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AN ESSAY ON MATHEMATICAL EDUCATION IN THE REPUBLIC OF SRPSKA, BOSNIA AND HERZEGOVINA

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Abstract: The European Society for Research in Mathematics Education was founded in 1998. Since then, not a single institution or an individual from the areas of the former Yugoslavia have, yet joined the society or participated in any of its activities.

This article has been written for the mathematics community of Bosnia and Herzegovina, and particularly for the Ministries of Education and Science, because of the need to begin the process of joining the European Society for Research in Mathematics Education. In particular, it is important that mathematicians become involved with research on mathematics education. In this article, the authors try to explain a number of problems related to mathematics education. Thus, we will consider the need for a change in the goals of mathematics education within the school system. On the other hand, we will explore the opinion that the Education ministry should consider the possibility of introducing different ways of teaching mathematics in primary and secondary schools, in order to follow, at least partially, the important reforms which are already taking place in this field.

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1. The aims of mathematics education

The first school in which the concept of mathematics education was formed was founded more than 1200 years ago (in 795, to be precise) in Aachen, during the reign of Charles the Great. It is known that, at his personal request, a monk called Alcuin of York was brought to Britain because mathematics was taught at that school. It seems that the monk wrote the first textbook in medieval Europe, titled "Problems to Sharpen the Yong" (in English) or "Propositiones ad Acuendos Juvenes" (in Latin). The 18th assignment in the book was a well-known problem called "on wolf, goat and cabbage". Since then, that assignment has been used in many mathematics texbooks. 'The sharping of the memory' is the aim of mathematics education at any level, especially in higher education in the areas of technical and humanistic sciences.

It is worth noting, in connection with the history of the first mathematics textbook, that on the 8th of August, 1900, David Hilbert read his famous paper "Mathematical problems" at the

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Second International Congress of Mathematicians, held in Paris. The paper began with the words [10]: "Who of us would not be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries? What particular goals will there be toward which the leading mathematical spirits of coming generations will strive? What new methods and new facts in the wide and rich field of mathematical thought will the new centuries disclose?" At this point, Hilbert had already formulated a sufficiently general view of the state of mathematics at the time, and set a number of unsolved mathematical problems that formed an outline of the considerable efforts being made by mathematicians across the world.

In the 20th century, no mathematician has repeated David Hilbert's attempt and nothing of a similar nature has been written since the beginning of the 21st century. At best, a few syntheses that referred to separate mathematical fields have been made. For example, International Meetings of Mathematicians, including the one held in Berlin in 1998, are testament to that fact. No mathematician has yet presented a general mathematical view at any of these meetings. This may be because of the complexity and clumsiness of mathematical disciplines. According to Victor Antonovich Sadovnichii, who presented a paper at the mathematical conference "Mathematics Education: The Present and Future", which was held in 2000 in Russia, the reasons lie elsewhere (The All-Russian Conference on Mathematical Education "Mathematics and Society: Mathematical Education at the Frontier of Centuries; Russia, September 2000; available on http://www.mccme.ru/conf2000/).

Mathematics is an international science. Gogol once said that Pushkin's name immediately sparks thoughts about his role as the national poet of Russia. A similar statement regarding any mathematician would make no sense. In contrast to national languages, mathematical language in an international language and mathematical truths have no national borders.

Mathematics has always been, and will always be an indivisible and essential part of human culture. According to many scientists, it is a vital part of human nature to get to know and understand one's natural surroundings, and also the social milieu in which one lives and works. This is the basis of scientific, technical and technological progress. Lately, a new quality has been added to mathematics: mathematics education is now considered to be an important component in the development of an individual.

Mathematics education is a treasure which every man has the right to, and it is the responsibility of the society (the national and international organizations) to provide an opportunity to every person to use that treasure.

With regard to mathematics education, it is necessary to perceive and to set the following: goals, principles and content of mathematics education. These principles and goals shall be analyzed in compliance with contemporary approaches in mathematical research (such as those seen in the following books: [2], [7], [10], [12], [21] and articles [3]-[6], [9], [11], [13]-[20].

Before we present our opinion on mathematics education, we will quote the results of some contemporary studies on the goals of mathematics education which were carried out in some European countries, the United States, Canada, China and Australia. The results of these investigations were announced to the mathematics community at the international conference entitled "Education, Science and Economy in the Third Millennium", which was held in Slovakia in 2000. An interesting question for this conversation was contained in the questionnaire: "What is the aim of mathematics education? (Arrange the following answers according to their significance):

- 1. Preparation for higher education,
- 2. Preparation for the future profession,
- 3. Intellectual development of an individual,

- 4. Formation of the scientific view on the world,
- 5. Orientation in surrounding,
- 6. Physical training of brain".

In the United States, Canada and some European countries where the studies were carried out, the greatest importance was attributed to the answer "preparation for a future profession". According to Vladimir Mihailovich Tihomirov's announcement, in Russia, priority was given to the answer "intellectual development", and the answer "preparation for higher education" was in the last place (We are unaware of any similar research having been carried out in the Balkans)

At the aforementioned conference, and according to the importance of these mathematical goals, the following order was established:

- 1. Intellectual development;
- 2. Orientation in one's surroundings;
- 3. The formation of the scientific view of the world,
- 4. Physical training of the brain;
- 5. Preparation for a future profession;
- 6. Preparation for higher education.

This is how teachers in the departments of the second $(4^{th} - 6^{th} \text{ grade})$ and third $(7^{th} - 9^{th})$ cycles in primary school, the ones who teach mathematics in the high school (students of 15-17 years old) and lecturers in mathematics courses in institution of higher education present the goals of mathematics education to themselves. When analyzing this problem – the aims of teaching mathematics in terms of the development of a fully educated personality – it is logical to ask a question that applies to every human being: when a girl or a boy, aged 14 to 16, thinks about his/her future, and then asks himself/herself the questions: What does mathematical education give me? Why do people learn mathematics? - what is the answer? These questions, which are highly appropriate, were asked at the Pan-Russian conference on mathematics education held in Dubna in 2000.

We would also be interested in the answers and the grouping of the answers to the aforementioned questions and their order in terms of importance. Following professor Tihomirov's presentation at the conference mentioned above, the ideas presented in this paper will contribute to such an understanding.

(1) Mathematics is found and used in all forms of life and work, and it is completely justified to assert that mathematical questions are necessary for everyone. Of course, it is true that most of our lives we need mathematical knowledge and skills to support us, and that we often use many mathematical concepts, such as our knowledge of geometric bodies and our comprehension of time passing, as well as many other mathematical subjects, without even noticing them. All knowledge that exceeds the mathematical knowledge we acquire at birth (according to the research by Israeli didactic mathematican, Uri Leron [16], a human being is born with some rudimentary mathematical knowledge and mathematical skills, is acquired during through formal schooling).

(2) Mathematical knowledge, skills and abilities are needed for all professions; this is particularly true for professions in which exact explanations are needed, and for professions connected to natural sciences, engineering, technology, and economics. Mathematics is the language of nature, engineering and technology. Therefore, the professions connected to natural sciences, engineering, technology and economics require knowledge, skills and habits based on an education in mathematics. Galileo spoke about this in his time, "Philosophy of nature is written in the magnificent book, which is before our eyes all the time, but its understanding is possible if we learn the language it was written by. It is written in the language of mathematics". But there is no doubt that mathematical knowledge, and especially

the logical ones, are needed to doctors, linguistics, historians, as well as to others who use objects mathematics deals with. Thus, mathematics and mathematical education are needed as a preparation for the future profession to every individual in mastering knowledge, skills and abilities. Knowledge of algebra, geometry, mathematical analysis, probability and statistics are needed for that.

(3) Explanations offered to us during our secondary school education we grasped in the following way: the goal of philosophy in terms of attaining a successful human civilization is a general understanding among its people, economic prosperity and successfully mastering the basic sciences. This is impossible without mathematics. Therefore, mathematics is needed for the formation of the general view of the world. ([9])

It is worth noting that the positions discussed above promote the ideas of Russian mathematician Andrei Nikolaevich Kolmogorov, when he offered these ideas as the reason for reform of school mathematics.

Here, at Banja Luka University, a significant number of the faculties do not attach any great importance to physical training for the body, or compulsory physical activities for the students 'in the function' of preparing students physically for the intellectual demands of learning. It seems that it is generally accepted by the University that physical training of the brain is not a prerequisite for success in studying. We have noted this because it appears that a significant numbers of courses at the University, including law, medicine, economy and various classes in the Faculty of Philosophy, have an insufficient number of classes in mathematics. How can it be possible for someone to complete their studies in philosophy and never hear anything about mathematical logic? How can it be possible that programs with no course of statistics exist at all? [18]. In this paper, we will present the opposite attitude: the physical training of the body and the brain is necessary for the realization of students' intentions – mastering the knowledge, skills and abilities necessary for their future in an appropriate profession. The physical training of the brain - intellectual gymnastics for the mind - is necessary for everyone (does it make any sense to deny it?). Logic and mathematics are making physical training of this kind possible (the fields of computer sciences and linguistics (the study of languages) have also seen success in this area, and are also based on mathematics.

(4) Obviously, preparation for university studies is one of the aims of mathematics education.

(5) There is an important assignment within mathematics education: this is the development of the ability to accurately comprehend the assignments, to think accurately, to draw logical conclusions and to adapt mathematical (algebraic and geometrical) thinking. Everybody needs to learn to conduct analyses, to differentiate between hypotheses and facts, to criticize, to understand set assignments, to be systematic, to present one's thoughts precisely, and so on. On the other hand, one must develop a sense of imagination, and strengthen one's intuition (spatial representation, the ability of anticipating results, and the ability to anticipate the correct way of solving an assignment) ([9]) In other words, mathematics is needed for the intellectual development of one's personality. The English philosopher, Roger Bacon³, said in 1267: "The one who does not know mathematics neither understands any other science nor can perceive his ignorance" (Roger Bacon: *The Opus Majus*, avaliable on http: // ecmd.nju.edu.cn/UploadFile/8/3712/mp10sc.doc)

There are two traditions of mathematics education [19]. One of them is formed on the idea that an individual has to know how to use the given methods in advance, and the other on the basis that, first of all, he/she can think independently about methods and their application. In the authors' universities, a significant number of people like to think that our educational tradition is based on the development of intellect. It would be good if that were so, and one of

³ Roger Bacon (1220-1292)

our educational goals should be preventing the disappearance of the local educational traditions. The arguments that motivate the goals of education are quoted in the aforementioned poll. These goals are frequently discussed.

Lately, the theory that mathematics is obliged to train an individual to accept ethical principles has been frequently discussed. Mathematics should aid the development of intellectual honesty, objectivity, and aspirations towards the truth.

It has already been stated that mathematics is a fundamental part of human culture, i.e., it is a part of the formation of the spiritual world of humanity. Therefore, every person should know some fragments of the history of mathematics, the names of the zealous workers in this field, the problems they were faced with and the mistakes they made.

2. The principles of mathematics education

One of the most important questions is: Does the one who realizes that teaching provides the opportunity to include the principle of limited freedom in educational principles or is he /she obligated to respect some elements of the state enforcement?

Our generation (born right after the WWII) was exposed to the education system when, it seemed, the main goal of a human life was to serve the state (such a concept was present in all countries of the Socialist Block). Thus, the state controlled almost all of the aspects of every individual's life. Education was provided equally for all, everybody learned from identical textbooks, while unique programmes and opportunities to choose were reduced to a minimum. (Of course, it is the same now, because the state does not allow anything outside of the regulated teaching plan and programme.) In many other countries, education is based on liberal principles, and it could be argued that it is based on the principle of limited freedom.

It seems that such extremes should be avoided in the new millennium, and that attempts should be made to offer education in a range of forms. In other words, the principle of personal freedom should be justified. These possibilities should be presented to the candidate (the student and his/her parents), leaving them a freedom to choose. Different forms of teaching make this possible. It should be noted that massive education is impossible without certain motivations and a stimulus for the acquisition of education and for mastering the knowledge accumulated through human efforts. Social consciousness, culture, education and open/mindedness (which is impossible without long-term and persistent efforts being made to acquire an education) are some of the most important criteria in the evaluation of personality. Judging by these criteria, there are important problems facing mankind, the solutions to which will considerably influence the future life of this planet. We think that humanity will not be able to solve these problems or to apply the solutions correctly without a significant number of educated and sophisticated people. We consider it to be natural that basic education, in its initial phase, as well as in the basic courses in higher education, to a great extent, should be common across all programmes of the same type, so that the necessity of such an education would be clear to everyone. Students could specialize in courses further on in their education.

In parallel with the principle of limited freedom, the principle of rational conservatism should be taken into consideration in the education system, in terms of what it means for the continuation of the system, how the results of this experiment should be taken, and in terms of the mathematization of other subjects in general education.

The differentiation of education, which is being discussed, is possible in two forms: **profile** differentiation and individual differentiation. This is particularly possible when mathematical education is in question. There are a few projects which deal with the introduction of differential teaching in mathematics education that have already been realized. It would be

useful to look at the results of the Mathematics Enhancement Programme (MEP) project (see, for example http://www.cimt.plymouth.ac.uk/projects/mep/default.htm). Romano [20]also wrote an article on the need for the differentiation of mathematics education in primary schools.

In addition, it has been understood that the principle of continuous education for the majority of students from school age to the completion of their higher education has to be accepted (or 'obligatory part' if the institution of higher education programme does not mean that mathematics is studied to the very end of education).

3. The structure of mathematics education

The previous sections can all be useful for conceptualizing the structure of mathematics education. It should begin when the child is of the pre-school age. First of all, children receive this education from their parents or in their nursery and kindergarten. There is an opinion that mathematics education at this stage should not be left in the chaotic state as it has been so far. Competent institutions should deal with the creation of appropriate literature for the training of teachers for this pre-school period. I think that there is no doubt that the formation of mathematical concepts is fairly important, and that they should not be left to laymen (our teachers in nurseries and kindergartens do not necessarily have any mathematical knowledge, or have gained only a modest amount during their teacher training. In our country, during the existence of the Pedagogical Academy, a significant part of the mathematics education for teachers was delivered by mathematically incompetent faculty. Unfortunately, this tendency continues at the Teacher's Department of the Faculty of Philosophy.)

Furthermore, we have three cycles of education in primary school, in which the manner of education of the teachers has not yet been defined. For now, the first cycle is realized by teachers with insufficient mathematics education. The programme for the mathematics education of teachers in our environment is very modest and, it seems, wrong in many aspects. At the 'teacher training institutions', two modest courses providing an introduction to mathematics are dedicated solely to mathematics, and there are also two modest courses in methods for teaching mathematics. As mathematics is a discipline where the continuity of knowledge is expressed unconditionally, and taking into consideration the estimate that high school graduates who apply for the teacher training actually apply because their knowledge of mathematics is modest, it is completely justified to question the validity of their knowledge, abilities and skills after finishing their teacher training. The other two cycles in primary school - the second (4-6 grades) and the third (7-9 grades) cycle - should resolve the omissions and mistakes of the first cycle. (For now, in our country, there is no general opinion about how teachers in the second and third cycles of primary school should be educated.) Is that possible? In the light of the previous comment, how to understand the tendency that the possibility that the teacher's faculty organizes and carries out the postgraduate studies from the teaching methods of mathematics is accepted without opposition. It is therefore justified to ask: What does postgraduate study in methods of teaching mathematics really mean in the teacher training institutions? Does it mean anything?

What is the status of mathematics education in high school? (There are several systems of high schools education in our country. For example, there exist classic high school, engineering high school, commercial high school, etc.) Literally speaking, it seems that quite a few aspects in this area are satisfactory. In essence, unusual things are happening: a considerable number of students finish high school with knowledge of mathematics which is more than modest. Consider the counter position to this assertion: either the children who are attending high school have a very modest coefficient of logical intelligence, or the problem is in the school system. The authors believe that the mathematics programme in high schools is very poorly conceived. In our opinion, the teaching methods applied in our high schools yield

results which are not necessary to anybody – neither to society nor to parents or their children. We think that a new relationship should be formed between the students and mathematics teachers in high schools, one in which the teachers put themselves in the role of the helper in the student's learning process, with the mandatory differentiation of teaching material, according to depth and width. This would provide better results than those currently being produced.

We recommend a similar scheme for higher education.

Finally, let us say a few words about the role of mathematics in the global development. Of course, only one part of its role will be maintained. It seems that humanity will only be able to survive in the future if it undertakes a well-thought-out general programme. Such a programme is impossible to imagine (without colossal intellectual effort) without the theory of algorithms, and by no means without mathematics. In the future, mathematics will be one of the most needed professions in the world.

4. What are the stimuli for the acquisition of mathematics education?

It is necessary that mathematics teachers in all three cycles in primary schools, secondary school teachers, university lecturers and the Ministers of Education and Science in the Republic of Srpska and Bosnia and Herzegovina, with international help become united in their efforts to increase the number of mathematics students for promotional purposes. For this reason, it is reasonable to use the experiences of other countries, which began attempting to resolve this problem earlier on, and where the competitive systems for solving mathematical assignments are well-developed. Mathematics is necessary to every state and humanity in general. We think that people in power (staff in the Ministry of education and culture, and in the Ministry of sciences and technology of the Republic of Srpska, B&H) should constantly emphasize that mathematics education is one of four equal pillars of every education (the other three are education in the native language, at least one foreign language and physical education) and that therefore it is necessary that they appreciate and support mathematics and mathematics education (see, for example [3]-[5], [12]-[14] and [17]).

We think that the opinion of Alfred Adler⁴ should be accepted, and that the thirst of youth for competition, comparison and self-esteem should be used ([1]). In many countries, the students in the school systems are not ranked in any way, nor is their invested energy evaluated in any way, which implies that it does not concern the society in which they live. By accepting that there are individuals who are more able than others, and who are ready to develop their abilities by investing their energy, the society in Bosnia and Herzegovina and in our environment should be able to give up the concept that these differences do not exist. In our opinion, the important characteristics of a person are creativity, an interest in the adoption of knowledge and a tendency to be instrumental. The creation of possibilities for the advancement of these abilities is a primary obligation of the society and its stake holders. It would be interesting to carry out a survey of public opinion in Republic of Srpska and in Bosnia and Herzegovina on the evaluation of success in education and scholarly work. It seems that the ranking of success (carefully done and precisely checked) should have already been introduced. The majority of people who have experience in primary and secondary school mathematics convinced themselves or were convinced by certain mathematics teachers that knowledge cannot be gained from mathematics.

⁴ Alfred Adler (February 7, 1870 - May 28, 1937) was an Austrian medical doctor, psychologist and founder of the school of individual psychology. In collaboration with Sigmund Freud and a small group of Freud's colleagues, Adler was among the co-founders of the psychoanalytic movement as a core member of the Vienna Psychoanalytic Society. [from Wikipedia]

5. The problems of mathematics education

The study of mathematics education has received constant attention from the international mathematical community over the last 30 years. Every year, a considerable number of mathematical meetings, dedicated to the study of mathematics education and teaching practices are held across the world. According to the practices of the European Society for Research in Mathematical Education (ERME), the following problems are constant topics discussed at these conferences (see, for example, the following books [2], [12], [21] and articles [3], [9], [11], [19]):

Group 1: The nature and content of mathematics and its connection to teaching and learning;

Group 1a: Relations between theory and practice in mathematics education;

Group 1b: The role of metaphors and visual aids in teaching and studying mathematics;

- Group 2: Tools and technologies in mathematical didactics;
- Group 3: From studying teaching practice to the education of teachers;

Group 3a: The creation of structures for mathematics education;

- Group 3b: Argumentation and proving;
- Group 4: Social interaction in mathematical education;

Group 4a: Teaching and learning in multicultural groups;

- Group 5: Mathematical thinking and learning as a cognitive process;
- Group 5a: Algebraic thinking;
- Group 5b: Geometric thinking;
- Group 5c: Statistical thinking;
- Group 6a: School algebra: epistemological and educational phenomena;
- Group 6b: School geometry: epistemological and educational phenomena,
- Group 6c: School statistics and probability,
- Group 7: Research into paradigms and methodology and their relations with the problems in mathematics education,
- Group 8: Education of mathematics teachers and research into their education;
- Group 9: Education of mathematics teachers and their conviction;
- Group 10: Education of teachers and research into their knowledge;
- Group 11: Education of teachers and research into their practice;
- Group 12: Education of teachers through teachers' research into their own practice;
- Group 13: Research into education for teachers: trends, future research and cooperation;
- Group 14: Evaluation of the success of teachers' work in teaching and the success of students' work in the teaching of mathematics.

For the sake of illustration, we will now quote the titles of presentations dealing with one of the aforementioned topics from the last conference of the European Society for Research in Mathematics Education (the Fourth Congress of the European Society for Research in Mathematics Education. Sant Feliu de Guixols, Spain, from 17th to 21st February 2005):

Working group 3: Building structures in mathematical knowledge

- 1. Building structures in mathematical knowledge
- 2. Building structure in mathematics within teaching and learning processes a study on teachers' input and students' achievement
- 3. The metaphor "contracts to deal with concepts" as a structuring tool in algebra
- 4. From experience, through generic models to abstract knowledge
- 5. Classification leading to structure
- 6. Discussing the challenge of categorising mathematical knowledge in mathematics research situations
- 7. On linguistic aspects of structure building
- 8. Approaching the distributive law with young pupils
- 9. The development of informal proportional thinking in primary school
- 10. 14 years old pupils' thought processes: a case study of constructing the triangular inequality

6. Conclusions

The ministries in charge of education should pay special attention to mathematics education. They must accept the need for international projects which will carry out research in the field of mathematics education. Let us remind ourselves that, in their time, Aleksandrov, Bogoljubov, Kolmogorov, Lavrentjev, Menjshov, Shmidt, Snireljman, Urison and others have played a significant role in establishing the world-famous Russian system of mathematics education.

Let us consider a few questions which are connected to mathematics education at university.

Mathematics education programs were developed, judging by the available evidence, between th 1930s and 1950s (there is a belief that a significant role was played by the Russian mathematician Andrei Nikolaevich Kolmogorov). Our generation failed to introduce corrections to mathematics education, so that mathematics education could develop over the last half a century. These failures are now the main obstacle preventing the development of mathematics education. The process, started by French mathematicians Z. Lere, A. Kartan, A. Weil, K. Chevalley and A. Grotendik, Z.–P. Serr and R. Tom, has reformed many aspects in this field. Significant innovations have appeared, such as, for example, many-valence logic, algebraic geometry, the theory of Lie groups, the theory of representation and many others, which have not yet become the topics of university courses.

A revision of university programmes is necessary in the areas of mathematics, information technology, technical-technology profiles, and social and humanistic sciences. In order to answer this challenge to university education, in accordance with contemporary scientific developments, a new type of experimental university appears to be needed where there would be a smaller number of students, and where contemporary and non-standard mathematical disciplines could be taught. Here, a new generation of mathematicians would be able to keep up with the development of new mathematical disciplines that should be cherished. On the contrary, the generation the authors belong to will be the last generation of mathematicians in Bosnia and Herzegovina which will be capable of following what is happening in a number of classical mathematical disciplines. There are indications that such priorities are now being understood in France and the United States.

Mathematics education has to include training in computers and contemporary information technologies, as well as mathematical disciplines which will form the basis for a new society, based on information technology (for example, the theory of chaos, the theory of catastrophes, the theory of fractals, and a new perspective on discrete mathematics).

One particular problem is the need to constantly adapt mathematics education to the results of contemporary research in the field. According to its definition, mathematics education is conservative. It differs considerably from mathematical science. In other words, mathematics is specific. The tumultuous growth of mathematics in the second half of the 20^{th} century provided mankind with special goals. In the last ten to twenty years there is the forethought across the whole world about standardization within mathematics education. For example, this can be seen in the MEP of the European Community (1995-2004) – a project on teaching mathematics – as well as in the announcements made by the Conference of the Mathematical Society for Research in Mathematical Education.

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