D(x)$*

(iii) *If $d(1) = 1$ then D is an identity right derivation*

PROOF. Let M be an idempotent of $\Gamma-$incline with unity 1, D be generalized right derivation of $\Gamma-$incline M associated with right derivation d of M, $x \in M$ and $\alpha \in \Gamma$.

(i) Let $x \in M$. Then there exists $\alpha \in \Gamma$ such that $x = 1\alpha x$.

$$\begin{aligned} D(x) = D(x\alpha 1) &= D(x)\alpha 1 + D(1)\alpha x, \text{ for all } \alpha \in \Gamma \\ d(1)\alpha x &\leqslant D(x)\alpha 1 + d(1)\alpha x \\ &= D(x), \text{ for all } \alpha \in \Gamma, x \in M. \\ d(1)\alpha x &\leqslant D(x). \end{aligned}$$

(ii) If $d(1) = 1$ then

$$\begin{aligned} &d(1)\alpha x \leqslant D(x) \text{ for all } \alpha \in \Gamma \\ \Rightarrow &1\alpha x \leqslant D(x), \text{ for all } \alpha \in \Gamma \\ \Rightarrow &x \leqslant D(x). \end{aligned}$$

(iii) By Theorem 2.1, We have $D(x) \leqslant x$. Therefore $D(x) = x$.
Hence the theorem holds.

□

Theorem 2.6. *Let M be an integral $\Gamma-$incline and generalized right derivation D of M associated with a non-zero right derivation d of M and $a \in M$. If $a\alpha D(x) = 0$, for all $x \in M, \alpha \in \Gamma$ then $a = 0$ or $d = 0$.*

Proof. Let M be an integral $\Gamma-$incline, generalized right derivation of M associated with a non-zero right derivation of $M, a \in M$ and $a\alpha D(x) = 0$, for all $x \in M$. Let $x, y \in M,\ \alpha \in \Gamma$. Then $a\alpha D(x\alpha y) = 0$

$$\begin{aligned}
&\Rightarrow a\alpha(D(x)\alpha y + d(y)\alpha x) = 0\\
&\Rightarrow a\alpha(D(x)\alpha y) + a\alpha(d(y)\alpha x) = 0\\
&\Rightarrow a\alpha(d(y)\alpha x) = 0.
\end{aligned}$$

Since M is an integral $\Gamma-$incline either $a = 0$ or $d(y)\alpha x = 0$, for all $\alpha \in \Gamma, x \in M$.
$\Rightarrow a = 0$ or $d(y)\alpha 1 = 0$, for all $\alpha \in \Gamma$
$\Rightarrow a = 0$ or $d(y) = 0$.
Therefore $a = 0$ or $d = 0$. $\square$

Corollary 2.3. *Let M be a commutative integral $\Gamma-$incline and D be a generalized right derivation of M associated with a nonzero right derivation d of M. If $D(x)\alpha a = 0$ then $a = 0$ or $d = 0$, for all $x \in M$.*

Theorem 2.7. *Let I be a proper ideal of integral $\Gamma-$incline M. If d is a nonzero right derivation of M then D is a nonzero generalized deviation on I.*

Proof. Let I be a proper ideal of integral $\Gamma-$incline M and d be a nonzero right derivation of M. Suppose $0 \neq x \in I$ and $D(x) = 0$, for all $x \in I$. Let $y \in M$ and $\alpha \in \Gamma$. Then

$$\begin{aligned}
&x\alpha y \leqslant x\\
&\Rightarrow x\alpha y \in I, \text{ since } I \text{ is an ideal of } M\\
&\Rightarrow D(x\alpha y) = 0\\
&\Rightarrow D(x)\alpha y + d(y)\alpha x = 0\\
&\Rightarrow d(y)\alpha x = 0\\
&\Rightarrow d(y) = 0, \text{ for all } \ y \in M.
\end{aligned}$$

This is a contradicts to $d \neq 0$ on M. Hence D is a nonzero generalized derivation on I. $\square$

Corollary 2.4. *Let D be a nonzero generalized right derivation of an integral $\Gamma-$incline M associated with a non-zero right derivation d of M. If I is a proper ideal of integral $\Gamma-$incline M and $a \in M$ such that $a\alpha D(I) = 0$ then $a = 0$.*

Theorem 2.8. *Let M be a $\Gamma-$incline. If I is an ideal of M if and only if I is a $k-$ideal of M*

Proof. Let M be a $\Gamma-$incline. Suppose I is an ideal of M and $x + y \in I$ and $y \in I$. Then $x + y = x + x + y = x + (x + y)$ and $x \leqslant x + y$. By definition of an ideal, $x \in I$. Hence I is a $k-$ideal.

Conversely suppose that I is a $k-$ideal. Let $y \in M, x \in I$ such that $y \leqslant x$. Thus $y + x = x \Rightarrow y + x \in I$ and $y \in I$. Hence I is an ideal of $\Gamma-$incline M. □

Let D be a generalized right derivation of $\Gamma-$incline M. The set $\{x \in M \mid D(x) = x\}$ is denoted by I_D.

THEOREM 2.9. *Let M be a $\Gamma-$incline and D be a generalized right derivation of M associated with a nonzero right derivation d of M. If M is an additively right cancellative then I_D is an ideal of $\Gamma-$incline M.*

PROOF. Let M be a $\Gamma-$incline and D be a generalized right derivation of M associated with a nonzero right derivation d of M, M is an additively right cancellative, $x, y \in I_D$ and $\alpha \in \Gamma$. Thus $D(x) = x, D(y) = y$ and $D(x + y) = D(x) + D(y) = x + y$. Hence $x + y \in I_D$. Further on

$$\begin{aligned} D(x\alpha y) &= D(x)\alpha y + d(y)\alpha x \\ &\geqslant D(x)\alpha y \\ &= x\alpha y. \end{aligned}$$

We have $D(x\alpha y) \leqslant x\alpha y$. Therefore $D(x\alpha y) = x\alpha y$. Hence $x\alpha y \in I_D$.
Suppose $x \in M, y \in I_D$ such that $x \leqslant y$. Then $D(y) = y$ and $D(x) \leqslant D(y)$.

$$\begin{aligned} \Rightarrow D(x) + D(y) &= D(y) \\ &= y \\ &= x + y \\ &= x + D(y) \\ \Rightarrow D(x) + D(y) &= x + D(y) \\ \Rightarrow D(x) &= x. \end{aligned}$$

Hence I_D is an ideal of $\Gamma-$incline M. □

THEOREM 2.10. *Let M be a $\Gamma-$incline and D_1, D_2 be generalized right derivations on M. If $I_{D_1} = I_{D_2}$ and $D_i(x) \in I_{D_i}, i = 1, 2$ then $D_1 = D_2$.*

PROOF. Let M be a $\Gamma-$incline and D_1, D_2 be generalized right derivations on M, $I_{D_1} = I_{D_2}$ and $D_i(x) \in I_{D_i}, i = 1, 2, x \in M$. Then

$$\begin{aligned} &D_1(x) \in I_{D_1} = I_{D_2} \\ &\Rightarrow D_2(D_1(x)) = D_1(x) \\ &\Rightarrow D_1(x) \in I_{D_2} = I_{D_1} \\ &\Rightarrow D_1(D_2(x)) = D_2(x). \\ \text{We have } \ &D_2(D_1(x)) \subseteq D_2(x), \text{ since } \ D_1(x) \leqslant x \\ &\Rightarrow D_2(D_1(x)) \leqslant D_2(x) = D_1(D_2(x)) \\ &\Rightarrow D_2D_1(x) \leqslant D_1D_2(x). \end{aligned}$$

Symmetrically we have $D_1D_2(x) \leqslant D_2D_1(x)$. Therefore $D_1D_2 = D_2D_1$.

$$\begin{aligned} D_2(x) &= D_1(D_2(x)) \\ &= D_1D_2(x) \\ &= D_2D_1(x) \\ &= D_1(x). \end{aligned}$$

Hence the theorem holds. □

THEOREM 2.11. *Let D be a generalized right derivation of $\Gamma-$semiring M associated with a non-zero right derivation d of M and the set $\{x \in M \mid D(x) = 0\}$ be denoted by $kerD$. Then $kerD$ is an ideal of $\Gamma-$semiring M.*

PROOF. Let D be a generalized right derivation of $\Gamma-$semiring M associated with a non-zero right derivation d of M and the set $kerD = \{x \in M \mid D(x) = 0\}$. Suppose $x, y \in kerD$.

$$\begin{aligned} &\Rightarrow D(x) = 0 = D(y) \\ &\Rightarrow D(x+y) = D(x) + D(y) = 0 + 0 = 0 \\ &\Rightarrow x + y \in kerD. \end{aligned}$$

$$\begin{aligned} D(x\alpha y) &= D(x)\alpha y + d(y)\alpha x \\ &= 0\alpha y + 0\alpha x \\ &= 0. \end{aligned}$$

Therefore $kerD$ is a $\Gamma-$subsemiring of M.
Let $x \in M$ and $y \in kerD$ such that $x \leqslant y$. Then

$$\begin{aligned} D(y) &= 0 \text{ and} x + y = y \\ D(x) &= D(x) + 0 \\ &= D(x) + D(y) \\ &= D(x+y) \\ &= D(y) = 0. \end{aligned}$$

Therefore $x \in kerD$. Hence $kerD$ is an ideal of $\Gamma-$incline M. □

DEFINITION 2.3. Let M be a $\Gamma-$incline and D be a generalized derivation of M. Then an ideal I of M is called a $D-$ideal if $D(I) = I$.

EXAMPLE 2.2. Since $D(0) = 0$, $\{0\}$ is an $D-$ideal.
If D is onto then $D(M) = M$ then M is a $D-$ideal.
If D is one-one then $kerD = \{0\}$. Therefore $kerD$ is a $D-$ideal.

THEOREM 2.12. *Let M be a $\Gamma-$incline. If I and J be are $D-$ideals of M then $I + J$ is a $D-$ideal.*

PROOF. Let M be a $\Gamma-$incline, I and J be are $D-$ideals of M. Now $I + J = D(I) + D(J) = D(I + J)$. Therefore $I + J$ is a $D-$ideal. □

Let D be a generalized right derivation of $\Gamma-$incline. Then the set $\{x \in M \mid D(x) = d(x)\}$ is denoted by $I_{D,d}$.

THEOREM 2.13. *Let M be an additively right cancellative $\Gamma-$incline. If D is a generalized right derivation of $\Gamma-$incline M associated with a non-zero right derivation d of M. then $I_{D,d}$ is an ideal of $\Gamma-$incline M.*

PROOF. Let M be an additively right cancellative $\Gamma-$incline, D be a generalized right derivation of $\Gamma-$incline M associated with a non-zero right derivation d of M, $x, y \in I_{D,d}$ and $\alpha \in \Gamma$. Then

$$\begin{aligned} D(x+y) &= D(x) + D(y) \\ &= d(x) + d(y) \\ &= d(x+y) \\ \Rightarrow x + y &\in I_{D,d}. \\ D(x\alpha y) &= D(x)\alpha y + d(y)\alpha x \\ &= d(x)\alpha y + d(y)\alpha x \\ &= d(x\alpha y) \\ \Rightarrow x\alpha y &\in I_{D,d}. \end{aligned}$$

Let $x \in M$ and $y \in I_{D,d}$ such that $x \leqslant y$.
Then $D(y) = d(y)$ and $x + y = y$. Therefore $D(x) \leqslant D(y)$.

$$\begin{aligned} \Rightarrow D(x) + D(y) &= D(y) \\ &= d(y) \\ &= d(x+y) \\ &= d(x) + d(y) \\ &= d(x) + D(y) \\ \Rightarrow D(x) &= d(x) \\ \Rightarrow x &\in I_{D,d}. \end{aligned}$$

Hence $I_{D,d}$ is an ideal of $\Gamma-$incline M. □

THEOREM 2.14. *Let D be a generalized derivation of $\Gamma-$incline M and $D(x) \in I_D$, for all $x \in M$. Then $D(x) = x$, for all $x \in M$ if and only if D is one-one or onto.*

PROOF. Let D be a generalized derivation of $\Gamma-$incline M and $D(x) \in I_D$, for all $x \in M$. Suppose $D(x) = x$, for all $x \in M$. Obviously D is a one-one and onto.

Conversely suppose that D is one-one. Since $D(x) \in I_D, D(D(x)) = D(x)$. Then $D(x) = x$,

Suppose D is an onto and $x \in M$. Then there exists $y \in M$ such that $D(y) = x$. Since $D(y) \in I_D$. Then $D(D(y) = D(y) = x$. Therefore $D(x) = x$.

Hence the theorem holds. □

THEOREM 2.15. *Let D_1 and D_2 be generalized right derivations of $\Gamma-$incline M associated with right derivations d_1 and d_2 of M respectively. If $D_1 + D_2(x) = D_1(x) + D_2(x)$ and $d_1 + d_2(x) = d_1(x) + d_2(x)$, for all $x \in M$ then $D_1 + D_2$ is a generalized right derivation of M.*

PROOF. Let D_1 and D_2 be generalized right derivations of $\Gamma-$incline M associated with right derivations d_1 and d_2 of M respectively and $a, b \in M, \alpha \in \Gamma$. Then

$$\begin{aligned}(D_1 + D_2)(a + b) &= D_1(a + b) + D_2(a + b)\\ &= D_1(a) + D_1(b) + D_2(a) + D_2(b)\\ &= (D_1 + D_2)(a) + (D_1 + D_2)(b).\\ (D_1 + D_2)(a\alpha b) &= D_1(a\alpha b) + D_2(a\alpha b)\\ &= D_1(a)\alpha b + d_1(b)\alpha a + D_2(a)\alpha b + d_2(b)\alpha a\\ &= (D_1 + D_2)(a)\alpha b + (d_1 + d_2)(b)\alpha a.\end{aligned}$$

Hence the theorem holds. □

DEFINITION 2.4. Let M be a $\Gamma-$incline, D_1 and D_2 be generalized right derivations of M associated with right derivation d_1 and d_2 of M respectively. We define

$$D_1D_2(x) = D_1(D_2(x)),\ D_2D_1(x) = D_2(D_1(x))$$

and

$$d_1d_2(x) = d_1(d_2(x))\ d_2d_1(x) = d_2(d_1(x)),$$

for all $x \in M$.

THEOREM 2.16. *If $D_1D_2 = 0$ and $D_i(x\alpha y) = D_i(y\alpha x)$ and $D_i(y)\alpha D_i(x) = D_i(x)\alpha D_i(y), i = 1, 2$. Then D_2D_1 is a generalized right derivative of $\Gamma-$incline M.*

PROOF. Suppose $D_1D_2 = 0, x, y \in M, \alpha \in \Gamma$. Then

$$\begin{aligned}&D_1D_2(x\alpha y) = 0\\ \Rightarrow &D_1(D_2(x)\alpha y + d_2(y)\alpha x) = 0\\ \Rightarrow &D_1(D_2(x)\alpha y) + D_1(d_2(y)\alpha x) = 0\\ \Rightarrow &D_1(D_2(x)\alpha y) + D_1(x\alpha d_2(y)) = 0\\ \Rightarrow &D_1D_2(x)\alpha y + d_1(y)\alpha D_2(x) + D_1(x)\alpha d_2(y) + d_1d_2(y)\alpha x = 0\\ \Rightarrow &d_1(y)\alpha D_2(x) + D_1(x)\alpha d_2(y) = 0\\ \Rightarrow &D_2(x)\alpha d_1(y) + d_2(y)\alpha D_1(x) = 0 \qquad \cdots (1).\end{aligned}$$

$$\begin{aligned}D_2D_1(x\alpha y) &= D_2(D_1(x)\alpha y + d_1(y)\alpha x)\\ &= D_2D_1(x)\alpha y + d_2d_1(y)\alpha x + d_2(y)\alpha D_2(x) + D_2(x)\alpha d_1(y)\\ &= D_2D_1(x)\alpha y + d_2d_1(y)\alpha x.\end{aligned}$$

Therefore D_2D_1 is a generalized right derivation of $\Gamma-$incline M. □

THEOREM 2.17. *Let M be an additively cancellative $\Gamma-$incline D_1 and D_2 be generalized right derivations of M associated with right derivations d_1 and d_2 of M and $D_1d_2 = d_1d_2$. If D_1D_2 is a generalized right derivation and d_1d_2 is a right derivation then*

(i) $d_2(x)\alpha d_1(y) + d_1(x)\alpha d_2(y) = 0$

(ii) $d_1(y)\alpha D_2(x) + d_1(x)\alpha d_2(y) = 0$, *for all* $x, y \in M, \alpha \in \Gamma$.

PROOF. Suppose D_1D_2 is a generalized right derivation of $\Gamma-$incline M where D_1 and D_2 are generalized right derivations of M associated with right derivation d_1 and d_2 of M respectively and $D_1d_2 = d_1d_2$.

$$\begin{aligned} d_1d_2(x\alpha y) &= d_1(d_2(x\alpha y)) \\ &= d_1(d_2(x)\alpha y + d_2(y)\alpha x) \\ &= d_1(d_2(x)\alpha y) + d_1(x\alpha d_2(y)) \\ &= d_1d_2(x)\alpha y + d_2(x)\alpha d_1(y) + d_1(x)\alpha d_2(y) + x\alpha d_1d_2(y) \end{aligned}$$

$\Rightarrow$ $d_1d_2(x)\alpha y + x\alpha d_1d_2(y) = d_1d_2(x)\alpha y + d_2(x)\alpha d_1(y) + d_1(x)\alpha d_2(y) + x\alpha d_1d_2(y)$.
Therefore $d_2(x)\alpha d_1(y) + d_1(x)\alpha d_2(y) = 0$, since M is a cancellative $\Gamma-$incline.

$$\begin{aligned} D_1D_2(x\alpha y) &= D_1(D_2(x)\alpha y + d_2(y)\alpha x) \\ &= D_1(D_2(x)\alpha y) + D_1(d_2(y)\alpha x) \\ &= D_1D_2(x)\alpha y + d_1(y)\alpha D_2(x) + D_1(d_2(y))\alpha x + d_1(x)\alpha d_2(y) \end{aligned}$$

$$\Rightarrow D_1D_2(x)\alpha y + d_1d_2(y)\alpha x =$$
$$D_1D_2(x)\alpha y + d_1(y)\alpha D_2(x) + D_1(d_2(y))\alpha x + d_1(x)\alpha d_2(y).$$

Therefore $d_1(y)\alpha D_2(x) + d_1(x)\alpha d_2(y) = 0$, Since M is a cancellative $\Gamma-$incline. Hence the theorem holds. $\square$

THEOREM 2.18. *Let M be a integral $\Gamma-$incline and D be a generalized derivation of M such that $D(x\alpha y) = D(y\alpha x)$ and $d(y)\alpha D(x) = D(x)\alpha d(y)$. Define $D^2(x) = D(D(x))$, for all $x \in M$. If $D^2 = 0$ then $D = 0$.*

PROOF. Let M be a integral $\Gamma-$incline and D be a generalized derivation of M such that $D(x\alpha y) = D(y\alpha x), d(y)\alpha D(x) = D(x)\alpha d(y)$ and $D^2(x) = D(D(x))$, for all $x \in M$. Let $x, y \in M$ and $\alpha \in \Gamma$.

$$\begin{aligned} &D^2(x\alpha y) = 0 \\ \Rightarrow &D(D(x\alpha y)) = 0 \\ \Rightarrow &D(D(x)\alpha y + d(y)\alpha x) = 0 \\ \Rightarrow &D(D(x)\alpha y) + D(d(y)\alpha x) = 0 \\ \Rightarrow &D^2(x)\alpha y + d(y)\alpha D(x) + D(x\alpha d(y)) = 0 \\ \Rightarrow &d(y)\alpha D(x) + +D(x)\alpha d(y) + d^2(y)\alpha x = 0 \\ \Rightarrow &D(x)\alpha d(y) = 0 \\ \Rightarrow &D(x) = 0 \text{ or } d(y) = 0, \text{ for all } x, y \in M. \end{aligned}$$

Hence the theorem holds. □

THEOREM 2.19. *If D is a non-zero generalized right derivation of $\Gamma-$incline M, $I \neq \{0\}$ be an ideal of M, $D(x\alpha y) = D(y\alpha x)$ and $D(x)\alpha d(y) = d(y)\alpha D(x)$. Then $D^2(I) \neq \{0\}$.*

PROOF. Suppose $D^2(I) = \{0\}$. Let $x, y \in I, \alpha \in \Gamma$. Then $D^2(x\alpha y) = 0$.

$$
\begin{aligned}
&\Rightarrow D(D(x\alpha y)) = 0\\
&\Rightarrow D(D(x)\alpha y + d(y)\alpha x) = 0\\
&\Rightarrow D(D(x)\alpha y) + D(d(y)\alpha x) = 0\\
&\Rightarrow D^2(x)\alpha y + d(y)\alpha d(x) + D(x\alpha d(y)) = 0\\
&\Rightarrow d(y)\alpha D(x) + D(x)\alpha d(y) + d^2(y)\alpha x = 0\\
&\Rightarrow d(y)\alpha D(x) = 0\\
&\Rightarrow d(y) = 0 \text{ or } D(x) = 0, \text{ for all } x, y \in I.
\end{aligned}
$$

In both cases, we have $D = 0$. This contradicts that D is a non-zero generalized right derivation. Hence $D^2(I) \neq \{0\}$. □

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