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POSSIBILITY PYTHAGOREAN NEUTROSOPHIC SOFT SETS AND ITS APPLICATION OF DECISION MAKING

M. Palanikumar, K. Arulmozhi and T. Gunasekar

Abstract. In the present communication, we discuss the theory of possibility Pythagorean neutrosophic soft set(shortly PPNSSS), possibility neutrosophic soft set(shortly PNSSS), and define some related operations such as complement, union, intersection, AND, and OR. Also, we study interact commutative law's, De Morgan's laws, associative laws, and distributive laws of holds for PPNSSSs. The possibility Pythagorean neutrosophic soft set is a new generalization of Pythagorean soft set and neutrosophic soft set. To compare PPNSSSs and PNSSSs for dealing with decision making problems and find a similarity measure is obtained. Practical examples are provided to strengthen our results.

1. Introduction

Decision making is defining the alternatives and choosing one of them by applying certain criteria. Decision making, in short, is to choose one from different alternatives. Effective decision making ability is closely linked with creative and critical thinking abilities. Creative thinking is needed to produce the necessary alternatives to choose from in decision making and critical thinking to evaluate these alternatives. Decision support consultants are employed or decision support systems are implemented in order to support decision-making in an organization. This assumes that the way in which decision making actually takes place in the organization is understood. Decision making is one of the most important abilities because people are always in the position of making decisions both in their private lives such as where to live, which job to choose, and in social issues such as which

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leader to elect and which team to support. Fuzzy set is used to the uncertainty using the membership grade [**30**], intuitionistic fuzzy set [**7**]. Neutrosophic set is used to uncertainty using the truth, indeterminacy and falsity membership grades by Smarandache [**25**] and Pythagorean fuzzy set [**29**]. Zadeh was introduced by fuzzy set suggests that decision makers are to be solving uncertain problems by considering membership degree. After, the concept of intuitionistic fuzzy set is introduced by Atanassov and is characterized by a degree of membership and non-membership satisfying the condition that sum of its membership degree and non membership degree is not exceed one [**7**]. However, we may interact a problem in decision making events where the sum of the degree of membership and non membership of a particular attribute is exceed one. So Yager was introduced by the concept of Pythagorean fuzzy sets. It has been to extended the intuitionistic fuzzy sets and characterized by the condition that square sum of its degree of membership and non membership is not exceed one. Molodtsov [**18**] proposed the theory of soft sets. In comparison with other uncertain theories, soft sets more accurately reflect the objectivity and complexity of decision making during actual situations. Moreover, the combination of soft sets with other mathematical models is also a critical research area. For example, Maji et al. proposed by the concept of fuzzy soft set [**16**] and intuitionistic fuzzy soft set [**17**]. These two theories are applied to solve various decision making problems. Alkhazaleh et al. [**1**] defined the concept of possibility fuzzy soft sets.

In recent years, Peng et al. [**24**] has extended fuzzy soft set to Pythagorean fuzzy soft set. This model solved a class of multi attribute decision making consists sum of the degree of membership and non membership value is exceed one but the sum of the squares is equal or not exceed one. In general, the possibility degree of belongingness of the elements should be considered in multi attribute decision making problems. However, Peng et al. [**24**] failed to do it. As for the problem, the purpose of this paper is to extend the concept of possibility Pythagorean fuzzy soft set to parameterization of possibility Pythagorean neutrosophic set. We obtain a possibility Pythagorean neutrosophic soft set model. We shall further establish a similarity measure method based on this model and apply it to decision making problems by suitable examples.

The paper is organized into six sections as follows. Section 1 is the introduction followed by Section 2 deal with basic concepts. Section 3 presents the possibility Pythagorean neutrosophic soft set of its properties with examples. Section 4 introduces the notion of similarity measure between PPNSSS. Section 5 is the comparative studies for PPNSSS and PNSSS. Concluding and further investigation is provided in Section 6.

2. Basic concepts

Definition 2.1. [**8**] A neutrosophic set A in the universe U is an object having the following form : $\widehat{A} = \{u, \xi_A^T(u), \xi_A^I(u), \xi_A^F(u) | u \in U\}$, where $\xi_A^T(u)$, $\xi_A^I(u)$ $\xi_A^F(u)$ represents the degree of truth-membership, degree of indeterminacy membership and degree of falsity-membership of A respectively. The mapping $\xi_A^T, \xi_A^I, \xi_A^F : U \to [0,1]$ and $0 \leqslant \xi_A^T(u) + \xi_A^I(u) + \xi_A^F(u) \leqslant 3$.

Definition 2.2. [**13**] A Pythagorean neutrosophic set (PNSS) A in U is of the form : $\widehat{A} = \{u, \xi_A^T(u), \xi_A^I(u), \xi_A^F(u) | u \in U\}$, where $\xi_A^T(u)$, $\xi_A^I(u)$ $\xi_A^F(u)$ represents the degree of truth-membership, degree of indeterminacy membership and degree of falsity-membership of A respectively. The mapping $\xi_A^T, \xi_A^I, \xi_A^F : U \to [0,1]$ and $0 \leqslant (\xi_A^T(u))^2 + (\xi_A^I(u))^2 + (\xi_A^F(u))^2 \leqslant 2$. Since $\widehat{A} = \langle \xi_A^T, \xi_A^I, \xi_A^F \rangle$ is called a Pythagorean neutrosophic number(PNSN).

Definition 2.3. [**8, 13**] Given that $\widehat{\beta_1} = \langle \xi_{\beta_1}^T, \xi_{\beta_1}^I, \xi_{\beta_1}^F \rangle$, $\widehat{\beta_2} = \langle \xi_{\beta_2}^T, \xi_{\beta_2}^I, \xi_{\beta_2}^F \rangle$ and $\widehat{\beta_3} = \langle \xi_{\beta_3}^T, \xi_{\beta_3}^I, \xi_{\beta_3}^F \rangle$ are any three PNSNs over (U, E). Then

(i) $\widehat{\beta_1}^c = \langle \xi_{\beta_1}^F, \xi_{\beta_1}^I, \xi_{\beta_1}^T \rangle$,

(ii) $\widehat{\beta_2} \sqcup \widehat{\beta_3} = \Big\langle \max(\xi_{\beta_2}^T, \xi_{\beta_3}^T), \min(\xi_{\beta_2}^I, \xi_{\beta_3}^I), \min(\xi_{\beta_2}^F, \xi_{\beta_3}^F) \Big\rangle$,

(iii) $\widehat{\beta_2} \sqcap \widehat{\beta_3} = \Big\langle \min(\xi_{\beta_2}^T, \xi_{\beta_3}^T), \max(\xi_{\beta_2}^I, \xi_{\beta_3}^I), \max(\xi_{\beta_2}^F, \xi_{\beta_3}^F) \Big\rangle$,

(iv) $\widehat{\beta_2} \geqslant \widehat{\beta_3}$ iff $\xi_{\beta_2}^T \geqslant \xi_{\beta_3}^T$ and $\xi_{\beta_2}^I \leqslant \xi_{\beta_3}^I$ and $\xi_{\beta_2}^F \leqslant \xi_{\beta_3}^F$,

(v) $\widehat{\beta_2} = \widehat{\beta_3}$ iff $\xi_{\beta_2}^T = \xi_{\beta_3}^T$ and $\xi_{\beta_2}^I = \xi_{\beta_3}^I$ and $\xi_{\beta_2}^F = \xi_{\beta_3}^F$.

Definition 2.4. Let U be a non-empty set of the universe and E be a set of parameter. The pair $(\widehat{\mathscr{F}}, A)$ is called a neutrosophic soft set(NSSS) on U if $A \sqsubseteq E$ and $\widehat{\mathscr{F}} : A \to \widehat{\mathscr{F}(U)}$, where $\widehat{\mathscr{F}(U)}$ is the set of all neutrosophic subsets of U.

Definition 2.5. [**24**] Let U be a non-empty set of the universe and E be a set of parameter. The pair $(\mathscr{F}, A)$ is called a Pythagorean fuzzy soft set(PFSS) on U if $A \sqsubseteq E$ and $\mathscr{F} : A \to P\mathscr{F}(U)$, where $P\mathscr{F}(U)$ is the set of all Pythagorean fuzzy subsets of U.

Definition 2.6. [**1**] Let U be a non-empty set of the universe and E be a set of parameter. The pair (U, E) is a soft universe. Consider the mapping $\mathscr{F} : E \to \mathscr{F}(U)$ and ξ be a fuzzy subset of E, ie. $\xi : E \to \mathscr{F}(U)$. Let $\mathscr{F}_\xi : E \to \mathscr{F}(U) \times \mathscr{F}(U)$ be a function defined as $\mathscr{F}_\xi(e) = (\mathscr{F}(e)(u), \xi(e)(u)), \forall u \in U$. Then $\mathscr{F}_\xi$ is called a possibility fuzzy soft set(PFSS) on (U, E).

3. Possibility Pythagorean neutrosophic soft set (PPNSSS)

Definition 3.1. Let U be a non-empty set of the universe and E be a set of parameter. The pair (U, E) is a soft universe. Consider the mapping $\widehat{\mathscr{F}} : E \to \widehat{\mathscr{F}(U)}$ and ξ be a neutrosophic subset of E, ie. $\widehat{\xi} : E \to \widehat{\mathscr{F}(U)}$. Let $\widehat{\mathscr{F}_\xi} : E \to \widehat{\mathscr{F}(U)} \times \widehat{\mathscr{F}(U)}$ be a function defined as $\widehat{\mathscr{F}_\xi(e)} = (\widehat{\mathscr{F}(e)(u)}, \widehat{\xi(e)(u)}), \forall u \in U$. Then $\widehat{\mathscr{F}_\xi}$ is called a PNSSS on (U, E).

Definition 3.2. Let U be a non-empty set of the universe and E be a set of parameter. The pair $(\widehat{\mathscr{F}}, A)$ is called a Pythagorean neutrosophic soft set on U if $A \sqsubseteq E$ and $\widehat{\mathscr{F}} : A \to \widehat{P\mathscr{F}(U)}$, where $\widehat{P\mathscr{F}(U)}$ is the set of all Pythagorean neutrosophic subsets of U.

EXAMPLE 3.1. A set of three patient's for cold infection $U = \{u_1, u_2, u_3\}$ and a set of parameter $E = \{e_1=$ Runny nose, $e_2=$ lung infection, $e_3=$ cough$\}$. Suppose that $\widehat{\mathscr{F}} : E \to \widehat{NSP\mathscr{F}}(U)$ is given by

$$\widehat{\mathscr{F}_p(e_1)} = \left\{ \begin{array}{c} \frac{u_1}{\langle 0.7,0.8,0.6\rangle} \\ \frac{u_2}{\langle 0.6,0.5,0.4\rangle} \\ \frac{u_3}{\langle 0.4,0.6,0.5\rangle} \end{array} \right\} \quad ; \widehat{\mathscr{F}_p(e_2)} = \left\{ \begin{array}{c} \frac{u_1}{\langle 0.4,0.8,0.6\rangle} \\ \frac{u_2}{\langle 0.5,0.6,0.8\rangle} \\ \frac{u_3}{\langle 0.5,0.8,0.7\rangle} \end{array} \right\} \quad ;$$

$$\widehat{\mathscr{F}_p(e_3)} = \left\{ \begin{array}{c} \frac{u_1}{\langle 0.3,0.6,0.8\rangle} \\ \frac{u_2}{\langle 0.7,0.6,0.9\rangle} \\ \frac{u_3}{\langle 0.6,0.4,0.7\rangle} \end{array} \right\}$$

Matrix form: $\begin{bmatrix} \langle 0.7, 0.8, 0.6\rangle & \langle 0.6, 0.5, 0.4\rangle & \langle 0.4, 0.6, 0.5\rangle \\ \langle 0.4, 0.8, 0.6\rangle & \langle 0.5, 0.6, 0.8\rangle & \langle 0.5, 0.8, 0.7\rangle \\ \langle 0.3, 0.6, 0.8\rangle & \langle 0.7, 0.6, 0.9\rangle & \langle 0.6, 0.4, 0.7\rangle \end{bmatrix}$

DEFINITION 3.3. Let U be a non-empty set of the universe and E be a set of parameter. The pair (U, E) is called a soft universe. Suppose that $\widehat{\mathscr{F}} : E \to \widehat{P\mathscr{F}(U)}$, and $\widehat{p}$ is a Pythagorean neutrosophic subset of E. That is $\widehat{p} : E \to \widehat{P\mathscr{F}(U)}$, where $\widehat{P\mathscr{F}(U)}$ denotes the collection of all Pythagorean neutrosophic subsets of U. If $\widehat{\mathscr{F}_p}$: $E \to \widehat{P\mathscr{F}(U)} \times \widehat{P\mathscr{F}(U)}$ is a function defined as $\widehat{\mathscr{F}_p}(e) = (\widehat{\mathscr{F}(e)(u)}, \widehat{p(e)(u)}), u \in U$, then $\widehat{\mathscr{F}_p}$ is a PPNSSS on (U, E). For each parameter e,
$\widehat{\mathscr{F}_p}(e) = \Big\{\Big\langle u, (\xi^T_{\mathscr{F}}(e)(u), \xi^I_{\mathscr{F}}(e)(u), \xi^F_{\mathscr{F}}(e)(u)), (\xi^T_p(e)(u), \xi^I_p(e)(u), \xi^F_p(e)(u))\Big\rangle,\ u \in U\Big\}$.

EXAMPLE 3.2. Let $U = \{u_1, u_2, u_3\}$ be a set of three heart patient's under treatment of a decision maker to heaviest heart effect, $E = \{e_1 =$ hyper tension, $e_2 =$ highly blood pressure, $e_3 =$ weight loss $\}$ is a set of parameters. Suppose that $\widehat{\mathscr{F}_p} : E \to \widehat{P\mathscr{F}(U)} \times \widehat{P\mathscr{F}(U)}$ is given by

$$\widehat{\mathscr{F}_p(e_1)} = \left\{ \begin{array}{c} \frac{u_1}{\langle (0.7,0.8,0.6),\ (0.8,0.7,0.6)\rangle} \\ \frac{u_2}{\langle (0.6,0.5,0.4),\ (0.5,0.4,0.1)\rangle} \\ \frac{u_3}{\langle (0.4,0.6,0.5),\ (0.6,0.4,0.2)\rangle} \end{array} \right\} \quad ;$$

$$\widehat{\mathscr{F}_p(e_2)} = \left\{ \begin{array}{c} \frac{u_1}{\langle (0.4,0.8,0.6),\ (0.7,0.5,0.4)\rangle} \\ \frac{u_2}{\langle (0.5,0.6,0.8),\ (0.5,0.3,0.2)\rangle} \\ \frac{u_3}{\langle (0.5,0.8,0.7),\ (0.8,0.6,0.5)\rangle} \end{array} \right\} \quad ;$$

$$\widehat{\mathscr{F}_p(e_3)} = \left\{ \begin{array}{c} \frac{u_1}{\langle (0.3,0.6,0.8),\ (0.9,0.7,0.5)\rangle} \\ \frac{u_2}{\langle (0.7,0.6,0.9),\ (0.8,0.6,0.4)\rangle} \\ \frac{u_3}{\langle (0.6,0.4,0.7),\ (0.5,0.4,0.3)\rangle} \end{array} \right\} \quad ;$$

Matrix form of $\widehat{\mathscr{F}_p}$ written as:

$$\begin{bmatrix} \langle(0.7,0.8,0.6),(0.8,0.7,0.6)\rangle & \langle(0.6,0.5,0.4),(0.5,0.4,0.1)\rangle & \langle(0.4,0.6,0.5),(0.6,0.4,0.2)\rangle \\ \langle(0.4,0.8,0.6),(0.7,0.5,0.4)\rangle & \langle(0.5,0.6,0.8),(0.5,0.3,0.2)\rangle & \langle(0.5,0.8,0.7),(0.8,0.6,0.5)\rangle \\ \langle(0.3,0.6,0.8),(0.9,0.7,0.5)\rangle & \langle(0.7,0.6,0.9),(0.8,0.6,0.4)\rangle & \langle(0.6,0.4,0.7),(0.5,0.4,0.3)\rangle \end{bmatrix}$$

DEFINITION 3.4. Let U be a non-empty set of the universe and E be a set of parameter. Suppose that $\widehat{\mathscr{F}_p}$ and $\widehat{\mathscr{G}_q}$ are two PPNSSSs on (U,E). Now $\widehat{\mathscr{G}_q}$ is a possibility Pythagorean neutrosophic soft subset of $\widehat{\mathscr{F}_p}$ (denoted by $\widehat{\mathscr{G}_q} \sqsubseteq \widehat{\mathscr{F}_p}$) if and only if

(i) $\widehat{\mathscr{G}(e)(u)} \sqsubseteq \widehat{\mathscr{F}(e)(u)}$ if $\xi^T_{\mathscr{F}}(e)(u) \geqslant \xi^T_{\mathscr{G}}(e)(u)$, $\xi^I_{\mathscr{F}}(e)(u) \leqslant \xi^I_{\mathscr{G}}(e)(u)$, $\xi^F_{\mathscr{F}}(e)(u) \leqslant \xi^F_{\mathscr{G}}(e)(u)$,
(ii) $\widehat{q(e)(u)} \sqsubseteq \widehat{p(e)(u)}$ if $\xi^T_p(e)(u) \geqslant \xi^T_q(e)(u)$, $\xi^I_p(e)(u) \leqslant \xi^I_q(e)(u)$, $\xi^F_p(e)(u) \leqslant \xi^F_q(e)(u)$, $\forall\, e \in E$ and $\forall\, u \in U$.

EXAMPLE 3.3. Consider the PPNSSS $\widehat{\mathscr{F}_p}$ over (U,E) in Example 3.2. Let $\widehat{\mathscr{G}_q}$ be another PPNSSS over (U,E) defined as:

$$\widehat{\mathscr{G}_q(e_1)} = \left\{ \frac{u_1}{\langle(0.6,0.9,0.8),\ (0.5,0.8,0.7)\rangle} \frac{u_2}{\langle(0.5,0.6,0.7),\ (0.4,0.6,0.5)\rangle} \frac{u_3}{\langle(0.4,0.8,0.6),\ (0.5,0.7,0.8)\rangle} \right\};$$

$$\widehat{\mathscr{G}_q(e_2)} = \left\{ \frac{u_1}{\langle(0.3,0.9,0.7),\ (0.6,0.8,0.7)\rangle} \frac{u_2}{\langle(0.4,0.7,0.9),\ (0.2,0.5,0.4)\rangle} \frac{u_3}{\langle(0.3,0.9,0.8),\ (0.4,0.7,0.9)\rangle} \right\};$$

$$\widehat{\mathscr{G}_q(e_3)} = \left\{ \frac{u_1}{\langle(0.2,0.8,0.9),\ (0.5,0.8,0.8)\rangle} \frac{u_2}{\langle(0.6,0.7,0.9),\ (0.3,0.7,0.6)\rangle} \frac{u_3}{\langle(0.4,0.6,0.8),\ (0.2,0.6,0.5)\rangle} \right\};$$

Matrix form of $\widehat{\mathscr{G}_q}$ written as:

$$\begin{bmatrix} \langle(0.6,0.9,0.8),\ (0.5,0.8,0.7)\rangle & \langle(0.5,0.6,0.7),\ (0.4,0.6,0.5)\rangle & \langle(0.4,0.8,0.6),(0.5,0.7,0.8)\rangle \\ \langle(0.3,0.9,0.7),\ (0.6,0.8,0.7)\rangle & \langle(0.4,0.7,0.9),(0.2,0.5,0.4)\rangle & \langle(0.3,0.9,0.8),(0.4,0.7,0.9)\rangle \\ \langle(0.2,0.8,0.9),\ (0.5,0.8,0.8)\rangle & \langle(0.6,0.7,0.9),(0.3,0.7,0.6)\rangle & \langle(0.4,0.6,0.8),(0.2,0.6,0.5)\rangle \end{bmatrix}.$$

DEFINITION 3.5. Let U be a non-empty set of the universe and E be a set of parameter. Suppose that $\widehat{\mathscr{F}_p}$ and $\widehat{\mathscr{G}_q}$ are two PPNSSSs on (U,E). Now $\widehat{\mathscr{F}_p}$ and $\widehat{\mathscr{G}_q}$ are PPNSSS equal (denoted by $\widehat{\mathscr{F}_p} = \widehat{\mathscr{G}_q}$) if and only if $\widehat{\mathscr{F}_p} \sqsubseteq \widehat{\mathscr{G}_q}$ and $\widehat{\mathscr{F}_p} \sqsupseteq \widehat{\mathscr{G}_q}$.

DEFINITION 3.6. Let U be a non-empty set of the universe and E be a set of parameter. Let $\widehat{\mathscr{F}_p}$ be a PPNSSS on (U,E). The complement of $\widehat{\mathscr{F}_p}$ is denoted by $\widehat{\mathscr{F}_p^c}$ and is defined by $\widehat{\mathscr{F}_p^c} = \left\langle \widehat{\mathscr{F}^c(e)(u)}, \widehat{p^c(e)(u)} \right\rangle$, where $\widehat{\mathscr{F}^c(e)(u)} = \left\langle \xi^F_{\mathscr{F}(e)}(u), \xi^I_{\mathscr{F}(e)}(u), \xi^T_{\mathscr{F}(e)}(u) \right\rangle$, $\widehat{p^c(e)(u)} = \left\langle \xi^F_p(e)(u), \xi^I_p(e)(u), \xi^T_p(e)(u) \right\rangle$. It is true that $(\widehat{\mathscr{F}_p^c})^c = \widehat{\mathscr{F}_p}$

DEFINITION 3.7. Let U be a non-empty set of the universe and E be a set of parameter. Let $\widehat{\mathscr{F}_p}$ and $\widehat{\mathscr{G}_q}$ be two PPNSSS on (U,E). Let $\widehat{\mathscr{F}_p}$ and $\widehat{\mathscr{G}_q}$ be two PPNSSSs on (U,E). The union and intersection of $\widehat{\mathscr{F}_p}$ and $\widehat{\mathscr{G}_q}$ over (U,E) are denoted by $\widehat{\mathscr{F}_p} \sqcup \widehat{\mathscr{G}_q}$ and $\widehat{\mathscr{F}_p} \sqcap \widehat{\mathscr{G}_q}$ respectively and is defined by $\widehat{J_j} : E \to \widehat{P\mathscr{F}(U)} \times \widehat{P\mathscr{F}(U)}$,

$\widehat{I_i} : E \to \widehat{P\mathscr{F}(U)} \times \widehat{P\mathscr{F}(U)}$ such that $\widehat{J_j}(e)(u) = \left\langle \widehat{J(e)(u)}, \widehat{j(e)(u)} \right\rangle$, $I_i\widehat{(e)(u)} = \left\langle \widehat{I(e)(u)}, \widehat{i(e)(u)} \right\rangle$, where $\widehat{J(e)(u)} = \widehat{\mathscr{F}(e)(u)} \sqcup \widehat{\mathscr{G}(e)(u)}$, $\widehat{j(e)(u)} = \widehat{p(e)(u)} \sqcup \widehat{q(e)(u)}$, $\widehat{I(e)(u)} = \widehat{\mathscr{F}(e)(u)} \sqcap \widehat{\mathscr{G}(e)(u)}$ and $\widehat{i(e)(u)} = \widehat{p(e)(u)} \sqcap \widehat{q(e)(u)}$, for all $u \in U$.

EXAMPLE 3.4. Let $\widehat{\mathscr{F}_p}$ and $\widehat{\mathscr{G}_q}$ be the two PPNSSSs on (U, E) is defined by

$$\widehat{\mathscr{F}_p(e_1)} = \left\{ \begin{array}{c} \frac{u_1}{\langle (0.5,0.7,0.6),(0.4,0.3,0.7) \rangle} \\ \frac{u_2}{\langle (0.5,0.6,0.4),(0.6,0.7,0.5) \rangle} \\ \frac{u_3}{\langle (0.7,0.5,0.6),(0.8,0.4,0.3) \rangle} \end{array} \right\} ; \widehat{\mathscr{F}_p(e_2)} = \left\{ \begin{array}{c} \frac{u_1}{\langle (0.6,0.8,0.7),(0.7,0.5,0.6) \rangle} \\ \frac{u_2}{\langle (0.6,0.4,0.8),(0.6,0.9,0.8) \rangle} \\ \frac{u_3}{\langle (0.7,0.5,0.3),(0.5,0.4,0.3) \rangle} \end{array} \right\} ;$$

$$\widehat{\mathscr{F}_p(e_3)} = \left\{ \begin{array}{c} \frac{u_1}{\langle (0.3,0.2,0.7),(0.3,0.6,0.8) \rangle} \\ \frac{u_2}{\langle (0.5,0.4,0.9),(0.8,0.7,0.3) \rangle} \\ \frac{u_3}{\langle (0.7,0.5,0.6),(0.4,0.6,0.5) \rangle} \end{array} \right\} ;$$

$$\widehat{\mathscr{G}_q(e_1)} = \left\{ \begin{array}{c} \frac{u_1}{\langle (0.4,0.3,0.5),(0.2,0.6,0.8) \rangle} \\ \frac{u_2}{\langle (0.8,0.9,0.6),(0.3,0.7,0.4) \rangle} \\ \frac{u_3}{\langle (0.6,0.4,0.7),(0.7,0.5,0.8) \rangle} \end{array} \right\} ; \widehat{\mathscr{G}_q(e_2)} = \left\{ \begin{array}{c} \frac{u_1}{\langle (0.7,0.6,0.3),(0.8,0.4,0.6) \rangle} \\ \frac{u_2}{\langle (0.8,0.3,0.6),(0.7,0.6,0.3) \rangle} \\ \frac{u_3}{\langle (0.3,0.6,0.4),(0.3,0.5,0.4) \rangle} \end{array} \right\} ;$$

$$\widehat{\mathscr{G}_q(e_3)} = \left\{ \begin{array}{c} \frac{u_1}{\langle (0.6,0.7,0.5),(0.7,0.4,0.9) \rangle} \\ \frac{u_2}{\langle (0.5,0.3,0.4),(0.6,0.7,0.4) \rangle} \\ \frac{u_3}{\langle (0.4,0.8,0.7),(0.5,0.8,0.3) \rangle} \end{array} \right\} ;$$

Matrix form of $\widehat{\mathscr{F}_p} \sqcup \widehat{\mathscr{G}_q}$ written as:

$$\begin{bmatrix} \langle (0.5,0.3,0.5),(0.4,0.3,0.7) \rangle & \langle (0.8,0.6,0.4),(0.6,0.7,0.4) \rangle & \langle (0.7,0.4,0.6),(0.8,0.4,0.3) \rangle \\ \langle (0.7,0.6,0.3),(0.8,0.4,0.6) \rangle & \langle (0.8,0.3,0.6),(0.7,0.6,0.3) \rangle & \langle (0.7,0.5,0.3),(0.5,0.4,0.3) \rangle \\ \langle (0.6,0.2,0.5),(0.7,0.4,0.8) \rangle & \langle (0.5,0.3,0.4),(0.8,0.7,0.3) \rangle & \langle (0.7,0.5,0.6),(0.5,0.6,0.3) \rangle \end{bmatrix}$$

Matrix form of $\widehat{\mathscr{F}_p} \sqcap \widehat{\mathscr{G}_q}$ written as:

$$\begin{bmatrix} \langle (0.4,0.7,0.6),(0.2,0.6,0.8) \rangle & \langle (0.5,0.9,0.6),(0.3,0.7,0.5) \rangle & \langle (0.6,0.5,0.7),(0.7,0.5,0.8) \rangle \\ \langle (0.6,0.8,0.7),(0.7,0.5,0.6) \rangle & \langle (0.6,0.4,0.8),(0.6,0.9,0.8) \rangle & \langle (0.3,0.6,0.4),(0.3,0.5,0.4) \rangle \\ \langle (0.3,0.7,0.7),(0.3,0.6,0.9) \rangle & \langle (0.5,0.4,0.9),(0.6,0.7,0.4) \rangle & \langle (0.4,0.8,0.7),(0.4,0.8,0.5) \rangle \end{bmatrix}.$$

DEFINITION 3.8. A PPNSSS $\emptyset_\theta\widehat{(e)(u)} = \left\langle \widehat{\emptyset(e)(u)}, \widehat{\theta(e)(u)} \right\rangle$ is said to a possibility null Pythagorean neutrosophic soft set $\widehat{\emptyset}_\theta : E \to \widehat{P\mathscr{F}(U)} \times \widehat{P\mathscr{F}(U)}$, where $\widehat{\emptyset(e)(u)} = (0, 1)$ and $\widehat{\theta(e)(u)} = (0, 1)$, $\forall\ u \in U$.

DEFINITION 3.9. A PPNSSS $\Omega_\Lambda\widehat{(e)(u)} = \left\langle \widehat{\Omega(e)(u)}, \widehat{\Lambda(e)(u)} \right\rangle$ is said to a possibility absolute Pythagorean neutrosophic soft set $\widehat{\Omega_\Lambda} : E \to \widehat{P\mathscr{F}(U)} \times \widehat{P\mathscr{F}(U)}$, where $\widehat{\Omega(e)(u)} = (1, 0)$ and $\widehat{\Lambda(e)(u)} = (1, 0)$, $\forall\ u \in U$.

THEOREM 3.1. *Let $\widehat{\mathscr{F}_p}$ be a PPNSSS on (U, E). Then the following properties are holds:*

(i) $\widehat{\mathscr{F}_p} = \widehat{\mathscr{F}_p} \sqcup \widehat{\mathscr{F}_p}$, $\widehat{\mathscr{F}_p} = \widehat{\mathscr{F}_p} \sqcap \widehat{\mathscr{F}_p}$,
(ii) $\widehat{\mathscr{F}_p} \sqsubseteq \widehat{\mathscr{F}_p} \sqcup \widehat{\mathscr{F}_p}$, $\widehat{\mathscr{F}_p} \sqsubseteq \widehat{\mathscr{F}_p} \sqcap \widehat{\mathscr{F}_p}$,
(iii) $\widehat{\mathscr{F}_p} \sqcup \widehat{\emptyset}_\theta = \widehat{\mathscr{F}_p}$, $\widehat{\mathscr{F}_p} \sqcap \widehat{\emptyset}_\theta = \widehat{\emptyset}_\theta$,
(iv) $\widehat{\mathscr{F}_p} \sqcup \widehat{\Omega_\Lambda} = \widehat{\Omega_\Lambda}$, $\widehat{\mathscr{F}_p} \sqcap \widehat{\Omega_\Lambda} = \widehat{\mathscr{F}_p}$.

REMARK 3.1. Let $\widehat{\mathscr{F}_p}$ be a PPNSSS on (U, E). If $\widehat{\mathscr{F}_p} \neq \widehat{\Omega_\Lambda}$ or $\widehat{\mathscr{F}_p} \neq \widehat{\emptyset_\theta}$, then $\widehat{\mathscr{F}_p} \sqcup \widehat{\mathscr{F}_p^c} \neq \widehat{\Omega_\Lambda}$ and $\widehat{\mathscr{F}_p} \sqcap \widehat{\mathscr{F}_p^c} \neq \widehat{\emptyset_\theta}$.

THEOREM 3.2. *Let $\widehat{\mathscr{F}_p}$, $\widehat{\mathscr{G}_q}$ and $\widehat{\mathscr{H}_r}$ are three PPNSSSs over (U, E). Then the commutative, De Morgan's laws, associative laws and distributive laws of PPNSSSs are holds:*
(i) $\widehat{\mathscr{F}_p} \sqcup \widehat{\mathscr{G}_q} = \widehat{\mathscr{G}_q} \sqcup \widehat{\mathscr{F}_p}$,
(ii) $\widehat{\mathscr{F}_p} \sqcap \widehat{\mathscr{G}_q} = \widehat{\mathscr{G}_q} \sqcap \widehat{\mathscr{F}_p}$,
(iii) $\widehat{\mathscr{F}_p} \sqcup (\widehat{\mathscr{G}_q} \sqcup \widehat{\mathscr{H}_r}) = (\widehat{\mathscr{F}_p} \sqcup \widehat{\mathscr{G}_q}) \sqcup \widehat{\mathscr{H}_r}$,
(iv) $\widehat{\mathscr{F}_p} \sqcap (\widehat{\mathscr{G}_q} \sqcap \widehat{\mathscr{H}_r}) = (\widehat{\mathscr{F}_p} \sqcap \widehat{\mathscr{G}_q}) \sqcap \widehat{\mathscr{H}_r}$,
(v) $(\widehat{\mathscr{F}_p} \sqcup \widehat{\mathscr{G}_q})^c = \widehat{\mathscr{F}_p^c} \sqcap \widehat{\mathscr{G}_q^c}$,
(vi) $(\widehat{\mathscr{F}_p} \sqcap \widehat{\mathscr{G}_q})^c = \widehat{\mathscr{F}_p^c} \sqcup \widehat{\mathscr{G}_q^c}$,
(vii) $(\widehat{\mathscr{F}_p} \sqcup \widehat{\mathscr{G}_q}) \sqcap \widehat{\mathscr{F}_p} = \widehat{\mathscr{F}_p}$,
(viii) $(\widehat{\mathscr{F}_p} \sqcap \widehat{\mathscr{G}_q}) \sqcup \widehat{\mathscr{F}_p} = \widehat{\mathscr{F}_p}$,
(ix) $\widehat{\mathscr{F}_p} \sqcup (\widehat{\mathscr{G}_q} \sqcap \widehat{\mathscr{H}_r}) = (\widehat{\mathscr{F}_p} \sqcup \widehat{\mathscr{G}_q}) \sqcap (\widehat{\mathscr{F}_p} \sqcup \widehat{\mathscr{H}_r})$,
(x) $\widehat{\mathscr{F}_p} \sqcap (\widehat{\mathscr{G}_q} \sqcup \widehat{\mathscr{H}_r}) = (\widehat{\mathscr{F}_p} \sqcap \widehat{\mathscr{G}_q}) \sqcup (\widehat{\mathscr{F}_p} \sqcap \widehat{\mathscr{H}_r})$.

Proof. The proof follows from Definition 3.6 and 3.7.

DEFINITION 3.10. Let $(\widehat{\mathscr{F}_p}, A)$ and $(\widehat{\mathscr{G}_q}, B)$ be two PPNSSSs on (U, E). Then the operations "$(\widehat{\mathscr{F}_p}, A)$ AND $(\widehat{\mathscr{G}_q}, B)$" is denoted by $(\widehat{\mathscr{F}_p}, A) \wedge (\widehat{\mathscr{G}_q}, B)$ and is defined $(\widehat{\mathscr{F}_p}, A) \wedge (\widehat{\mathscr{G}_q}, B) = (\widehat{\mathscr{H}_r}, A \times B)$, where $\widehat{\mathscr{H}_r(\alpha, \beta)} = \left\langle \widehat{\mathscr{H}(\alpha, \beta)(u)}, r\widehat{(\alpha, \beta)(u)} \right\rangle$ such that $\widehat{\mathscr{H}(\alpha, \beta)} = \widehat{\mathscr{F}(\alpha)} \sqcap \widehat{\mathscr{G}(\beta)}$ and $\widehat{r(\alpha, \beta)} = \widehat{p(\alpha)} \sqcap \widehat{q(\beta)}$, for all $(\alpha, \beta) \in A \times B$.

DEFINITION 3.11. Let $(\widehat{\mathscr{F}_p}, A)$ and $(\widehat{\mathscr{G}_q}, B)$ be two PPNSSSs on (U, E). Then the operations "$(\widehat{\mathscr{F}_p}, A)$ OR $(\widehat{\mathscr{G}_q}, B)$" is denoted by $(\widehat{\mathscr{F}_p}, A) \vee (\widehat{\mathscr{G}_q}, B)$ and is defined by $(\widehat{\mathscr{F}_p}, A) \vee (\widehat{\mathscr{G}_q}, B) = (\widehat{\mathscr{H}_r}, A \times B)$, where $\widehat{\mathscr{H}_r(\alpha, \beta)} = \left\langle \widehat{\mathscr{H}(\alpha, \beta)(u)}, r\widehat{(\alpha, \beta)(u)} \right\rangle$ such that $\widehat{\mathscr{H}(\alpha, \beta)} = \widehat{\mathscr{F}(\alpha)} \sqcup \widehat{\mathscr{G}(\beta)}$ and $\widehat{r(\alpha, \beta)} = \widehat{p(\alpha)} \sqcup \widehat{q(\beta)}$, for all $(\alpha, \beta) \in A \times B$.

THEOREM 3.3. *Let $(\widehat{\mathscr{F}_p}, A)$ and $(\widehat{\mathscr{G}_q}, B)$ be two PPNSSSs on (U, E), then*
(i) $((\widehat{\mathscr{F}_p}, A) \wedge (\widehat{\mathscr{G}_q}, B))^c = (\widehat{\mathscr{F}_p}, A)^c \vee (\widehat{\mathscr{G}_q}, B)^c$,
(ii) $((\widehat{\mathscr{F}_p}, A) \vee (\widehat{\mathscr{G}_q}, B))^c = (\widehat{\mathscr{F}_p}, A)^c \wedge (\widehat{\mathscr{G}_q}, B)^c$.

Proof. (i) Suppose that $(\widehat{\mathscr{F}_p}, A) \wedge (\widehat{\mathscr{G}_q}, B) = (\widehat{\mathscr{H}_r}, A \times B)$ and $(\widehat{\mathscr{F}_p}, A) \wedge (\widehat{\mathscr{G}_q}, B)^c = (\widehat{\mathscr{H}_r^c}, A \times B)$. Now, $\widehat{\mathscr{H}_r^c(\alpha, \beta)} = \left\langle \mathscr{H}^c\widehat{(\alpha, \beta)(u)}, r^c\widehat{(\alpha, \beta)(u)} \right\rangle$, for all $(\alpha, \beta) \in A \times B$. By Theorem 3.2 and Definition 3.10, $\mathscr{H}^c\widehat{(\alpha, \beta)} = (\widehat{\mathscr{F}(\alpha)} \sqcap \widehat{\mathscr{G}(\beta)})^c = \widehat{\mathscr{F}^c(\alpha)} \sqcup \widehat{\mathscr{G}^c(\beta)}$ and $r^c\widehat{(\alpha, \beta)} = (\widehat{p(\alpha)} \sqcap \widehat{q(\beta)})^c = \widehat{p^c(\alpha)} \sqcup \widehat{q^c(\beta)}$. Also, $(\widehat{\mathscr{F}_p}, A)^c \vee (\widehat{\mathscr{G}_q}, B)^c = (\widehat{\Lambda_o}, A \times B)$, where $\Lambda_o\widehat{(\alpha, \beta)} = \left\langle \Lambda\widehat{(\alpha, \beta)(u)}, o\widehat{(\alpha, \beta)(u)} \right\rangle$ such that $\Lambda\widehat{(\alpha, \beta)} = \widehat{\mathscr{F}^c(\alpha)} \sqcup \widehat{\mathscr{G}^c(\beta)}$ and $o\widehat{(\alpha, \beta)} = \widehat{p^c(\alpha)} \sqcup \widehat{q^c(\beta)}$ for all $(\alpha, \beta) \in A \times B$. Thus, $\widehat{\mathscr{H}_r^c} = \widehat{\Lambda_o}$. Hence $((\widehat{\mathscr{F}_p}, A) \wedge (\widehat{\mathscr{G}_q}, B))^c = (\widehat{\mathscr{F}_p}, A)^c \vee (\widehat{\mathscr{G}_q}, B)^c$. Similarly to prove other part.

4. Similarity measure between PPNSSSs

In this section, the concept of similarity measure between PPNSSSs is introduced as follows.

DEFINITION 4.1. Let U be a non-empty set of the universe and E be a set of parameter. Suppose that $\widehat{\mathscr{F}_p}$ and $\widehat{\mathscr{G}_q}$ are two PPNSSSs on (U, E). The similarity measure between two PPNSSSs $\widehat{\mathscr{F}_p}$ and $\widehat{\mathscr{G}_q}$ is denoted by $Sim(\widehat{\mathscr{F}_p}, \widehat{\mathscr{G}_q})$ and is defined as $Sim(\widehat{\mathscr{F}_p}, \widehat{\mathscr{G}_q}) = \varphi(\widehat{\mathscr{F}}, \widehat{\mathscr{G}}) \cdot \psi(\widehat{p}, \widehat{q})$ such that

$$\varphi(\widehat{\mathscr{F}}, \widehat{\mathscr{G}}) = \frac{T_1(\widehat{\mathscr{F}(e)(u)}, \widehat{\mathscr{G}(e)(u)}) + \ T_2(\widehat{\mathscr{F}(e)(u)}, \widehat{\mathscr{G}(e)(u)}) + S(\widehat{\mathscr{F}(e)(u)}, \widehat{\mathscr{G}(e)(u)})}{3}$$

$$\psi(\widehat{p}, \widehat{q}) = 1 - \frac{\sum |(\alpha_{1i} + \alpha_{2i}) - (\beta_{1i} + \beta_{2i})|}{\sum |(\alpha_{1i} + \alpha_{2i}) + (\beta_{1i} + \beta_{2i})|},$$

where

$$T_1(\widehat{\mathscr{F}(e)(u)}, \widehat{\mathscr{G}(e)(u)}) = \frac{\sum_{i=1}^{n} (\xi^T_{\mathscr{F}(e_i)}(u) \ \cdot \ \xi^T_{\mathscr{G}(e_i)}(u))}{\sum_{i=1}^{n} (\, 1 - \sqrt{(1 - \xi^{2T}_{\mathscr{F}(e_i)}(u)) \ \cdot \ (1 - \xi^{2T}_{\mathscr{G}(e_i)}(u))}\,)},$$

$$T_2(\widehat{\mathscr{F}(e)(u)}, \widehat{\mathscr{G}(e)(u)}) = \frac{\sum_{i=1}^{n} (\xi^I_{\mathscr{F}(e_i)}(u) \ \cdot \ \xi^I_{\mathscr{G}(e_i)}(u))}{\sum_{i=1}^{n} (\, 1 - \sqrt{(1 - \xi^{2I}_{\mathscr{F}(e_i)}(u)) \ \cdot \ (1 - \xi^{2I}_{\mathscr{G}(e_i)}(u))}\,)},$$

$$S(\widehat{\mathscr{F}(e)(u)}, \widehat{\mathscr{G}(e)(u)}) = \sqrt{1 - \frac{\sum_{i=1}^{n} |\xi^{2F}_{\mathscr{F}(e_i)}(u) \ - \ \xi^{2F}_{\mathscr{G}(e_i)}(u)|}{\sum_{i=1}^{n} 1 + ((\xi^{2F}_{\mathscr{F}(e_i)}(u)) \ \cdot \ (\xi^{2F}_{\mathscr{G}(e_i)}(u))\,)}},$$

$$\alpha_{1i} = \frac{\xi^{2T}_{p(e_i)}(u)}{\xi^{2T}_{p(e_i)}(u) \ + \ \xi^{2F}_{p(e_i)}(u)}, \ \alpha_{2i} = \frac{\xi^{2T}_{p(e_i)}(u)}{\xi^{2T}_{p(e_i)}(u) \ + \ \xi^{2I}_{p(e_i)}(u)},$$

$$\beta_{1i} = \frac{\xi^{2T}_{q(e_i)}(u)}{\xi^{2T}_{q(e_i)}(u) \ + \ \xi^{2F}_{q(e_i)}(u)}, \ \beta_{2i} = \frac{\xi^{2T}_{q(e_i)}(u)}{\xi^{2T}_{q(e_i)}(u) \ + \ \xi^{2I}_{q(e_i)}(u)}.$$

THEOREM 4.1. *Let $\widehat{\mathscr{F}_p}$, $\widehat{\mathscr{G}_q}$ and $\widehat{\mathscr{H}_r}$ be the any three PPNSSSs over (U, E). Then*
(i) $Sim(\widehat{\mathscr{F}_p}, \widehat{\mathscr{G}_q}) = Sim(\widehat{\mathscr{G}_q}, \widehat{\mathscr{F}_p})$,
(ii) $0 \leqslant Sim(\widehat{\mathscr{F}_p}, \widehat{\mathscr{G}_q}) \leqslant 1$,
(iii) $\widehat{\mathscr{F}_p} = \widehat{\mathscr{G}_q} \implies Sim(\widehat{\mathscr{F}_p}, \widehat{\mathscr{G}_q}) = 1$,
(iv) $\widehat{\mathscr{F}_p} \sqsubseteq \widehat{\mathscr{G}_q} \sqsubseteq \widehat{\mathscr{H}_r} \implies Sim(\widehat{\mathscr{F}_p}, \widehat{\mathscr{H}_r}) \leqslant Sim(\widehat{\mathscr{G}_q}, \widehat{\mathscr{H}_r})$,
(v) $\widehat{\mathscr{F}_p} \sqcap \widehat{\mathscr{G}_q} = \{\phi\} \Leftrightarrow Sim(\widehat{\mathscr{F}_p}, \widehat{\mathscr{G}_q}) = 0$.

Proof. The proof (i), (ii) and (v) are trivial. (iii) Suppose that $\widehat{\mathscr{F}_p} = \widehat{\mathscr{G}_q}$ implies that $\xi^T_{\mathscr{F}(e)}(u) = \xi^T_{\mathscr{G}(e)}(u)$, $\xi^I_{\mathscr{F}(e)}(u) = \xi^I_{\mathscr{G}(e)}(u)$, $\xi^F_{\mathscr{F}(e)}(u) = \xi^F_{\mathscr{G}(e)}(u)$, $\xi^T_{p(e)}(u) = \xi^T_{q(e)}(u)$, $\xi^I_{p(e)}(u) = \xi^I_{q(e)}(u)$ and $\xi^F_{p(e)}(u) = \xi^F_{q(e)}(u)$. Now,

$$T_1(\widehat{\mathscr{F}(e)(u)}, \widehat{\mathscr{G}(e)(u)}) = \frac{\sum_{i=1}^n (\xi^T_{\mathscr{F}(e_i)}(u))^2}{\sum_{i=1}^n (1 - 1 + (\xi^T_{\mathscr{F}(e_i)}(u))^2)} = \frac{\sum_{i=1}^n (\xi^T_{\mathscr{F}(e_i)}(u))^2}{\sum_{i=1}^n (\xi^T_{\mathscr{F}(e_i)}(u))^2} = 1$$

$$T_2(\widehat{\mathscr{F}(e)(u)}, \widehat{\mathscr{G}(e)(u)}) = \frac{\sum_{i=1}^n (\xi^I_{\mathscr{F}(e_i)}(u))^2}{\sum_{i=1}^n (1 - 1 + (\xi^I_{\mathscr{F}(e_i)}(u))^2)} = \frac{\sum_{i=1}^n (\xi^I_{\mathscr{F}(e_i)}(u))^2}{\sum_{i=1}^n (\xi^I_{\mathscr{F}(e_i)}(u))^2} = 1$$

$$S(\widehat{\mathscr{F}(e)(u)}, \widehat{\mathscr{G}(e)(u)}) = \sqrt{(1-0)} = 1.$$

Thus,

$$\varphi(\widehat{\mathscr{F}}, \widehat{\mathscr{F}}) = \frac{1+1+1}{3} = 1.$$

and

$$\psi(\widehat{p}, \widehat{p}) = 1.$$

Hence,

$$Sim(\widehat{\mathscr{F}_p}, \widehat{\mathscr{F}_p}) = 1.$$

(iv) Clearly, $\xi^T_{\mathscr{F}(e)}(u) \cdot \xi^T_{\mathscr{H}(e)}(u) \leqslant \xi^T_{\mathscr{G}(e)}(u) \cdot \xi^T_{\mathscr{H}(e)}(u)$ implies that

$$\sum_{i=1}^{n} (\xi^T_{\mathscr{F}(e_i)}(u) \cdot \xi^T_{\mathscr{H}(e_i)}(u)) \leqslant \sum_{i=1}^{n} (\xi^T_{\mathscr{G}(e_i)}(u) \cdot \xi^T_{\mathscr{H}(e_i)}(u)) \tag{4.1}$$

Clearly, $(\xi^{2T}_{\mathscr{F}(e)}(u)) \leqslant (\xi^{2T}_{\mathscr{G}(e)}(u))$ implies that $-(\xi^{2T}_{\mathscr{F}(e)}(u)) \geqslant -(\xi^{2T}_{\mathscr{G}(e)}(u))$ and

$$(1 - (\xi^{2T}_{\mathscr{F}(e)}(u))) \cdot (1 - (\xi^{2T}_{\mathscr{H}(e)}(u))) \geqslant (1 - (\xi^{2T}_{\mathscr{G}(e)}(u))) \cdot (1 - (\xi^{2T}_{\mathscr{H}(e)}(u)))$$

$$\sqrt{(1 - (\xi^{2T}_{\mathscr{F}(e)}(u))) \cdot (1 - (\xi^{2T}_{\mathscr{H}(e)}(u)))} \geqslant \sqrt{(1 - (\xi^{2T}_{\mathscr{G}(e)}(u))) \cdot (1 - (\xi^{2T}_{\mathscr{H}(e)}(u)))}$$

$$1 - \sqrt{(1 - (\xi^{2T}_{\mathscr{F}(e)}(u))) \cdot (1 - (\xi^{2T}_{\mathscr{H}(e)}(u)))} \leqslant 1 - \sqrt{(1 - (\xi^{2T}_{\mathscr{G}(e)}(u))) \cdot (1 - (\xi^{2T}_{\mathscr{H}(e)}(u)))} \text{ and}$$

$$\sum_{i=1}^{n} 1 - \sqrt{(1 - (\xi^{2T}_{\mathscr{F}(e_i)}(u))) \cdot (1 - (\xi^{2T}_{\mathscr{H}(e_i)}(u)))} \leqslant \sum_{i=1}^{n} 1 - \sqrt{(1 - (\xi^{2T}_{\mathscr{G}(e_i)}(u))) \cdot (1 - (\xi^{2T}_{\mathscr{H}(e_i)}(u)))} \tag{4.2}$$

Equations (4.1) and (4.2), we get

$$\frac{\sum_{i=1}^n (\xi^T_{\mathscr{F}(e_i)}(u) \cdot \xi^T_{\mathscr{H}(e_i)}(u))}{\sum_{i=1}^n 1 - \sqrt{(1 - (\xi^{2T}_{\mathscr{F}(e_i)}(u))) \cdot (1 - (\xi^{2T}_{\mathscr{H}(e_i)}(u)))}} \leqslant \frac{\sum_{i=1}^n (\xi^T_{\mathscr{G}(e_i)}(u) \cdot \xi^T_{\mathscr{H}(e_i)}(u))}{\sum_{i=1}^n 1 - \sqrt{(1 - (\xi^{2T}_{\mathscr{G}(e_i)}(u))) \cdot (1 - (\xi^{2T}_{\mathscr{H}(e_i)}(u)))}} \tag{4.3}$$

Clearly, $\xi^I_{\mathscr{F}(e)}(u) \cdot \xi^I_{\mathscr{H}(e)}(u) \leqslant \xi^I_{\mathscr{G}(e)}(u) \cdot \xi^I_{\mathscr{H}(e)}(u)$ implies that

$$\sum_{i=1}^{n}(\xi^{I}_{\mathscr{F}(e_i)}(u)\cdot\xi^{I}_{\mathscr{H}(e_i)}(u))\leqslant\sum_{i=1}^{n}(\xi^{I}_{\mathscr{G}(e_i)}(u)\cdot\xi^{I}_{\mathscr{H}(e_i)}(u)) \tag{4.4}$$

Clearly, $(\xi^{2I}_{\mathscr{F}(e)}(u))\leqslant(\xi^{2I}_{\mathscr{G}(e)}(u))$ implies that $-(\xi^{2I}_{\mathscr{F}(e)}(u))\geqslant-(\xi^{2I}_{\mathscr{G}(e)}(u))$ and

$$(1-(\xi^{2I}_{\mathscr{F}(e)}(u)))\cdot(1-(\xi^{2I}_{\mathscr{H}(e)}(u)))\geqslant(1-(\xi^{2I}_{\mathscr{G}(e)}(u)))\cdot(1-(\xi^{2I}_{\mathscr{H}(e)}(u)))$$

$$\sqrt{(1-(\xi^{2I}_{\mathscr{F}(e)}(u)))\cdot(1-(\xi^{2I}_{\mathscr{H}(e)}(u)))}\geqslant\sqrt{(1-(\xi^{2I}_{\mathscr{G}(e)}(u)))\cdot(1-(\xi^{2I}_{\mathscr{H}(e)}(u)))}$$

$$\sum_{i=1}^{n}1-\sqrt{(1-(\xi^{2I}_{\mathscr{F}(e_i)}(u)))\cdot(1-(\xi^{2I}_{\mathscr{H}(e_i)}(u)))}\leqslant\sum_{i=1}^{n}1-\sqrt{(1-(\xi^{2I}_{\mathscr{G}(e_i)}(u)))\cdot(1-(\xi^{2I}_{\mathscr{H}(e_i)}(u)))} \tag{4.5}$$

Equations (4.4) and (4.5), we get

$$\frac{\sum_{i=1}^{n}(\xi^{I}_{\mathscr{F}(e_i)}(u)\cdot\xi^{I}_{\mathscr{H}(e_i)}(u))}{\sum_{i=1}^{n}1-\sqrt{(1-(\xi^{2I}_{\mathscr{F}(e_i)}(u)))\cdot(1-(\xi^{2I}_{\mathscr{H}(e_i)}(u)))}}\leqslant\frac{\sum_{i=1}^{n}(\xi^{I}_{\mathscr{G}(e_i)}(u)\cdot\xi^{I}_{\mathscr{H}(e_i)}(u))}{\sum_{i=1}^{n}1-\sqrt{(1-(\xi^{2I}_{\mathscr{G}(e_i)}(u)))\cdot(1-(\xi^{2I}_{\mathscr{H}(e_i)}(u)))}} \tag{4.6}$$

Clearly, $\xi^{2F}_{\mathscr{F}(e)}(u)\geqslant\xi^{2F}_{\mathscr{G}(e)}(u)$ and $\xi^{2F}_{\mathscr{F}(e)}(u)-\xi^{2F}_{\mathscr{H}(e)}(u)\geqslant\xi^{2F}_{\mathscr{G}(e)}(u)-\xi^{2F}_{\mathscr{H}(e)}(u)$. Hence

$$\sum_{i=1}^{n}\left|\xi^{2F}_{\mathscr{F}(e_i)}(u)-\xi^{2F}_{\mathscr{H}(e_i)}(u)\right|\geqslant\sum_{i=1}^{n}\left|\xi^{2F}_{\mathscr{G}(e_i)}(u)-\xi^{2F}_{\mathscr{H}(e_i)}(u)\right| \tag{4.7}$$

Also, $(\xi^{2F}_{\mathscr{F}(e)}(u)\cdot\xi^{2F}_{\mathscr{H}(e)}(u))\geqslant(\xi^{2F}_{\mathscr{G}(e)}(u)\cdot\xi^{2F}_{\mathscr{H}(e)}(u))$ implies that

$$\sum_{i=1}^{n}1+(\xi^{2F}_{\mathscr{F}(e_i)}(u)\cdot\xi^{2F}_{\mathscr{H}(e_i)}(u))\geqslant\sum_{i=1}^{n}1+(\xi^{2F}_{\mathscr{G}(e_i)}(u)\cdot\xi^{2F}_{\mathscr{H}(e_i)}(u)) \tag{4.8}$$

Equations (4.7) and (4.8), we get

$$\frac{\sum_{i=1}^{n}|\xi^{2F}_{\mathscr{F}(e_i)}(u)-\xi^{2F}_{\mathscr{H}(e_i)}(u)|}{\sum_{i=1}^{n}1+(\xi^{2F}_{\mathscr{F}(e_i)}(u)\cdot\xi^{2F}_{\mathscr{H}(e_i)}(u))}\geqslant\frac{\sum_{i=1}^{n}|\xi^{2F}_{\mathscr{G}(e_i)}(u)-\xi^{2F}_{\mathscr{H}(e_i)}(u)|}{\sum_{i=1}^{n}1+(\xi^{2F}_{\mathscr{G}(e_i)}(u)\cdot\xi^{2F}_{\mathscr{H}(e_i)}(u))}$$

$$1-\frac{\sum_{i=1}^{n}|\xi^{2F}_{\mathscr{F}(e_i)}(u)-\xi^{2F}_{\mathscr{H}(e_i)}(u)|}{\sum_{i=1}^{n}1+(\xi^{2F}_{\mathscr{F}(e_i)}(u)\cdot\xi^{2F}_{\mathscr{H}(e_i)}(u))}\leqslant1-\frac{\sum_{i=1}^{n}|\xi^{2F}_{\mathscr{G}(e_i)}(u)-\xi^{2F}_{\mathscr{H}(e_i)}(u)|}{\sum_{i=1}^{n}1+(\xi^{2F}_{\mathscr{G}(e_i)}(u)\cdot\xi^{2F}_{\mathscr{H}(e_i)}(u))}$$

$$\sqrt{1-\frac{\sum_{i=1}^{n}|\xi^{2F}_{\mathscr{F}(e_i)}(u)-\xi^{2F}_{\mathscr{H}(e_i)}(u)|}{\sum_{i=1}^{n}1+(\xi^{2F}_{\mathscr{F}(e_i)}(u)\cdot\xi^{2F}_{\mathscr{H}(e_i)}(u))}}\leqslant\sqrt{1-\frac{\sum_{i=1}^{n}|\xi^{2F}_{\mathscr{G}(e_i)}(u)-\xi^{2F}_{\mathscr{H}(e_i)}(u)|}{\sum_{i=1}^{n}1+(\xi^{2F}_{\mathscr{G}(e_i)}(u)\cdot\xi^{2F}_{\mathscr{H}(e_i)}(u))}} \tag{4.9}$$

Equations (4.3),(4.6) and (4.9), we get

$$\varphi(\widehat{\mathscr{F}}, \widehat{\mathscr{H}}) \leqslant \varphi(\widehat{\mathscr{G}}, \widehat{\mathscr{H}}) \tag{4.10}$$

Clearly $\alpha_{1i} \leqslant \beta_{1i} \leqslant \gamma_{1i}$ and $\alpha_{2i} \leqslant \beta_{2i} \leqslant \gamma_{2i}$, where

$$\alpha_{1i} = \frac{\xi^{2T}_{p(e_i)}(u)}{\xi^{2T}_{p(e_i)}(u) \ + \ \xi^{2F}_{p(e_i)}(u)},\ \alpha_{2i} = \frac{\xi^{2T}_{p(e_i)}(u)}{\xi^{2T}_{p(e_i)}(u) \ + \ \xi^{2I}_{p(e_i)}(u)},$$

$$\beta_{1i} = \frac{\xi^{2T}_{q(e_i)}(u)}{\xi^{2T}_{q(e_i)}(u) \ + \ \xi^{2F}_{q(e_i)}(u)},\ \beta_{2i} = \frac{\xi^{2T}_{q(e_i)}(u)}{\xi^{2T}_{q(e_i)}(u) \ + \ \xi^{2I}_{q(e_i)}(u)},$$

$$\gamma_{1i} = \frac{\xi^{2T}_{r(e_i)}(u)}{\xi^{2T}_{r(e_i)}(u) \ + \ \xi^{2F}_{r(e_i)}(u)},\ \gamma_{2i} = \frac{\xi^{2T}_{r(e_i)}(u)}{\xi^{2T}_{r(e_i)}(u) \ + \ \xi^{2I}_{r(e_i)}(u)}.$$

Clearly,

$$(\alpha_{1i} + \alpha_{2i}) \leqslant (\beta_{1i} + \beta_{2i}) \leqslant (\gamma_{1i} + \gamma_{2i})$$

and

$$(\alpha_{1i} + \alpha_{2i}) - (\gamma_{1i} + \gamma_{2i}) \leqslant (\beta_{1i} + \beta_{2i}) - (\gamma_{1i} + \gamma_{2i}).$$

Hence,

$$\Big|(\beta_{1i} + \beta_{2i}) - (\gamma_{1i} + \gamma_{2i})\Big| \leqslant \Big|(\alpha_{1i} + \alpha_{2i}) - (\beta_{1i} + \beta_{2i})\Big|$$

and

$$-\Big|(\alpha_{1i} + \alpha_{2i}) - (\gamma_{1i} + \gamma_{2i})\Big| \leqslant -\Big|(\beta_{1i} + \beta_{2i}) - (\gamma_{1i} + \gamma_{2i})\Big| \tag{4.11}$$

$$\Big|(\alpha_{1i} + \alpha_{2i}) + (\gamma_{1i} + \gamma_{2i})\Big| \leqslant \Big|(\beta_{1i} + \beta_{2i}) + (\gamma_{1i} + \gamma_{2i})\Big| \tag{4.12}$$

Equations (4.11) and (4.12), we get

$$-\frac{\Big|(\alpha_{1i} + \alpha_{2i}) - (\gamma_{1i} + \gamma_{2i})\Big|}{\Big|(\alpha_{1i} + \alpha_{2i}) + (\gamma_{1i} + \gamma_{2i})\Big|} \leqslant -\frac{\Big|(\beta_{1i} + \beta_{2i}) - (\gamma_{1i} + \gamma_{2i})\Big|}{\Big|(\beta_{1i} + \beta_{2i}) + (\gamma_{1i} + \gamma_{2i})\Big|}$$

$$1 - \frac{\Big|(\alpha_{1i} + \alpha_{2i}) - (\gamma_{1i} + \gamma_{2i})\Big|}{\Big|(\alpha_{1i} + \alpha_{2i}) + (\gamma_{1i} + \gamma_{2i})\Big|} \leqslant 1 - \frac{\Big|(\beta_{1i} + \beta_{2i}) - (\gamma_{1i} + \gamma_{2i})\Big|}{\Big|(\beta_{1i} + \beta_{2i}) + (\gamma_{1i} + \gamma_{2i})\Big|}.$$

Hence

$$\psi(\widehat{p}, \widehat{r}) \leqslant \psi(\widehat{q}, \widehat{r}) \tag{4.13}$$

Equations (4.10) and (4.13), we get

$$\varphi(\widehat{\mathscr{F}}, \widehat{\mathscr{H}}) \cdot \psi(\widehat{p}, \widehat{r}) \leqslant \varphi(\widehat{\mathscr{G}}, \widehat{\mathscr{H}}) \cdot \psi(\widehat{q}, \widehat{r}).$$

Hence

$$Sim(\widehat{\mathscr{F}}_p, \widehat{\mathscr{H}}_r) \leqslant Sim(\widehat{\mathscr{G}}_q, \widehat{\mathscr{H}}_r).$$

This proves (iv).

Example 4.1. Calculate the similarity measure between the two PPNSSSs, $\widehat{\mathscr{F}_p}$ and $\widehat{\mathscr{G}_q}$. We choose the first sample of $\widehat{\mathscr{F}_p}$ and $\widehat{\mathscr{G}_q}$, $E = \{e_1, e_2, e_3, e_4\}$ can be defined as below:

$\widehat{\mathscr{F}_p(e)}$	e_1	e_2	e_3	e_4
$\widehat{\mathscr{F}(e)}$	$\langle 0.5, 0.4, 0.6\rangle$	$\langle 0.6, 0.7, 0.9\rangle$	$\langle 0.7, 0.3, 0.6\rangle$	$\langle 0.4, 0.5, 0.8\rangle$
$\widehat{p(e)}$	$\langle 0.6, 0.2, 0.7\rangle$	$\langle 0.7, 0.5, 0.8\rangle$	$\langle 0.8, 0.4, 0.5\rangle$	$\langle 0.5, 0.3, 0.2\rangle$

$\widehat{\mathscr{G}_q(e)}$	e_1	e_2	e_3	e_4
$\widehat{\mathscr{G}(e)}$	$\langle 0.4, 0.6, 0.3\rangle$	$\langle 0.5, 0.7, 0.6\rangle$	$\langle 0.3, 0.6, 0.2\rangle$	$\langle 0.6, 0.3, 0.5\rangle$
$\widehat{q(e)}$	$\langle 0.6, 0.5, 0.7\rangle$	$\langle 0.8, 0.6, 0.9\rangle$	$\langle 0.4, 0.3, 0.7\rangle$	$\langle 0.7, 0.6, 0.5\rangle$

Using Definition 4.1 and routine calculation, we get

$$T_1(\widehat{\mathscr{F}(e)(u)}, \widehat{\mathscr{G}(e)(u)}) = \frac{0.20 + 0.30 + 0.21 + 0.24}{1.098993319} = \frac{0.95}{1.098993319} = 0.864427457.$$

$$T_2(\widehat{\mathscr{F}(e)(u)}, \widehat{\mathscr{G}(e)(u)}) = \frac{0.24 + 0.49 + 0.18 + 0.15}{1.056352587} = \frac{1.06}{1.167500946} = 0.907922177.$$

$$S(\widehat{\mathscr{F}(e)(u)}, \widehat{\mathscr{G}(e)(u)}) = \sqrt{1 - \frac{1.43}{4.4984}} = 0.825899022.$$

$$\varphi(\widehat{\mathscr{F}}, \widehat{\mathscr{G}}) = \frac{0.864427457 + 0.907922177 + 0.825899022}{3} = 0.866082885.$$

$$\psi(\widehat{p}, \widehat{q}) = 1 - \frac{1.315924846}{9.755643352} = 0.865111423.$$

$$Sim(\widehat{\mathscr{F}_p}, \widehat{\mathscr{G}_q}) = 0.866082885 \times 0.865111423 = 0.749258198.$$

5. Similarity measure in decision making for parental choice of colleges

In our daily life we face problems in decision making such as education, economy, management, politics and technology. The results for education to choose the best college education. In the selection of college teaching education, the evaluation of teacher education is carried out according to various standards of experts. There are various studies, primarily conducted that have investigated the reasons why parents select a college, which they think best suite their college students needs and parental aspirations for their college student. We identify a factor regarded as parental decision making: Academic Factor - divided into five identified elements namely Campus Environment, Overall Cost, Academic Quality, Student/Faculty relationship and Career Opportunities. Our goal is to select the optimal one out of a great number of alternatives based on the assessment of experts against the criteria.

5.1. Survey study. A parent intends to choose the popular college education. Here we intends to choose five colleges are nominated. The score of the college education evaluated by the experts is represented by $E = \{e_1$: Campus Environment, e_2: Overall Cost, e_3 : Academic Quality, e_4 : Student/Faculty relationship, e_5: Career Opportunities}.

Table 1

PPNSSS for the ideal college education property

$\widehat{\mathscr{L}_{p(e)}}$	e_1	e_2	e_3	e_4	e_5
$\widehat{\mathscr{L}(e)}$	$\langle 0.8, 0.6, 0.3\rangle$	$\langle 0.75, 0.7, 0.2\rangle$	$\langle 0.9, 0.8, 0.3\rangle$	$\langle 0.8, 0.65, 0.35\rangle$	$\langle 0.7, 0.65, 0.3\rangle$
$\widehat{p(e)}$	$\langle 1, 0, 0\rangle$	$\langle 1, 0, 0\rangle$	$\langle 1, 0, 0\rangle$	$\langle 1, 0, 0\rangle$	$\langle 1, 0, 0\rangle$

Table 2

PPNSSS for the first college education property

$\widehat{\mathscr{A}_{p_1(e)}}$	e_1	e_2	e_3	e_4	e_5
$\widehat{\mathscr{A}(e)}$	$\langle 0.5, 0.75, 0.4\rangle$	$\langle 0.6, 0.85, 0.3\rangle$	$\langle 0.7, 0.85, 0.45\rangle$	$\langle 0.6, 0.8, 0.5\rangle$	$\langle 0.5, 0.8, 0.6\rangle$
$\widehat{p_1(e)}$	$\langle 0.9, 0.6, 0.7\rangle$	$\langle 0.6, 0.5, 0.4\rangle$	$\langle 0.65, 0.7, 0.75\rangle$	$\langle 0.7, 0.6, 0.8\rangle$	$\langle 0.8, 0.6, 0.4\rangle$

Table 3

PPNSSS for the second college education property

$\widehat{\mathscr{B}_{p_2(e)}}$	e_1	e_2	e_3	e_4	e_5
$\widehat{\mathscr{B}(e)}$	$\langle 0.7, 0.8, 0.5\rangle$	$\langle 0.6, 0.9, 0.5\rangle$	$\langle 0.85, 0.8, 0.4\rangle$	$\langle 0.5, 0.9, 0.6\rangle$	$\langle 0.55, 0.8, 0.6\rangle$
$\widehat{p_2(e)}$	$\langle 0.8, 0.7, 0.4\rangle$	$\langle 0.7, 0.8, 0.6\rangle$	$\langle 0.75, 0.5, 0.4\rangle$	$\langle 0.6, 0.4, 0.5\rangle$	$\langle 0.85, 0.7, 0.6\rangle$

Table 4

PPNSSS for the third college education property

$\widehat{\mathscr{C}_{p_3(e)}}$	e_1	e_2	e_3	e_4	e_5
$\widehat{\mathscr{C}(e)}$	$\langle 0.6, 0.7, 0.4\rangle$	$\langle 0.75, 0.85, 0.3\rangle$	$\langle 0.65, 0.85, 0.45\rangle$	$\langle 0.55, 0.8, 0.5\rangle$	$\langle 0.7, 0.8, 0.6\rangle$
$\widehat{p_3(e)}$	$\langle 0.5, 0.55, 0.65\rangle$	$\langle 0.8, 0.4, 0.3\rangle$	$\langle 0.85, 0.6, 0.5\rangle$	$\langle 0.7, 0.6, 0.5\rangle$	$\langle 0.9, 0.4, 0.6\rangle$

Table 5

PPNSSS for the fourth college education property

$\widehat{\mathscr{D}_{p_4(e)}}$	e_1	e_2	e_3	e_4	e_5
$\widehat{\mathscr{D}(e)}$	$\langle 0.7, 0.8, 0.35\rangle$	$\langle 0.55, 0.9, 0.4\rangle$	$\langle 0.7, 0.85, 0.5\rangle$	$\langle 0.6, 0.8, 0.45\rangle$	$\langle 0.45, 0.8, 0.6\rangle$
$\widehat{p_4(e)}$	$\langle 0.85, 0.45, 0.55\rangle$	$\langle 0.8, 0.6, 0.65\rangle$	$\langle 0.65, 0.75, 0.5\rangle$	$\langle 0.9, 0.7, 0.6\rangle$	$\langle 0.7, 0.55, 0.6\rangle$

Table 6

PPNSSS for the fifth college education property

$\widehat{\mathscr{E}_{p_5(e)}}$	e_1	e_2	e_3	e_4	e_5
$\widehat{\mathscr{E}(e)}$	$\langle 0.8, 0.75, 0.6\rangle$	$\langle 0.7, 0.85, 0.5\rangle$	$\langle 0.75, 0.85, 0.5\rangle$	$\langle 0.5, 0.75, 0.6\rangle$	$\langle 0.65, 0.8, 0.6\rangle$
$\widehat{p_5(e)}$	$\langle 0.9, 0.7, 0.6\rangle$	$\langle 0.65, 0.5, 0.55\rangle$	$\langle 0.7, 0.6, 0.8\rangle$	$\langle 0.75, 0.6, 0.7\rangle$	$\langle 0.8, 0.4, 0.6\rangle$

To find the college education property which is closest to the ideal college education property, we should calculate the similarity measure of PPNSSSs in Tables 2-6 with the one in Table 1 based on Definition 4.1. The threshold of the similarity should rely on the college property. Calculating the similarity measure for the 1-5 colleges education property is given below the table.

	T_1	T_2	S	φ	ψ	$Similarity$
$\widehat{(\mathscr{L},\mathscr{A})}$	0.908919	0.964537	0.936190	0.936549	0.745287	0.697998
$\widehat{(\mathscr{L},\mathscr{B})}$	0.949307	0.932548	0.902809	0.928222	0.779975	0.723990
$\widehat{(\mathscr{L},\mathscr{C})}$	0.933342	0.968314	0.936190	0.945949	0.800766	**0.757483**
$\widehat{(\mathscr{L},\mathscr{D})}$	0.925652	0.949434	0.932751	0.935946	0.772679	0.723185
$\widehat{(\mathscr{L},\mathscr{E})}$	0.956689	0.968981	0.881369	0.935680	0.759359	0.710516

From the above results, we find that the **third college** education property is closest to the ideal college education property with the highest value of the similarity measure is $\mathbf{0.757483}$.

5.2. Comparison of PPNSSS approach with PNSSS approach without the generalization parameter. We investigate the above mentioned survey study using the PNSSS approach to consider the effect of the possibility parameter. Calculating the similarity measure for the mention above 1-5 colleges education property as follows. We have

	T_1	T_2	S	$Similarity$
$\widehat{(\mathscr{L},\mathscr{A})}$	0.908919	0.964537	0.936190	0.936549
$\widehat{(\mathscr{L},\mathscr{B})}$	0.949307	0.932548	0.902809	0.928222
$\widehat{(\mathscr{L},\mathscr{C})}$	0.933342	0.968314	0.936190	**0.945949**
$\widehat{(\mathscr{L},\mathscr{D})}$	0.925652	0.949434	0.932751	0.935946
$\widehat{(\mathscr{L},\mathscr{E})}$	0.956689	0.968981	0.881369	0.935680

From the above results, the parameter has a significant impact on the calculation of the similarity measure of PPNSSSs. It is observed that the **first, second, four and fifth** colleges education property from the perspective of similarity measure are quite away from the ideal college education property. If the college education property unit chooses the threshold $\langle \mathbf{0.6}, \mathbf{0.7}, \mathbf{0.4} \rangle$, we should choose the **third college** education property as a potential college. On the contrary, when using PNSSS approach without the generalization parameter, we can not distinguish which the colleges education property is the best one. So the possibility parameter has an important influence to the similarity measure of the **third college** education property. Therefore, PPNSSS approach is more scientific and reasonable than PNSSS approach without the generalization parameter in the process of decision-making.

6. Conclusion and future directions

The main goal of this work is to present a possibility Pythagorean neutrosophic soft set to solve the phenomena related to decision making. From the above discussion possibility parameter has an important influence to the similarity measure. Therefore, PPNSSS approach is more scientific and reasonable than PNSSS approach in the process of decision-making. To illustrate the validity of this similarity measure, possibility Pythagorean neutrosophic soft set is applied to decision making problems. So in future, we should consider the possibility Pythagorean cubic soft sets and bipolar soft sets theory.

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M. Palanikumar, Department of Mathematics, Annamalai University, Annamalainagar 608002, India.
Email address: palanimaths86@gmail.com

K. Arulmozhi, Department of Mathematics, Bharath Institute of Higher Education and Research, (Deemed to be University), Chennai-600073, India.
Email address: arulmozhiems@gmail.com

T. Gunasekar, Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R and D Institute of Science and Technology (Deemed to be University), Avadi, Cennai, Tamilnadu, India.
Email address: tguna84@gmail.com