NEUTROSOPHIC SETS IN UP-ALGEBRAS BY MEANS OF INTERVAL-VALUED FUZZY SETS

Metawee Songsaeng and Aiyared Iampan

Abstract. In this paper, we introduce the notion of interval-valued neutrosophic UP-subalgebras (resp., interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals) of UP-algebras, proved some results, and their generalizations. Furthermore, we discuss the relations between interval-valued neutrosophic UP-subalgebras (resp., interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals) and their level subsets.

1. Introduction and Preliminaries

Among many algebraic structures, algebras of logic form important class of algebras. Examples of these are BCK-algebras [8], BCI-algebras [9], B-algebras [21], UP-algebras [5] and others. They are strong connected with logic. For example, BCI-algebras introduced by Išiški [9] in 1966 have connections with BCI-logic being the BCI-system in combinatory logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Išiški [8, 9] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.

A fuzzy set \( f \) in a nonempty set \( S \) is a function from \( S \) to the closed interval \([0, 1] \). The concept of a fuzzy set in a nonempty set was first considered by Zadeh [32]. The fuzzy set theories developed by Zadeh and others have found many applications in the domain of mathematics and elsewhere. Zadeh [33] was introduced an interval-value fuzzy sets. An interval-valued fuzzy set is defined by an interval-valued membership function. Wang et al. [31] introduced the concept of interval-valued neutrosophic sets. The interval-valued neutrosophic set is an instance of neutrosophic set which can be used in real scientific and engineering applications. Jun et al. [10] introduced the notion of interval-valued neutrosophic sets with applications in BCK/BCI-algebra, they also introduced the notion of interval-valued neutrosophic length of an interval-valued neutrosophic set, and investigate their properties and relations. In 2018-2019, Muhiuddin et al. [15, 16, 17, 18, 19, 20] applied the notion of neutrosophic sets to semigroups, BCK/BCI-algebras.

In this paper, we apply the concept of interval-valued neutrosophic sets to UP-algebras. We introduce the notion of interval-valued neutrosophic UP-subalgebras (resp., interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals) in UP-algebras, proved some results, and their generalizations. Furthermore, we discuss the relations between interval-valued neutrosophic UP-subalgebras (resp., interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals) and their level subsets.

Before we begin our study, we will give the definition of a UP-algebra.

**Definition 1.1.** ([5]) An algebra \( X = (X, \cdot, 0) \) of type \((2, 0)\) is called a UP-algebra, where \( X \) is a nonempty set, \( \cdot \) is a binary operation on \( X \), and 0 is a fixed element of \( X \) (i.e., a nullary operation) if it satisfies the following axioms: for any \( x, y, z \in X \),

\[
\text{(UP-1): } (y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0,
\]

\[
\text{(UP-2): } 0 \cdot x = x,
\]

\[
\text{(UP-3): } x \cdot 0 = 0, \text{ and}
\]

\[
\text{(UP-4): } x \cdot y = 0 \text{ and } y \cdot x = 0 \text{ imply } x = y.
\]

From [5], the binary relation \( \leq \) on a UP-algebra \( X = (X, \cdot, 0) \) defined as follows:

\[
(\forall x, y \in X)(x \leq y \Leftrightarrow x \cdot y = 0).
\]
EXAMPLE 1.1. [23] Let $X$ be a universal set and let $\Omega \in \mathcal{P}(X)$, where $\mathcal{P}(X)$ means the power set of $X$. Let $\mathcal{P}_\Omega(X) = \{ A \in \mathcal{P}(X) \mid \Omega \subseteq A \}$. Define a binary operation $\cdot$ on $\mathcal{P}_\Omega(X)$ by putting $A \cdot B = B' \cap (A' \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(X)$, where $A'$ means the complement of a subset $A$. Then $(\mathcal{P}_\Omega(X), \cdot, \Omega)$ is a UP-algebra and we shall call it the \textit{generalized power UP-algebra of type 1 with respect to} $\Omega$. Let $\mathcal{P}_\Omega^0(X) = \{ A \in \mathcal{P}(X) \mid A \subseteq \Omega \}$. Define a binary operation $*$ on $\mathcal{P}_\Omega^0(X)$ by putting $A * B = B \cup (A' \cap \Omega)$ for all $A, B \in \mathcal{P}_\Omega^0(X)$. Then $(\mathcal{P}_\Omega^0(X), *, \Omega)$ is a UP-algebra and we shall call it the \textit{generalized power UP-algebra of type 2 with respect to} $\Omega$. In particular, $(\mathcal{P}(X), \cdot, \emptyset)$ is a UP-algebra and we shall call it the power UP-algebra of type 1, and $(\mathcal{P}(X), *, \emptyset)$ is a UP-algebra and we shall call it the \textit{power UP-algebra of type 2}.

EXAMPLE 1.2. ([3]) Let $\mathbb{N}$ be the set of all natural numbers with two binary operations $\circ$ and $\bullet$ defined by

$$x \circ y = \begin{cases} y & \text{if } x < y, \\ 0 & \text{otherwise} \end{cases}$$

and

$$x \bullet y = \begin{cases} y & \text{if } x > y \text{ or } x = 0, \\ 0 & \text{otherwise}. \end{cases}$$

Then $(\mathbb{N}, \circ, 0)$ and $(\mathbb{N}, \bullet, 0)$ are UP-algebras.

For more examples of UP-algebras, see [1, 2, 6, 22, 23, 24, 25].

In a UP-algebra $X = (X, \cdot, 0)$, the following assertions are valid (see [5, 6]).

(1.1) \hfill $(\forall x \in X)(x \cdot x = 0)$,

(1.2) \hfill $(\forall x, y, z \in X)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0)$,

(1.3) \hfill $(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0)$,

(1.4) \hfill $(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0)$,

(1.5) \hfill $(\forall x, y \in X)(x \cdot (y \cdot x) = 0)$,

(1.6) \hfill $(\forall x, y \in X)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x)$,

(1.7) \hfill $(\forall x, y \in X)(x \cdot (y \cdot y) = 0)$,

(1.8) \hfill $(\forall a, x, y, z \in X)(((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0)$,

(1.9) \hfill $(\forall a, x, y, z \in X)(((a \cdot x) \cdot (a \cdot y)) \cdot (x \cdot (y \cdot z)) = 0)$,

(1.10) \hfill $(\forall x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot z) = 0)$,

(1.11) \hfill $(\forall x, y, z \in X)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0)$,

(1.12) \hfill $(\forall x, y, z \in X)((x \cdot y) \cdot z) \cdot (x \cdot (y \cdot z)) = 0)$, and

(1.13) \hfill $(\forall a, x, y, z \in X)(((x \cdot y) \cdot z) \cdot (y \cdot (a \cdot z)) = 0)$.

DEFINITION 1.2. ([5, 26, 4, 7]) A nonempty subset $S$ of a UP-algebra $X = (X, \cdot, 0)$ is called

1. a UP-subalgebra of $X$ if $(\forall x, y \in S)(x \cdot y \in S)$.
2. a near UP-filter of $X$ if it satisfies the following properties:
(i) the constant 0 of $X$ is in $S$, and 
(ii) $(\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S)$.

(3) a $UP$-filter of $X$ if it satisfies the following properties:
(i) the constant 0 of $X$ is in $S$, and 
(ii) $(\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S)$.

(4) a $UP$-ideal of $X$ if it satisfies the following properties:
(i) the constant 0 of $X$ is in $S$, and 
(ii) $(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S)$.

(5) a strong $UP$-ideal (renamed from a strongly UP-ideal) of $X$ if it satisfies the following properties:
(i) the constant 0 of $X$ is in $S$, and 
(ii) $(\forall x, y, z \in X)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S)$.

Guntasow et al. [4] proved that the notion of UP-subalgebras is a generalization of near UP-filters, near UP-filters is a generalization of UP-filters, UP-filters is a generalization of UP-ideals, and UP-ideals is a generalization of strong UP-ideals. Moreover, they also proved that a UP-algebra $X$ is the only one strong UP-ideal of itself.

In 1965, the concept of a fuzzy set in a nonempty set was first considered by Zadeh [32] as the following definition.

**Definition 1.3.** A fuzzy set (briefly, FS) in a nonempty set $X$ (or a fuzzy subset of $X$) is defined to be a function $\lambda : X \rightarrow [0, 1]$, where $[0, 1]$ is the unit segment of the real line. Denote by $[0, 1]^X$ the collection of all fuzzy sets in $X$. Define a binary relation $\leq$ on $[0, 1]^X$ as follows:

\[ (\forall \lambda, \mu \in [0, 1]^X)(\lambda \leq \mu \Leftrightarrow (\forall x \in X)(\lambda(x) \leq \mu(x))). \]

**Definition 1.4.** ([26]) Let $\lambda$ be a fuzzy set in a nonempty set $X$. The complement of $\lambda$, denoted by $\lambda^c$, is defined by

\[ (\forall x \in X)(\lambda^c(x) = 1 - \lambda(x)). \]

**Definition 1.5.** ([14]) Let $\{\lambda_i \mid i \in J\}$ be a family of fuzzy sets in a nonempty set $X$. We define the join and the meet of $\{\lambda_i \mid i \in J\}$, denoted by $\vee_{i \in J} \lambda_i$ and $\wedge_{i \in J} \lambda_i$, respectively, as follows:

\begin{align*}
(\forall x \in X)((\vee_{i \in J} \lambda_i)(x) &= \sup_{i \in J}{\lambda_i(x)}), \\
(\forall x \in X)((\wedge_{i \in J} \lambda_i)(x) &= \inf_{i \in J}{\lambda_i(x)}).
\end{align*}

In particular, if $\lambda$ and $\mu$ be fuzzy sets in $X$, we have the join and meet of $\lambda$ and $\mu$ as follows:

\begin{align*}
(\forall x \in X)((\lambda \vee \mu)(x) &= \max\{\lambda(x), \mu(x)\}), \\
(\forall x \in X)((\lambda \wedge \mu)(x) &= \min\{\lambda(x), \mu(x)\}),
\end{align*}

respectively.
An interval number we mean a close subinterval \( \bar{a} = [a^-, a^+] \) of \([0, 1]\), where \(0 \leq a^- \leq a^+ \leq 1\). The interval number \( \bar{a} = [a^-, a^+] \) with \( a^- = a^+ \) is denoted by \( a \). Denote by \([0, 1]\) the set of all interval numbers.

**Definition 1.6.** ([11]) Let \( \{\bar{a}_i \mid i \in J\} \) be a family of interval numbers. We define the refined infimum and the refined supremum of \( \{\bar{a}_i \mid i \in J\} \), denoted by \( \rinf_{i \in J} \bar{a}_i \) and \( \rsup_{i \in J} \bar{a}_i \), respectively, as follows:

\[
\begin{align*}
\rinf_{i \in J} \{\bar{a}_i\} &= [\inf_{i \in J} \{a_i^-\}, \inf_{i \in J} \{a_i^+\}], \\
\rsup_{i \in J} \{\bar{a}_i\} &= [\sup_{i \in J} \{a_i^-\}, \sup_{i \in J} \{a_i^+\}].
\end{align*}
\]

In particular, if \( \bar{a}_1 \) and \( \bar{a}_2 \) are interval numbers, we define the refined minimum and the refined maximum of \( \bar{a}_1 \) and \( \bar{a}_2 \), denoted by \( \rmin\{\bar{a}_1, \bar{a}_2\} \) and \( \rmax\{\bar{a}_1, \bar{a}_2\} \), respectively, as follows:

\[
\begin{align*}
\rmin\{\bar{a}_1, \bar{a}_2\} &= [\min\{a_1^-, a_2^+\}, \min\{a_1^+, a_2^+\}], \\
\rmax\{\bar{a}_1, \bar{a}_2\} &= [\max\{a_1^-, a_2^-\}, \max\{a_1^+, a_2^+\}].
\end{align*}
\]

**Definition 1.7.** ([11]) Let \( \bar{a}_1 \) and \( \bar{a}_2 \) be interval numbers. We define the symbols \( \preceq \), \( \succeq \), \( \preceq \) in case of \( \bar{a}_1 \) and \( \bar{a}_2 \) as follows:

\[
\bar{a}_1 \preceq \bar{a}_2 \Leftrightarrow a_1^- \preceq a_2^- \text{ and } a_1^+ \succeq a_2^+,
\]

and similarly we may have \( \bar{a}_1 \succeq \bar{a}_2 \) and \( \bar{a}_1 = \bar{a}_2 \). To say \( \bar{a}_1 \succ \bar{a}_2 \) (resp., \( \bar{a}_1 \prec \bar{a}_2 \)) we mean \( a_1^+ \preceq a_2^- \) and \( a_1^- \succeq a_2^+ \) (resp., \( a_1^+ \succeq a_2^- \) and \( a_1^- \preceq a_2^+ \)).

**Definition 1.8.** ([33]) Let \( \bar{a} \) be an interval number. The complement of \( \bar{a} \), denoted by \( \bar{a}^C \), is defined by the interval number

\[
\bar{a}^C = [1 - a^+, 1 - a^-].
\]

In the \([0, 1]\), the following assertions are valid (see [29]).

\[
\begin{align*}
(\forall \bar{a} \in [0, 1]) & ((\bar{a}^C)^C = \bar{a}), \\
(\forall \bar{a} \in [0, 1]) & (\rmax\{\bar{a}, \bar{a}\} = \bar{a} \text{ and } \rmin\{\bar{a}, \bar{a}\} = \bar{a}), \\
(\forall \bar{a}_1, \bar{a}_2 \in [0, 1]) & (\rmax\{\bar{a}_1, \bar{a}_2\} = \rmax\{\bar{a}_2, \bar{a}_1\} \text{ and } \rmin\{\bar{a}_1, \bar{a}_2\} = \rmin\{\bar{a}_2, \bar{a}_1\}), \\
(\forall \bar{a}_1, \bar{a}_2 \in [0, 1]) & (\rmax\{\bar{a}_1, \bar{a}_2\} \succeq \bar{a}_1 \text{ and } \bar{a}_2 \succeq \rmin\{\bar{a}_1, \bar{a}_2\}), \\
(\forall \bar{a}_1, \bar{a}_2 \in [0, 1]) & (\bar{a}_1 \succeq \bar{a}_2 \Leftrightarrow \bar{a}_1^C \preceq \bar{a}_2^C), \\
(\forall \bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4 \in [0, 1]) & (\bar{a}_1 \succeq \bar{a}_2, \bar{a}_3 \succeq \bar{a}_4 \Rightarrow \rmin\{\bar{a}_1, \bar{a}_3\} \succeq \rmin\{\bar{a}_2, \bar{a}_4\}).
\end{align*}
\]
We deﬁne the symbols \(~\) of an element \(x\) in \(X\) as follows:

\[
(\forall x \in X)(A \subseteq B \iff A(x) \leq B(x)),
\]

and similarly we may have \(A \supseteq B\) and \(A = B\).

**Definition 1.11.** ([33]) Let \(A\) be an interval-valued fuzzy set in a nonempty set \(X\). The complement of \(A\), denoted by \(A^C\), is deﬁned as follows: \(A^C(x) = A(x)^C\) for all \(x \in X\), that is,

\[
(\forall x \in X)(A^C(x) = [1 - A^+(x), 1 - A^-(x))].
\]
We note that $A^{-}\times(x) = 1 - A^{+}(x)$ and $A^{+}\times(x) = 1 - A^{-}(x)$ for all $x \in X$.

**Definition 1.12.** ([33]) Let $\{A_i \mid i \in J\}$ be a family of interval-valued fuzzy sets in a nonempty set $X$. We define the intersection and the union of $\{A_i \mid i \in J\}$, denoted by $\cap_{i \in J}A_i$ and $\cup_{i \in J}A_i$, respectively, as follows:

$$\forall x \in X)((\cap_{i \in J}A_i)(x) = \inf_{i \in J}\{A_i(x)\}),$$

$$\forall x \in X)((\cup_{i \in J}A_i)(x) = \sup_{i \in J}\{A_i(x)\}).$$

We note that

$$\forall x \in X)((\cap_{i \in J}A_i)(x) = (\land_{i \in J}A_i^{-})(x) = \inf_{i \in J}\{A_i^{-}(x)\}))$$

and

$$\forall x \in X)((\cup_{i \in J}A_i)(x) = (\land_{i \in J}A_i^{+})(x) = \inf_{i \in J}\{A_i^{+}(x)\}).$$

Similarly,

$$\forall x \in X)((\cap_{i \in J}A_i^{+})(x) = (\land_{i \in J}A_i^{-})(x) = \sup_{i \in J}\{A_i^{-}(x)\}))$$

and

$$\forall x \in X)((\cup_{i \in J}A_i^{+})(x) = (\land_{i \in J}A_i^{+})(x) = \sup_{i \in J}\{A_i^{+}(x)\}).$$

In particular, if $A_1$ and $A_2$ are interval-valued fuzzy sets in $X$, we have the intersection and the union of $A_1$ and $A_2$ as follows:

$$\forall x \in X)((A_1 \cap A_2)(x) = \min\{A_1(x), A_2(x)\}),$$

$$\forall x \in X)((A_1 \cup A_2)(x) = \max\{A_1(x), A_2(x)\}).$$

### 2. Interval-Valued Neutrosophic Sets in UP-Algebras

In 2005, the concept of an interval-valued neutrosophic set was first considered by Wang et al. [31] as the following definition.

An **interval-valued neutrosophic set** (briefly, IVNS) in a nonempty set $X$ is a structure of the form:

$$A := \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\},$$

where $A_T$, $A_I$, and $A_F$ are interval-valued fuzzy sets in $X$, which are called a **truth membership function**, an **indeterminacy membership function** and a **falsity membership function**, respectively.

For our convenience, we will denote an IVNS as

$$A = (X, A_T, A_I, A_F) = (X, A_T, A_I, A_F) = \{(x, A_T(x), A_I(x), A_F(x)) \mid x \in X\}.$$

Now, we introduce the notions of interval-valued neutrosophic UP-subalgebras, interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals of UP-algebras, provide the necessary examples, investigate their properties, and prove their generalizations.

In what follows, let $X$ denote a UP-algebra $(X, \cdot, 0)$ unless otherwise specified.
Definition 2.1. An IVNS $A$ in $X$ is called an interval-valued neutrosophic UP-subalgebra of $X$ if it holds the following conditions:

\[(\forall x, y \in X)(A_T(x \cdot y) \geq \text{rmin}\{A_T(x), A_T(y)\}),\]
\[(\forall x, y \in X)(A_I(x \cdot y) \leq \text{rmax}\{A_I(x), A_I(y)\}),\]
\[(\forall x, y \in X)(A_F(x \cdot y) \geq \text{rmin}\{A_F(x), A_F(y)\}).\]

Proposition 2.1. If $A$ is an interval-valued neutrosophic UP-subalgebra of $X$, then

\[(\forall x \in X)(A_T(0) \geq A_T(x)),\]
\[(\forall x \in X)(A_I(0) \leq A_I(x)),\]
\[(\forall x \in X)(A_F(0) \geq A_F(x)).\]

Proof. Let $A$ be an interval-valued neutrosophic UP-subalgebra of $X$. By (1.1), we have

\[
(\forall x \in X) \begin{pmatrix} A_T(0) = A_T(x \cdot x) \geq \text{rmin}\{A_T(x), A_T(x)\} = A_T(x), \\
A_I(0) = A_I(x \cdot x) \leq \text{rmin}\{A_I(x), A_I(x)\} = A_I(x), \\
A_F(0) = A_F(x \cdot x) \geq \text{rmin}\{A_F(x), A_F(x)\} = A_F(x) \end{pmatrix}.
\]

Example 2.1. Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element $0$ and a binary operation $\cdot$ defined by the following Cayley table:

<table>
<thead>
<tr>
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<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<td>3</td>
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</tr>
</tbody>
</table>

We define an IVNS $A$ in $X$ as follows:

\[
A_T = \begin{pmatrix} 0 & 1 & 2 & 3 \\
[0.9, 1] & [0.2, 0.5] & [0.3, 0.4] & [0.3, 0.4] \end{pmatrix},
A_I = \begin{pmatrix} 0 & 1 & 2 & 3 \\
[0.0, 0.3] & [0.7, 0.8] & [0.2, 0.3] & [0.8, 0.9] \end{pmatrix},
A_F = \begin{pmatrix} 0 & 1 & 2 & 3 \\
[0.7, 1] & [0.1, 0.3] & [0.5, 0.7] & [0.6, 0.7] \end{pmatrix}.
\]

Then $A$ is an interval-valued neutrosophic UP-subalgebra of $X$.

Definition 2.2. An IVNS $A$ in $X$ is called an interval-valued neutrosophic near UP-filter of $X$ if it holds the following conditions: (2.4), (2.5), (2.6), and

\[(\forall x, y \in X)(A_T(x \cdot y) \geq A_T(y)),\]
\[(\forall x, y \in X)(A_I(x \cdot y) \leq A_I(y)),\]
\[(\forall x, y \in X)(A_F(x \cdot y) \geq A_F(y)).\]
Example 2.2. Let \( X = \{0, 1, 2, 3\} \) be a UP-algebra with a fixed element 0 and a binary operation \( \cdot \) defined by the following Cayley table:

\[
\begin{array}{c|cccc}
  & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 1 & 2 & 3 \\
1 & 0 & 0 & 2 & 0 \\
2 & 0 & 1 & 0 & 3 \\
3 & 0 & 1 & 2 & 0 
\end{array}
\]

We define an IVNS \( A \) in \( X \) as follows:

\[
A_T = \left( \begin{array}{cccc}
0 & 1 & 2 & 3 \\
[0.9,1] & [0.6,0.8] & [0.5,0.6] & [0.4,0.6] \\
\end{array} \right), \\
A_I = \left( \begin{array}{cccc}
0 & 1 & 2 & 3 \\
[0.0,1] & [0.1,0.3] & [0.3,0.4] & [0.5,0.8] \\
\end{array} \right), \\
A_F = \left( \begin{array}{cccc}
0 & 1 & 2 & 3 \\
[0.8,0.9] & [0.6,0.8] & [0.5,0.7] & [0.4,0.6] \\
\end{array} \right).
\]

Then \( A \) is an interval-valued neutrosophic near UP-filter of \( X \).

Definition 2.3. An IVNS \( A \) in \( X \) is called an interval-valued neutrosophic UP-filter of \( X \) if it holds the following conditions: (2.4), (2.5), (2.6), and

\[
\begin{align*}
(2.10) & & (\forall x, y \in X) (A_T(y) \geq \text{rmin}\{A_T(x \cdot y), A_T(x)\}), \\
(2.11) & & (\forall x, y \in X) (A_I(y) \leq \text{rmax}\{A_I(x \cdot y), A_I(x)\}), \\
(2.12) & & (\forall x, y \in X) (A_F(y) \geq \text{rmin}\{A_F(x \cdot y), A_F(x)\}).
\end{align*}
\]

Example 2.3. Let \( X = \{0, 1, 2, 3\} \) be a UP-algebra with a fixed element 0 and a binary operation \( \cdot \) defined by the following Cayley table:

\[
\begin{array}{c|cccc}
  & 0 & 1 & 2 & 3 \\
\hline
0 & 0 & 1 & 2 & 3 \\
1 & 0 & 0 & 3 & 3 \\
2 & 0 & 1 & 0 & 0 \\
3 & 0 & 1 & 2 & 0 
\end{array}
\]

We define an IVNS \( A \) in \( X \) as follows:

\[
A_T = \left( \begin{array}{cccc}
0 & 1 & 2 & 3 \\
[0.9,1] & [0.5,0.8] & [0.3,0.6] & [0.3,0.6] \\
\end{array} \right), \\
A_I = \left( \begin{array}{cccc}
0 & 1 & 2 & 3 \\
[0.0,1] & [0.2,0.3] & [0.6,0.8] & [0.6,0.8] \\
\end{array} \right), \\
A_F = \left( \begin{array}{cccc}
0 & 1 & 2 & 3 \\
[0.8,0.9] & [0.4,0.5] & [0.3,0.4] & [0.3,0.4] \\
\end{array} \right).
\]

Then \( A \) is an interval-valued neutrosophic UP-filter of \( X \).
Definition 2.4. An IVNS $A$ in $X$ is called an interval-valued neutrosophic UP-ideal of $X$ if it holds the following conditions: (2.4), (2.5), (2.6), and
\[
\begin{align*}
(2.13) & \quad (\forall x, y, z \in X) (A_T(x \cdot z) \geq \min\{A_T(x \cdot (y \cdot z)), A_T(x)\}), \\
(2.14) & \quad (\forall x, y, z \in X) (A_I(x \cdot z) \leq \max\{A_I(x \cdot (y \cdot z)), A_I(x)\}), \\
(2.15) & \quad (\forall x, y, z \in X) (A_F(x \cdot z) \geq \min\{A_F(x \cdot (y \cdot z)), A_F(x)\}).
\end{align*}
\]

Example 2.4. Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

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We define an IVNS $A$ in $X$ as follows:
\[
\begin{align*}
A_T &= \begin{pmatrix}
0 & 1 & 2 & 3 \\
0.9 & 0.7 & 0.9 & 0.6 & 0.8 & 0.6 & 0.9
\end{pmatrix}, \\
A_I &= \begin{pmatrix}
0 & 1 & 2 & 3 \\
0.1 & 0.3 & 0.5 & 0.4 & 0.7 & 0.3 & 0.6
\end{pmatrix}, \\
A_F &= \begin{pmatrix}
0 & 1 & 2 & 3 \\
0.8 & 0.9 & 0.5 & 0.9 & 0.4 & 0.6 & 0.5 & 0.8
\end{pmatrix}.
\]

Then $A$ is an interval-valued neutrosophic UP-ideal of $X$.

Definition 2.5. An IVNS $A$ in $X$ is called an interval-valued neutrosophic strong UP-ideal of $X$ if it holds the following conditions: (2.4), (2.5), (2.6), and
\[
\begin{align*}
(2.16) & \quad (\forall x, y, z \in X) (A_T(x) \geq \min\{A_T((z \cdot y) \cdot (z \cdot x)), A_T(y)\}), \\
(2.17) & \quad (\forall x, y, z \in X) (A_I(x) \leq \max\{A_I((z \cdot y) \cdot (z \cdot x)), A_I(y)\}), \\
(2.18) & \quad (\forall x, y, z \in X) (A_F(x) \geq \min\{A_F((z \cdot y) \cdot (z \cdot x)), A_F(y)\}).
\end{align*}
\]

Example 2.5. Let $X = \{0, 1, 2, 3\}$ be a UP-algebra with a fixed element 0 and a binary operation $\cdot$ defined by the following Cayley table:

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We define an IVNS $A$ in $X$ as follows:
\[
\begin{align*}
(\forall x \in X) & \quad A_T(x) = [0.7, 0.9], \\
& \quad A_I(x) = [0.3, 0.5], \\
& \quad A_F(x) = [0.5, 0.9].
\end{align*}
\]

Then $A$ is an interval-valued neutrosophic strong UP-ideal of $X$. 
Definition 2.6. An IVNS $A$ in a nonempty set $X$ is said to be constant if $A$ is a constant function from $X$ to $[[0, 1]]^3$. That is, $A_T$, $A_I$, and $A_F$ are constant functions from $X$ to $[[0, 1]]$.

Theorem 2.1. An IVNS $A$ in $X$ is constant if and only if it is an interval-valued neutrosophic strong UP-ideal of $X$.

Proof. Assume that an IVNS $A$ is constant in $X$. Then $A_T(x) = A_T(0)$, $A_I(x) = A_I(0)$, and $A_F(x) = A_F(0)$ for all $x \in X$. Then for all $x \in X$, $A_T(0) \succeq A_T(x)$, $A_I(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$, and for all $x, y, z \in X$,

$$\min\{A_T((z \cdot y) \cdot (z \cdot x)), A_T(y)\} = \min\{A_T(0), A_T(0)\}\quad(1.27)$$
$$\max\{A_I((z \cdot y) \cdot (z \cdot x)), A_I(y)\} = \max\{A_I(0), A_I(0)\}\quad(1.27)$$
$$\min\{A_F((z \cdot y) \cdot (z \cdot x)), A_F(y)\} = \min\{A_F(0), A_F(0)\}\quad(1.27)$$

Hence, $A$ is an interval-valued neutrosophic strong UP-ideal of $X$.

Conversely, assume that $A$ is an interval-valued neutrosophic strong UP-ideal of $X$. Then for all $x \in X$,

$$A_T(x) \succeq \min\{A_T((x \cdot 0) \cdot (x \cdot x)), A_T(0)\}\quad((\text{UP}-3))$$
$$= \min\{A_T(0 \cdot (x \cdot x)), A_T(0)\}\quad((\text{UP}-2))$$
$$= \min\{A_T(x \cdot x), A_T(0)\}\quad((1.1))$$
$$= A_T(0)\quad((1.27))$$
$$\succeq A_T(x),\quad(1.27)$$

$$A_I(x) \preceq \max\{A_I((x \cdot 0) \cdot (x \cdot x)), A_I(0)\}\quad((\text{UP}-3))$$
$$= \max\{A_I(0 \cdot (x \cdot x)), A_I(0)\}\quad((\text{UP}-2))$$
$$= \max\{A_I(x \cdot x), A_I(0)\}\quad((1.1))$$
$$= A_I(0)\quad((1.27))$$
$$\preceq A_I(x),\quad(1.27)$$
\[ A_F(x) \geq \text{rmin}\{A_F((x \cdot 0) \cdot (x \cdot x)), A_F(0)\} \]

((UP-3))
\[ = \text{rmin}\{A_F(0 \cdot (x \cdot x)), A_F(0)\} \]

((UP-2))
\[ = \text{rmin}\{A_F(x \cdot x), A_F(0)\} \]

((1.1))
\[ = \text{rmin}\{A_F(0), A_F(0)\} \]

((1.27))
\[ = A_F(0) \]

\[ \geq A_F(x). \]

Thus \( A_T(0) = A_T(x), A_I(0) = A_I(x), \) and \( A_F(0) = A_F(x) \) for all \( x \in X \). Hence, \( A \) is constant.

\textbf{Theorem 2.2.} Every interval-valued neutrosophic strong UP-ideal of \( X \) is an interval-valued neutrosophic UP-ideal.

\textbf{Proof.} Assume that \( A \) is an interval-valued neutrosophic strong UP-ideal of \( X \). Then for all \( x \in X, A_T(0) \supseteq A_T(x), A_T(0) \supseteq A_I(x), \) and \( A_F(0) \supseteq A_F(x) \). Let \( x, y, z \in X \). Then

\[ A_T(x \cdot z) \supseteq \text{rmin}\{A_T((z \cdot y) \cdot (z \cdot (x \cdot z))), A_T(y)\} \]

((1.5))
\[ = \text{rmin}\{A_T((z \cdot y) \cdot 0), A_T(y)\} \]

((UP-3))
\[ = \text{rmin}\{A_T(0), A_T(y)\} \]

\[ = A_T(y) \]

\[ \geq \text{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}, \]

\[ A_I(x \cdot z) \supseteq \text{rmax}\{A_I((z \cdot y) \cdot (z \cdot (x \cdot z))), A_I(y)\} \]

((1.5))
\[ = \text{rmax}\{A_I((z \cdot y) \cdot 0), A_I(y)\} \]

((UP-3))
\[ = \text{rmax}\{A_I(0), A_I(y)\} \]

\[ = A_I(y) \]

\[ \leq \text{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}, \]

\[ A_F(x \cdot z) \supseteq \text{rmin}\{A_F((z \cdot y) \cdot (z \cdot (x \cdot z))), A_F(y)\} \]

((1.5))
\[ = \text{rmin}\{A_F((z \cdot y) \cdot 0), A_F(y)\} \]

((UP-3))
\[ = \text{rmin}\{A_F(0), A_F(y)\} \]

\[ = A_F(0) \]

\[ \geq A_F(y). \]

Hence, \( A \) is an interval-valued neutrosophic UP-ideal of \( X \). \qed

The following example show that the converse of Theorem 2.2 is not true.

\begin{example}
From Example 2.4, we have \( A \) is an interval-valued neutrosophic UP-ideal of \( X \). Since \( A_T(1) = [0.7, 0.9] \not\supseteq [0.9, 1] = \text{rmin}\{A_T((2 \cdot 0) \cdot (2 \cdot 1)), A_T(0)\} \), we have \( A \) is not an interval-valued neutrosophic strong UP-ideal of \( X \).
\end{example}

\textbf{Theorem 2.3.} Every interval-valued neutrosophic UP-ideal of \( X \) is an interval-valued neutrosophic UP-filter.
Proof. Assume that $A$ is an interval-valued neutrosophic UP-ideal of $X$. Then for all $x \in X$, $A_I(0) \supseteq A_I(x)$, $A_T(0) \subseteq A_T(x)$, and $A_F(0) \supseteq A_F(x)$. Let $x, y \in X$. Then

\begin{align*}
((UP-2)) \quad A_T(y) &= A_T(0 \cdot y) \\
&\supseteq \min\{A_T(0 \cdot (x \cdot y)), A_T(x)\} \\
&= \min\{A_T(x \cdot y), A_T(x)\}, \\
((UP-2)) \quad A_I(y) &= A_I(0 \cdot y) \\
&\subseteq \max\{A_I(0 \cdot (x \cdot y)), A_I(x)\} \\
&= \max\{A_I(x \cdot y), A_I(x)\}, \\
((UP-2)) \quad A_F(y) &= A_F(0 \cdot y) \\
&\subseteq \min\{A_F(0 \cdot (x \cdot y)), A_F(x)\} \\
&= \min\{A_F(x \cdot y), A_F(x)\}.
\end{align*}

Hence, $A$ is an interval-valued neutrosophic UP-filter of $X$. \qed

The following example show that the converse of Theorem 2.3 is not true.

Example 2.7. From Example 2.3, we have $A$ is an interval-valued neutrosophic UP-filter of $X$. Since $A_I(3 \cdot 2) = [0.6, 0.8] \not\supseteq [0.2, 0.3] = \max\{A_I(3 \cdot (1 \cdot 2)), A_I(1)\}$, we have $A$ is not an interval-valued neutrosophic UP-ideal of $X$.

Theorem 2.4. Every interval-valued neutrosophic UP-filter of $X$ is an interval-valued neutrosophic near UP-filter.

Proof. Assume that $A$ is an interval-valued neutrosophic UP-filter of $X$. Then for all $x \in X$, $A_I(0) \supseteq A_I(x)$, $A_T(0) \subseteq A_T(x)$, and $A_F(0) \supseteq A_F(x)$. Let $x, y \in X$. Then

\begin{align*}
((1.5)) \quad A_T(x \cdot y) &\supseteq \min\{A_T(y \cdot (x \cdot y)), A_T(y)\} \\
&= \min\{A_T(y), A_T(y)\} \\
&= A_T(y), \\
((1.5)) \quad A_I(x \cdot y) &\subseteq \max\{A_I(y \cdot (x \cdot y)), A_I(y)\} \\
&= \max\{A_I(y), A_I(y)\} \\
&= A_I(y), \\
((1.5)) \quad A_F(x \cdot y) &\supseteq \min\{A_F(y \cdot (x \cdot y)), A_F(y)\} \\
&= \min\{A_F(y), A_F(y)\} \\
&= A_F(y).
\end{align*}

Hence, $A$ is an interval-valued neutrosophic near UP-filter of $X$. \qed

The following example show that the converse of Theorem 2.4 is not true.

Example 2.8. From Example 2.2, we have $A$ is an interval-valued neutrosophic near UP-filter of $X$. Since $A_F(3) = [0.4, 0.6] \not\supseteq [0.6, 0.8] = \min\{A_F(1 \cdot 3), A_F(1)\}$, we have $A$ is not an interval-valued neutrosophic UP-filter of $X$.
Theorem 2.5. Every interval-valued neutrosophic near UP-filter of $X$ is an interval-valued neutrosophic UP-subalgebra.

Proof. Assume that $A$ is an interval-valued neutrosophic near UP-filter of $X$. Then for all $x \in X$, $A_T(0) \succeq A_T(x)$, $A_T(0) \preceq A_I(x)$, and $A_F(0) \succeq A_F(x)$. Let $x, y \in X$. By (1.29), we have

$$A_T(x \cdot y) \succeq A_T(y) \succeq r_{\min}\{A_T(x), A_T(y)\},$$

$$A_I(x \cdot y) \preceq A_I(y) \preceq r_{\max}\{A_I(x), A_I(y)\},$$

$$A_F(x \cdot y) \succeq A_F(y) \succeq r_{\min}\{A_F(x), A_F(y)\}.$$ 

Hence, $A$ is an interval-valued neutrosophic UP-subalgebra of $X$. 

The following example show that the converse of Theorem 2.5 is not true.

Example 2.9. From Example 2.1, we have $A$ is an interval-valued neutrosophic UP-subalgebra of $X$. Since $A_F(1 \cdot 3) = [0.5, 0.7] \not\succeq [0.6, 0.8] = A_F(3)$, we have $A$ is not an interval-valued neutrosophic near UP-filter of $X$.

By Theorems 2.2, 2.3, 2.4, and 2.5 and Examples 2.6, 2.7, 2.8, and 2.9, we have that the notion of interval-valued neutrosophic UP-subalgebras is a generalization of interval-valued neutrosophic near UP-filters, interval-valued neutrosophic near UP-filters is a generalization of interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-filters is a generalization of interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic UP-ideals is a generalization of interval-valued neutrosophic strong UP-ideals. Moreover, by Theorem 2.1, we obtain that interval-valued neutrosophic strong UP-ideals and constant interval-valued neutrosophic set coincide.

Theorem 2.6. If $A$ is an interval-valued neutrosophic UP-subalgebra of $X$ satisfying the following condition:

$$(\forall x, y \in X) \left( x \cdot y \neq 0 \Rightarrow \begin{cases} A_T(x) \succeq A_T(y) \\ A_I(x) \preceq A_I(y) \\ A_F(x) \succeq A_F(y) \end{cases} \right),$$

then $A$ is an interval-valued neutrosophic near UP-filter of $X$.

Proof. Assume that $A$ is an interval-valued neutrosophic UP-subalgebra of $X$ satisfying the condition (2.19). By Theorem 2.1, we have $A$ satisfies the conditions (2.4), (2.5), and (2.6). Next, let $x, y \in X$.

Case 1: $x \cdot y = 0$. Then

$$A_T(x \cdot y) = A_T(0) \succeq A_T(y),$$

$$A_I(x \cdot y) = A_I(0) \preceq A_I(y),$$

$$A_F(x \cdot y) = A_F(0) \succeq A_F(y).$$
Case 2: \( x \cdot y \neq 0 \). By (2.19), it follows that

\[
\begin{align*}
A_T(x \cdot y) &\geq \text{rmin}\{A_T(x), A_T(y)\} \\
= A_T(y), \\
A_I(x \cdot y) &\leq \text{rmax}\{A_I(x), A_I(y)\} \\
= A_I(y), \\
A_F(x \cdot y) &\geq \text{rmin}\{A_F(x), A_F(y)\} \\
= A_F(y).
\end{align*}
\]

Hence, \( A \) is an interval-valued neutrosophic near UP-filter of \( X \).

\[\square\]

**Theorem 2.7.** If \( A \) is an interval-valued neutrosophic near UP-filter of \( X \) satisfying the following condition:

\[
A_T = A_I = A_F,
\]

then \( A \) is an interval-valued neutrosophic UP-filter of \( X \).

**Proof.** Assume that \( A \) is an interval-valued neutrosophic near UP-filter of \( X \) satisfying the condition (2.20). Then \( A \) satisfies the conditions (2.4), (2.5), and (2.6). Next, let \( x, y \in X \). Then

\[
\begin{align*}
\text{rmin}\{A_T(x \cdot y), A_T(x)\} &\geq \text{rmin}\{A_I(x \cdot y), A_I(x)\} \\
&= \text{rmin}\{A_T(y), A_T(x)\} \\
&\leq A_I(y), \\
\text{rmax}\{A_I(x \cdot y), A_I(x)\} &\geq \text{rmax}\{A_T(x \cdot y), A_T(x)\} \\
&= \text{rmax}\{A_I(y), A_I(x)\} \\
&\geq A_I(y), \\
\text{rmin}\{A_F(x \cdot y), A_F(x)\} &\geq \text{rmin}\{A_I(x \cdot y), A_I(x)\} \\
&= \text{rmin}\{A_F(y), A_F(x)\} \\
&\leq A_F(y).
\end{align*}
\]

Hence, \( A \) is an interval-valued neutrosophic UP-filter of \( X \).

\[\square\]

**Theorem 2.8.** If \( A \) is an interval-valued neutrosophic UP-filter of \( X \) satisfying the following condition:

\[
(\forall x, y, z \in X) \quad \begin{cases} 
A_T(y \cdot (x \cdot z)) = A_T(x \cdot (y \cdot z)) \\
A_I(y \cdot (x \cdot z)) = A_I(x \cdot (y \cdot z)) \\
A_F(y \cdot (x \cdot z)) = A_F(x \cdot (y \cdot z)) 
\end{cases}
\]

then \( A \) is an interval-valued neutrosophic UP-ideal of \( X \).
Proof. Assume that $A$ is an interval-valued neutrosophic UP-filter of $X$ satisfying the condition (2.21). Then $A$ satisfies the conditions (2.4), (2.5), and (2.6). Next, let $x, y, z \in X$. Then

\begin{align*}
\text{(2.10)} & \quad A_T(x \cdot z) \geq \operatorname{rmin}\{A_T(y \cdot (x \cdot z)), A_T(y)\} \\
\text{(2.21) for } A_T & \quad = \operatorname{rmin}\{A_T(x \cdot (y \cdot z)), A_T(y)\}, \\
\text{(2.11)} & \quad A_I(x \cdot z) \leq \operatorname{rmax}\{A_I(y \cdot (x \cdot z)), A_I(y)\} \\
\text{(2.21) for } A_I & \quad = \operatorname{rmax}\{A_I(x \cdot (y \cdot z)), A_I(y)\}, \\
\text{(2.12)} & \quad A_F(x \cdot z) \geq \operatorname{rmin}\{A_F(y \cdot (x \cdot z)), A_F(y)\} \\
\text{(2.21) for } A_F & \quad = \operatorname{rmin}\{A_F(x \cdot (y \cdot z)), A_F(y)\}.
\end{align*}

Hence, $A$ is an interval-valued neutrosophic UP-ideal of $X$. \hfill \Box

**Theorem 2.9.** If $A$ is an IVNS in $X$ satisfying the following condition:

\begin{equation}
(2.22) \quad (\forall x, y, z \in X) \left( \begin{array}{ll}
\text{z} & \leq x \cdot y \Rightarrow \\
A_T(z) & \geq \operatorname{rmin}\{A_T(x), A_T(y)\} \\
A_I(z) & \leq \operatorname{rmax}\{A_I(x), A_I(y)\} \\
A_F(z) & \geq \operatorname{rmin}\{A_F(x), A_F(y)\}
\end{array} \right)
\end{equation}

then $A$ is an interval-valued neutrosophic UP-subalgebra of $X$.

Proof. Assume that $A$ is an IVNS in $X$ satisfying the condition (2.22). Let $x, y \in X$. By (1.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (2.22) that

\begin{align*}
A_T(x \cdot y) & \geq \operatorname{rmin}\{A_T(x), A_T(y)\}, \\
A_I(x \cdot y) & \leq \operatorname{rmax}\{A_I(x), A_I(y)\}, \\
A_F(x \cdot y) & \geq \operatorname{rmin}\{A_F(x), A_F(y)\}.
\end{align*}

Hence, $A$ is an interval-valued neutrosophic UP-subalgebra of $X$. \hfill \Box

**Theorem 2.10.** If $A$ is an IVNS in $X$ satisfying the following condition:

\begin{equation}
(2.23) \quad (\forall x, y, z \in X) \left( \begin{array}{ll}
\text{z} & \leq x \cdot y \Rightarrow \\
A_T(z) & \geq A_T(y) \\
A_I(z) & \leq A_I(y) \\
A_F(z) & \geq A_F(y)
\end{array} \right)
\end{equation}

then $A$ is an interval-valued neutrosophic near UP-filter of $X$.

Proof. Assume that $A$ is an IVNS in $X$ satisfying the condition (2.23). Let $x \in X$. By (UP-2) and (1.1), we have $0 \cdot (x \cdot x) = 0$, that is, $0 \leq x \cdot x$. It follows from (2.23) that $A_T(0) \geq A_T(x), A_I(0) \leq A_I(x)$, and $A_F(0) \geq A_F(x)$. Next, let $x, y \in X$. By (1.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (2.23) that $A_T(x \cdot y) \geq A_T(y), A_I(x \cdot y) \leq A_I(y)$, and $A_F(x \cdot y) \geq A_F(y)$. Hence, $A$ is an interval-valued neutrosophic near UP-filter of $X$. \hfill \Box
Theorem 2.11. If $A$ is an IVNS in $X$ satisfying the following condition:

$$(2.24) \quad (\forall x, y, z \in X) \left\{ \begin{array}{ll}
    z \leq x \cdot y \Rightarrow \\
    A_T(y) \geq \min\{A_T(z), A_T(x)\}, \\
    A_I(y) \leq \max\{A_I(z), A_I(x)\}, \\
    A_F(y) \geq \min\{A_F(z), A_F(x)\}.
\end{array} \right.$$ 

then $A$ is an interval-valued neutrosophic UP-filter of $X$.

Proof. Assume that $A$ is an IVNS in $X$ satisfying the condition (2.24). Let $x \in X$. By (UP-3), we have $x \cdot (x \cdot 0) = 0$, that is, $x \leq x \cdot 0$. It follows from (2.24) and (1.27) that

$$
\begin{align*}
A_T(0) &\geq \min\{A_T(x), A_T(x)\} = A_T(x), \\
A_I(0) &\leq \max\{A_I(x), A_I(x)\} = A_I(x), \\
A_F(0) &\geq \min\{A_F(x), A_F(x)\} = A_F(x).
\end{align*}
$$

Next, let $x, y \in X$. By (1.1), we have $(x \cdot y) \cdot (x \cdot y) = 0$, that is, $x \cdot y \leq x \cdot y$. It follows from (2.24) that

$$
\begin{align*}
A_T(y) &\geq \min\{A_T(x \cdot y), A_T(x)\}, \\
A_I(y) &\leq \max\{A_I(x \cdot y), A_I(x)\}, \\
A_F(y) &\geq \min\{A_F(x \cdot y), A_F(x)\}.
\end{align*}
$$

Hence, $A$ is an interval-valued neutrosophic UP-filter of $X$. \hfill \square

Theorem 2.12. If $A$ is an IVNS in $X$ satisfying the following condition:

$$(2.25) \quad (\forall a, x, y, z \in X) \left\{ a \leq x \cdot (y \cdot z) \Rightarrow \begin{array}{ll}
    A_T(x \cdot z) \geq \min\{A_T(a), A_T(y)\}, \\
    A_I(x \cdot z) \leq \max\{A_I(a), A_I(y)\}, \\
    A_F(x \cdot z) \geq \min\{A_F(a), A_F(y)\}.
\end{array} \right.$$ 

then $A$ is an interval-valued neutrosophic UP-ideal of $X$.

Proof. Assume that $A$ is an IVNS in $X$ satisfying the condition (2.25). Let $x \in X$. By (UP-3), we have $x \cdot (0 \cdot (x \cdot 0)) = 0$, that is, $x \leq 0 \cdot (x \cdot 0)$. It follows from (2.25) and (1.27) that

$$
\begin{align*}
\text{(UP-2)} \quad A_T(0) &\geq \min\{A_T(x), A_T(x)\} = A_T(x), \\
\text{(UP-2)} \quad A_I(0) &\leq \max\{A_I(x), A_I(x)\} = A_I(x), \\
\text{(UP-2)} \quad A_F(0) &\geq \min\{A_F(x), A_F(x)\} = A_F(x).
\end{align*}
$$

Next, let $x, y, z \in X$. By (1.1), we have $(x \cdot (y \cdot z)) \cdot (x \cdot (y \cdot z)) = 0$, that is, $x \cdot (y \cdot z) \leq x \cdot (y \cdot z)$. It follows from (2.25) that

$$
\begin{align*}
A_T(x \cdot z) &\geq \min\{A_T(x \cdot (y \cdot z)), A_T(y)\}, \\
A_I(x \cdot z) &\leq \max\{A_I(x \cdot (y \cdot z)), A_I(y)\}, \\
A_F(x \cdot z) &\geq \min\{A_F(x \cdot (y \cdot z)), A_F(y)\}.
\end{align*}
$$

Hence, $A$ is an interval-valued neutrosophic UP-ideal of $X$. \hfill \square
For any fixed interval numbers $\tilde{a}^+, \tilde{a}^-, \tilde{b}^+, \tilde{b}^-, \tilde{c}^+, \tilde{c}^- \in [[0, 1]]$ such that $\tilde{a}^+ \succ \tilde{a}^-, \tilde{b}^+ \succ \tilde{b}^-, \tilde{c}^+ \succ \tilde{c}^-$ and a nonempty subset $G$ of $X$, the IVNS $A^G_{\tilde{a}^+ \tilde{b}^-, \tilde{c}^+} = (X, A^G_{[\tilde{a}^+]}, A^G_{[\tilde{b}^-]}, A^G_{[\tilde{c}^+]})$ in $X$, where $A^G_{[\tilde{a}^+]}, A^G_{[\tilde{b}^-]},$ and $A^G_{[\tilde{c}^+]}$ are IVFSs in $X$ which are given as follows:

$$A^G_{\tilde{a}^+}(x) = \begin{cases} \tilde{a}^+ & \text{if } x \in G, \\ \tilde{a}^- & \text{otherwise,} \end{cases}$$

$$A^G_{\tilde{b}^-}(x) = \begin{cases} \tilde{b}^- & \text{if } x \in G, \\ \tilde{b}^+ & \text{otherwise,} \end{cases}$$

$$A^G_{\tilde{c}^+}(x) = \begin{cases} \tilde{c}^+ & \text{if } x \in G, \\ \tilde{c}^- & \text{otherwise.} \end{cases}$$

**Lemma 2.1.** If the constant $0$ of $X$ is in a nonempty subset $G$ of $X$, then the IVNS $A^G_{\tilde{a}^+ \tilde{b}^-, \tilde{c}^+}$ in $X$ satisfies the conditions (2.4), (2.5), and (2.6).

**Proof.** If $0 \in G$, then $A^G_{[\tilde{a}^+]}(0) = \tilde{a}^+, A^G_{[\tilde{b}^-]}(0) = \tilde{b}^-, \text{ and } A^G_{[\tilde{c}^+]}(0) = \tilde{c}^+$.

Thus

$$\forall x \in X \left( \begin{array}{l} A^G_{\tilde{a}^+}(0) = \tilde{a}^+ \succeq A^G_{[\tilde{a}^+]}(x) \\ A^G_{\tilde{b}^-}(0) = \tilde{b}^- \preceq A^G_{[\tilde{b}^-]}(x) \\ A^G_{\tilde{c}^+}(0) = \tilde{c}^+ \succeq A^G_{[\tilde{c}^+]}(x) \end{array} \right).$$

Hence, $A^G_{\tilde{a}^+ \tilde{b}^-, \tilde{c}^+}$ satisfies the conditions (2.4), (2.5), and (2.6). $\square$

**Lemma 2.2.** If the IVNS $A^G_{\tilde{a}^+ \tilde{b}^-, \tilde{c}^+}$ in $X$ satisfies the condition (2.4) (resp., (2.5), (2.6)), then the constant $0$ of $X$ is in a nonempty subset $G$ of $X$.

**Proof.** Assume that the IVNS $A^G_{\tilde{a}^+ \tilde{b}^-, \tilde{c}^+}$ in $X$ satisfies the condition (2.4).

Then $A^G_{[\tilde{a}^+]}(0) \succeq A^G_{[\tilde{a}^+]}(g)$ for all $x \in X$. Since $G$ is nonempty, there exists $g \in G$. Thus $A^G_{[\tilde{a}^+]}(g) = \tilde{a}^+$ and so $A^G_{[\tilde{a}^+]}(0) \succeq A^G_{[\tilde{a}^+]}(g) = \tilde{a}^+ \succeq A^G_{[\tilde{a}^+]}(0)$, that is, $A^G_{[\tilde{a}^+]}(0) = \tilde{a}^+$. Hence, $0 \in G$. $\square$

**Theorem 2.13.** The IVNS $A^G_{\tilde{a}^+ \tilde{b}^-, \tilde{c}^+}$ in $X$ is an interval-valued neutrosophic UP-subalgebra of $X$ if and only if a nonempty subset $G$ of $X$ is a UP-subalgebra of $X$. 
Proof. Assume that \( A_{[a^+, b^-; c^-]} \) is an interval-valued neutrosophic UP-subalgebra of \( X \). Let \( x, y \in G \). Then \( A_{[a^+, b^-; c^-]}(x) = a^+ = A_{[a^+, b^-; c^-]}(y) \). Thus

\[
\begin{align*}
((2.1)) & \quad A_{[a^+, b^-; c^-]}(x, y) \geq \min\{A_{[a^+, b^-; c^-]}(x), A_{[a^+, b^-; c^-]}(y)\} \\
& = \min\{a^+, a^+\} \\
((1.27)) & \quad = \bar{a}^+ \\
& \geq A_{[a^+, b^-; c^-]}(x, y)
\end{align*}
\]

and so \( A_{[a^+, b^-; c^-]}(x, y) = \bar{a}^+ \). Thus \( x \cdot y \in G \). Hence, \( G \) is a UP-subalgebra of \( X \).

Conversely, assume that \( G \) is a UP-subalgebra of \( X \). Let \( x, y \in X \).

**Case 1:** \( x, y \in G \). Then

\[
\begin{align*}
A_{[a^+, b^-; c^-]}(x) & = \bar{a}^+ = A_{[a^+, b^-; c^-]}(y), \\
A_{[b^+, c^-]}(x) & = \bar{b}^+ = A_{[b^+, c^-]}(y), \\
A_{[c^+, d^-]}(x) & = \bar{c}^+ = A_{[c^+, d^-]}(y).
\end{align*}
\]

Since \( G \) is a UP-subalgebra of \( X \), we have \( x \cdot y \in G \) and so \( A_{[a^+, b^-; c^-]}(x, y) = \bar{a}^+, A_{[b^+, c^-]}(x, y) = \bar{b}^+, \) and \( A_{[c^+, d^-]}(x, y) = \bar{c}^+ \). By (1.27), it follows that

\[
\begin{align*}
A_{[a^+, b^-; c^-]}(x, y) = \bar{a}^+ & \geq \bar{a}^+ = \min\{a^+, a^+\} = \min\{A_{[a^+, b^-; c^-]}(x), A_{[a^+, b^-; c^-]}(y)\}, \\
A_{[b^+, c^-]}(x, y) = \bar{b}^- & \leq \bar{b}^- = \max\{b^-, b^-\} = \max\{A_{[b^+, c^-]}(x), A_{[b^+, c^-]}(y)\}, \\
A_{[c^+, d^-]}(x, y) = \bar{c}^+ & \geq \bar{c}^+ = \min\{c^+, c^+\} = \min\{A_{[c^+, d^-]}(x), A_{[c^+, d^-]}(y)\}.
\end{align*}
\]

**Case 2:** \( x \notin G \) or \( y \notin G \). Then

\[
\begin{align*}
A_{[a^+, b^-; c^-]}(x) & = \bar{a}^- \quad \text{or} \quad A_{[a^+, b^-; c^-]}(y) = \bar{a}^-, \\
A_{[b^+, c^-]}(x) & = \bar{b}^- \quad \text{or} \quad A_{[b^+, c^-]}(y) = \bar{b}^-, \\
A_{[c^+, d^-]}(x) & = \bar{c}^- \quad \text{or} \quad A_{[c^+, d^-]}(y) = \bar{c}^-.
\end{align*}
\]

By (1.27), it follows that

\[
\begin{align*}
\min\{A_{[a^+, b^-; c^-]}(x), A_{[a^+, b^-; c^-]}(y)\} & = \min\{\bar{a}^-, \bar{a}^-\} = \bar{a}^-, \\
\max\{A_{[b^+, c^-]}(x), A_{[b^+, c^-]}(y)\} & = \max\{\bar{b}^-, \bar{b}^-\} = \bar{b}^-, \\
\min\{A_{[c^+, d^-]}(x), A_{[c^+, d^-]}(y)\} & = \min\{\bar{c}^-, \bar{c}^-\} = \bar{c}^-.
\end{align*}
\]

Therefore,

\[
\begin{align*}
A_{[a^+, b^-; c^-]}(x, y) & \geq \bar{a}^- = \min\{A_{[a^+, b^-; c^-]}(x), A_{[a^+, b^-; c^-]}(y)\}, \\
A_{[b^+, c^-]}(x, y) & \geq \bar{b}^- = \max\{A_{[b^+, c^-]}(x), A_{[b^+, c^-]}(y)\}, \\
A_{[c^+, d^-]}(x, y) & \geq \bar{c}^- = \min\{A_{[c^+, d^-]}(x), A_{[c^+, d^-]}(y)\}.
\end{align*}
\]

Hence, \( A_{[a^+, b^-; c^-]} \) is an interval-valued neutrosophic UP-subalgebra of \( X \). \( \square \)
Theorem 2.14. The IVNS $A^G[a_\bar{a}^+, \bar{b}^-\bar{c}^+]$ in $X$ is an interval-valued neutrosophic near UP-filter of $X$ if and only if a nonempty subset $G$ of $X$ is a near UP-filter of $X$.

Proof. Assume that $A^G[a_\bar{a}^+, \bar{b}^-\bar{c}^+]$ is an interval-valued neutrosophic near UP-filter of $X$. Since $A^G[a_\bar{a}^+, \bar{b}^-\bar{c}^+]$ satisfies the condition (2.4), it follows from Lemma 2.2 that $0 \in G$. Next, let $x \in X$ and $y \in G$. Then $A^G_G[a_\bar{a}^+](y) = \bar{a}^+$. By (2.7)

$$A^G_G[a_\bar{a}^+](x \cdot y) \succeq A^G_G[a_\bar{a}^+](y) = \bar{a}^+ \succeq A^G_G[a_\bar{a}^+](x \cdot y)$$

and so $A^G_G[a_\bar{a}^+](x \cdot y) = \bar{a}^+$. Thus $x \cdot y \in G$. Hence, $G$ is a near UP-filter of $X$.

Conversely, assume that $G$ is a near UP-filter of $X$. Since $0 \in G$, it follows from Lemma 2.1 that $A^G_G[a_\bar{a}^+, \bar{b}^-\bar{c}^+]$ satisfies the conditions (2.4), (2.5), and (2.6). Next, let $x, y \in X$.

Case 1: $y \in G$. Then $A^G_G[a_\bar{a}^+](y) = \bar{a}^+, A^G_G[b_\bar{b}^-](y) = \bar{b}^-$, and $A^G_G[c_\bar{c}^+](y) = \bar{c}^+$. Since $G$ is a near UP-filter of $X$, we have $x \cdot y \in G$ and so $A^G_G[a_\bar{a}^+](x \cdot y) = \bar{a}^+, A^G_G[b_\bar{b}^-](x \cdot y) = \bar{b}^-$, and $A^G_G[c_\bar{c}^+](x \cdot y) = \bar{c}^+$. Thus

$$A^G_G[a_\bar{a}^+](x \cdot y) = \bar{a}^+ \succeq \bar{a}^+ = A^G_G[a_\bar{a}^+](y),$$

$$A^G_G[b_\bar{b}^-](x \cdot y) = \bar{b}^- \preceq \bar{b}^- = A^G_G[b_\bar{b}^-](y),$$

$$A^G_G[c_\bar{c}^+](x \cdot y) = \bar{c}^+ \succeq \bar{c}^+ = A^G_G[c_\bar{c}^+](y).$$

Case 2: $y \notin G$. Then $A^G_G[a_\bar{a}^+](y) = \bar{a}^-, A^G_G[b_\bar{b}^-](y) = \bar{b}^+$, and $A^G_G[c_\bar{c}^+](y) = \bar{c}^-$. Thus

$$A^G_G[a_\bar{a}^+](x \cdot y) = \bar{a}^- \succeq \bar{a}^- = A^G_G[a_\bar{a}^+](y),$$

$$A^G_G[b_\bar{b}^-](x \cdot y) = \bar{b}^+ \preceq \bar{b}^+ = A^G_G[b_\bar{b}^-](y),$$

$$A^G_G[c_\bar{c}^+](x \cdot y) = \bar{c}^- \succeq \bar{c}^- = A^G_G[c_\bar{c}^+](y).$$

Hence, $A^G_G[a_\bar{a}^+, \bar{b}^-\bar{c}^+]$ is an interval-valued neutrosophic near UP-filter of $X$. \qed

Theorem 2.15. The IVNS $A^G_G[a_\bar{a}^+, \bar{b}^-\bar{c}^+]$ in $X$ is an interval-valued neutrosophic UP-filter of $X$ if and only if a nonempty subset $G$ of $X$ is a UP-filter of $X$.

Proof. Assume that $A^G_G[a_\bar{a}^+, \bar{b}^-\bar{c}^+]$ is an interval-valued neutrosophic UP-filter of $X$. Since $A^G_G[a_\bar{a}^+, \bar{b}^-\bar{c}^+]$ satisfies the condition (2.4), it follows from Lemma 2.2 that $0 \in G$. Next, let $x, y \in X$ be such that $x \cdot y \in G$ and $x \in G$. Then
Therefore, by (1.27), it follows that
\[ A^G_T[\bar{\alpha}^+](y) \geq r\min\{A^G_T[\bar{\alpha}^+](x \cdot y), A^G_T[\bar{\alpha}^+](x)\} \]
\[ = r\min\{\bar{a}^+, \bar{a}^+\} \]
\[ = \bar{a}^+ \]
\[ \geq A^G_T[\bar{a}^+](y) \]
and so \( A^G_T[\bar{a}^+](y) = \bar{a}^+ \). Thus \( y \in G \). Hence, \( G \) is a UP-filter of \( X \).

Conversely, assume that \( G \) is a UP-filter of \( X \). Since \( 0 \in G \), it follows from Lemma 2.1 that \( A^G_T[\bar{a}^+, \bar{b}^-, \bar{c}^+] \) satisfies the conditions (2.4), (2.5), and (2.6). Next, let \( x, y \in X \).

**Case 1:** \( x \cdot y \in G \) and \( x \in G \). Then
\[ A^G_T[\bar{a}^+](x \cdot y) = \bar{a}^+ = A^G_T[\bar{a}^+](x), \]
\[ A^G_T[\bar{b}^+](x \cdot y) = \bar{b}^+ = A^G_T[\bar{b}^+](x), \]
\[ A^G_T[\bar{c}^+](x \cdot y) = \bar{c}^+ = A^G_T[\bar{c}^+](x). \]
Since \( G \) is a UP-filter of \( X \), we have \( y \in G \) and so \( A^G_T[\bar{a}^+](y) = \bar{a}^+, A^G_T[\bar{b}^+](y) = \bar{b}^-, \)
and \( A^G_T[\bar{c}^+](y) = \bar{c}^+ \). By (1.27), it follows that
\[ A^G_T[\bar{a}^+](y) = \bar{a}^+ \geq \bar{a}^+ = r\min\{\bar{a}^+, \bar{a}^+\} = r\min\{A^G_T[\bar{a}^+](x \cdot y), A^G_T[\bar{a}^+](x)\}, \]
\[ A^G_T[\bar{b}^+](y) = \bar{b}^- \geq \bar{b}^- = r\max\{\bar{b}^-, \bar{b}^-\} = r\max\{A^G_T[\bar{b}^+](x \cdot y), A^G_T[\bar{b}^+](x)\}, \]
\[ A^G_T[\bar{c}^+](y) = \bar{c}^+ \geq \bar{c}^+ = r\min\{\bar{c}^+, \bar{c}^+\} = r\min\{A^G_T[\bar{c}^+](x \cdot y), A^G_T[\bar{c}^+](x)\}. \]

**Case 2:** \( x \cdot y \notin G \) or \( x \notin G \). Then
\[ A^G_T[\bar{a}^+](x \cdot y) = \bar{a}^- \text{ or } A^G_T[\bar{a}^+](x) = \bar{a}^-, \]
\[ A^G_T[\bar{b}^+](x \cdot y) = \bar{b}^+ \text{ or } A^G_T[\bar{b}^+](x) = \bar{b}^+, \]
\[ A^G_T[\bar{c}^+](x \cdot y) = \bar{c}^+ \text{ or } A^G_T[\bar{c}^+](x) = \bar{c}^-. \]
By (1.27), it follows that
\[ r\min\{A^G_T[\bar{a}^+](x \cdot y), A^G_T[\bar{a}^-+](x)\} = r\min\{\bar{a}^-, \bar{a}^-\} = \bar{a}^-, \]
\[ r\max\{A^G_T[\bar{b}^+](x \cdot y), A^G_T[\bar{b}^-+](x)\} = r\max\{\bar{b}^+, \bar{b}^+\} = \bar{b}^+, \]
\[ r\min\{A^G_T[\bar{c}^+](x \cdot y), A^G_T[\bar{c}^-+](x)\} = r\min\{\bar{c}^-, \bar{c}^-\} = \bar{c}^- . \]
Therefore,
\[ A^G_T[\bar{a}^-+](y) \geq \bar{a}^- = \min\{A^G_T[\bar{a}^-+](x \cdot y), A^G_T[\bar{a}^-+](x)\}, \]
\[ A^G_T[\bar{b}^-+](y) \geq \bar{b}^+ = \max\{A^G_T[\bar{b}^-+](x \cdot y), A^G_T[\bar{b}^-+](x)\}, \]
\[ A^G_T[\bar{c}^-+](y) \geq \bar{c}^- = \min\{A^G_T[\bar{c}^-+](x \cdot y), A^G_T[\bar{c}^-+](x)\}. \]
Hence, $A^{G[\tilde{a},\tilde{b},\tilde{c}^{+}]}$ is an interval-valued neutrosophic UP-filter of $X$. \hfill \Box

\textbf{THEOREM 2.16.} The IVNS $A^{G[\tilde{a},\tilde{b},\tilde{c}^{+}]}$ in $X$ is an interval-valued neutrosophic UP-ideal of $X$ if and only if a nonempty subset $G$ of $X$ is a UP-ideal of $X$.

\textbf{Proof.} Assume that $A^{G[\tilde{a},\tilde{b},\tilde{c}^{+}]}$ is an interval-valued neutrosophic UP-ideal of $X$. Since $A^{G[\tilde{a},\tilde{b},\tilde{c}^{+}]}$ satisfies the condition (2.4), it follows from Lemma 2.2 that $0 \in G$. Next, let $x, y, z \in X$ be such that $x \cdot (y \cdot z) \in G$ and $y \in G$. Then $A^{G[\tilde{u}^{+}]}[x \cdot (y \cdot z)] = \tilde{a}^{+} = A^{G[\tilde{u}^{+}]}(y)$. Thus

\begin{align*}
A^{G[\tilde{u}^{+}]}(x \cdot z) &\geq \min\{A^{G[\tilde{u}^{+}]}(x \cdot (y \cdot z)), A^{G[\tilde{u}^{+}]}(y)\} \\
&= \min\{\tilde{a}^{+}, \tilde{a}^{+}\} \\
&= \tilde{a}^{+} \\
&\geq A^{G[\tilde{u}^{+}]}(x \cdot z)
\end{align*}

and so $A^{G[\tilde{u}^{+}]}(x \cdot z) = \tilde{a}^{+}$. Thus $x \cdot z \in G$. Hence, $G$ is a UP-ideal of $X$.

Conversely, assume that $G$ is a UP-ideal of $X$. Since $0 \in G$, it follows from Lemma 2.1 that $A^{G[\tilde{u},\tilde{b},\tilde{c}^{+}]}$ satisfies the conditions (2.4), (2.5), and (2.6). Next, let $x, y, z \in X$.

\textbf{Case 1:} $x \cdot (y \cdot z) \in G$ and $y \in G$. Then

\begin{align*}
A^{G[\tilde{u}^{+}]}(x \cdot (y \cdot z)) &= \tilde{a}^{+} = A^{G[\tilde{u}^{+}]}(y), \\
A^{G[\tilde{b}^{+}]}(x \cdot (y \cdot z)) &= \tilde{b}^{-} = A^{G[\tilde{b}^{+}]}(y), \\
A^{G[\tilde{c}^{+}]}(x \cdot (y \cdot z)) &= \tilde{c}^{+} = A^{G[\tilde{c}^{+}]}(y).
\end{align*}

Since $G$ is a UP-ideal of $X$, we have $x \cdot z \in G$ and so $A^{G[\tilde{u}^{+}]}(x \cdot z) = \tilde{a}^{+}, A^{G[\tilde{b}^{+}]}(x \cdot z) = \tilde{b}^{-}$, and $A^{G[\tilde{c}^{+}]}(x \cdot z) = \tilde{c}^{+}$. By (1.27), it follows that

\begin{align*}
A^{G[\tilde{u}^{+}]}(x \cdot z) &= \tilde{a}^{+} \geq \tilde{a}^{+} = \min\{\tilde{a}^{+}, \tilde{a}^{+}\} = \min\{A^{G[\tilde{u}^{+}]}(x \cdot (y \cdot z)), A^{G[\tilde{u}^{+}]}(y)\}, \\
A^{G[\tilde{b}^{+}]}(x \cdot z) &= \tilde{b}^{-} \leq \tilde{b}^{-} = \max\{\tilde{b}^{-}, \tilde{b}^{-}\} = \max\{A^{G[\tilde{b}^{+}]}(x \cdot (y \cdot z)), A^{G[\tilde{b}^{+}]}(y)\}, \\
A^{G[\tilde{c}^{+}]}(x \cdot z) &= \tilde{c}^{+} \leq \tilde{c}^{+} = \min\{\tilde{c}^{+}, \tilde{c}^{+}\} = \min\{A^{G[\tilde{c}^{+}]}(x \cdot (y \cdot z)), A^{G[\tilde{c}^{+}]}(y)\}.
\end{align*}

\textbf{Case 2:} $x \cdot (y \cdot z) \not\in G$ or $y \not\in G$. Then

\begin{align*}
A^{G[\tilde{u}^{+}]}(x \cdot (y \cdot z)) &= \tilde{a}^{-} \text{ or } A^{G[\tilde{u}^{+}]}(y) = \tilde{a}^{-}, \\
A^{G[\tilde{b}^{+}]}(x \cdot (y \cdot z)) &= \tilde{b}^{+} \text{ or } A^{G[\tilde{b}^{+}]}(y) = \tilde{b}^{+}, \\
A^{G[\tilde{c}^{+}]}(x \cdot (y \cdot z)) &= \tilde{c}^{-} \text{ or } A^{G[\tilde{c}^{+}]}(y) = \tilde{c}^{-}.
\end{align*}
By (1.27), it follows that
\[
\begin{align*}
\text{rmin}\{A_G^G[\tilde{a}^+]|(x \cdot (y \cdot z)), A_G^G[\tilde{a}^+]|(y)\} = \text{rmin}\{\tilde{a}^-, \tilde{a}^-\} = \tilde{a}^-,
\text{rmax}\{A_G^G[\tilde{b}^+]|(x \cdot (y \cdot z)), A_G^G[\tilde{b}^+]|(y)\} = \text{rmax}\{\tilde{b}^+, \tilde{b}^+\} = \tilde{b}^+,
\text{rmin}\{A_G^G[\tilde{c}^-]|(x \cdot (y \cdot z)), A_G^G[\tilde{c}^-]|(y)\} = \text{rmin}\{\tilde{c}^-, \tilde{c}^-\} = \tilde{c}^-.
\end{align*}
\]
Therefore,
\[
\begin{align*}
A_G^G[\tilde{a}^+]|(x \cdot z) \succeq \tilde{a}^- &= \text{rmin}\{A_G^G[\tilde{a}^+]|(x \cdot (y \cdot z)), A_G^G[\tilde{a}^+]|(y)\},
A_G^G[\tilde{b}^-]|(x \cdot z) \preceq \tilde{b}^+ &= \text{rmax}\{A_G^G[\tilde{b}^+]|(x \cdot (y \cdot z)), A_G^G[\tilde{b}^+]|(y)\},
A_G^G[\tilde{c}^-]|(x \cdot z) \succeq \tilde{c}^- &= \text{rmin}\{A_G^G[\tilde{c}^-]|(x \cdot (y \cdot z)), A_G^G[\tilde{c}^-]|(y)\}.
\end{align*}
\]
Hence, \(A_G^G[\tilde{a}^+ \tilde{b}^- \tilde{c}^+]|_{\tilde{a}^- \tilde{b}^+ \tilde{c}^-}\) is an interval-valued neutrosophic strong UP-ideal of \(X\).

**Theorem 2.17.** The IVNS \(A_G^G[\tilde{a}^+ \tilde{b}^- \tilde{c}^+]|_{\tilde{a}^- \tilde{b}^+ \tilde{c}^-}\) in \(X\) is an interval-valued neutrosophic strong UP-ideal of \(X\) if and only if \(G\) is a nonempty subset of \(X\) is a strong UP-ideal of \(X\).

**Proof.** Assume that \(A_G^G[\tilde{a}^+ \tilde{b}^- \tilde{c}^+]|_{\tilde{a}^- \tilde{b}^+ \tilde{c}^-}\) is an interval-valued neutrosophic strong UP-ideal of \(X\). By Theorem 2.1, we have \(A_G^G[\tilde{a}^+ \tilde{b}^- \tilde{c}^+]|_{\tilde{a}^- \tilde{b}^+ \tilde{c}^-}\) is constant, that is, \(A_G^G[\tilde{a}^+]|_{\tilde{a}^- \tilde{b}^+ \tilde{c}^-}\) is constant. Since \(G\) is nonempty, we have \(A_G^G[\tilde{a}^+]|(x) = \tilde{a}^+\) for all \(x \in X\). Thus \(G = X\). Hence, \(G\) is a strong UP-ideal of \(X\).

Conversely, assume that \(G\) is a strong UP-ideal of \(X\). Then \(G = X\), so
\[
(\forall x \in X) \left( A_G^G[\tilde{a}^+]|(x) = \tilde{a}^+, A_G^G[\tilde{b}^-]|(x) = \tilde{b}^-, A_G^G[\tilde{c}^+]|(x) = \tilde{c}^+ \right).
\]
Thus \(A_G^G[\tilde{a}^+], A_G^G[\tilde{b}^-], \) and \(A_G^G[\tilde{c}^+]\) are constant, that is, \(A_G^G[\tilde{a}^+ \tilde{b}^- \tilde{c}^+]|_{\tilde{a}^- \tilde{b}^+ \tilde{c}^-}\) is constant. By Theorem 2.1, we have \(A_G^G[\tilde{a}^+ \tilde{b}^- \tilde{c}^+]|_{\tilde{a}^- \tilde{b}^+ \tilde{c}^-}\) is an interval-valued neutrosophic strong UP-ideal of \(X\).

**3. Level Subsets of Interval-Valued Neutrosophic Sets**

In this section, we discuss the relationships among interval-valued neutrosophic UP-subalgebras (resp., interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, interval-valued neutrosophic strong UP-ideals) of UP-algebras and their level subsets.
Definition 3.1. Let $A$ be an IVFS in a nonempty set $X$. For any $\tilde{a} \in [[0,1]]$, the sets

\begin{align}
(3.1) & \quad U(A; \tilde{a}) = \{x \in X \mid A(x) \geq \tilde{a}\}, \\
(3.2) & \quad L(A; \tilde{a}) = \{x \in X \mid A(x) \leq \tilde{a}\}, \\
(3.3) & \quad E(A; \tilde{a}) = \{x \in X \mid A(x) = \tilde{a}\}
\end{align}

are called an upper $\tilde{a}$-level subset, a lower $\tilde{a}$-level subset, and an equal $\tilde{a}$-level subset of $A$, respectively.

Theorem 3.1. An IVFS $A$ in $X$ is an interval-valued neutrosophic UP-subalgebra of $X$ if and only if for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, the sets $U(A_T; \tilde{a})$, $L(A_T; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or UP-subalgebras of $X$.

Proof. Assume that $A$ is an interval-valued neutrosophic UP-subalgebra of $X$. Let $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$ be such that $U(A_T; \tilde{a})$, $L(A_T; \tilde{b})$, and $U(A_F; \tilde{c})$ are nonempty.

Let $x, y \in U(A_T; \tilde{a})$. Then $A_T(x) \geq \tilde{a}$ and $A_T(y) \geq \tilde{a}$. Since $A$ is an interval-valued neutrosophic UP-subalgebra of $X$, by (1.29), we have

$A_T(x \cdot y) \geq \text{rmin}\{A_T(x), A_T(y)\} \geq \tilde{a}$.

Thus $x \cdot y \in U(A_T; \tilde{a})$.

Let $x, y \in L(A_T; \tilde{b})$. Then $A_T(x) \leq \tilde{b}$ and $A_T(y) \leq \tilde{b}$. Since $A$ is an interval-valued neutrosophic UP-subalgebra of $X$, by (1.34), we have

$A_T(x \cdot y) \leq \text{rmax}\{A_T(x), A_T(y)\} \leq \tilde{b}$.

Thus $x \cdot y \in L(A_T; \tilde{b})$.

Similarly, we can show that $x \cdot y \in U(A_F; \tilde{c})$.

Conversely, assume that for all $\tilde{a}, \tilde{b}, \tilde{c} \in [[0,1]]$, the sets $U(A_T; \tilde{a})$, $L(A_T; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or UP-subalgebras of $X$.

Let $x, y \in X$. By (1.29), we have $A_T(x) \geq \text{rmin}\{A_T(x), A_T(y)\}$ and $A_T(y) \geq \text{rmin}\{A_T(x), A_T(y)\}$. Thus $x, y \in U(A_T; \text{rmin}\{A_T(x), A_T(y)\})$. By assumption, we have $U(A_T; \text{rmin}\{A_T(x), A_T(y)\})$ is a UP-subalgebra of $X$. Then $x \cdot y \in U(A_T; \text{rmin}\{A_T(x), A_T(y)\})$. Thus $A_T(x \cdot y) \geq \text{rmin}\{A_T(x), A_T(y)\}$.

Let $x, y \in X$. By (1.29), we have $A_F(x) \leq \text{rmax}\{A_F(x), A_F(y)\}$ and $A_F(y) \leq \text{rmax}\{A_F(x), A_F(y)\}$. Thus $x, y \in L(A_F; \text{rmax}\{A_F(x), A_F(y)\})$. By assumption, we have $L(A_F; \text{rmax}\{A_F(x), A_F(y)\})$ is a UP-subalgebra of $X$. Then $x \cdot y \in U(A_F; \text{rmin}\{A_F(x), A_F(y)\})$. Thus $A_F(x \cdot y) \geq \text{rmin}\{A_F(x), A_F(y)\}$.

Hence, $A$ is an interval-valued neutrosophic UP-subalgebra of $X$. $\square$
THEOREM 3.2. An IVNS $A$ in $X$ is an interval-valued neutrosophic near UP-filter of $X$ if and only if for all $\tilde{a}, \tilde{b}, \tilde{c} \in [0, 1]$, the sets $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or near UP-filters of $X$.

**Proof.** Assume that $A$ is an interval-valued neutrosophic near UP-filter of $X$. Let $\tilde{a}, \tilde{b}, \tilde{c} \in [0, 1]$ be such that $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are nonempty.

Let $x \in U(A_T; \tilde{a}), y \in L(A_I; \tilde{b}), z \in U(A_F; \tilde{c})$. Since $A$ is an interval-valued neutrosophic near UP-filter of $X$, we have

$$A_T(0) \succeq A_T(x) \succeq \tilde{a}, \quad A_I(0) \preceq A_I(y) \preceq \tilde{b}, \quad A_F(0) \succeq A_F(z) \succeq \tilde{c}.$$  

Thus $0 \in U(A_T; \tilde{a}), 0 \in L(A_I; \tilde{b})$, and $0 \in U(A_T; \tilde{a})$.

Let $x \in X$ and $y \in U(A_T; \tilde{a})$. Then $A_T(y) \succeq \tilde{a}$. Since $A$ is an interval-valued neutrosophic near UP-filter of $X$, we have

$$A_I(x \cdot y) \preceq A_I(y) \preceq \tilde{b}.$$  

Thus $x \cdot y \in L(A_I; \tilde{b})$.

Let $x \in X$ and $y \in U(A_F; \tilde{c})$. Then $A_F(y) \preceq \tilde{c}$. Since $A$ is an interval-valued neutrosophic near UP-filter of $X$, we have

$$A_F(x \cdot y) \preceq A_F(y) \preceq \tilde{c}.$$  

Thus $x \cdot y \in U(A_F; \tilde{c})$.

Hence, $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are near UP-filters of $X$.

Conversely, assume that for all $\tilde{a}, \tilde{b}, \tilde{c} \in [0, 1]$, the sets $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or near UP-filters of $X$.

Let $x \in X$. Then $x \in U(A_T; A_T(x)) \neq \emptyset, x \in L(A_I; A_I(x)) \neq \emptyset,$ and $x \in U(A_T; A_T(x)) \neq \emptyset$. By assumption, we have $U(A_T; A_T(x)), L(A_I; A_I(x))$, and $U(A_F; A_F(x))$ are near UP-filters of $X$. Then $0 \in U(A_T; A_T(x)), 0 \in L(A_I; A_I(x))$, and $0 \in U(A_F; A_F(x))$. Thus $A_T(0) \succeq A_T(x), A_I(0) \preceq A_I(x),$ and $A_F(0) \succeq A_F(x)$.

Let $x, y \in X$. Then $y \in U(A_T; A_T(y)) \neq \emptyset$. By assumption, we have $U(A_T; A_T(y))$ is a near UP-filter of $X$. Then $x \cdot y \in U(A_T; A_T(y)).$ Thus $A_T(x \cdot y) \succeq A_T(y)$.

Let $x, y \in X$. Then $y \in L(A_I; A_I(y)) \neq \emptyset$. By assumption, we have $L(A_I; A_I(y))$ is a near UP-filter of $X$. Then $x \cdot y \in L(A_I; A_I(y)).$ Thus $A_I(x \cdot y) \preceq A_I(y)$.

Let $x, y \in X$. Then $y \in U(A_F; A_F(y)) \neq \emptyset$. By assumption, we have $U(A_F; A_F(y))$ is a near UP-filter of $X$. Then $x \cdot y \in U(A_F; A_F(y)).$ Thus $A_F(x \cdot y) \preceq A_F(y)$.

Hence, $A$ is an interval-valued neutrosophic near UP-filter of $X$. $\square$

THEOREM 3.3. An IVNS $A$ in $X$ is an interval-valued neutrosophic UP-filter of $X$ if and only if for all $\tilde{a}, \tilde{b}, \tilde{c} \in [0, 1]$, the sets $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or UP-filters of $X$. 


By assumption, we have
$$A_T(0) \geq A_T(x) \geq \tilde{a}, \quad A_T(0) \leq A_T(y) \leq \tilde{b}, \quad A_T(0) \geq A_T(z) \geq \tilde{c}.$$ 

Thus $0 \in U(A_T; \tilde{a})$, $0 \in L(A_T; \tilde{b})$, and $0 \in U(A_T; \tilde{a})$.

Let $x, y \in X$ be such that $x \cdot y, x \in U(A_T; \tilde{a})$. Then $A_T(x \cdot y) \geq \tilde{a}$ and $A_T(x) \geq \tilde{a}$. Since $A$ is an interval-valued neutrosophic UP-filter of $X$, we have
$$A_T(y) \geq \min\{A_T(x \cdot y), A_T(x)\} \geq \tilde{a}.$$

Thus $y \in U(A_T; \tilde{a})$.

Let $x, y \in X$ be such that $x \cdot y, x \in L(A_T; \tilde{b})$. Then $A_T(x \cdot y) \leq \tilde{b}$ and $A_T(x) \leq \tilde{b}$. Since $A$ is an interval-valued neutrosophic UP-filter of $X$, we have
$$A_T(y) \leq \max\{A_T(x \cdot y), A_T(x)\} \leq \tilde{b}.$$

Thus $y \in L(A_T; \tilde{b})$.

Let $x, y \in X$ be such that $x \cdot y, x \in U(A_T; \tilde{c})$. Then $A_T(x \cdot y) \geq \tilde{c}$ and $A_T(x) \geq \tilde{c}$. Since $A$ is an interval-valued neutrosophic UP-filter of $X$, we have
$$A_T(y) \geq \min\{A_T(x \cdot y), A_T(x)\} \geq \tilde{c}.$$

Thus $y \in U(A_T; \tilde{c})$.

Hence, $U(A_T; \tilde{a}), L(A_T; \tilde{b})$, and $U(A_T; \tilde{c})$ are UP-filters of $X$.

Conversely, assume that for all $\tilde{a}, \tilde{b}, \tilde{c} \in [0, 1]$, the sets $U(A_T; \tilde{a}), L(A_T; \tilde{b})$, and $U(A_T; \tilde{c})$ are either empty or UP-filters of $X$.

Let $x \in X$. Then $x \in U(A_T; A_T(x)) \neq \emptyset, x \in L(A_T; A_T(x)) \neq \emptyset$, and $x \in U(A_T; A_T(x)) \neq \emptyset$. By assumption, we have $U(A_T; A_T(x)), L(A_T; A_T(x)), \text{and } U(A_T; A_T(x))$ are UP-filters of $X$. Then $0 \in U(A_T; A_T(x)), 0 \in L(A_T; A_T(x))$, and $0 \in U(A_T; A_T(x))$. Thus $A_T(0) \geq A_T(x), A_T(0) \leq A_T(x)$, and $A_T(0) \geq A_T(x)$.

Let $x, y \in X$. By (1.29), we have $A_T(x \cdot y) \geq \min\{A_T(x \cdot y), A_T(x)\}$ and $A_T(x \cdot y) \geq \max\{A_T(x \cdot y), A_T(x)\}$. Thus $x \cdot y, x \in U(A_T; \min\{A_T(x \cdot y), A_T(x)\})$. By assumption, we have $U(A_T; \min\{A_T(x \cdot y), A_T(x)\})$ is a UP-filter of $X$. Then $y \in U(A_T; \min\{A_T(x \cdot y), A_T(x)\})$. Thus $A_T(y) \geq \min\{A_T(x \cdot y), A_T(x)\}$.

Let $x, y \in X$. By (1.29), we have $A_T(x \cdot y) \leq \max\{A_T(x \cdot y), A_T(x)\}$ and $A_T(x \cdot y) \leq \max\{A_T(x \cdot y), A_T(x)\}$. Thus $x \cdot y, x \in L(A_T; \max\{A_T(x \cdot y), A_T(x)\})$. By assumption, we have $L(A_T; \max\{A_T(x \cdot y), A_T(x)\})$ is a UP-filter of $X$. Then $y \in L(A_T; \max\{A_T(x \cdot y), A_T(x)\})$. Thus $A_T(y) \geq \max\{A_T(x \cdot y), A_T(x)\}$.

Let $x, y \in X$. By (1.29), we have $A_T(x \cdot y) \geq \min\{A_T(x \cdot y), A_T(x)\}$ and $A_T(x \cdot y) \geq \min\{A_T(x \cdot y), A_T(x)\}$. Thus $x \cdot y, x \in U(A_T; \min\{A_T(x \cdot y), A_T(x)\})$. By assumption, we have $U(A_T; \min\{A_T(x \cdot y), A_T(x)\})$ is a UP-filter of $X$. Then $y \in U(A_T; \min\{A_T(x \cdot y), A_T(x)\})$. Thus $A_T(y) \geq \min\{A_T(x \cdot y), A_T(x)\}$.

Hence, $A$ is an interval-valued neutrosophic UP-filter of $X$.

\[\square\]
THEOREM 3.4. An IVNS $A$ in $X$ is an interval-valued neutrosophic UP-ideal of $X$ if and only if for all $\tilde{a}, \tilde{b}, \tilde{c} \in [0, 1]$, the sets $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or UP-ideals of $X$.

PROOF. Assume that $A$ is an interval-valued neutrosophic UP-ideal of $X$. Let $\tilde{a}, \tilde{b}, \tilde{c} \in [0, 1]$ be such that $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are nonempty.

Let $x, y, z \in X$ be such that $x \cdot (y \cdot z), y \in U(A_T; \tilde{a})$. Then $A_T(x \cdot (y \cdot z)) \geq \tilde{a}$ and $A_T(y) \geq \tilde{b}$. Since $A$ is an interval-valued neutrosophic UP-ideal of $X$, we have

$$A_T(0) \geq A_T(x) \geq \tilde{a}, \quad A_T(0) \leq A_T(y) \leq \tilde{b}, \quad A_F(0) \geq A_F(z) \geq \tilde{c}.$$ 

Thus $0 \in U(A_T; \tilde{a}), 0 \in L(A_I; \tilde{b})$, and $0 \in U(A_F; \tilde{c})$.

Let $x, y, z \in X$ be such that $x \cdot (y \cdot z), y \in U(A_T; \tilde{a})$. Then $A_T(x \cdot (y \cdot z)) \geq \tilde{a}$ and $A_T(y) \geq \tilde{b}$. Since $A$ is an interval-valued neutrosophic UP-ideal of $X$, we have

$$A_T(x \cdot z) \geq \min\{A_T(x \cdot (y \cdot z)), A_T(y)\} \geq \tilde{a}.$$ 

Thus $x \cdot z \in U(A_T; \tilde{a})$.

Let $x, y, z \in X$ be such that $x \cdot (y \cdot z), y \in U(A_F; \tilde{c})$. Then $A_F(x \cdot (y \cdot z)) \geq \tilde{c}$ and $A_F(y) \geq \tilde{c}$. Since $A$ is an interval-valued neutrosophic UP-ideal of $X$, we have

$$A_F(x \cdot z) \geq \min\{A_F(x \cdot (y \cdot z)), A_F(y)\} \geq \tilde{c}.$$ 

Thus $x \cdot z \in U(A_F; \tilde{c})$.

Hence, $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are UP-ideals of $X$.

Conversely, assume that for all $\tilde{a}, \tilde{b}, \tilde{c} \in [0, 1]$, the sets $U(A_T; \tilde{a}), L(A_I; \tilde{b})$, and $U(A_F; \tilde{c})$ are either empty or UP-ideals of $X$.

Let $x, y \in X$. Then $x \in U(A_T; A_T(x)) \neq \emptyset, x \in L(A_I; A_I(x)) \neq \emptyset$, and $x \in U(A_F; A_F(x)) \neq \emptyset$. By assumption, we have $U(A_T; A_T(x)), L(A_I; A_I(x))$, and $U(A_F; A_F(x))$ are UP-ideals of $X$. Then $0 \in U(A_T; A_T(x)), 0 \in L(A_I; A_I(x))$, and $0 \in U(A_F; A_F(x))$. Thus $A_T(0) \geq A_T(x), A_I(0) \leq A_I(x)$, and $A_F(0) \geq A_F(x)$.

Let $x, y \in X$. By (1.29), we have $A_T(x \cdot (y \cdot z)) \geq \min\{A_T(x \cdot (y \cdot z)), A_T(y)\}$ and $A_T(y) \geq \min\{A_T(x \cdot (y \cdot z), A_T(y)\})$. Thus $x \cdot (y \cdot z), y \in U(A_T; \min\{A_T(x \cdot (y \cdot z), A_T(y)\})$.

By assumption, we have $U(A_T; \min\{A_T(x \cdot (y \cdot z), A_T(y)\})$ is a UP-ideal of $X$. Thus $x \cdot z \in U(A_T; \min\{A_T(x \cdot (y \cdot z), A_T(y)\})$. Thus $A_T(x \cdot z) \geq \min\{A_T(x \cdot (y \cdot z), A_T(y)\})$.

Let $x, y \in X$. By (1.29), we have $A_T(x \cdot (y \cdot z)) \geq \min\{A_T(x \cdot (y \cdot z), A_T(y)\}$ and $A_T(y) \geq \min\{A_T(x \cdot (y \cdot z), A_T(y)\}$. Thus $x \cdot (y \cdot z), y \in L(A_I; \max\{A_I(x \cdot (y \cdot z), A_I(y)\})$.

By assumption, we have $L(A_I; \max\{A_I(x \cdot (y \cdot z), A_I(y)\})$ is a UP-ideal of $X$. Thus $x \cdot z \in L(A_I; \max\{A_I(x \cdot (y \cdot z), A_I(y)\})$. Thus $A_I(x \cdot z) \leq \min\{A_I(x \cdot (y \cdot z), A_I(y)\}$.

Let $x, y \in X$. By (1.29), we have $A_F(x \cdot (y \cdot z)) \geq \min\{A_F(x \cdot (y \cdot z), A_F(y)\}$ and $A_F(y) \geq \min\{A_F(x \cdot (y \cdot z), A_F(y)\}$. Thus $x \cdot (y \cdot z), y \in U(A_F; \min\{A_F(x \cdot (y \cdot z), A_F(y)\}$.
Hence, $A$ is a UP-ideal of $X$ if and only if for all $x \cdot z \in U(A_F; r\min\{A_F(x \cdot (y \cdot z)), A_F(y)\})$. Thus $A_F(x \cdot z) \supseteq r\min\{A_F(x \cdot (y \cdot z)), A_F(y)\}$.

Hence, $A$ is an interval-valued neutrosophic UP-ideal of $X$. \hfill \Box

**Theorem 3.5.** An IVNS $A$ in $X$ is an interval-valued neutrosophic strong UP-ideal if and only if for all $\bar{a}, \bar{b}, \bar{c} \in [0, 1]$, the sets $E(A_T; A_T(0))$, $E(A_I; A_I(0))$, and $E(A_F; A_F(0))$ are strong UP-ideals of $X$.

**Proof.** Assume that $A$ is an interval-valued neutrosophic strong UP-ideal of $X$. By Theorem 2.1, we have $A$ is constant, that is, $A_T$, $A_I$, $A_F$ are constant. Thus

$$(\forall x \in X) \begin{pmatrix} A_T(x) = A_T(0) \\ A_I(x) = A_I(0) \\ A_F(x) = A_F(0) \end{pmatrix}.$$ 

Hence, $E(A_T; A_T(0)) = X, E(A_I; A_I(0)) = X$, and $E(A_F; A_F(0)) = X$ and so $E(A_T; A_T(0)), E(A_I; A_I(0)), \text{ and } E(A_F; A_F(0))$ are strong UP-ideals of $X$.

Conversely, assume that $E(A_T; A_T(0)), E(A_I; A_I(0)), \text{ and } E(A_F; A_F(0))$ are strong UP-ideals of $X$. Then $E(A_T; A_T(0)) = X, E(A_I; A_I(0)) = X$, and $E(A_F; A_F(0)) = X$ and so

$$(\forall x \in X) \begin{pmatrix} A_T(x) = A_T(0) \\ A_I(x) = A_I(0) \\ A_F(x) = A_F(0) \end{pmatrix}.$$ 

Thus $A_T, A_I, A_F$ are constant, that is, $A$ is constant. By Theorem 2.1, we have $A$ is an interval-valued neutrosophic strong UP-ideal of $X$. \hfill \Box

**4. Conclusions and Future Work**

In this paper, we have introduced the notions of interval-valued neutrosophic UP-subalgebras, interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals of UP-algebras and investigated some of their important properties. Then, we get the diagram of generalization of IVNSs in UP-algebras as shown in Figure 1.

In our future study, we will apply this notions/results to other type of IVNSs in UP-algebras. Also, we will study the soft set theory of interval-valued neutrosophic UP-subalgebras, interval-valued neutrosophic near UP-filters, interval-valued neutrosophic UP-filters, interval-valued neutrosophic UP-ideals, and interval-valued neutrosophic strong UP-ideals.

**References**


Figure 1. IVNSs in UP-algebras