JOURNAL OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4866, ISSN (o) 2303-4947
www.imvibl.org /JOURNALS / JOURNAL
J. Int. Math. Virtual Inst., **13**(1)(2023), 55-71
DOI: 10.7251/JIMVI2301055M

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

CHARACTERIZING HYPERGROUPOIDS THROUGH M-RELATIONS AND M-CONSISTENCIES

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Abstract. In this article, we define the m-right consistent/m-left consistent and m-consistent hypergroupoids. We also define the m-intra-consistent hypergroupoid. Along these line, we define the Green's m-relations namely m-right relation, m-left relations, and m-relation. The other three relations, namely m-reflexive, m-symmetric and m-transitive, are also defined. The idea of m-equivalence relation is also given. We present different characterization of hypergroupoids in the article through these concepts.

1. Introduction

The idea of the hyperstructure was given by Marty [**11**] in 1934. Among all the hyperstructures, a hypergroupoid is the simplest one consisting of a non-empty set together with a hyperoperation [**5**]. Hypergroupoids are studied by their characterizations through the properties of their hyperideals. Kehayopulu characterized the hypergroupoids through the properties of their fuzzy prime and fuzzy semiprime hyperideals [**6**]. Suebsung et al characterized the semihypergroupoids through the properties of almost hyperideals [**23**]. Hasankhani studied the hyperideals with respect to Greens's relations in hypersemigroups [**2**].

The hypergroupoids have been in use to study the problems in different scientific fields. Haidari et al studied the chemical salt reactions with the help of ideals in the hyper structures in [**3**]. In a recent article, Munir et. al., introduced the m-hyperideals in the hypergroupoid, and used them to study the genetics of the blood groups system [**12**].

2010 *Mathematics Subject Classification.* Primary 20M14; Secondary 20M30.
Key words and phrases. m-consistent, m-relation, m-equivalence.
Communicated by Dusko Bogdanic.

In this article, we basically generalize the hyperideals of hypergroupoids by an integer m, and then discuss the concepts arising out as their consequences with examples and applications. This work is a continuation of our work on the generalizations of ideals in the semigroup theory([**13**], [**14**], [**15**], [**16**], [**17**], [**18**]) and semiring theory([**19**], [**20**]) through a positive integer m. Moreover, this work is also based on the generalization of ideals through three integers in semigroups([**9**], [**22**], [**1**], [**10**]) and in fuzzy sets [**21**].

The contents of this paper are divided into four sections. In Sections 1, we have presented the introduction of the article, and in Section 2, the preliminary ideas from the literature have been presented. In Section 3, we present the idea of the m-consistent hypergroupoid and their important characterizations. In Section 4, we present the Green's m-relations on the hypergroupoid with examples and important characterizations.

2. Preliminaries

We present some preliminary ideas from the literature of hypergroupoids which will be necessary for our onward work.

Definition 2.1. Following [**4**], [**7**] and [**8**], let H be a nonempty set, then the map $\circ : H \times H \to P^*(H)$ is called a hyperoperation, where $P^*(H)$ is the family of all nonempty subsets of H.

Definition 2.2. We define a hypergroupoid as an ordered pair $(H, \circ)$ consisting of a nonempty set H and a hyperoperation $\circ : H \times H \to P^*(H)$, where $P^*(H)$ is the set of all nonempty subsets of H, defined by

$$S \circ K = \bigcup_{a \in S, b \in K} (a \circ b), \tag{2.1}$$

for all non-empty subsets S and K of H.

We shall denote the hypergroupoid $(H, \circ)$ simply by H. It follows from the definition that $a \circ H = \{a\} \circ H$ and $H \circ a = H \circ \{a\}$. e is called left identity if $a \circ e = e$, e is called right identity if $e \circ a = \{a\}$ for al $a \in H$. e is called scalar identity or identity if $a \circ e = e \circ a = \{a\}$ for al $a \in H$. The identity of H is unique by definition. An element $a \in H$ is idempotent if $a \circ a = a$. H is said to be commutative if $a \circ b = b \circ a$, for all $a, b \in H$. A hypergroupoid $(H, \circ)$ is said to be a hypersemigroup if the hyperoperation $\circ$ follows the associative law on all elements of H, i.e., if $a \circ (b \circ c) = (a \circ b) \circ c$ for every $a, b, c \in H$.

A non-empty subset S of H is known as its subhypergroupoid if $a, b \in S$ implies $a \circ b \subseteq S$, comparably, $S \circ S \subseteq S$ infers S is subhypergroupoid of H. A non-empty subset S of a hypergroupoid H is said to be a right(left) hyperideal of H if $S \circ H \subseteq S$ $(H \circ S \subseteq S)$. A nonempty set, T, is called a two-sided hyperideal or simply a hyperideal of H if it is both a left hyperideal and a right hyperideal of H, that is $H \circ T \circ H \subseteq T$.

For a positive integer m, $H^m = \underbrace{H \circ H \circ H \circ \cdots \circ H}_{m\text{-times}}$. Moreover, $H^2 = H \circ H \subseteq H$(given by Definition 2.2). Continuing in this manner, we have $H^l \subseteq H^n$ for l, m to be two positive integers such that $l \geqslant m$. H^m is a subhypergroupoid of H for all values of m. $H^{m+n} = H^n \circ H^m$, for any two positive integers m, and n. Any subset of $H^m \times H^m$ is called an m-relation or binary m-relation.
The condition $S \circ H \subseteq S$ ($H \circ S \subseteq S$) is equivalent to $s \circ h \subseteq S$ ($h \circ s \subseteq S$) for every $s \in S$ and $h \in H$.

Definition 2.3. A hypergroupoid H is termed as right consistent if $(a \circ b) \circ H = a \circ (b \circ H)$ for all $a, b \in H$. H is called left consistent if $H \circ (a \circ b) = (H \circ a) \circ b$ for all $a, b \in H$. If H is both left and right consistent, then it is called a consistent hypergroupoid [**7**].

Definition 2.4. A hypergroupoid H is said to be intra-consistent if $(a \circ H) \circ b = a \circ (H \circ b)$ for all $a, b \in H$.

Definition 2.5. An element a of a hypergroupoid H is said to be a *weak right (left) magnifying* element of H if there exists a proper subset K of H such that $H = K \circ a (H = a \circ K)$. a is said to be a *strong right(left) magnifying* element of H if there exists a proper subhypergroupoid S of H such that $H = S \circ a (H = a \circ S)$ [**24**].

The identity and the regular elements of a hypergroupoid are not weak right or weak left magnifying elements [**12**].

Definition 2.6. A nonempty subset $L(R)$ of a hypergroupoid $(H, \circ)$ is called an m-left hyperideal (m-right hyperideal) of H if $H^m \circ L \subseteq L(R \circ H^m \subseteq R)$, where m is a positive number. Equivalently, if for each $a \in L$ ($a \in R$) and each $h \in H^m$, we have $a \circ h \subseteq L$ ($h \circ a \subseteq R$). A nonempty set T of H is called an m-two-sided hyperideal or simply m-hyperideal of H in the event that it is both m-left hyperideal and m-right hyperideal of H, that is, $H^m \circ T \circ H^m \subseteq T$.

Remark 2.1. Every left hyperideal is an m-left hyperideal of H (for $m = 1$), but the converse does not follows. Similar statements hold for the m-right hyperideals and m-hyperideals of H.

Definition 2.7. The principal m-left hyperideal generated by an element $a \in H$ is the m-left hyperideal $H^m \circ a$. The principal m-right ideal generated by a is characterized to be $a \circ H^m$. The principal two-sided m-ideal generated by a is characterized to be $H^m \circ a \circ H^m$.

3. m-consistent hypergroupoids

In this section, we first define the m-right(m-left) consistent and m-consistent hypergroupoids, and then explain their basic properties along with examples.

Definition 3.1. A hypergroupoid H is said to be m-right consistent if, for every $a, b \in H$,

$$(a \circ b) \circ H^m = a \circ (b \circ H^m).$$

∘	u	v	w
u	{u}	{u, v}	{u}
v	{u}	{v}	{u}
w	{v}	{v}	{v}

TABLE 1

∘	u	v	w	x
u	{u}	{u}	{w}	{u}
v	{w}	{v}	{u}	{w}
w	{w}	{v}	{u}	{v}
x	{w}	{v}	{w}	{u}

TABLE 2

H is called m-left consistent if, for every $a, b \in H^m$,

$$H^m \circ (a \circ b) = (H^m \circ a) \circ b.$$

If H is both m-left consistent and m-right consistent, then it is called an m-consistent hypergroupoid [**12**].

DEFINITION 3.2. A hypergroupoid H is said to be m-intra-consistent if the following proposition follows

$$\text{“ } (a \circ H^m) \circ b = a \circ (H^m \circ b) \text{ for all } a, b \in H\text{”.}$$

This is to be noted that right consistent, left consistent and consistent hypergroupoids are m-right consistent, m-left consistent and m-consistent hypergroupoids for all m, a positive integer; but the converse does not follow except for $m = 1$. This is evident from the proceeding Example 3.1.

EXAMPLE 3.1. Consider the hypergroupoid $H = \{u, v, w\}$ with the hyperoperation $\circ$ defined in Table 1. If we take $m = 2$, then $H^2 = \{u, v\}$. H is 2-right consistent, 2-left consistent and therefore 2-consistent hypergroupoid. H is also 2-intra-consistent hypergroupoid, but not right consistent as $(v \circ w) \circ H = v \circ (w \circ H)$ gives $\{u, v\} \neq \{v\}$ which is not possible.

The following Example 3.2 shows that all hypergoupoids are not m-right(m-left) consistent, m-consistent or the m-intra-consistent.

EXAMPLE 3.2. The hypergroupoid $H = \{u, v, w, x\}$ with the hyperoperation $\circ$ is descibed in Table 2. For $m = 2$, $H^2 = \{u, v, w\}$. H is not 2-right consistent because $(w \circ v) \circ H^2 = H^2$ and $w \circ \big(\{v\} \circ H^2\big) = \{v, w\}$. It is not 2-left consistent since $H^2 \circ (u \circ v) = \{u, w\}$ and $(H^2 \circ u) \circ v = \{u, v\}$ and not 2-intra-consistent as $\big(u \circ H^2\big) \circ v = \{u, v\}$ and $u \circ \big(H^2 \circ v\big) = \{u\}$.

EXAMPLE 3.3. The hypergroupoid $H = \{u, v, w, x, y\}$ with hyperoperation $\circ$ defined by Table 3 is m-right consistent, m-left consistent and m-intra-consistent for $m = 2$.

$\circ$	u	v	w	x	y
u	{u}	{u, v}	{u, w}	{u, x}	{u, v}
v	{u, v}	{u, v}	{u, v, w}	{u, v, x}	{u, v}
w	{u, w}	{u, v, w}	{u, w}	{v, w, x}	{u, v, x}
x	{u, x}	{u, v, x}	{v, w, x}	{w, x}	{u, v}
y	{w}	{w, x}	{u, w, x}	{u, v, x}	{u, v, w}

TABLE 3

4. Green m-relations in hypergroupoids

On the pattern of Green Relations [**7**], we define the following three relations in the context of m-hyperideals in order to characterize the m-consistent hypergroupoids.

DEFINITION 4.1. For a hypergroupoid H, we define the following five relations on it.

(1) A relation $\mathcal{R}^m$ on H is said to an m-right relation if for $a, b \in H$, $a\mathcal{R}^m b$ implies either $a = b$ or there exist $s, t \in H^m$ such that $a \in b \circ s$ and $b \in a \circ t$.
(2) A relation $\mathcal{L}^m$ is said to an m-left relation if for $a, b \in H$, $a\mathcal{L}^m b$ implies either $a = b$ or there exist $s, t \in H^m$ such that $a \in s \circ b$ and $b \in t \circ a$.
(3) The relation $\mathcal{T}^m$ on H is said to be an m-relation if it both an m-right relation and m-left relation. That is, it is defined by the proposition "$a\mathcal{T}^m b$ if and only if $a\mathcal{L}^m b$ and $a\mathcal{R}^m b$".

PROPOSITION 4.1. *Let H be a hypergroupoid and $a, b \in H$, then the following results follow:*

(1) If $(a \circ H^m) \cup \{a\} = (b \circ H^m) \cup \{b\}$, then $a\mathcal{R}^m b$.
(2) If $(H^m \circ a) \cup \{a\} = (H^m \circ b) \cup \{b\}$, then $a\mathcal{L}^m b$.

PROOF. If $a = b$, then $a\mathcal{R}^m b$ and $a\mathcal{L}^m b$ clearly. Suppose $a \neq b$.

(1) Then, let $(a \circ H^m) \cup \{a\} = (b \circ H^m) \cup \{b\}$. Since $a \in (b \circ H^m) \cup \{b\}$ and $a \neq b$, there exists $s \in H^m$ such that $a \in b \circ s$. Since $b \in (a \circ H^m) \cup \{a\}$ and $b \neq a$, there exists $t \in H^m$ such that $b \in a \circ t$. Since $s, t \in H^m$ such that $a \in b \circ s$ and $b \in a \circ t$, we have $a\mathcal{R}^m b$.
(2) Similarly.

□

The converse of above Proposition 4.1 follows for H to be a hypersemigroup as explained below.

PROPOSITION 4.2. *Let H be a hypersemigroup and $a\mathcal{R}^m b$ [**7**], then*

(1) if $a\mathcal{R}^m b$, then $(a \circ H^m) \cup \{a\} = (b \circ H^m) \cup \{b\}$ and
(2) if $a\mathcal{L}^m b$, then $(H^m \circ a) \cup \{a\} = (H^m \circ b) \cup \{b\}$.

PROOF. The proofs are given in the following paragraphs.

(1) Let $a\mathcal{R}^m b$ and $u \in (a \circ H^m) \cup \{a\}$. Since $a\mathcal{R}^m b$, $a = b$ or there exist $s, t \in H^m$ such that $a \in b \circ s$ and $b \in a \circ t$. If $a = b$, then (1) holds. Let $a \neq b$ and $a \in b \circ s$, $b \in a \circ t$ for some $s, t \in H^m$. Since $u \in (a \circ H^m) \cup \{a\}$, we have $u \in a \circ v$ for some $v \in H^m$ or $u = a$. We have the following two cases:

(I) $a \in b \circ s$, $b \in a \circ t$ and $u \in a \circ v$. Since H is an hypersemigroup, so $u \in a \circ v \subseteq (b \circ s) \circ \{v\} = \{b\} \circ (s \circ v)$. Since $s, v \in H^m$, $s \circ v \subseteq H^m$, and then $u \in \{b\} \circ H^m \subseteq (b \circ H^m) \cup \{b\}$.
Therefore,

$$(a \circ H^m) \cup \{a\} \subseteq (b \circ H^m) \cup \{b\}. \tag{4.1}$$

(II) $a \in b \circ s$, $b \in a \circ t$ and $u = a$. Then we have $u = a \in b \circ s \subseteq \{b\} \circ H^m \subseteq (b \circ H^m) \cup \{b\}$. So again we get $(a \circ H^m) \subseteq \{a\} \subseteq (b \circ H^m) \cup \{b\}$.

By symmetry, we get

$$(b \circ H^m) \cup \{b\} \subseteq (a \circ H^m) \cup \{a\}. \tag{4.2}$$

Joining (1I) and (1), we get

$$(a \circ H^m) \cup \{a\} = (b \circ H^m) \cup \{b\}.$$

(2) Similarly.

□

Definition 4.2. We define now the m-reflexive, m-symmetric, m-transitive and m-equivalence relations on a hypergroupoid H as follow:

(1) An m-relation $\mathcal{A}^m$ on H is said to be m-reflexive if for all $a \in H$, $a\mathcal{A}^m a$.
(2) An m-relation $\mathcal{A}^m$ is said to be m-symmetric if for $a, b \in H$, $a\mathcal{A}^m b$ implies $b\mathcal{A}^m a$.
(3) An m-relation $\mathcal{A}^m$ is said to be m-transitive if for $a, b, c \in H$, $a\mathcal{A}^m b$ and $b\mathcal{A}^m c$ implies $a\mathcal{A}^m c$.
(4) An m-relation $\mathcal{A}^m$ is said to be m-equivalence if it is also m-reflexive, m-symmetric, and m-transitive.

Remark 4.1. (1) An m-reflexive relation or reflexive m-relation are used to connote the same concepts. Therefore, we use them for the same meaning. Similar statements hold for m-symmetric, m-transitive and m-equivalence relations.
(2) This is obvious to verify that reflexive, symmetric and transitive relations are respectively m-reflexive, m-symmetric and m-transitive for all positive integer m, but the converse does not follow except for $m = 1$.

Proposition 4.3. [**7**]. *If H is a hypersemigroup, then the m-relations $\mathcal{R}^m$ and $\mathcal{L}^m$ are equivalence m-relations.*

Proof. An m-relation is an m-equivalence relation if it is m-reflexive, m-symmetric and m-transitive relation. We prove for the case of $\mathcal{R}^m$, the proof for $\mathcal{L}^m$ follows analogously.

(1) $\mathcal{R}^m$ is m-reflexive as $s\mathcal{R}^m s$ as $s = s$ for all $s \in H^m$.
(2) $a\mathcal{R}^m b$ implies either $a = b$ or there exist $s, t \in H^m$ such that $a \in b \circ s$ and $b \in a \circ t$, which clearly implies that $b\mathcal{R}^m a$. $\mathcal{R}^m$ is m-symmetric.
(3) If $a\mathcal{R}^m b$ and $b\mathcal{R}^m c$, the either $a = b$ or there exist $s, t \in H^m$ such that $a \in b \circ s$ and $b \in a \circ t$, and either $b = c$ or there exist $s, t \in H^m$ such that $b \in c \circ s$ and $c \in b \circ t$. These two results jointly imply that $a = c$ or $a \in b \circ s \subseteq (a \circ t) \circ s = a \circ (t \circ s) = a \circ l$, for some $l = t \circ s$, $l \in H^m$. Similarly, $c \in a \circ n$, for some $n \in H^m$. Therefore, $\mathcal{R}^m$ is m-transitive. Consequently, $\mathcal{R}^m$ is an m-equivalence relation.

□

Since the hypergroupoids are the natural extension of the groupoids, we can find them from the groupoids by defining a relation between the binary operation $\cdot$ and the hyperoperation $\circ$ [**5**]. In the following lines, we characterize the hypergroupoids using groupoids and *vice versa* in relation to the m-consistencies. Before this, we first formally define m-consistent and m-intra-consistent groupoids.

Definition 4.3. A groupoid $(G, \cdot)$ is said to an m-right (m-left) consistent if for any $a, b \in G$, $(ab)G^m = a(bG^m)$ $(G^m(ab) = (G^m a)b)$, for a positive integer m. The groupoid G is said to be an m-consistent if it is both an m-right and an m-left consistent.

Definition 4.4. A groupoid $(G, \cdot)$ is called an m-intra-consistent if $(aG^m)b = a(G^m b)$ for every $a, b \in G$, and m is a positive integer.

The following proposition explains how groupoids are converted into hypergroupoids using the ideas of m-right(m-left) consistencies.

Proposition 4.4. *Let $(G, \cdot)$ be an m-right (m-left) consistent or m-intra-consistent groupoid and $\circ$ is the hyperoperation on G defined by $a \circ b = \{ab\}$, then*
(1) $(G, \circ)$ is an m-right (m-left) consistent hypergroupoid,
(2) $(G, \circ)$ is an m-intra-consistent hypergroupoid.

Proof. The proofs are given below:
(1) Let $(G, \cdot)$ be an m-right consistent groupoid. We shall show that $(a \circ b) \circ G^m = \{a\} \circ (b \circ G^m)$ for all $a, b \in G$. Let $t \in (a \circ b) \circ G^m$, then $t \in u \circ h$ for some $u \in a \circ b, h \in G^m$. Moreover, $t = uh$ and $u = ab$. Since G is m-right consistent, $t = (ab)h \in (ab)G^m = a(bG^m)$. We again have $t = a(bk)$ for some $k \in G^m$. By the same reason, $t \in a(bk) = a \circ (bk) = \{a\} \circ \{bk\} = \{a\} \circ (b \circ k) \subseteq \{a\} \circ (b \circ G)$, and so

$$(a \circ b) \circ G^m \subseteq \{a\} \circ (b \circ G^m). \tag{4.3}$$

Similarly,

$$\{a\} \circ (b \circ G^m) \subseteq (a \circ b) \circ G^m. \tag{4.4}$$

Joining (4.3) and (4.4), we get

$$(a \circ b) \circ G^m = \{a\} \circ (b \circ G^m).$$

·	u	v	w	x	y
u	u	u	v	v	u
v	v	v	u	u	w
w	w	w	x	x	x
x	x	x	w	w	v
y	u	w	v	x	u

TABLE 4

Thus $(G, \circ)$ is m-right consistent. Similarly, if $(G, \cdot)$ is an m-left consistent, then we can prove that $(G, \circ)$ is m-left consistent as well.

(2) Suppose that $(G, \cdot)$ is an m-intra-consistent groupoid. In order to show that $(G, \circ)$ is an m-intra-consistent hypergroupoid, we shall show that $\{a\} \circ (G^m \circ b) \subseteq (a \circ G^m) \circ \{b\}$ for all $a, b \in G$. If $t \in (a \circ G^m) \circ \{b\}$, then $t \in u \circ b$ for some $u \in a \circ G^m$ and $u \in a \circ v$ for some $v \in G^m$. Since G is m-intra-consistent, so $t = ub = (av)b \in (aG^m)b = a(G^m b)$. Again since there exists $h \in G$ such that $t = a(hb)$, we get $t \in \{a(hb)\} = a \circ (hb) = \{a\} \circ \{hb\} = \{a\} \circ (h \circ b) \subseteq \{a\} \circ (G^m \circ b)$, and $(a \circ G^m) \circ \{b\} \subseteq \{a\} \circ (G^m \circ b)$. In a similar way, we prove that $\{a\} \circ (G^m \circ b) \subseteq (a \circ G^m) \circ \{b\}$, and so the equality holds; therefore result follows.

□

EXAMPLE 4.1. We consider the groupoid $G = \{u, v, w, x, y\}$ with the multiplication $\cdot$ defined by Table 4. In this case, $G^2 = \{u, v, w, x\}$. Taking $m = 2$, G is a 2-left consistent groupoid, as $G^m x = G^m$ for any $x \in G$ then, for any $a, b \in G$, we have $G^m(ab) = G^m$ and $(G^m a)b = G^m b = G^m$, so $G^m(ab) = (G^m a)b$ and $(G, \cdot)$ is a 2-left consistent. By Proposition 4.4, the set G with the hyperoperation defined by Table 5 is a 2-left consistent hypergroupoid. In addition, this is an example of a 2-left consistent hypergroupoid which is not 2-right consistent. In fact, we have $(v \circ x) \circ G^2 = \{1, v\}$ but $\{v\} \circ (x \circ G^2) = \{a\}$.

By interchanging rows and columns in Table 4, we get the groupoid $(G, \cdot)$ given by Table 6. We have $aG^2 = G^2$ for any $a \in G$. Thus, for any $a, b \in G$, we have $(ab)G^2 = G^2$ and $a(bG^2) = aG^2 = G^2$. Then we have $(ab)G^2 = a(bG^2)$, and G is 2-right consistent. By Proposition 4.4, the hypergroupoid $(G, \circ)$ given by Table 7 is 2-right consistent. But this is not 2-left consistent as $G^2 \circ (v \circ x) = \{w, x\}$ and $(G^2 \circ v) \circ \{x\} = \{x\}$. By Proposition 4.4, the hypergroupoids defined in Tables 5 and 7 are 2-intra-consistent also.

For the hypersemigroups, we the following proposition.

PROPOSITION 4.5. *If H is an hypersemigroup and $a, b \in H^m$, then we have*

(1) $aR^m b \iff (a * H^m) \cup \{a\} = (b * H^) \cup \{b\}$,
(2) $aL^m b \iff (H^m * a) \cup \{a\} = (H^m * b) \cup \{b\}$.

By Corollary 2.4 we have the following

∘	u	v	w	x	y
u	{u}	{u}	{v}	{v}	{u}
v	{v}	{v}	{u}	{u}	{w}
w	{w}	{w}	{x}	{x}	{x}
x	{x}	{x}	{w}	{w}	{v}
y	{u}	{w}	{v}	{x}	{u}

Table 5

·	u	v	w	x	y
u	u	v	w	x	u
v	u	v	w	x	w
w	v	u	x	w	v
x	v	u	x	w	x
y	u	w	x	v	u

Table 6

∘	u	v	w	x	y
u	{u}	{v}	{w}	{x}	{u}
v	{u}	{v}	{w}	{x}	{w}
w	{v}	{u}	{x}	{w}	{v}
x	{v}	{u}	{x}	{w}	{x}
y	{u}	{w}	{x}	{v}	{u}

Table 7

Proposition 4.6. *If H is an hypersemigroup, then the relations R^m and L^m are equivalence relations on H^m.*

Proof. (1) R^m is reflexive as xR^mx as $x = x$ for all $x \in H^m$.

(2) aR^mb implies either $a = b$ or there exist $x, y \in H^m$ such that $a \in b \circ x$ and $b \in a \circ y$, which clearly implies that bR^ma.

(3) aR^mb and bR^mc. The either $a = b$ or there exist $x, y \in H^m$ such that $a \in b \circ x$ and $b \in a \circ y$, and either $b = c$ or there exist $s, t \in H^m$ such that $b \in c \circ s$ and $c \in b \circ t$. These two results jointly impy that $a = c$ or $a \in b \circ x \subseteq (a \circ y) \circ x = a \circ (y \circ x) = a \circ l$, for some $l = y \circ x$. Similarly, $c \in a \circ n$, for some $n \in H^m$.

□

Remark 4.2. A right consistent hypergroupoid is an m-right consistent, but the converse does not follow. This is evident from the following Example 4.2.

Example 4.2. Consider the hypergroupoid $H = \{a, b, c\}$ with the hyperoperation $\circ$ defined in the following Table 8. Here $H^2 = \{a, b\}$. This is a 2-right consistent, 2-left consistent and 2-intra-consistent hypergroupoid, but it is not right consistent as $(b \circ c) * H = \{b\} * (\{c\} * H)$ gives $\{a, b\} \neq \{b\}$, which is not possible.

∘	a	b	c
a	{a}	{a, b}	{a}
b	{a}	{b}	{a}
c	{b}	{b}	{b}

Table 8

∘	a	b	c	d
a	{a}	{a}	{c}	{a}
b	{c}	{b}	{a}	{c}
c	{c}	{b}	{a}	{b}
d	{c}	{b}	{c}	{a}

Table 9

∘	a	b	c	d	e
a	{a}	{a, b}	{a, c}	{a, d}	{a, b}
b	{a, b}	{a, b}	{a, b, c}	{a, b, d}	{a, b}
c	{a, c}	{a, b, c}	{a, c}	{b, c, d}	{a, b, d}
d	{a, d}	{a, b, d}	{b, c, d}	{c, d}	{a, b}
e	{c}	{c, d}	{a, c, d}	{a, b, d}	{a, b, c}

Table 10

Example 4.3. Consider the hypergroupoid $H = \{a, b, c, d\}$ with the hyperoperation defined in Table 9 which makes it an hypergroupoid. Here $H^2 = \{a, b, c\}$. This is not 2-right consistent because $(c \circ b) * H^2 = H^2$ and $\{c\} * \big(\{b\} * H^2\big) = \{b, c\}$, not 2-left consistent since $H^2 * (a \circ b) = \{a, c\}$ and $(H^* \{a\}) * \{b\} = \{a, b\}$ and not 2-intra-consistent as $\big(\{a\} * H^2\big) * \{b\} = \{a, b\}$ and $\{a\} * \big(H^2 * \{b\}\big) = \{a\}$.

Example 4.4. The hypergroupoid defined by Table 10 is m-right consistent, m-left consistent and m-intra-consistent for $m = 2$.

Definition 4.5. A groupoid $(G, \cdot)$ is said to be m-right (resp. m-left) consistent if for any $x, y \in G$, we have $(xy)G^m = x(yG^m)$ (resp. $G^m(xy) = (G^m x)y$). G is called m-intra-consistent if $(xG^m)y = x(G^m y)$ for every $x, y \in G$.

Example 4.5. We consider the groupoid $G = \{a, b, c, d\}$ with the multiplication $\cdot$ defined by the following Table 11.

This is a 2-left consistent groupoid, as $G^m x = G^m$ for any $x \in G$ then, for any $x, y \in G$, we have $G^m(xy) = G^m$ and $(G^m x)y = G^m y = G^m$, so $G^m(xy) = (G^m x)y$ and $(G, \cdot)$ is a 2-left consistent. By Proposition 4.4, the set G with the hyperoperation defined by Table 12 is a 2-left consistent hypergroupoid.

In addition, this is an example of a 2-left consistent hypergroupoid which is not 2-right consistent. In fact, we have $(b \circ d) * G^2 = \{a, b\}$ but $\{b\} * (d * G^2) = \{a\}$.

·	a	b	c	d	e
a	a	a	b	b	a
b	b	b	a	a	c
c	c	c	d	d	d
d	d	d	c	c	b
e	a	c	b	d	a

TABLE 11

∘	a	b	c	d	e
a	{a}	{a}	{b}	{b}	{a}
b	{b}	{b}	{a}	{a}	{c}
c	{c}	{c}	{d}	{d}	{d}
d	{d}	{d}	{c}	{c}	{b}
e	{a}	{c}	{b}	{d}	{a}

TABLE 12

·	a	b	c	d	e
a	a	b	c	d	a
b	a	b	c	d	c
c	b	a	d	c	b
d	b	a	d	c	d
e	a	c	d	b	a

TABLE 13

∘	a	b	c	d	e
a	{a}	{b}	{c}	{d}	{a}
b	{a}	{b}	{c}	{d}	{c}
c	{b}	{a}	{d}	{c}	{b}
d	{b}	{a}	{d}	{c}	{d}
e	{a}	{c}	{d}	{b}	{a}

TABLE 14

By interchanging rows and columns in Table 11, we get the groupoid $(G, \cdot)$ given by the following Table 13.

We have $xG^2 = G^2$ for any $x \in G$. Thus, for any $x, y \in G$, we have $(xy)G^2 = G^2$ and $x(yG^2) = xG^2 = G^2$. Then we have$(xy)G^2 = x(yG^2)$, and G is 2-right consistent. By Proposition 4.4, the hypergroupoid $(G, \circ)$ given by Table 14 is 2-right consistent.

But this is not 2-left consistent as $G^2 * (b \circ d) = \{c, d\}$ and $(G^2 * b) * \{d\} = \{d\}$. By Proposition 4.4, the hypergroupoids defined in Tables 12 and 14 are 2-intra-consistent also.

PROPOSITION 4.7. *Let H be an m-right consistent hypergroupoid and $aR^m b$. Then $(a * H^m) \cup \{a\} = (b * H^m) \cup \{b\}$.*

PROOF. The result follows for $a = b$ immediately. For $a \neq b$, then since $aR^m b$, there exist $x, y \in H^m$ such that $a \in b \circ x$ and $b \in a \circ y$. Then $a * H^m \subseteq (b \circ x) * H^m = \{b\} * (x * H^m) \subseteq \{b\} * H^m \subseteq (a \circ y) * H^m = \{a\} * (y * H^m) \subseteq \{a\} * H^m$, and so $a * H^m = b * H^m$.
Hence we obtain $(a * H^m) \cup \{a\} = (b * H^m) \cup \{a\} \subseteq (b * H^m) \cup (b * H^m) = b * H^m \subseteq (b * H^m) \cup \{b\}$.
and
$(b * H^m) \cup \{b\} = (a * H^m) \cup \{b\} \subseteq (a * H^m) \cup (a * H^m) = a * H^m \subseteq (a * H^m) \cup \{a\}$.
and then $(a * H^m) \cup \{a\} = (b * H^m) \cup \{b\}$. □

COROLLARY 4.1. *If H is an m-left consistent hypergroupoid and aL^m, then $(H^m * a) \subseteq \{a\} = (H^m * b) \subseteq \{b\}(*)$*

COROLLARY 4.2. *If H is an m-right (resp. an m-left) consistent hypergroupoid then, for every $a, b \in H$, then $aR^m b$ if and only if $(a * H^m) \subseteq \{a\} = (b * H^m) \subseteq \{b\}$ (resp. $aL^m b$ if and only if $(H^m * a) \subseteq \{a\} = (H^m * b) \subseteq \{b\}$).*

PROPOSITION 4.8. *Let H be an m-intra-consistent hypergroupoid and $aR^m b$. Then $(a * H^m) \subseteq \{a\} = (b * H^m) \subseteq \{b\}$*

PROOF. The result holds for $a = b$. If $a \neq b$, then there exist $x, y \in H^m$ such that $a \in b \circ x$ and $b \in a \circ y$. If $t \in a * H^m$, then $t \in a \circ u$ for some $u \in H^m$, then

$$
\begin{aligned}
t \in a \circ u &\subseteq (b \circ x) * \{u\} \subseteq (b * H^m) * \{u\} \\
&= \{b\} * (H^m * u)(\text{since } H \text{ is } m\text{-intra-consistent}). \\
&\subseteq \{b\} * (H^m * H^m) \subseteq b * H^m
\end{aligned}
$$

so $a * H^m \subseteq b * H^m$. Similarly $b * H^m \subseteq a * H^m$ and so $a * H^m = b * H^m$. Then as before in the proof of Proposition 4.7, we have $(a * H^m) \cup \{a\} = (b * H^m) \cup \{b\}$. □

COROLLARY 4.3. *Let H be an m-intra-consistent hypergroupoid. Then we have $aR^m b$ if and only if $(a * H^m) \cup \{a\} = (b * H^m) \cup \{b\}$.*

PROPOSITION 4.9. *If H is a m-right (resp. m-left) consistent hypergroupoid then, for every $a \in H$, the set $a * H^m$ (resp. $H^m * a$) is a subgroupoid of H.*

PROOF. Let H be an m-right consistent hypergroupoid and $x, y \in a * H^m$. Then $x \in a \circ u$ and $y \in a \circ v$ for some $u, v \in H^m$. Then we have

$$
\begin{aligned}
x \circ y &\subseteq (a \circ u) * (a \circ v) \subseteq (a \circ u) * (H^m * H^m) \subseteq (a \circ u) * H^m \\
&= \{a\} * (u * H^m)(\text{since } H \text{ is } m\text{-right consistent}) \\
&\subseteq \{a\} * (H^m * H^m) \subseteq a * H^m
\end{aligned}
$$

so $a * H^m$ is a subgroupoid of H. □

PROPOSITION 4.10. *If an hypergroupoid H is m-right consistent (resp. m-left consistent) then, for any nonempty subset A of H, the set $A \cup (A * H^m)$ (resp. $A \cup (H^m * A)$) is the m-right (resp. m-left) ideal of H generated by A.*

PROOF. Let H be m-right consistent. Clearly, the set $A \cup (A * H^m)$ is a nonempty subset of H containing A. Moreover, $(A \cup (A * H^m)) * H^m \subseteq A \cup (A * H^m)$. Let $t \in (A \cup (A * H^m) * H^m$. Then $t \in u \circ h$ for some $u \in A \cup (A * Hm)$, $h \in H^m$. If $u \in A$, then $t \in u \circ h \subseteq A * H^m \subseteq A \cup (A * H^m)$. Let $u \in A * H^m$. Then $u \in a \circ v$ for some $a \in A$, $v \in H^m$. Then we have $t \in u \circ h \subseteq (a \circ v) * H^m = \{a\} * (v * H)^m$ (since H is m-right consistent) $\subseteq A * (H^m * H^m) \subseteq A * H^m \subseteq A \cup (A * H^m)$; thus $A \cup (A * H^m)$ is an m-right ideal of H. Let now T be an m-right ideal of H such that $T \supseteq A$. Then we have $A \cup (A * H^m) \subseteq T \cup (T * H^m) \subseteq T$ and so the set $A \cup (A * H^m)$ is the m-right ideal of H generated by A. If H is m-left consistent, then in a similar way we prove that the set $A \cup (H^m * A)$ is the m-left ideal of H generated by A. □

COROLLARY 4.4. *If an hypergroupoid H is m-right consistent (resp. m-left consistent) then, for any $a \in H$, the set $\{a\} \cup (a * H^m)$ (resp. $\{a\} \cup (H^m * a)$) is the m- right (resp. m-left) ideal of H generated by A.*

As a result, if H is an m-right or an m-left consistent hypergroupoid then, for every $a \in H$, the sets $\{a\} \cup (a * H^m)$ and $\{a\} \cup (H^m * a)$ are subgroupoids of H.

REMARK 4.3. We denote by $R^m(A)$ (resp. $L^m(A)$) the m-right (resp. m-left) ideal of H generated by A. For $A = \{a\}$, we write $R^m(a)$, $L^m(a)$ instead of $R^m(\{a\})$, $L^m(\{a\})$.

COROLLARY 4.5. *If an hypergroupoid H is m-right consistent (resp. m-left consistent) then, for every nonempty subset A of H, we have $R^m(A) = A \cup (A * H^m)$ (resp. $L^m(A) = A \cup (H^m * A)$).*

COROLLARY 4.6. *If H is a m-right consistent or m-intra-consistent hypergroupoid, then aR^mb if and only if $R^m(a) = R^m(b)$. If H is a m-left consistent hypergroupoid, then aL^mb if and only if $L^m(a) = L^m(b)$.*

By Propositions 3.9, 3.10 and 3.12 or by Corollary 3.19. we have the following

THEOREM 4.1. *If an hypergroupoid H is m-right (m-left) consistent or m-intra-consistent, then the relations R^m and L^m are equivalence relations on H^m.*

PROPOSITION 4.11. *A commutative hypergroupoid H is m-right consistent if and only if it is m-left consistent and therefore m-consistent.*

PROOF. Let H be m-right consistent and $x, y \in H$. Then we have

$$\begin{aligned} H^m * (x \circ y) &= (x \circ y) * H^m \\ &= (y \circ x) * H^m \text{(since } H \text{ is commutative)} \\ &= \{y\} * (x * H) \text{(since } H \text{ is } m\text{-right consistent)} \\ &= \{y\} * (H * x) = (H * x) * \{y\} \text{(since } H \text{ is commutative)} \end{aligned}$$

·	a	b	c	d
a	c	b	c	c
b	b	b	c	b
c	c	c	c	c
d	c	b	c	b

TABLE 15

so $H * (x \circ y) = (H * x) * \{y\}$, and H is m-left consistent.
Conversely, let Hbe an m-left consistent and $x, y \in H$. Then we have $(x \circ y) * H = H * (y \circ x) = (H * y) * \{x\} = \{x\} * (y * H)$, so $(x \circ y) * H = \{x\} * (y * H)$, and H is m-right consistent. □

PROPOSITION 4.12. *Let H be a commutative hypergroupoid. If H is an m-right (resp. m-left) consistent, then H is an m-intra-consistent. The converse statement does not hold in general. However, there are commutative m-intra-consistent hypergroupoids that are m-consistent.*

PROOF. Let H be an m-right consistent and $x, y \in H$. Then we have

$$\begin{aligned}(x * H^m) * \{y\} &= \{y\} * (x * H^m)(\text{since } H \text{ is commutative})\\ &= (y \circ x) * H^m(\text{since } H \text{ is } m\text{-right consistent})\\ &= (x \circ y) * H^m(\text{since } H \text{ is commutative})\\ &= \{x\} * (y * H^m)(\text{since } H \text{ is } m\text{-right consistent})\\ &= \{x\} * (H^m * y)(\text{since } H \text{ is commutative}),\end{aligned}$$

so $(x * H^m) * \{y\} = \{x\} * (H^m * y)$, and H is m-intra-consistent. Let now H be m-left consistent and $xy \in H$. Then we have
$(x * H^m) * \{y\} = (H^m * x) * \{y\} = H^m * (x \circ y) = H^m * (y \circ x)$
$= (H^m * y) * \{x\} = \{x\} * (H^m * y)$,
and again H is m-intra-consistent. □

For the converse statement we give the following example.

EXAMPLE 4.6. We consider the commutative groupoid $G = \{a, b, c, d\}$ given by Table 15. For $m = 2$, $G^2 = \{a, b, c\}$

One can check 16 cases to see that this is an m-intra-consistent groupoid. According to Proposition 3.7, the set G with the hyperoperation defined by Table 16 is an m-intra-consistent commutative hypergroupoid.

This is not m-right regular because $(a \circ a) * H^2 = \{c\}$ and $\{a\} * (a * H^2) = \{b, c\}$. As $(G, \circ)$ is commutative, this is not m-left consistent as well.

We prove the last part of the proposition by the following example.

EXAMPLE 4.7. Applying Proposition 4.4 in the groupoid given in following example, we get the hypergroupoid $G = \{a, b, c, d\}$ defined by Table 17. This is a commutative, m-intra-consistent and m-consistent hypergroupoid.

○	a	b	c	d
a	{c}	{b}	{c}	{c}
b	{b}	{b}	{c}	{b}
c	{c}	{c}	{c}	{c}
d	{c}	{b}	{c}	{b}

Table 16

○	a	b	c	d
a	{c}	{c}	{a}	{b}
b	{c}	{a}	{a}	{b}
c	{a}	{a}	{c}	{c}
d	{b}	{b}	{c}	{c}

Table 17

Lemma 4.1. *If H is an hypergroupoid then, for any nonempty subsets A, B, C of H, we*

(1) $A * (B \cup C) = (A * B) \cup (A * C)$ *and*
(2) $(A \cup B) * C = (A * C) \cup (B * C)$.

Proof. (1) Since $B \cup C \supseteq B, C$, we have $A * (B \cup C) \supseteq A * B, A * C$, so $A * (B \cup C) \supseteq (A * B) \cup (A * C)$. Let $x \in (A * B) \cup (A * C)$. If $x \in A * B$, then $x \in a \circ b$ for some $a \in A, b \in B \subseteq B \cup C$, so $x \in a \circ b \subseteq A * (B \cup C)$. If $x \in A * C$, then $x \in a \circ c$ for some $a \in A, c \in C$, then again $x \in A * (B \cup C)$. So result follows.
(2) The proof is similar.

□

Proposition 4.13. *If an hypergroupoid H is m-right (resp. m-left) consistent, $(a, b) \in R^m$ (resp. $(a, b) \in L^m$) and $c \in H$, then we have $R^m(c \circ a) = R^m(c \circ b)$ (resp. $L^m(a \circ c) = L^m(b \circ c)$).*

Proposition 4.14. *Let H be m-right consistent, $(a, b) \in R^m$ and $c \in H$. Then $R^m(c \circ a) = R^m(c \circ b)$. Indeed:*

$$\begin{aligned} R(c \circ a) &= (c \circ a) \cup ((c \circ a) * H^m) \text{ (by Corollary 3.18)} \\ &= (\{c\} * \{a\}(\cup (\{c\} * (a * H^m)) \text{ (since } H \text{ is right consistent)} \\ &= \{c\} * (\{a\} \cup (a * H^m)) \text{ (by Lemma 3.23)} \\ &= \{c\} * (\{b\} \cup (b * H^m)) \text{ (by Proposition 3.9)} \\ &= (\{c\} * \{b\}(\cup (\{c\} * (b * H^m)) \text{ (by Lemma 3.23)} \\ &= (c \circ b) \cup ((c \circ b) * H^m) \text{ (since } H \text{ is m-right consistent)} \\ &= R(c \circ b) \text{(by Corollary 3.18).} \end{aligned}$$

PROPOSITION 4.15. *If H is a m-right consistent hypergroupoid such that $x \in x \circ x$ for every $x \in H$ then, for every $a \in H$, we have $R^m(a) = R^m(a \circ a)$.*

PROOF. Let $a \in H$. Then, we have

$$\begin{aligned} R^m(a \circ a) &= (a \circ a) \cup ((a \circ a) * H^m) \text{ (by Corollary 3.18)} \\ &= (\{a\} * \{a\}(\cup (\{a\} * (a * H^m)) \text{ (since } H \text{ is right consistent)} \\ &\subseteq (a * H^m) \cup (\{a\} * (H^m * H^m)) \subseteq (a * H^m) \cup (a * H^m) \\ &= (a * H^m) \subseteq \{a\} \cup (a * H^m) \\ &= R^m(a) \end{aligned}$$

and $R^m(a) = \{a\} \cup (a * H^m) \subseteq (a \circ a) \cup ((a \circ a) * H^m) = R^m(a \circ a)$, so $R^m(a) = R^m(a \circ a)$. □

In a similar way we have the following

PROPOSITION 4.16. *If H is a m-left consistent hypergroupoid such that $x \in x \circ x$ for every $x \in H$ then, for every $a \in H$, we have $L^m(a) = L^m(a \circ a)$.*

REMARK 4.4. If H is a commutative hypergroupoid and $a, b \in H$, then we have $R^m(a) = L^m(a)$ and $R^m(a \circ b) = (a \circ b) \cup (a \circ b) * H^m = (b \circ a) \cup (b \circ a) * H^= R^m(b \circ a)$

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Received by editors 8.8.2022; Revised version 19.5.2023; Available online 11.6.2023.

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