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ON FILTERS IN UP-ALGEBRAS, A REVIEW AND SOME NEW REFLECTIONS

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Abstract. UP-algebra was introduced by Iampan 2017 as a generalization of KU-algebra. In such logical-algebraic structures, many types of UP-filters have been introduced and analyzed in 2019, such as implicative, comparative, allied and shift UP-filters by Iampan and Jun. Also, the concept of weak implicative UP-filters was introduced in 2020 by Romano and Jun. Further, normal UP-filters were analyzed by Romano. The concept of near UP-filters, as a generation of UP-filters, was designed in 2021 by A. Iampan and he showed that this notion lies between the notion of UP-subalgebras and the notion of UP-filters. Moreover, the concept of meet-commutative UP–algebras was introduced by Sawika et al. Furthermore, in such special UP-algebras, the notions of prime and irreducible UP-filters in UP-algebras are discussed by Muhiuddin et al. As a follow up, Romano introduced the concepts of prime UP-filters of the second and third kinds and he analyzed interrelationships of these three types of prime UP-filters. In addition, Romano introduced in 2021 the concept of weakly irreducible UP-filters in meet-commutative UP-algebras. Finally, the concept of semi-prime UP-filters in such algebras is discussed by Iampan and Romano. This paper gives a brief overview of the mentioned UP-filters and describes their established interconnections.

1. Introduction

In 1966, Y. Imai and K. Iseki [**13**] defined algebras of type $(2, 0)$, also known as BCI- and BCK-algebra, as a generalization of the notion of algebra sets with the subtraction set with the only a fundamental, non-nullary operation and the notion of implication algebra [**14**] on the other hand. Most of the commutative algebras

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of logic (such as residuated lattices, Boolean algebras, MV-algebras, BE-algebras, Wajsberg algebras, BL algebras, Hilbert algebras, Heyting algebras, NM algebras, MTL algebras, weak-R0 algebras, etc.) can be expressed as particular cases of BCK-algebras and the BCK-algebras are particular cases of BCI-algebras. Several generalizations of a BCI/BCK-algebra were extensively investigated by many researchers, and their properties have been considered systematically. Y. Komori introduced in [**18**] the new class of algebras called BCC-algebras.

In order to study algebraic properties of the implication operation in Boolean algebras J. C. Abbott introduced the notion of implication algebras ([**1, 2**]). He analyzed filters in [**3**] in such algebraic structures. He showed that these algebras are in one-to-one correspondence with join semi-lattices with 1 the principal filters of which are Boolean algebras. In her well-known book [**24**], Rasiowa states without proof that in implicative algebras there is a one-to-one correspondence between kernels of epimorphisms and the so-called 'special implicative filters', and that in the logic whose algebraic counterpart is the class of implicative algebras the deductive filters coincide with the special implicative filters. Although J. M. Font showed (see [**5**]) that the previous observation is not correct, interest in various types of filters in logical algebras has increased.

The concept of KU-algebras was introduced in 2009 by Prabpayak and Leerawat in the article [**23**]. In the article [**26**], the authors (Rezaei, Saeod and Borzooei) proved that a KU-algebra is equivalent to a commutative self-distributive BE-algebra. (A BE-algebra A is self-distributive if $x \cdot (y \cdot z) = (z \cdot y) \cdot (x \cdot z)$ for all $x, y, z \in A$.) Additionally, they proved that every KU-algebra is a BE-algebra ([**26**], Theorem 3.4), every Hilbert algebra is a KU-algebra ([**26**], Theorem 3.5) and a self-distributive KU-algebra is equivalent to a Hilbert algebra ([**26**]).

Iampan introduced in [**8**] the concept of UP-algebras as a generalization of KU-algebras. This class of algebras has recently been in the focus of interest of researchers (for example, [**4, 7, 9, 10, 20, 21, 29, 37, 40**]). In [**41**], Somjanta et al. introduced the notion of filters in this class of algebra. Proper UP-filter in a UP-algebra was introduced by Romano 2018 ([**27, 28**]). Jun and Iampan then introduced and analyzed several classes of filters in UP algebras such as implicative, comparative, allied and shift UP-filters (see, for example, [**15, 16, 17**]). The concept of weak implicative UP-filters in a UP-algebra was introduced and analyzed by Romano and Jun ([**34**]). As the last class of filters in UP-algebras, the so-called normal UP-filters, was analyzed by Romano in [**36**]. The concept of near UP-filters, as a generation of UP-filters, was designed in [**12**] by A. Iampan and he showed that this notion lies between the notion of UP-subalgebras and the notion of UP-filters.

The concept of meet-commutative UP-algebras was introduced in article [**39**]. In article [**22**], a number of important properties of meet-commutative UP-algebras are given by Muhiuddin et al. In addition, in such UP-algebras, the concept of prime UP-filters (of the first kind) and the concept of irreducible UP-filters were introduced and analyzed. Then, Romano introduces the notions of prime UP-filters of the second kind ([**34**]) and prime filter of the third kind ([**32**]) in meet-commutative UP-algebras and connects them with the prime UP-filters of the first type. Romano also introduces ([**35**]) weakly irreducible UP-filters in meet-commutative UP-algebra by weakening the conditions that determine the concept of irreducible UP-filters. Iampan and Romano have described semi-prime UP-filters in such algebras in article [**36**].

This paper summarizes the mentioned UP-filters and describes their established interconnections. As a review for all filters in UP-algebras (including meet-commutative), we think that this report can provide convenience and a good starting point to researchers in this field.

2. Preliminaries: UP-algebras

In this section, taking from the literature, we will repeat some concepts and statements of interest for this research.

An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra* (see [**8**]) if it satisfies the following axioms:

(UP-1) $(\forall x, y, \in A)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0)$,
(UP-2) $(\forall x \in A)(0 \cdot x = x)$,
(UP-3) $(\forall x \in A)(x \cdot 0 = 0)$,
(UP-4) $(\forall x, y \in A)((x \cdot y = 0 \wedge y \cdot x = 0) \implies x = y)$.

Example 2.1. ([**8**], Example 1.4) Let X be a set. Define a binary operation '$\cdot$' on the power set $\mathcal{P}(X)$ of X by putting $A \cdot B := B \cap A^c = B \smallsetminus A$ for all $A, B \in \mathcal{P}(X)$. Then $(\mathcal{P}(X), \cdot, \emptyset)$ is a UP-algebra.

Example 2.2. ([**37**], Theorem 2.3) Let X be a nonempty set and let t be an arbitrary element in X. Define a binary operation '$\cdot$' on X by: for all $x, y \in X$, let it be $x \cdot y = y$ if $x > y$ or $x = t$, and $x \cdot y = t$ in other cases. Then $(X, \cdot, t)$ is a UP-algebra.

Example 2.3. ([**8**], Example 1.6) Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	0	0	0
b	0	a	0	c
c	0	a	b	0

Then $A = (A, \cdot, 0)$ is a UP-algebra.

In a UP-algebra, the order relation '$\leqslant$' is defined as follows

$$(\forall x, y \in A)(x \leqslant y \iff x \cdot y = 0).$$

The following proposition gives the main properties of the order relation in UP-algebras.

Proposition 2.1 ([**8**], Proposition 1.8). *In a UP-algebra A, the following properties hold:*

(1) $x \leqslant x$,

(2) $x \leqslant y \wedge y \leqslant x \implies x = y$,
(3) $x \leqslant y \wedge y \leqslant z \implies x \leqslant z$,
(4) $x \leqslant y \implies z \cdot x \leqslant z \cdot y$,
(5) $x \leqslant y \implies y \cdot z \leqslant x \cdot z$,
(6) $x \leqslant y \cdot x$, *and*
(7) $x \leqslant y \cdot y$

for any $x, y, z \in A$,

Thus, the relation '$\leqslant$' is an order relation in UP-algebras such that the multiplication on the left is monotonic and the multiplication on the right is inversely monotonic.

In this logical-algebraic environment, substructures (subalgebras, ideals and filters) and some processes with them (relations between them) are considered, such as homomorphisms ([**7, 8, 9, 20**]), hipper-structures ([**10, 30**]) and bi-algebras ([**21, 31**]).

3. Some Filters in UP-algebras

Filters theory plays an important role in studying any class of logical algebras. From the logical point of view, various filters correspond to various sets of valid formulas in an appropriate logic. On the other hand, designing different types of filters in some logical algebra is also interesting from an algebraic aspect.

A subset F of a UP-algebra A is called a *UP-filter* of A (see [**41**]) if it satisfies the following conditions:
(F-1) $0 \in F$,
(F-2) $(\forall x, y \in A)((x \in F \wedge x \cdot y \in F) \implies y \in F)$.
It is clear that every UP-filter F of a UP-algebra A satisfies:
(8) $(\forall x, y \in A)((x \in F \wedge x \leqslant y) \implies y \in F)$.

The family of all UP-filters of a UP-algebra A is denoted by $\mathfrak{F}(A)$. It is easy to verify that the intersection of UP-filters of a UP-algebra A is also a UP-filter of A. For any subset S of A, let $F(S) := \bigcap\{F \in \mathfrak{F}(A) : S \subseteq F\}$. Then $F(S)$ is the smallest UP-filter of A containing S. Therefore, if S and T are UP-filters in a UP-algebra A, then $S \sqcap T := S \cap T$ and $S \sqcup T := F(S \cup T)$ are UP-filters in A. So, $(\mathfrak{F}(A), \sqcap, \sqcup)$ is a complete lattice.

3.1. Implicative UP-filter.

The concept of implicative UP-filter was introduced in [**15**] by Jun and Iampan.

Definition 3.1. ([**15**], Definition 1) A subset F of a UP-algebra A is called an *implicative UP-filter* of A if it satisfies the following conditions:
(F-1) $0 \in F$ and
(IF) $(\forall x, y, z \in A)((x \cdot (y \cdot z) \in F \wedge x \cdot y \in F) \implies x \cdot z \in F)$.

Every implicative UP-filter is a UP-filter ([**15**], Theorem 1). The converse is not true in general as seen in the following example.

Example 3.1. ([**15**], Example 2). Consider a UP-algebra $A = \{0, 1, 2, 3\}$ with the binary operation '$\cdot$' which is given in the following table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	1	2
2	0	0	0	2
3	0	0	0	0

Then $\{0\}$ is a UP-filter of A, but it is not an implicative UP-filter since $2 \cdot (2 \cdot 3) = 0 \in \{0\}$ and $2 \cdot 2 = 0 \in \{0\}$, but $2 \cdot 3 = 2 \notin \{0\}$.

On the other hand, for a UP-filter F in a UP- algebra A to be an implicative UP-filter, as shown in [**15**], Theorem 5, it suffices that F satisfies the following conditions:
(9) $(\forall x, y, z \in A)(x \cdot (y \cdot z) = y \cdot (x \cdot z))$ and
(10) $(\forall x, y \in A)(x \cdot (x \cdot y) = x \cdot y)$.
The following example shows that, in the general case, a UP-algebra does not have to satisfy condition (9).

Example 3.2. Let $S = \{0, a, b, c, d\}$ and operation '$\cdot$' defined on A as follows:

$\cdot$	0	a	b	c	d
0	0	a	b	c	d
a	0	0	0	0	0
b	0	b	0	0	0
c	0	b	b	0	0
d	0	b	b	c	0

Then $A = (A, \cdot, 0)$ is a UP-algebra ([**8**], Example 1.12). Since $c \cdot (b \cdot a) = c \cdot b = b$ but $b \cdot (c \cdot a) = b \cdot b = 0$ we have that this UP-algebra does not satisfy the condition (9).

3.2. Comparative UP-filter. The concept of comparative UP-filter was introduced in [**16**] by Jun and Iampan

Definition 3.2. ([**16**], Definition 2) A subset F of a UP-algebra A is called a *comparative UP-filter* of A if it satisfies the following conditions:
(F-1) $0 \in F$ and
(CF) $(\forall x, y, z \in A)((x \cdot ((y \cdot z) \cdot y) \in F \wedge x \in F) \implies y \in F)$.

Example 3.3. ([**16**], Example 1) Let A be as in Example 3.1. Then the subset $\{0, 1, 2\}$ is a comparative UP-filter of the UP-algebra A.

Every comparative UP-filter is a UP-filter ([**16**], Theorem 1). The converse is not true in general as seen in the following example.

Example 3.4. ([**16**], Example 2) Let $A = \{0, 1, 2, 3\}$ be the UP-algebra in Example 3.1. Then $\{0\}$ is a UP-filter of A, but it is not a comparative UP-filter of A since $0 \cdot ((1 \cdot 2) \cdot 1) = 0 \cdot (1 \cdot 1) = 0 \cdot 0 = 0 \in \{0\}$ and $0 \in \{0\}$ but $1 \notin \{0\}$.

However, as shown in Theorem 2 in article [**16**], for a UP-filter F of a UP-algebra A to be a comparative UP-filter it is necessary and sufficient to be valid

(11) $(\forall x, y \in A)((x \cdot y) \cdot x \in F \implies x \in F)$.

In the general case, a comparative UP-filter does not have to be an implicative UP-filter and vice versa.

Example 3.5. Let A be as in Example 2.3. By direct verification it is routine to verify that subsets $\{0\}$, $\{0; b\}$, $\{0, c\}$ and $\{0, b, c\}$ are implicative UP-filters of A (see [**15**]). On the other hand, $F := \{0, b\}$ is not a comparative UP-filter of A. since $b \cdot ((c \cdot a) \cdot c) = b \cdot (a \cdot c) = b \cdot 0 = 0 \in F$ and $b \in F$ but $c \notin F$.

Example 3.6. ([**16**]) Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	0	a	b
b	0	0	0	b
c	0	a	0	0

Then $A = (A, \cdot, 0)$ is a UP-algebra and the subset $F := \{0, a, b\}$ is a comparative UP-filter of A but it is not an implicative UP-filter of A since for example for $x = 0$, $y = a$ and $z = c$, we have $x \cdot (y \cdot z) = 0 \cdot (a \cdot c) = 0 \cdot b = b \in F$ and $x \cdot y = 0 \cdot a = a \in F$ but $x \cdot z = 0 \cdot c = c \notin F$.

However, there is one special situation for UP-algebras when implicative and comparative UP-filters coincide. Such a situation is described in [**16**], Theorem 3. If one UP-algebra satisfies condition (9) and the following condition

(12) $(\forall x, y \in A)((x \cdot y) \cdot y = (y \cdot x) \cdot x)$,

then in such a UP-algebra these two types of UP-filters coincide.

3.3. Allied UP-filter. In [**16**], Jun and Iampan introduced the notion of allied UP-filters in UP-algebras. The concept of allied UP-filters with respect to the element x in A is given by the following definition.

Definition 3.3. ([**16**], Definition 4) Let x be a fixed element of A. A subset F of A is called an *allied UP-filter* of A with respect to x (briefly, x*-allied UP-filter* of A) if it satisfies the conditions

(F-1) $0 \in F$ and

(FA) $(\forall y, z \in A)((x \cdot (y \cdot z) \in F \wedge x \cdot y \in F) \implies z \in F)$.

Example 3.7. ([**16**], Example 5) Let $A = \{0, 1, 2, ; 3\}$ be a set with the binary operation '$\cdot$' which is given in the following table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	1	3
2	0	0	0	3
3	0	0	0	0

Then the subset $\{0\}$ is a 0-allied UP-filter of A. The set $\{0, 1, 2\}$ is an allied UP-filter of A with respect to 0, 1 and 2. But UP-filter $\{0\}$ is not a 3-allied UP-filter of A since $3 \cdot (3 \cdot 3) = 3 \cdot 0 = 0 \in \{0\}$ and $3 \cdot 3 = 0 \in \{0\}$ but $3 \notin \{0\}$.

Every 0-allied UP-filter is a UP-filter, and vice versa ([**16**], Theorem 7). In the general case, for any UP-filter F of a UP-algebra A there exists an element $a \in A$ such that F is not an a-allied UP-filter ([**16**]).

3.4. Shift UP-filter. The concept of shifting UP-filters in UP-algebras was introduced in [**17**] by Jun and Iampan.

Definition 3.4. ([**17**], Definition 4.1) A subset F of a UP-algebra A is called a *shift UP-filter* of X if it satisfies the conditions
(F-1) $0 \in F$ and
(SF) $(\forall x, y, z \in A)((x \cdot (y \cdot z) \in F \wedge x \in F) \Longrightarrow ((z \cdot y) \cdot y) \cdot z \in F)$

Example 3.8. ([**17**], Example 4.2) Consider a UP-algebra $A = \{0, 1, 2, 3\}$ with the binary operation '$\cdot$' which is given in the following

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	0	3
2	0	1	0	3
3	0	1	2	0

Then the subset $\{0, 2\}$ is a shift UP-filter of A.

Every shift UP-filter is a UP-filter([**17**], Theorem 4.3). The following example shows that the converse is not true.

Example 3.9. Let A be as in Example 2.3. Then A is a UP-algebra, and it is routine to verify that $\{0, b\}$ is a UP-filter of A. But it is not a shift UP-filter of A since $b \cdot (a \cdot c) = b \cdot 0 = 0 \in \{0, b\}$ and $b \in \{0, b\}$, but $((c \cdot a) \cdot a) \cdot c = (a \cdot a) \cdot c = 0 \cdot c = c \notin \{0, b\}$.

The following example shows that a comparative UP-filter does not have to be a shift UP-filter.

Example 3.10. Let A be a UP-algebra as in Example 3.1. Then the subset $F := \{0, 1, 2\}$ is a comparative UP-filter of A. But it is is not a shift UP-filter of A since $1 \in F$ and $1 \cdot (0 \cdot 3) = 1 \cdot 3 = 2 \in F$ but $((3 \cdot 0) \cdot 0) \cdot 3 = (0 \cdot 0) \cdot 3 = 0 \cdot 3 = 3 \notin F$.

In [**17**] it is shown (Theorem 4.10) that if the UP-algebra A satisfies the condition (9), then every comparative UP-filter in A is a shift UP-filter in A.

3.5. Normal UP-filter. The concept of normal UP-filters in UP-algebras was introduced in [**36**] by Romano.

Definition 3.5. ([**36**]) Let F be a UP-filter of a UP-algebra A. F is called a *normal UP-filter* of A if it satisfies:
(NF) $(\forall x, y, z \in A)((z \in F \wedge z \cdot ((y \cdot x) \cdot x) \in F) \Longrightarrow (x \cdot y) \cdot y \in F)$.

It is obvious that the following statement is true: Any normal UP-filter of a UP-algebra is a UP-filter of it. It is shown ([**36**], Theorem 3.5) that if the UP-algebra A satisfies the condition (12), then any UP-filter in A is a normal UP-filter in A. Also, in this article it is shown that the UP-filter of a UP-algebra A is a normal UP-filter in A if and only if it satisfies the following condition

(13) $(\forall x, y \in A)((y \cdot x) \cdot x \in F \implies (x \cdot y) \cdot y \in F)$.

In the following example, we show one example of a normal UP-filter and show that, generally speaking, a UP-filter does not have to be a normal UP-filter.

EXAMPLE 3.11. Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

·	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	0	0	0
c	0	a	c	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. The subsets $F := \{0\}$ and $G := \{0, a\}$ are UP-filters of A. The filter F is not a normal UP-filter of A because, for example, for $z = 0$, $y = a$ and $x = b$, we have $z \cdot ((y \cdot x) \cdot x) = 0 \cdot (a \cdot b) \cdot b) = (a \cdot b) \cdot b = 0 \in F$ but $(x \cdot y) \cdot y = (b \cdot a) \cdot a = 0 \cdot a = a \notin F$. By direct verification it can be proved that G is a normal UP-filter of A.

Any normal UP-filter need not be an implicative UP-filter.

EXAMPLE 3.12. Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

·	0	a	b	c
0	0	a	b	c
a	0	0	0	a
b	0	a	0	c
c	0	0	0	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. Subset $F := \{0, b\}$ is a normal UP-filter of A while it is not an implicative UP-filter, because for $x = a$, $y = a$ and $z = c$ we have $a \cdot (a \cdot c) = a \cdot a = 0 \in F$ and $a \cdot a = 0 \in F$ but $a \cdot c = a \notin F$.

A normal UP-filter of a UP-algebra does not have to be a comparative UP-filter.

EXAMPLE 3.13. Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

·	0	a	b	c
0	0	a	b	c
a	0	0	0	b
b	0	b	0	a
c	0	0	0	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. Subset $F := \{0\}$ is a normal UP-filter of A while it is not a comparative UP-filter, since for $x = 0$, $y = b$ and $z = c$ we have $0 \in F$ and $0 \cdot ((b \cdot c) \cdot b) = a \cdot b = 0 \in F$ but $b \notin F$.

However, if the UP-algebra A satisfies the condition (9), then any comparative UP-filter in A is a normal UP-filter in A ([**36**], Theorem 3.10).

Finally, we state one connection between implicative, comparative and normal filters: ([**36**], Theorem 3.15) Let F be an implicative UP-filter of a UP-algebra A which additionally satisfies the condition (9). Then F is a comparative UP-filter of A if and only if it is a normal UP-filter.

3.6. Weak implicative UP-filter. The concept of weak implicative UP-filter in UP-algebras was introduced in [**33**] by Romano and Jun.

Definition 3.6. ([**33**]) A subset F of A is called a *weak implicative UP-filter* of A if it satisfies the following conditions:
(F-1) $0 \in F$,
(WIF) $(\forall x, y, z \in A)((x \cdot (y \cdot z) \in F \wedge x \cdot y \in F) \implies x \cdot (x \cdot z) \in F)$.

Example 3.14. Let $A = \{0, 1, 2, 3, 4\}$ be a set with the operation '$\cdot$' given by

$\cdot$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	0	0	0
2	0	2	0	0	0
3	0	2	2	0	0
4	0	2	2	4	0

Then A is a UP-algebra (see Example 1.12 in [**8**]), and $F := \{0, 2\}$ is a weak implicative UP-filter of A.

Every weak implicative UP-filter is a UP-filter ([**33**], Theorem 1). The following example shows that the converse does not have to be true.

Example 3.15. Let $A = \{0, 1, 2, 3\}$ be a set with the binary operation '$\cdot$' which is given by the following table.

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	1	1
2	0	0	0	1
3	0	0	0	0

Then A is a UP-algebra (see Example 3 in [**16**]). The subset $F := \{0, 1, 2\}$ is a UP-filter of A, but it is not a weak implicative UP-filter of A because $0 \cdot (2 \cdot 3) = 1 \in F$ and $0 \cdot 2 = 2 \in F$ but $0 \cdot (0 \cdot 3) = 3 \notin F$.

Every implicative UP-filter is a weak implicative UP-filter ([**33**], Theorem 2). The converse does not have to be true as seen in the following example.

Example 3.16. Consider a UP-algebra $A = \{0, 1, 2, 3\}$ ([**8**], Example 2) as in Example 3.1. Then the subset $\{0\}$ is a weak implicative UP-filter of A, but it is not an implicative UP-filter since $2 \cdot (2 \cdot 3) = 0 \in \{0\}$ and $2 \cdot 2 = 0 \in \{0\}$, but $2 \cdot 3 = 2 \notin \{0\}$.

In [**33**] has been shown (Theorem 3) that if the weak implicative UP-filter F of a UP-algebra A satisfies the condition

$$(\forall x, y \in A)(x \cdot (x \cdot y) \in F \iff x \cdot y \in F),$$

then F is an implicative UP-filter in A. Also, in this paper it is shown (Theorem 4) that if the UP-filter F of a UP-algebra A satisfies the condition

$$(\forall x, y, z \in A)(x \cdot (y \cdot z) \in F \implies (x \cdot y) \cdot (x \cdot (x \cdot z)) \in F),$$

then F is a weak implicative UP-filter in A.

3.7. Near UP-filters. In the articles [**12, 38**], the concept of near-filters in UP-algebras as a generalization of the concept of UP-filters in these algebras is introduced and analyzed by A. Iampan (the first paper) and Satired et al. (in the second paper).

Definition 3.7. ([**12**], Definition 3.25; [**38**], Definition 1.15) A nonempty subset S of a UP-algebra $(A, \cdot, 0)$ is called a *near UP-filter* of A if it satisfies the following properties:
(i) $0 \in S$, and
(ii) $(\forall x, y \in A)((x \in A \land y \in S) \implies x \cdot y \in S)$.

In [**12**] it is shown (Theorem 3.26) that every near UP-filter is a UP-subalgebra and (Theorem 3.27) that every UP-filter is a near UP-filter. However, a near UP-filter does not have to be a UP-filter as shown in the following example.

Example 3.17. ([**12**], Example 3.8) Consider a UP-algebra $A = \{0, 1, 2, 3\}$ with the binary operation '$\cdot$' which is given in the following table:

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	1	3
2	0	0	0	3
3	0	1	1	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. The subset $S := \{0, 1\}$ is a near UP-filter of A. Since $1 \cdot 2 = 1 \in S$ and $1 \in S$ but $2 \notin S$, we have that S is not a UP-filter of A. The subset $T := \{0, 2\}$ is a UP-subalgebra of A. Since $3 \in A$ and $2 \in T$ but $3 \cdot 2 = 1 \notin T$, we have that T is not a near UP-filter of A.

The mentioned theorems in [**12**] and this example show that the notion of UP-subalgebras is a generalization of near UP-filters and the notion of near UP-filters is a generalization of UP-filters.

4. Filters in meet-commutative UP-algebras

In this section we will look at some new concepts of UP-filters in a special class of UP-algebras. In some of the previous considerations about filters in UP-algebras, the condition (12) appeared as an additional determination of the environment in which the filters are observed. Since the research reports of this class of UP-algebras appear during the design of this review, we designed this section somewhat differently than the previous sections.

A UP-algebra A is said to be meet-commutative (see [**39**], Definition 1.15) if it satisfies the condition

$$(\forall x, y \in A)((x \cdot y) \cdot y = (y \cdot x) \cdot x).$$

This term also appears in the paper [**16**], Definition 3.

The following example shows that a UP-algebra does not have to be a meet-commutative.

Example 4.1. ([**16**], Example 3) Let $S = \{0, a, b, c\}$ and operation '$\cdot$' defined on A as follows:

$\cdot$	1	a	b	c
0	0	a	b	c
a	0	0	a	a
b	0	0	0	a
c	0	0	0	0

Then $A = (A, \cdot, 0)$ is a UP-algebra which does not satisfy the condition (12). For example, for $x = b$ and $y = c$, we have $(x \cdot y) \cdot y = (b \cdot c) \cdot c = a \cdot c = a$ but $(y \cdot x) \cdot x = (c \cdot b) \cdot b = 0 \cdot b = b$.

We first characterize the meet-commutative UP-algebras.

Theorem 4.1 ([**22**], Theorem 3.1). *Let A be a meet-commutative UP-algebra. Then the following holds*

(14) $(\forall x, y \in A)(x \leqslant y \implies y = (y \cdot x) \cdot x)$.

An important specificity of this class of UP-algebras is described in the following theorem:

Theorem 4.2 ([**22**], Theorem 3.2). *Let A be a meet-commutative UP-algebra. For any $x, y \in A$, the element $x \sqcup y := (x \cdot y) \cdot y = (y \cdot x) \cdot x$ is the least upper bound of x and y.*

Thus, the previous theorem allows us to look at meet-commutative UP-algebras as semi-lattices. Some of the properties of this semi-lattice are given below.

Proposition 4.1 ([**22**], Proposition 3.1; [**34**], Proposition 2.2). *Let A be a meet-commutative UP-algebra. Then*

(15) $(\forall x, y \in A)(0 \sqcup x = x,\ x \sqcup 0 = 0,\ x \sqcup x = x,\ \text{and}\ x \sqcup y = y \sqcup x)$.
(16) $(\forall x, y, z \in A)((x \sqcup y) \sqcup z = (x \sqcup z) \sqcup (y \sqcup z))$.
(17) $(\forall x, y, z \in A)((z \cdot x) \sqcup (z \cdot y) \leqslant z \cdot (x \sqcup y))$.
(18) $(\forall x, y, z \in A)((x \sqcup y) \cdot z \leqslant (x \cdot z) \sqcup (y \cdot z))$.
(19) $(\forall x, y \in A)(x \sqcup y \leqslant (y \cdot x) \sqcup (x \cdot y))$.

The semi-lattice $(A, \sqcup)$ can be considered as a join distributive semi-lattice due to the validity of the formula (16). Note that the notion of semi-lattice distributivity used here differs from the notion of join semi-lattice in the book [**6**]. It is quite justified to ask a question, and try to find some answer to it:

'What are the properties of this semi-lattice designed in this way?'

4.1. Prime UP-filters. The notion of prime UP-filters of a meet-commutative UP-algebra was introduced in article [**22**]. For the purposes of this paper, we will recognize such a UP-filter as a 'prime UP-filter of the first kind'.

Definition 4.1. Let F be a UP-filter of a meet-commutative UP-algebra A. Then F is said to be a *prime UP-filter of the first kind* of A if the following holds

(PF1) $(\forall x, y \in A)(x \sqcup y \in F \implies (x \in F \vee y \in F))$.

Example 4.2. Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	0	c	0
b	0	c	0	c
c	0	b	b	0

Then $A = (A, \cdot, 0)$ is a meet-commutative UP-algebra. Subsets $\{0\}$, $\{0, b\}$ and $\{0, c\}$ are UP-filters of A. It is not difficult to verify that UP-filters $\{0, b\}$ and $\{0, c\}$ are prime of the first kind. It is clear that $\{0\}$ is not a prime UP-filter of the first kind of A because $b \sqcup c = 0 \in \{0\}$ but $b \notin \{0\}$ and $c \notin \{0\}$.

The following definition gives another type of a prime UP-filter in meet-commutative UP-algebras.

Definition 4.2. ([**34**], Definition 3.2) Let F be a UP-filter of a meet-commutative UP-algebra A. Then F is said to be a *prime UP-filter of the second kind* of A if the following holds

(PF2) $(\forall x, y \in A)(x \cdot y \in F \vee y \cdot x \in F)$.

Example 4.3. Let A be as in Example 4.2. Then $A = (A, \cdot, 0)$ is a meet-commutative UP-algebra. The subsets $\{0, c\}$ is a prime UP-filter of the second kind of A. The subset $F := \{0, b\}$ is a prime UP-filter of the first kind but it is not a prime UP-filter of the second kind because, for example, it holds $a \cdot b = c \notin F$ and $b \cdot a = c \notin F$.

In the previous example it was shown that a UP-filter can be a prime UP-filter of the first kind but it does not have to be a prime UP-filter of the second kind. However, it has been shown ([**34**], Theorem 3.1) that if F satisfies the condition (PF2), then it satisfies the condition (PF1) also, i.e. any prime UP-filter of the second kind in a meet-commutative UP-algebra A is a prime UP-filter of the first kind in A.

The following definition introduces the term 'prime UP–filter of the third kind' in meet-commutative UP–algebras.

Definition 4.3. ([**32**]) Let F be a UP–filter of a meet-commutative UP-algebra A. Then F is said to be a *prime UP-filter of the third kind* of A if the following holds

(PF3) $(\forall x, y \in A)((x \cdot y) \sqcup (y \cdot x) \in F)$.

The following example shows that a UP-filter in a meet-commutative UP algebra does not have to be a prime UP-filter of the third kind.

EXAMPLE 4.4. Let A be as in Example 4.2. Then $A = (A, \cdot, 0)$ is a meet-commutative UP-algebra. The subset $F := \{0\}$ is a UP-filter of A but it is not a prime UP-filter of the third kind of A because, for example, we have $(a \cdot b) \sqcup (b \cdot a) = c \notin F$. The subset $G := \{0, b\}$ is not a prime UP-filter of the third kind in A also because, for example, $(a \cdot b) \sqcup (b \cdot a) = c \notin G$ holds. The subset $\{0, c\}$ is a prime UP-filter of the third type and A.

The following example shows that a UP-filter of a meet-commutative algebra can be a prime UP-filter of the third kind and neither a UP-filter of the first kind nor a prime UP-filter of the second kind.

EXAMPLE 4.5. Let $A = \{1, a, b, c, d\}$ and operation '$\cdot$' on A as follows:

$\cdot$	0	a	b	c	d
0	0	a	b	c	d
a	0	0	0	0	0
b	0	b	0	0	0
c	0	c	c	0	d
d	0	d	d	c	0

Then $(A, \cdot, 0)$ is a meet-commutative UP-algebra. Here it is $(c \cdot d) \cdot d = d \cdot d = 0$, and $(d \cdot c) \cdot c = c \cdot c = 0$. So $c \sqcup d = 0$. The subset $F := \{0\}$ is a UP-filter of A. Obviously, this filter is not a prime filter of the first kind because $c \sqcup d = 0 \in F$ but $c \notin F$ and $d \notin F$. It can be shown by direct verification that F is a prime filter of the third kind of A. Also, this filter is not a prime filter of the second kind, because for $x = c$ and $y = d$ we have $x \cdot y = c \cdot d = d \notin F$ and $y \cdot x = d \cdot c = c \notin F$.

EXAMPLE 4.6. Let $A = \{0, a, b, c\}$ and operation '$\cdot$' is defined on A as follows:

$\cdot$	0	a	b	c
0	0	a	b	c
a	0	0	0	0
b	0	c	0	0
c	0	b	c	0

Then $A = (A, \cdot, 0)$ is a meet-commutative UP-algebra. Subsets $\{0\}$ and $\{0, c\}$ are UP-filters of A. Due to the linearity of the order relation in A, both UP-filters are prime UP-filters of the first / second / third kind in A.

4.2. Irreducible UP-filters. The concept of irreducible UP-filters in the class of meet-commutative UP-algebras was introduced and discussed in [**22**] by Muhiuddin et al.

DEFINITION 4.4. A UP-filter F of a UP-algebra A is said to be an *irreducible UP-filter* of A if for any UP-filters S and T of A the following implication holds

$$F = S \cap T \implies (S = F \vee T = F).$$

Any prime UP-filter of a meet-commutative UP-algebra is an irreducible UP-filter ([**22**]).

In an effort to better understand the properties of this class of logical algebras, Romano analyzed the possibility of weakening the hypothesis in the previous implication. The consequence of the $S \cap T \subseteq F$ option is considered. As a consequence of this choice, a new class of UP-filters in meet-commutative UP-algebras was obtained.

Definition 4.5. ([**35**], Definition 4.1) Let F be a UP-filter of A. F is a *weakly irreducible UP-filter* in A if and only if for all UP-filters S and T of A such that $S \cap T \subseteq F$ the following holds $S \subseteq F$ or $T \subseteq F$.

Example 4.7. Let A be as in the Example 4.2. Then A is a meet-commutative UP-algebra. For the UP-filter $F := \{0, b\}$ we have $\{0, b\} \cap \{0, a, b, c\} \subseteq F$ and $\{0, b\} \subseteq F$. Thus, F is a weakly irreducible UP-filter in A.

The following theorem gives one important characterization of weakly irreducible UP-filters in meet-commutative UP-algebras.

Theorem 4.3 ([**35**], Theorem 4.1). *Let F be a UP-filter in A. Then F is weakly irreducible if and only if the following conditions hold*

(WIrF) $(\forall x, y \in A)((x \notin F \wedge y \notin F) \implies (\exists z \notin F)(x \leqslant z \wedge y \leqslant z))$.

Every weakly irreducible UP-filter in a meet-commutative UP-algebra is a prime UP-filter ([**34**], Theorem 4.2). So, every weakly irreducible UP-filter of a meet-commutative UP-algebra is an irreducible UP-filter ([**34**], Theorem 4.3).

As shown in [**22**], any prime UP-filter (of the first kind) in a meet-commutative UP-algebra is an irreducible UP-filter. In this paper it is shown that the weakly irreducible UP-filter is a prime UP-filter (of the first kind) and, therefore, the irreducible UP-filter. It is quite natural to ask the question: When will the reverse be true?

One of the answers can be recognized immediately:

Theorem 4.4. *If the lattice $\mathfrak{F}(A)$ of a meet-commutative UP-algebra A is distributive, then any irreducible UP-filter in A is a weakly irreducible UP-filter in A.*

4.3. Semi-prime UP-filters. This subsection presents the concept of semi-prime UP-filters and some of its basic features introduced by Iampan and Romano in [**12**].

The term semi-prime ideal was first used by W. Krull in his famous paper [**19**] (pp. 735). (Cited according to [**25**], page 107.). Y. Rav adapted this term to general lattices ([**25**]). In addition, in the mentioned article, Rav introduced and analyzed the concept of semi-prime filters in general semi-lattices, also.

Definition 4.6. ([**12**]) Let A be a meet-commutative UP-algebra. A UP-filter F of A is said to be a *semi-prime UP-filter* of A if the following holds

$$(\forall x, y, z \in A)((x \sqcup y \in F \wedge x \sqcup z \in F) \implies (\exists d \in A)(d \leqslant y \wedge d \leqslant z)(x \sqcup d \in F)).$$

The following example shows that one UP-filter in a meet-commutative UP-algebra can be a prime UP-filter and a semi-prime UP-filter at the same time.

Example 4.8. Let A be a meet-commutative UP-algebra as in Example 4.2. By direct checking, it can be proved that the subsets $F := \{0, b\}$ and $G := \{0, c\}$ are semi-prime UP-filters in A and they are prime UP filters.

A UP-filter in a meet-commutative UP-algebra can be a semi-prime UP-filter and that it is not a prime UP-filter, as the following example shows.

Example 4.9. Let A be a meet-commutative UP-algebra as in Example 4.2. By direct checking, it can be proved that the subset $\{0\}$ is a semi-prime UP-filter in A and it is not a prime UP-filter as is shown in Example 4.2.

While in a meet-commutative UP-algebra the intersection of prime UP-filters does not have to be a prime UP-filter, the intersection of semi-prime UP-filters in such a UP-algebra is always semi-prime UP- filter

Theorem 4.5. *The intersection of two semi-prime UP-filters of a meet-commutative UP-algebra A is a semi-prime UP-filter in A.*

The following example shows that a prime UP-filter in a meet-commutative UP-algebra does not have to be a semi-prime UP-filter.

Example 4.10. Let $A = \{0, a, b, c, d\}$ and operation '$\cdot$' defined on A as follows:

$\cdot$	0	a	b	c	d
0	0	a	b	c	d
a	0	0	0	0	d
b	0	c	0	c	d
c	0	b	b	0	d
d	0	a	b	c	0

Then $A = (A, \cdot, 0)$ is a meet-commutative UP-algebra. Subsets $\{0\}$, $\{0, b\}$, $\{0, c\}$, $\{0, d\}$ and $\{0, a, b, c, d\}$ are UP-filters of A. In doing so, the following is valid:

- The UP-filters $\{0\}$, $\{0, b\}$ and $\{0, c\}$ are not prime and they are not semi-prime;
- The UP-filter $F := \{0, d\}$ is a prime UP-filter in A but it is not a semi-prime.

5. Final comments and possible further research in this domain

There is a belief in the academic community of mathematicians that an important task of artificial intelligence is to make a computer simulate a human being in dealing with certainty and understanding of uncertainty in information. They are convinced that in the human endeavor to solve this task logic can offer a foundation for these intentions. Until recently, information processing dealing with certain information relied on classical logic. Non-classical logic including many-valued logic such as Intuitionist logic and fuzzy logic takes the advantage of classical logic to handle information with various faces of uncertainty, such as fuzziness and randomness. Therefore, non-classical logic has become a formal and useful tool for computer science to deal with fuzzy information and uncertain information. Such logics have been represented as algebras, that is, sets with one, two, or more algebraic operations satisfying some conditions inspired by these logics. In the past and this century, have been designed, among others, BCK-algebras, BCI-algebras and BL-algebras and a basic logic as well as many others such as KU-algebras, for example.

UP-algebra is designed in 2017 as a generalization of KU-algebra by A. Iampan ([**8**]). Then, several other authors took part in analyzing the properties of this class of logical algebras. Some of them have analyzed UP-filters in these algebras. The logical milieu of this research sometimes was the fuzzy environment. The concepts of implicative, comparative, allied, shift and normal filters have been designed by several authors. The concept of near UP-filters in UP-algebras, as a generalization of the notion of UP-filters is discussed in the papers [**12, 38**]. This paper pretends to be a sublimation of the research of UP-filter types in UP-algebras.

The concept of meet-commutative UP-algebras was introduced in the article [**39**] by Sawika et al. In such a UP-algebra, the concept of prime UP-filter (of the first kind) was introduced by Muhiuddin et al. ([**22**]). In papers [**32, 34**], the concepts of prime UP-filter of the second kind and third kind are introduced and the relationship of these three types of prime UP-filters in these UP-algebras is considered. In this paper, the authors sought to present a range of filters designed so far in the class of meet-commutative UP-algebras.

For a meet-commutative UP-algebra A that satisfies the condition

$$(\forall x, y \in A)((x \cdot y) \sqcup (y \cdot x) = 0),$$

the term 'pre-linear meet-commutative UP-algebra' can be used by looking at the MTL-algebra (for example, [**42**]). Analyzing the properties of this special class of meet-commutative UP-algebras and their connection with MTL-algebras could be the subject of one of the following researches. On the other hand, since $(A, \sqcup)$ is a semi-lattice, one might investigate whether it is possible and how to design a modification of Stone's theorem from 1936 (M. H. Stone. The theory of representation for Boolean algebras. Trans. Am. Math. Soc., 40 (1) (1936), 37-111).

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