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Heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium

K. DAS
Department of mathematics, Kalyani Govt. Engineering College
Kalyani, West Bengal. e-mail : kd_kgec@rediffmail.com

S. JANA
Department of mathematics, Kalyani Govt. Engineering College
Kalyani, West Bengal.

Abstract. *The present study deals with the effect of heat and mass transfer on free convection flow near an infinite vertical plate embedded in porous medium, which moves with time dependent velocity in a viscous, electrically conducting incompressible fluid, under the influence of uniform magnetic field, applied normal to the plate. The problem is solved analytically in closed form by Laplace transform technique and the expressions for velocity, temperature, concentration , skin friction, rate of heat and mass transfer has been obtained. Some important applications of physical interest for different type motion of the plate are discussed. The results obtained have also been presented numerically through graphs to observe the effects of various parameters and the physical aspects of the problem .*

Key words: Free convection, MHD flow, heat transfer, mass transfer, Laplace transforms.

AMS subject classification:76W05,76S05.

1. Introduction

In nature, there exist flows which are caused not only by the temperature differences but also by concentration differences . These mass transfer differences do effect the rate of heat transfer. In industries, many transport processes exist in which heat and mass transfer takes place simultaneously as a result of combined buoyancy affect of thermal diffusion and diffusion through chemical species. The phenomenon of heat and mass transfer frequently exist in chemically processed industries such as food processing and polymer production. Free convection flow involving coupled heat and mass transfer occurs frequently in nature. For this flow, the driving forces arise due to the temperature and concentration variations in the fluid. For example, in atmospheric flows, thermal convection resulting from heating of the earth by sunlight is affected by differences in water vapour concentration. Magnetohydrodynamics has attracted the attention of a large number of scholars due to its diversified applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, interstellar matter, radio propagation through the ionosphere etc.. In engineering it finds its application in MHD pumps, MHD bearing etc. The study of effects of magnetic field on free convection flow is important in liquid-metals, electrolytes and ionized gases. The thermal physics of hydromagnetic problems with mass transfer is of interest in power engineering and metallurgy. Moreover, there are several engineering situations wherein combined heat and mass transport arise viz. humidifiers, dehumidifiers, desert coolers, chemical reactors etc. The usual way to study these phenomena is to consider a characteristic moving continuous surface.

Free convection flow near a vertical plate or surface with different conditions has been extensively studied by various authors like Vedhanayagam *et al.*[22], Martynenko *et al.*[10], Kolar *et al.*[9], Ramanaiah et al. [13] and Camargo *et al.*[3]. Free convection flow with mass transfer past a vertical moving plate has been studied by Soundalgeker[19], Revankar[15], Soundalgeker *et al.*[20], Das et al. [5], Muthukumaraswamy et al. [11] and Panda *et al.*[12]. Many workers like Revankar[14], Anwar [2], Sahoo *et al.*[16] worked on hydromagnetic natural convection flow past a vertical surface under different conditions.The effects of heat and mass transfer on a free convection flow near an infinite vertical porous plate has been extensively investigated by Takhar *et al.*[18], Hossain *et al.* [7], Israel *et al.*[8], Sahoo *et al*[17] ,Ali[1], Chaudhary and Jain[4]. But most of them solved the problem numerically. Hence it appears that the analytical solution of this problem will be of greater interest.Recently Das[22] developed the problem by considering the magnetic effect on free convection flow in presence of thermal radiation.

The aim of the present investigation is to analyse the effect of heat and mass transfer on the unsteady free convection flow of a viscous, electrically conducting incompressible fluid near an infinite vertical plate embedded in porous medium which moves with time dependent velocity under the influence of uniform magnetic field, applied normal to the plate. A general exact solution of the governing partial differential equation is obtained by using Laplace transform technique. Furthermore, this general solution is applied to consider some important cases of the flow: (i) motion of the plate with uniform velocity ,(ii) the single accelerated motion of the plate and (iii) the plate with periodic acceleration.

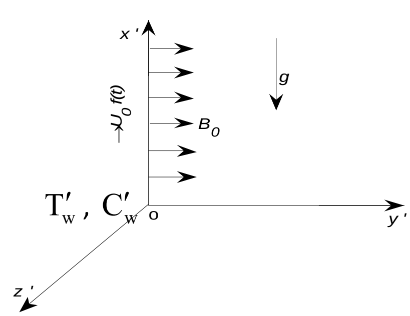

Fig.1 A schematic of the problem and coordinate system

2. Formulation of the problem

Let us consider unsteady free convection and mass transfer flow of a viscous incompressible and electrically conducting fluid along an infinite non-conducting vertical flat plate (or surface) through a porous medium in presence of a uniform transverse magnetic field B_0 applied on this plate. On this plate an arbitrary point has been chosen as the origin of a Cartesian co-ordinate system with the x'-axis along the plate in the upward direction and the y'-axis normal to the plate (Fig.1).

Initially, for time $t' \leq 0$, the plate and the fluid are maintained at the same constant temperature T'_∞ in a stationary condition with the same species concentration C'_∞ at all points. Subsequently(t'>0), the plate is assumed to be accelerating with a velocity $U_0 f(t')$ in its own plane along the x'-axis, instantaneously the temperature of the plate and the concentration are raised to T'_w and C'_w respectively, which are hereafter regarded as constant.

For free convection flow, we also assume that : (i) All the physical properties of the fluid such as coefficient of viscosity(μ), kinematic coefficient of viscosity(ν), specific heat at constant pressure(C_P), thermal conductivity(κ), volumetric coefficient of thermal expansion(β_T^*),volumetric coefficient of expansion for concentration(β_C^*), chemical molecular diffusivity(D), etc., remain constant. (ii) The effect of variations of density(ρ) (with temperature) and species concentration are considered only on the body force term, in accordance with the usual Boussinesq approximation ([6],[11]).(iii) In the energy equation, the term due to the viscous dissipation can be neglected in comparison with the conducting term ([4],[6]).(iv) The thermal-diffusion (Soret) and diffusion thermal (Dufour) effects in the energy equation and in the concentration equation, can be ignored, as the level of concentration is usually assumed to be very low in free convection flows ([6]). (v) Since the flow of the fluid is assumed to be in the direction of the x-axis, so the physical quantities are functions of the space co-ordinate y' and time t' only.

Under the above assumptions, the governing equations for the two dimensional flow can be expressed as follows ([7]).

Momentum equation:

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u}{\partial y'} + g\beta_T (T' - T'_\infty) + g\beta_c (C' - C'_\infty) - \frac{\sigma B_0^2 u'}{\rho} - \frac{\nu u'}{K'}, \tag{2.1}$$

Energy equation:

$$\frac{\partial T'}{\partial t'}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}, \tag{2.2}$$

Concentration equation:

$$\frac{\partial C'}{\partial t'}=D\frac{\partial^2 C'}{\partial y'^2}, \tag{2.3}$$

where u' is the velocity, T' is the temperature, C' is the species concentration and g is the acceleration due to gravity

The initial and boundary conditions corresponding to the present problem are :

$$u'(y',t')=0, T'(y',t')=T'_\infty, C'(y',t')=C'_\infty \text{ for } y'\geq 0 \text{ and } t'\leq 0, \tag{2.4}$$

$$u'(0,t')=U_0 f(t'), T'(0,t')=T'_w, C'(0,t')=C'_w \text{ for } t'>0 \tag{2.5}$$

$$u'(\infty,t')\to 0, T'(\infty,t')\to T'_\infty, C'(\infty,t')\to C'_\infty \text{ for } t'>0$$

To reduce the above equations into non-dimensional form for convenience, let us introduce the following dimensionless variables and parameters:

$$u=\frac{u'}{U'_0},\ t=\frac{t'U_0^2}{\nu},\ y=\frac{Uy'}{\nu},\ K=\frac{K'U_0^2}{\nu^2},\ M=\frac{\sigma B_0^2\nu}{\rho U_0^2},\ Pr=\frac{\mu C_p}{\kappa},\ \omega=\frac{\omega'\nu}{U_0^2},$$

$$Gr=\frac{\nu g\beta_T(T'_w-T'_\infty)}{U_0^3},\ Gm=\frac{\nu g\beta_C(C'_w-C'_\infty)}{U_0^3},$$

$$S_c=\frac{\nu}{D},\ \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty},\ C=\frac{C'-C'_\infty}{C'_w-C'_\infty}, \tag{2.6}$$

where Gr is the thermal Grashof number, Gm is the mass Grashof number, K is the permeability parameter, M is the magnetic parameter, Pr is Prandtl number, Sc is Schmidt number, β_T is thermal expansion coefficient, β_C is concentration expansion coefficient and ω is frequency of oscillation. Other physical variables have their usual meanings.

With the help of (2.6), the governing equations (2.1) to (2.3) reduce to

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+Gr\theta+Gm\phi-(M+\frac{1}{K})u, \tag{2.7}$$

$$Pr\frac{\partial \theta}{\partial t}=\frac{\partial^2\theta}{\partial y^2}, \tag{2.8}$$

$$Sc\frac{\partial C}{\partial t}=\frac{\partial^2 C}{\partial y^2}, \tag{2.9}$$

The corresponding initial and boundary conditions in non-dimensional form are :

$$\begin{aligned} &u(y,t) = 0, \quad \theta(y,t) = 0, \quad C(y,t) = 0, \quad \text{for } y \geq 0, \text{ and } t \leq 0,\\ &u(0,t) = f(t), \quad \theta(0,t) = 1, \quad C(0,t) = 1, \quad \text{for } t > 0, \\ &u(\infty,t) \to 0, \quad \theta(\infty,t) \to 0, \quad C(\infty,t) \to 0, \quad \text{for } t > 0, \end{aligned} \tag{2.10}$$

3. Solution of the problem

In order to obtain the analytical solutions of the system of differential equations (2.7) to (2.9), we shell use the Laplace transform technique.

Applying the Laplace transform (with respect to time t) to equations (2.7) to (2.10), we get

$$\frac{d^2\bar{u}}{dy^2} - q^2\bar{u} = -Gr\bar{\theta} - Gm\bar{C}, \tag{3.1}$$

$$\frac{d^2\bar{\theta}}{dy^2} - sPr\bar{\theta} = 0, \tag{3.2}$$

$$\frac{d^2\bar{C}}{dy^2} - sSc\bar{C} = 0, \tag{3.3}$$

where $q^2 = s + M + \frac{1}{K}$ and s is the Laplace transformation parameter.

Also

$$\begin{aligned} &\bar{u}(y,s) = \bar{f}(s), \quad \bar{\theta} = \bar{C} = \frac{1}{s} \qquad \text{at } y = 0, \quad t > 0 \\ &\bar{u}(\infty,s) \to 0, \quad \bar{\theta}(\infty,s) \to 0, \quad \bar{C}(\infty,s) \to 0, \; t > 0 \end{aligned} \tag{3.4}$$

Solving equations (3.1) to (3.3) with the help of equation (3.4), we get

$$\bar{\theta}(y,s) = \frac{1}{s}e^{-ry}, \tag{3.5}$$

$$\bar{C}(y,s) = \frac{1}{s}e^{-my}, \tag{3.6}$$

$$\bar{u}(y,s) = \bar{f}(s)e^{-qy} + \bar{A}(y,Pr,s) + \bar{B}(y,Sc,s), \tag{3.7}$$

where

$$\bar{A} = \frac{Gr}{s(r^2 - q^2)}(e^{-qy} - e^{-ry}), \qquad \text{for} \quad Pr \neq 1$$

$$= \frac{Gr}{sM'}(e^{-y\sqrt{s}} - e^{-qy}), \qquad \text{for} \quad Pr = 1$$

$$\bar{B} = \frac{Gm}{s(m^2 - q^2)}(e^{-qy} - e^{-my}), \qquad \text{for} \quad Sc \neq 1$$

$$= \frac{Gm}{sM'}(e^{-y\sqrt{s}} - e^{-qy}), \qquad \text{for} \quad Sc = 1$$

with the abbreviations

$$r = \sqrt{s\,Pr}, m = \sqrt{sSc}, q = \sqrt{s+M'}, M' = M+\frac{1}{K}$$

Then, inverting equations (3.5), (3.6) and (3.7) in the usual way we get the general solution of the problem for the temperature θ(y,t), the species concentration C(y,t) and the velocity u(y,t) for t > 0 in non-dimensional form as

$$\theta(y,t) = \mathrm{erfc}(\eta\sqrt{Pr}), \qquad (3.8)$$

$$C(y,t) = \mathrm{erfc}(\eta\sqrt{Sc}), \qquad (3.9)$$

$$u(y,t) = \Phi(y,t) + A(y,Pr,t) + B(y,Sc,t), \qquad (3.10)$$

where

$$\Phi(y,t) = L^{-1}\left[\bar{f}(s)e^{-qy}\right], \qquad (3.11)$$

$$A(y,Pr,t) = GrL^{-1}\left[\frac{1}{s(r^2-q^2)}(e^{-qy}-e^{-ry})\right]$$

$$= \frac{Gr}{2M'}\exp\left(\frac{M't}{Pr-1}\right)\Big[\exp\left(-2\eta\sqrt{\frac{M'Prt}{Pr-1}}\right)\Big\{\mathrm{erfc}\left(\eta-\sqrt{\frac{M'Prt}{Pr-1}}\right)$$

$$-\mathrm{erfc}\left(\eta\sqrt{Pr}-\sqrt{\frac{M't}{Pr-1}}\right)\Big\}$$

$$+\exp\left(2\eta\sqrt{\frac{M'Prt}{Pr-1}}\right)\Big\{\mathrm{erfc}\left(\eta+\sqrt{\frac{M'Prt}{Pr-1}}\right)-\mathrm{erfc}\left(\eta\sqrt{Pr}+\sqrt{\frac{M't}{Pr-1}}\right)\Big\}\Big]$$

$$-\frac{Gr}{2M'}\left\{\exp\left(-2\eta\sqrt{M't}\right)\mathrm{erfc}\left(\eta-\sqrt{M't}\right)+\exp\left(2\eta\sqrt{M't}\right)\mathrm{erfc}\left(\eta+\sqrt{M't}\right)\right\}+\frac{Gr}{M'}\mathrm{erfc}\left(\eta\sqrt{Pr}\right),$$

(3.12a)

and for $Pr \neq 1$

$$A(y,Pr,t) = \frac{Gm}{2M'}\left\{2\mathrm{erfc}(\eta)-\exp(-2\eta\sqrt{M't})\mathrm{erfc}(\eta-\sqrt{M't})-\exp(2\eta\sqrt{M't})\mathrm{erfc}(\eta+\sqrt{M't})\right\}, \quad (3.12b)$$

for Pr =1

$$B(y,Sc,t) = GmL^{-1}\left[\frac{1}{s(m^2-q^2)}(e^{-qy}-e^{-my})\right]$$

$$=\frac{Gm}{2M'}\exp\left(\frac{M't}{Sc-1}\right)\Big[\exp\left(-2\eta\sqrt{\frac{M'Sct}{Sc-1}}\right)\left\{\operatorname{erfc}\left(\eta-\sqrt{\frac{M'Sct}{Sc-1}}\right)-\operatorname{erfc}\left(\eta\sqrt{Sc}-\sqrt{\frac{M't}{Sc-1}}\right)\right\}$$

$$+\exp\left(2\eta\sqrt{\frac{M'Sct}{Sc-1}}\right)\left\{\operatorname{erfc}\left(\eta+\sqrt{\frac{M'Sct}{Sc-1}}\right)-\operatorname{erfc}\left(\eta\sqrt{Sc}+\sqrt{\frac{M't}{Sc-1}}\right)\right\}\Big]$$

$$-\frac{Gm}{2M'}\left\{\exp\left(-2\eta\sqrt{M't}\right)\operatorname{erfc}\left(\eta-\sqrt{M't}\right)+\exp\left(2\eta\sqrt{M't}\right)\operatorname{erfc}\left(\eta+\sqrt{M't}\right)\right\}+\frac{Gm}{M'}\operatorname{erfc}\left(\eta\sqrt{Sc}\right), \quad (3.13a)$$

for $Sc \neq 1$

$$B(y,Sc,t)=\frac{Gr}{2M'}\left\{2\operatorname{erfc}(\eta)-\exp(-2\eta\sqrt{M't})\operatorname{erfc}(\eta-\sqrt{M't})-\exp(2\eta\sqrt{M't})\operatorname{erfc}(\eta+\sqrt{M't})\right\}, \quad (3.13b)$$

for Sc = 1

where $\eta=\frac{y}{2\sqrt{t}}$

Thus the expressions (3.8), (3.9) and (3.10) are the general solution of the present problem.These general solutions include the effects of heating (cf. term A), the diffusion (cf. term B) and the motion of the plate. Since the non-dimensional temperature θ(y,t), non-dimensional species concentration C(y,t) are clearly described in (3.8) and (3.9), so we shell confine ourself to non-dimensional velocity u(y,t) for various types of f(t).

4. Applications of the general solution

In this section, we now consider some important cases of flow as given below:

Case (i) *Motion of the plate with uniform velocity*

Let f(t) = H(t), the Heaviside unit function

Then $\overline{f}(t)=\frac{1}{s}$.

In this case, we observe that the results (18) and (19) for θ(y,t) and C(y,t) are unaffected and the expression for u(y,t) is reduced to

$$u(y,t)=\frac{1}{2}\left\{\exp(2\eta\sqrt{Mt})\operatorname{erfc}(\eta+\sqrt{Mt})+\exp(-2\eta\sqrt{Mt})\operatorname{erfc}(\eta-\sqrt{Mt})\right\}+A(y,Pr,t)+B(y,Sc,t), \quad (4.1)$$

where A(y,Pr,t), B(y,Sc,t) are given from equations (3.12), (3.13) respectively for various values of Pr and Sc.

Case (ii) *Motion of the plate with a given acceleration* **.**

Let f(t) = tH(t)

Then $\bar{f}(t)=\frac{1}{s^2}$

In this case, also we observe that the results (3.8) and (3.9) for θ(y,t) and C(y,t) remain in the same form but the expression (3.10) for u(y,t) reduces to the following analytical form:

$$u(y,t)=\frac{1}{2}\left\{(t+\sqrt{\frac{t}{M'}}\eta)\exp(2\eta\sqrt{M't})\operatorname{erfc}(\eta+\sqrt{M't})+(t-\sqrt{\frac{t}{M'}}\eta)\exp(-2\eta\sqrt{M't})\operatorname{erfc}(\eta-\sqrt{M't})\right\}$$
$$+A(y,Pr,t)+B(y,Sc,t)\,, \tag{4.2}$$

where A(y,Pr,t), B(y,Sc,t) are given from the equations (3.12), (3.13) respectively for various values of Pr and Sc.

Case (iii) *Motion of the plate with periodic acceleration* .

For this case, let $f(t)=H(t)\cos\omega t$

Then $\bar{f}(t)=\frac{s}{s^2+\omega^2}$

In this case also the expressions (3.8) and (3.9) remain again in the same form. Then, instead of the solution (3.10), we get the following analytical expression:

$$u(y,t)=\frac{1}{4}\exp(i\omega t)\left\{\exp(2\eta\sqrt{(M'+i\omega)t})\operatorname{erfc}(\eta+\sqrt{(M'+i\omega)t})+\exp(-2\eta\sqrt{(M'+i\omega)t})\operatorname{erfc}(\eta-\sqrt{(M'+i\omega)t}\right\}$$

$$+\frac{1}{4}\exp(-i\omega t)\left\{\exp(2\eta\sqrt{(M'-i\omega)t})\operatorname{erfc}(\eta+\sqrt{(M'-i\omega)t})+\exp(-2\eta\sqrt{(M'-i\omega)t})\operatorname{erfc}(\eta-\sqrt{(M'\text{-}i\omega)t})\right\}$$

$$+A(y,Pr,t)+B(y,Sc,t)\,, \tag{4.3}$$

Here also the term A(y,Pr,t) and B(y,Sc,t) are given from the equations (3.12) and (3.13) respectively for various values of Pr and Sc.

It should be noted that our results of case (ii) (cf. Equation (4.3)) are in close agreement with those of Chowdhury and Jain[4].

5.Skin-friction

Knowing the velocity field, we now study the effects of time, Pr, Sc, M, K etc. on the skin-friction.

It is given by

$$\tau' = -\mu\left(\frac{\partial u'}{\partial y'}\right)_{y'=0}$$

which by virtue of (2.6) reduces to

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

For Pr $\neq 1$, Sc $\neq$ 1,

$$\tau = -\left(\frac{\partial \Phi}{\partial y}\right)_{y=0} - \frac{Gr+Gm}{M'}\sqrt{M'}\,\mathrm{erf}\sqrt{M't} + \frac{Gr}{M'}\sqrt{\frac{M'Pr}{Pr-1}}\exp\left(\frac{M't}{Pr-1}\right)\left\{\mathrm{erf}\sqrt{\frac{M'Prt}{Pr-1}} - \mathrm{erf}\sqrt{\frac{M't}{Pr-1}}\right\}$$

$$+\frac{Gm}{M'}\sqrt{\frac{M'Sc}{Sc-1}}\exp\left(\frac{M't}{Sc-1}\right)\left\{\mathrm{erf}\sqrt{\frac{M'Sct}{Sc-1}} - \mathrm{erf}\sqrt{\frac{M't}{Sc-1}}\right\},$$

For Pr = Sc = 1,

$$\tau = -\left(\frac{\partial \Phi}{\partial y}\right)_{y=0} + \frac{1}{\sqrt{\pi t}}\exp(-M't) - \frac{Gr+Gm}{M'}\sqrt{M'}\,\mathrm{erf}\sqrt{M't}$$

When the plate is moving with uniform velocity, then

$$\left(\frac{\partial \Phi}{\partial y}\right)_{y=0} = -[\sqrt{M'}\,\mathrm{erf}\sqrt{M't} + \frac{1}{\sqrt{\pi t}}\exp(-M't)]$$

Again if the plate moves with single acceleration, then

$$\left(\frac{\partial \Phi}{\partial y}\right)_{y=0} = -[\sqrt{M'}\left(t + \frac{1}{2M'}\right)\mathrm{erf}\sqrt{M't} + \sqrt{\frac{t}{\pi}}\exp(-M't)]$$

Lastly when the plate is moving with periodic acceleration, then

$$\left(\frac{\partial \Phi}{\partial y}\right)_{y=0} = \frac{1}{2\sqrt{t}}\left\{\exp(i\omega t)\sqrt{(M'+i\omega)t}\,\mathrm{erfc}\sqrt{(M'+i\omega)t} + \exp(-i\omega t)\sqrt{(M'-i\omega)t}\,\mathrm{erfc}\sqrt{(M'-i\omega)t}\right\} + \frac{1}{\sqrt{\pi t}}\exp(-M't)$$

6.Nusselt number

An important phenomenon in this study is to understand the effects of t, Pr, on the Nusselt number. In non-dimensional form, the rate of heat transfer is given by

$$N_u = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \sqrt{\frac{Pr}{\pi t}}$$

7.Sherwood number

Another important physical quantities of interest is the Sherwood number which in non-dimensional form is

$$S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = \sqrt{\frac{Sc}{\pi t}}$$

We study the effects of t, Sc on Sherwood number numerically in the next section for better understanding.

8. Numerical discussions:

To understand the physical meaning of the problem, we have computed the expression for u, θ, C, τ, N_u and S_h for different values of Prandtl number Pr, magnetic field parameter M, Grashof number Gr, modified Grashof number Gm, Schmidt number Sc, permeability parameter K. The purpose of the numerical result given here is to assess the effects of different parameters upon the nature of the flow, temperature and concentration etc..

The values of the Prandtl number are chosen Pr = 7(water) and Pr = 0.71(air). The values of the Schmidt number are chosen to represent the presence of species by hydrogen(0.22), water vapor(0.60), ammonia(0.78) and carbon dioxide(0.96).

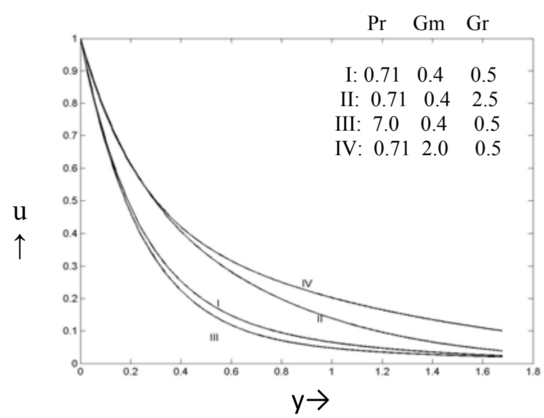

Fig.2 Velocity profile when the plate moves with the plate moves with uniform velocity. (taking t = 0.8, Sc = 0.2, M = 5, K = 1)

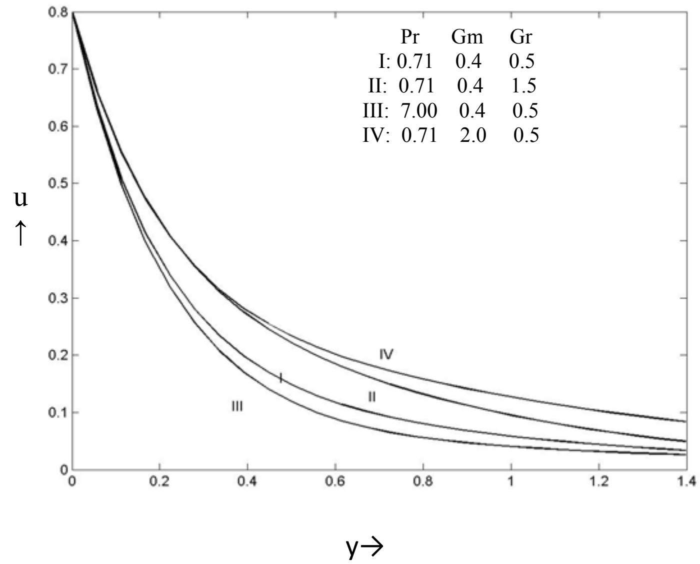

Fig.3 Velocity profile when the plate moves with single acceleration.
(taking t = 0.8, Sc = 0.2, M = 5, K=1)

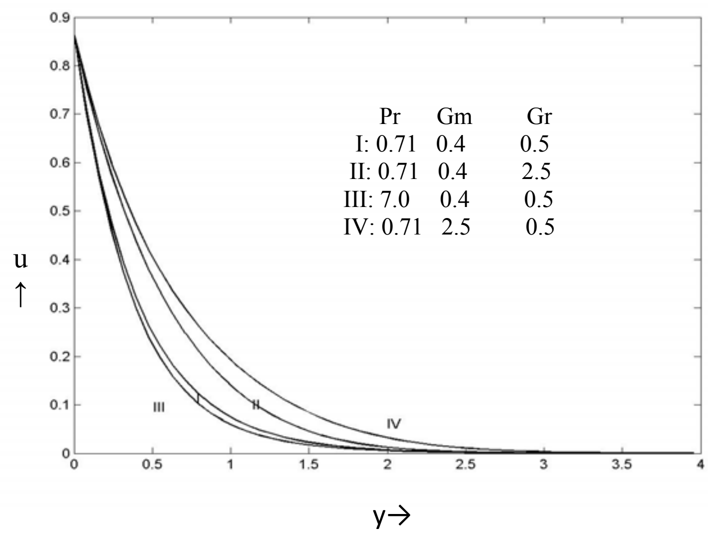

Fig.4 Velocity profile when the plate moves with periodic acceleration.
(taking t = 0.2, Sc = 0.4, M = 5, K = 1, $\omega t = \pi/6$)

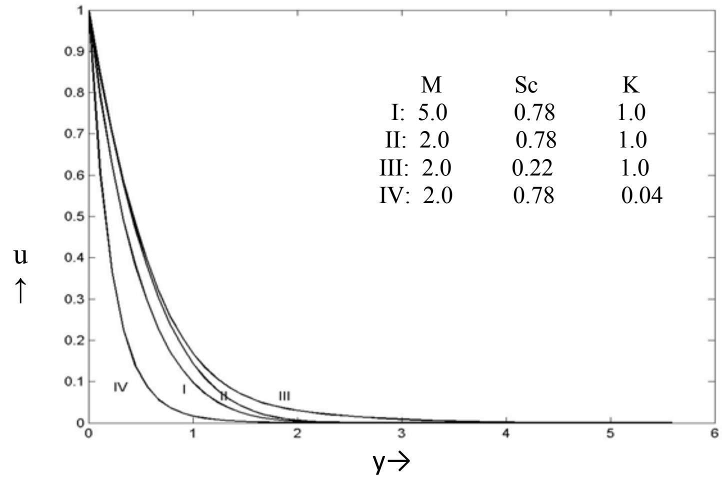

Fig.5 Velocity profile when the plate moves with uniform velocity.
(taking t = 0.2, Pr = 0.71, Gm = 0.4, Gr = 0.5)

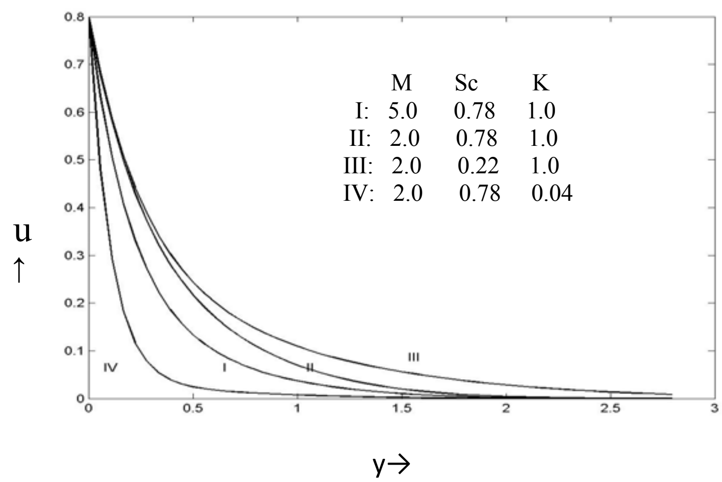

Fig.6 Vlocity profile when the plate moves with single acceleration.
(taking t = 0.2, Pr = 0.71, Gm = 0.4, Gr = 0.5)

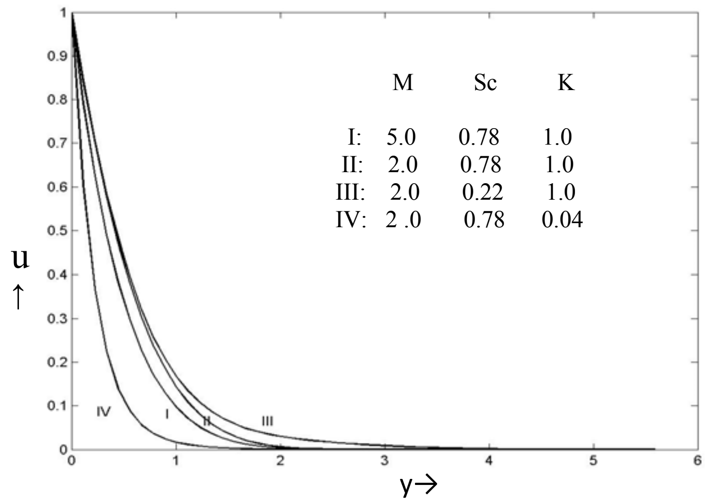

Fig.7 Vlocity profile when the plate moves with periodic acceleration.
(takingt = 0.2, Pr = 0.71, Gm = 0.4, Gr =0.5, ωt = $\pi/6$)

The velocity profiles are shown in figs.2 to 7 for water(Pr = 7) and air (Pr = 0.71). It is observed that the velocity decreases with increasing M, Pr but increases with increasing, Sc, Gr, K,Gm. Also it is seen from fig.8 that the velocity decreases with increasing phase angle (ωt) .

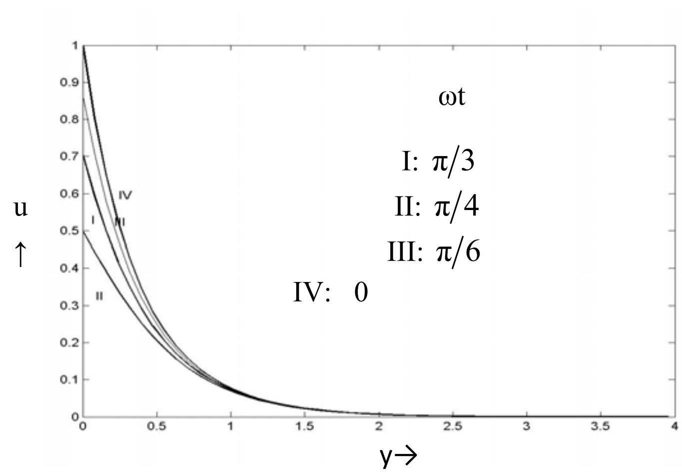

Fig.8 Velocity profile when the plate moves with periodic acceleration.
(taking t = 0.4, Pr = 0.71, Sc = 0.4, Gm = 0.4, Gr = 0.5, M = 5, K = 1, ωt = $\pi/6$)

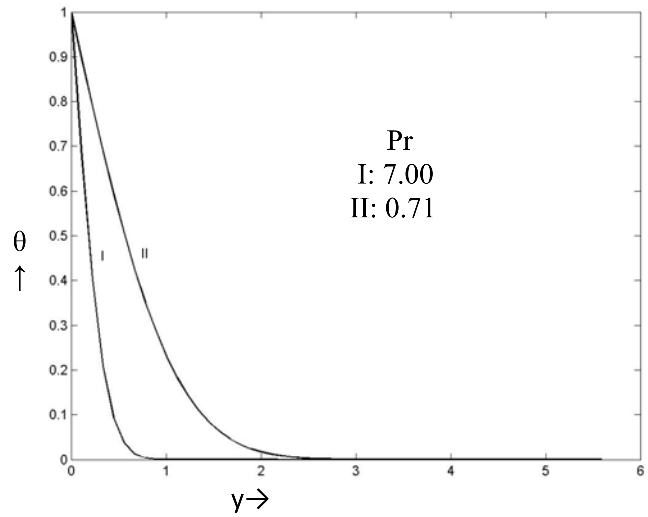

Fig.9 Temperature profile**.**

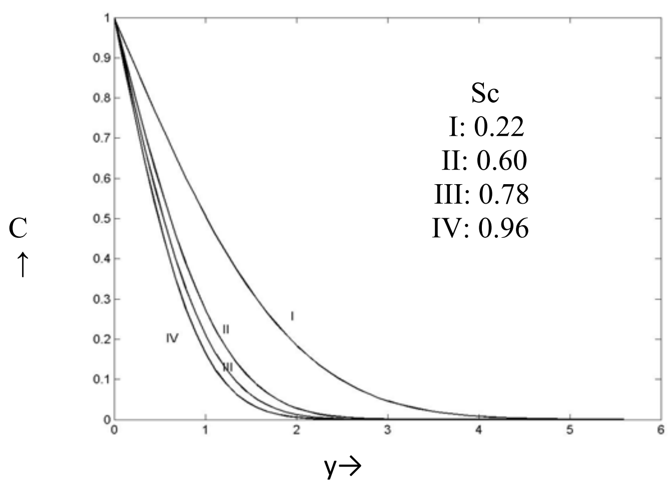

Fig.10 Concentration profile.

Fig.9 depicts the temperature profiles against n (distance from plate). We observe that the temperature for air is greater then that of water, which is due to the fact that thermal conductivity of fluid decreases with increasing Pr. For various values of Schmidt number(Sc), the concentration profiles are shown in fig.10. The numerical values of Sc are chosen such that they represent a reality in case of air. These values of Sc are 0.22(H_2), 0.60(H_2O) 0.78(NH_3) and 0.96 (CO_2). It is seen from figure that an increase in the Schmidt number leads to an increase in the concentration of air.

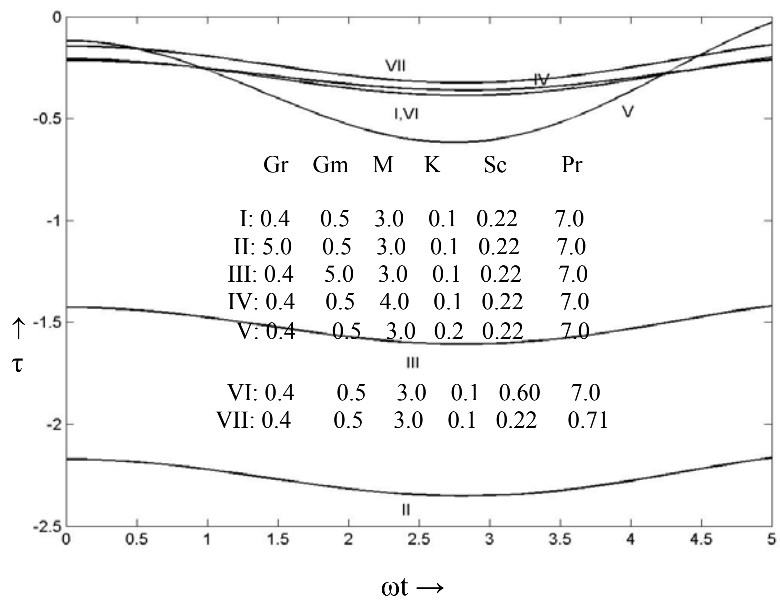

Fig.11 Skin-friction.

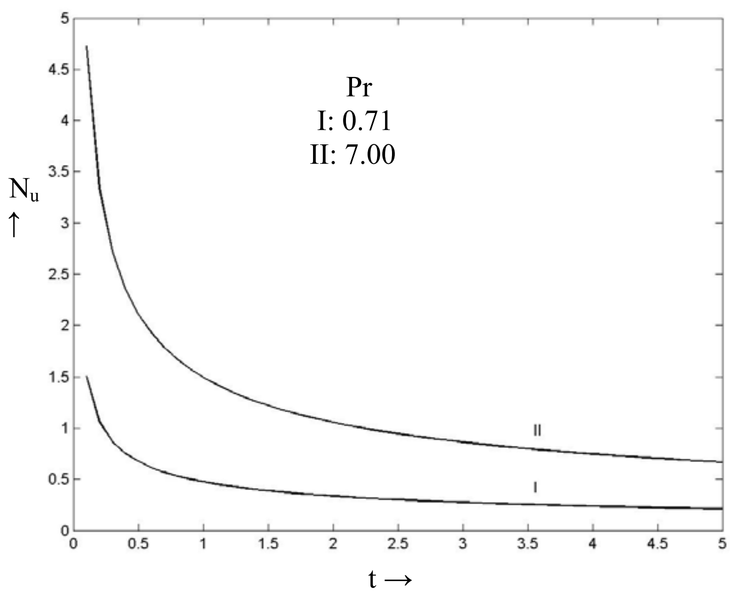

Fig.12 Nusselt number

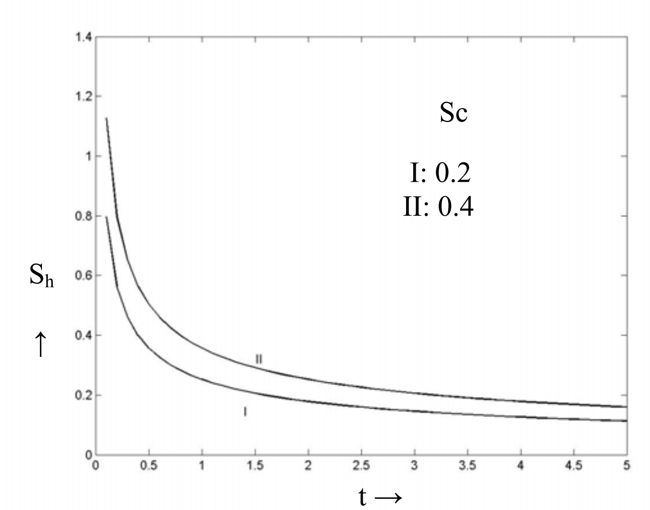

Fig.13 Sherwood number

Fig.11. depicts skin-friction against time t for different values of parameters. It is inferred from the figure, skin-friction decreases with increasing values of K, Gm and Gr but it increase with increasing Sc, M,Pr It is also observed that the magnitude of skin-friction for Pr = 0.71 is less than that of Pr = 7.Nusselt number is presented in fig.12 against time. It is observed that the Nusselt number decreases with time. Also Nusselt number for Pr = 7 is higher than that of Pr = 0.71. In fig.13, Sherwood number is presented against time t for different values of Schmidt numbers. We observed that Sherwood number decreases/ increases with increasing Schmidt numbers.

9. Conclusion:

In this study, a general analytical solution for the problem of unsteady MHD free convection flow with heat and mass transfer near a moving vertical plate has been determined. Some important application from the point of view of physical interest was discussed. Also we investigate some physical examples for evaluation of the numerical values of the velocity, temperature, concentration etc. of water (Pr = 7) and air (Pr = 0.71). To our knowledge, this study gives in close form the actual analytical solution of MHD free convection flow with heat and mass transfer problem which have wide application in different fields of Engineering.

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