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ARTERIAL MHD PULSATILE FLOW OF BLOOD UNDER PERIODIC BODY ACCELERATION

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Abstract: A mathematical model for pulsatile flow of blood through a stenosed porous medium with periodic body acceleration under the influence of a uniform transverse magnetic field has been developed by considering the blood to be a Newtonian and incompressible fluid. Using finite Hankel and Laplace transforms, analytical expressions for velocity profile, volumetric flow rate and wall shear stress have been obtained and their natures are portrayed graphically for different parameters such as Hartmann number, phase angle, time etc. It is observed that the velocity and maximum value of volumetric flow rate decreases with increase in Hartmann number and for a particular value of phase angle, the maximum value of wall shear stress increases with increase in Hartmann number but the effect is reverse for a fixed value of time. The present study seems to be useful in various field of biomedical engineering.

Keywords: Pulsatile blood flow, Body acceleration, Magnetic field, Hartmann number, Phase angle, Cardiovascular diseases, Stenosis.

2000 Mathematics subject classification: **76Z05**

INTRODUCTION

Under normal conditions, blood flow in the human circulatory system depends upon the pumping action of the heart and this produces a pressure gradient throughout the arterial and venous network. Also pressure gradient consists of two components, one of which is constant or non-fluctuating and the other is fluctuating or pulsatile. The investigations of blood flow through arteries are of considerable importance in many cardiovascular diseases particularly atherosclerosis. The normal pattern of blood flow is disturbed due to some abnormal growths like stenosis in the lumen of the artery. The actual reason for formation of stenosis is yet unknown, but its effect over the flow characteristics has been studied by many research workers. In some pathological situations, the distribution of fatty cholesterol and artery-clogging blood clod in the lumen of the coronary artery can be considered as equivalent to a fictitious porous medium (Dash *et.al* 1996). In daily life, humans often face some external body accelerations (or vibrations), such as traveling in high vehicles, aircrafts etc. Again in various sports it needs a high acceleration suddenly. These type of situations undoubtedly effects the normal flow of blood which may lead to many health problems like, headache, abdominal pain, vomiting tendency, loss of vision, abnormality in pulse rate etc. This could be due to dangerous combinations of body acceleration and pressure gradient of blood flow. It is, therefore, desirable to maintain such type of body accelerations to avoid these types of health hazards.

Many mathematical models have already been investigated by several research workers to explore the nature of blood flow under the influence of external acceleration. Flow under similar and other conditions has been analysed by Womersly [1]. Sometimes human being suffering from cardiogenic or anoxic shock may deliberately be subjected to whole body acceleration as a therapeutic measure as suggested by Arntzenius *et.al* [2] and Verdouw *et.al* [3]. Sud *et.al* [5] studied the characteristics of blood flow under body accelerations. Sud and Sekhon [6, 7, 8] considered various types of body accelerations and studied different characteristics of blood flow according to the nature of accelerations. Chaturani and Palanisamy [9, 10, 11] discussed the flow characteristics of blood under external body acceleration assuming blood to be a Newtonian fluid, Casson fluid and power law fluid respectively. Again Chaturani and Upadhya [12] studied the gravity flow of fluid with couple stress along an inclined plane with application to blood flow. Assuming blood to be a couple stress fluid, Sanyal *et.al* [13] investigated the effect of magnetic field on pulsatile motion of blood through an inclined circular tube with periodic body acceleration. Dash *et.al* [14] considered Casson fluid flow in a pipe filled with a homogeneous porous medium. Recently Moustafa [15] and Bhardwaj *et.al* [16] extended this problem to consider the pulsatile flow of blood through a stenosed porous medium under periodic body acceleration without considering magnetic effect.

Our object in the present paper is to study the effect of magnetic field on pulsatile flow of blood through a stenosed porous medium with periodic body acceleration. The analytical solutions for the velocity, volumetric flow rate and wall shear stress are obtained using finite Hankel and Laplace transforms and their natures are shown graphically for different values of involved parameters. Discussions drawn from the results may be important from medical points of view.

MATHEMATICAL FORMULATION

Let us consider the axially symmetric and fully developed pulsatile flow of blood through a stenosed porous circular artery with body acceleration under the influence of an external uniform transverse magnetic field. Blood is assumed to be Newtonian and incompressible fluid. Also for mathematical model, we take the artery to be a long cylindrical tube with the axis along the z-axis. The pressure gradient and body acceleration are respectively given by

$$-\frac{\partial P}{\partial z} = A_0 + A_1 \cos\left(\omega_P t\right) \tag{1}$$

$$G = a_0 \cos\left(\omega_b t + \phi\right) \tag{2}$$

where A_0 and A_1 are pressure gradient of steady flow and amplitude of oscillatory part respectively, a_0 is the amplitude of body acceleration, $\omega_P = 2\pi f_P$, $\omega_b = 2\pi f_b$ with f_P is the pulse frequency and f_b is body acceleration frequency, ϕ is the phase angle of body acceleration G with respect to the pressure gradient and t is time.

The governing equation of motion for flow in cylindrical polar coordinates is given by

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial z} + \rho G + \mu \nabla^2 u - \frac{\mu}{k} u - \sigma B_0 u \tag{3}$$

where u is the axial velocity of blood; P, blood pressure; $\frac{\partial P}{\partial z}$, pressure gradient; ρ, density of blood; μ, the viscosity of blood; k, the permeability of the isotropic porous medium; B_0, the external magnetic field along the radial direction and σ is the conductivity of the blood.

The geometry of the stenosis is shown in figure- 1.

$$R(z) = \begin{cases} a - \delta\left(1 + \cos\frac{\pi z}{2z_0}\right), & -2z_0 \le z \le 2z_0 \\ a, \text{ otherwise,} & \end{cases}$$

where $R(z)$ is the radius of stenosed artery, a is the radius of artery, $4z_0$ is the length of stenosis and 2δ is the maximum protuberance of the stenotic from of the artery wall.

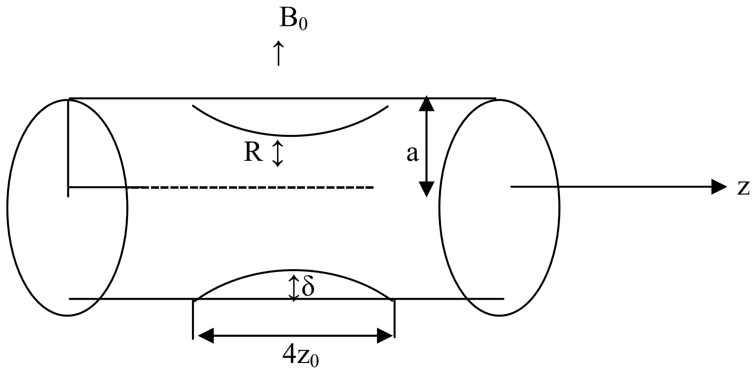

Fig. 1. Geometry of artery with stenosis.

Let us introduce a radial coordinate transformation by

$$\xi = \frac{r}{R(z)},$$

where $R(z)$ depends on δ.

Then equation (3) becomes

$$\rho\frac{\partial u}{\partial t} = A_0 + A_1\cos(\omega_P t) + \rho a_0 \cos(\omega_b t + \phi) + \frac{\mu}{R^2}\left[\frac{\partial^2 u}{\partial \xi^2} + \frac{1}{\xi}\frac{\partial u}{\partial \xi}\right] - \mu C^2 u \qquad (4)$$

where $C = \sqrt{\frac{1}{k} + \frac{M^2}{R^2}}$, $M = \sqrt{\frac{\sigma}{\mu}}.RB_0$ (Hartmann number).

We assumed that at $t < 0$ only the pumping action of the heart is present and at $t = 0$ the flow in the artery corresponds to the instantaneous pressure gradient i.e.,

$$-\frac{\partial P}{\partial z} = A_0 + A_1$$

As a result, the flow velocity at $t = 0$ is given by

$$u(\xi, 0) = \frac{A_0 + A_1}{\mu C^2}\left[1 - \frac{I_0(CR\xi)}{I_0(CR)}\right] \qquad (5)$$

where I_0 is modified Bessel function of first kind of order zero.

The initial and boundary conditions for the problem are

$$u(\xi,0)=\frac{A_0+A_1}{\mu C^2}\left[1-\frac{I_0(CR\xi)}{I_0(CR)}\right]$$
$$u=0 \ at\ \xi=1 \tag{6}$$
$$u \text{ is finite at } \xi=0$$

SOLUTIONS

Applying Laplace transform to equation (4) and first boundary condition of (6), we get

$$\rho s\bar{u}-\frac{\rho(A_0+A_1)}{\mu C^2}\left[1-\frac{I_0(CR\xi)}{I_0(CR)}\right]=\frac{A_0}{s}+\frac{A_1 s}{\left(s^2+\omega_p^2\right)}+\frac{\rho a_0\left(s\cos\phi-\omega_b\sin\phi\right)}{\left(s^2+\omega_b^2\right)}+\frac{\mu}{R^2}\left[\frac{\partial^2\bar{u}}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial\bar{u}}{\partial\xi}\right]-\mu C^2\bar{u}$$

where $\bar{u}(\xi,s)=\int_0^\infty e^{-st}u(\xi,t)\ dt\ \ (s>0)$ (7)

Then applying the finite Hankel transform to equation (7), we obtain

$$\bar{u}^*(\lambda_n,s)=\frac{J_1(\lambda_n)R^2}{\lambda_n\left[\rho sR^2+\mu\left(C^2R^2+\lambda_n^2\right)\right]}\left[\frac{A_0}{s}+\frac{A_1 s}{\left(s^2+\omega_p^2\right)}+\frac{\rho a_0\left(s\cos\phi-\omega_b\sin\phi\right)}{\left(s^2+\omega_b^2\right)}.\frac{\rho(A_0+A_1)R^2}{\mu\left(C^2R^2+\lambda_n^2\right)}\right] \tag{8}$$

where $\bar{u}^*(\lambda_n,s)=\int_0^1 r\ u(r,s)\ \mathrm{J}_0(r\lambda_n)\ dr$ and λ_n are zeros of J_0, Bessel function of first kind and $\nu=\frac{\mu}{\rho}$.

The Laplace and Hankel inversions of equation (8) give the final solution for blood velocity as

$$
\begin{aligned}
u(\xi,t) = 2\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\xi)}{\lambda_n J_1(\lambda_n)}\Bigg[\Bigg\{&\frac{A_0R^2}{\mu(\lambda_n^2+C^2R^2)}+\frac{A_1R^2\left[\nu(\lambda_n^2+C^2R^2)\cos\omega_pt+\omega_pR^2\sin\omega_pt\right]}{\rho\left[R^4\omega_p^2+\nu^2(\lambda_n^2+C^2R^2)^2\right]}\\
&+\frac{a_0R^2\left[\nu(\lambda_n^2+C^2R^2)\cos(\omega_bt+\phi)+\omega_bR^2\sin(\omega_bt+\phi)\right]}{R^4\omega_b^2+\nu^2(\lambda_n^2+C^2R^2)^2}\Bigg\}\\
&-e^{-\left(\frac{\nu}{R^2}\right)(\lambda_n^2+C^2R^2)t}\Bigg\{\frac{-A_1\omega_p^2R^6}{\mu(\lambda_n^2+C^2R^2)\left[R^4\omega_p^2+\nu^2(\lambda_n^2+C^2R^2)^2\right]}\\
&+\frac{a_0R^2\left[\nu(\lambda_n^2+C^2R^2)\cos\phi+\omega_bR^2\sin\phi\right]}{R^4\omega_b^2+\nu^2(\lambda_n^2+C^2R^2)^2}\Bigg\}\Bigg]
\end{aligned}
\tag{9}
$$

which can be written in the form

$$
\begin{aligned}
u(\xi,t)=\frac{2A_0R^2}{\mu}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\xi)}{\lambda_nJ_1(\lambda_n)}\Bigg[\Bigg\{&\frac{1}{(\lambda_n^2+C^2R^2)}+\frac{\varepsilon(\lambda_n^2+C^2R^2)\cos\omega_pt+\alpha^2\sin\omega_pt}{(\lambda_n^2+C^2R^2)^2+\alpha^4}\Bigg\}\\
&+\frac{\rho a_0}{A_0}\left\{\frac{(\lambda_n^2+C^2R^2)\cos(\omega_bt+\phi)+\beta^2\sin(\omega_bt+\phi)}{(\lambda_n^2+C^2R^2)^2+\beta^4}\right\}\\
&-e^{-\left(\frac{\nu}{R^2}\right)(\lambda_n^2+C^2R^2)t}\left\{\frac{-\varepsilon\alpha^4}{(\lambda_n^2+C^2R^2)\left[\alpha^4+(\lambda_n^2+C^2R^2)^2\right]}+\frac{\frac{\rho a_0}{A_0}\{(\lambda_n^2+C^2R^2)\cos\phi+\beta^2\sin\phi\}}{(\lambda_n^2+C^2R^2)^2+\beta^4}\right\}\Bigg]
\end{aligned}
\tag{10}
$$

where $\alpha^2=\dfrac{\omega_PR^2}{\nu}=\mathrm{Re}_P$, $\beta^2=\dfrac{\omega_bR^2}{\nu}=\mathrm{Re}_b$, $\varepsilon=\dfrac{A_1}{A_0}$

The analytical expression of u consists of four parts. The first and second parts correspond to steady and oscillatory parts of pressure gradient, the third term indicates body acceleration and the last term is the transient term. As $t\to\infty$, the transient term approaches to zero. Then from equation (10), we get

$$u(\xi,t)=\frac{2A_0R^2}{\mu}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\xi)}{\lambda_nJ_1(\lambda_n)}\left[\left\{\frac{1}{\left(\lambda_n^{\ 2}+C^2R^2\right)}+\frac{\varepsilon\left(\lambda_n^{\ 2}+C^2R^2\right)\cos\omega_Pt+\alpha^2\sin\omega_Pt}{\left(\lambda_n^{\ 2}+C^2R^2\right)^2+\alpha^4}\right\}\right.$$
$$\left.+\frac{\rho a_0}{A_0}\left\{\frac{\left(\lambda_n^{\ 2}+C^2R^2\right)\cos(\omega_bt+\phi)+\beta^2\sin(\omega_bt+\phi)}{\left(\lambda_n^{\ 2}+C^2R^2\right)^2+\beta^4}\right\}\right] \tag{11}$$

The volumetric flow rate Q is given by

$$Q(\xi,t)=2\pi\int_0^R ru\ dr$$
$$=\frac{4\pi A_0R^4}{\mu}\sum_{n=0}^{\infty}\frac{1}{\lambda_n^{\ 2}}\left[\left\{\frac{1}{\left(\lambda_n^{\ 2}+C^2R^2\right)}+\frac{\varepsilon\left(\lambda_n^{\ 2}+C^2R^2\right)\cos\omega_Pt+\alpha^2\sin\omega_Pt}{\left(\lambda_n^{\ 2}+C^2R^2\right)^2+\alpha^4}\right\}\right.$$
$$\left.+\frac{\rho a_0}{A_0}\left\{\frac{\left(\lambda_n^{\ 2}+C^2R^2\right)\cos(\omega_bt+\phi)+\beta^2\sin(\omega_bt+\phi)}{\left(\lambda_n^{\ 2}+C^2R^2\right)^2+\beta^4}\right\}\right] \tag{12}$$

The fluid acceleration F is given by

$$F(\xi,t)=\frac{\partial u}{\partial t}$$
$$=\frac{2A_0}{\rho}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\xi)}{\lambda_nJ_1(\lambda_n)}\left[\left\{\frac{\alpha^2\left\{-\varepsilon\left(\lambda_n^{\ 2}+C^2R^2\right)\sin\omega_Pt+\alpha^2\cos\omega_Pt\right\}}{\left(\lambda_n^{\ 2}+C^2R^2\right)^2+\alpha^4}\right\}\right.$$
$$\left.+\frac{\rho a_0\beta^2}{A_0}\left\{\frac{-\left(\lambda_n^{\ 2}+C^2R^2\right)\sin(\omega_bt+\phi)+\beta^2\cos(\omega_bt+\phi)}{\left(\lambda_n^{\ 2}+C^2R^2\right)^2+\beta^4}\right\}\right] \tag{13}$$

The expression for the wall shear stress τ_w can be obtained from

$$\tau_w=\mu\left(\frac{\partial u}{\partial r}\right)_{r=R}$$

as

$$\tau_w(\xi,t) = -2A_0R\sum_{n=1}^{\infty}\left[\left\{\frac{1}{\left(\lambda_n{}^2 + C^2R^2\right)} + \frac{\varepsilon\left(\lambda_n{}^2 + C^2R^2\right)\cos\omega_P t + \alpha^2\sin\omega_P t}{\left(\lambda_n{}^2 + C^2R^2\right)^2 + \alpha^4}\right\}\right.$$
$$\left. + \frac{\rho a_0}{A_0}\left\{\frac{\left(\lambda_n{}^2 + C^2R^2\right)\cos\left(\omega_b t + \phi\right) + \beta^2\sin\left(\omega_b t + \phi\right)}{\left(\lambda_n{}^2 + C^2R^2\right)^2 + \beta^4}\right\}\right] \quad (14)$$

NUMERICAL RESULTS

For a fixed steady state pressure gradient A_0, fixed values of k and non-zero values of a_0, the variations of the physiologically important fluid dynamic quantities, viz. velocity, volumetric flow rate, wall shear stress etc. are shown graphically in figures 2(a) to 9(c) for different values of Hartmann number (M), phase angle (ϕ), time (t) etc. For numerical calculations, we choose

$$f_P = 1.2,\ f_b = 1.2,\ A_1 = 0.2A_0,\ \omega_P = 2.4\pi$$

and the radius of different arteries are given below:

Blood vessels	Radius(cm)
Aorta	1.0
Femorat	0.5
Carotid	0.4
Coronary	0.15
Arteriole	0.008

The expression for velocity profile computed in equation (10) has been depicted in figures 2(a) to 5(c) by plotting $\frac{r}{R}$ versus u in presence/absence of Hartmann number (M), for different values of phase angle (ϕ) and time t. It is observed that velocity decreases with increasing Hartmann number (M) and is blunted near the axis of the artery and decreases rapidly with respect to $\frac{r}{R}$ i.e. with respect to r.

Again the position of maximum of axial velocity is dependent on tube diameter, i.e. the velocity increases as tube diameter increases from arteriole to aorta. Also it is seen that velocity of blood decreases with increase in phase angle (ϕ) for different arteries.

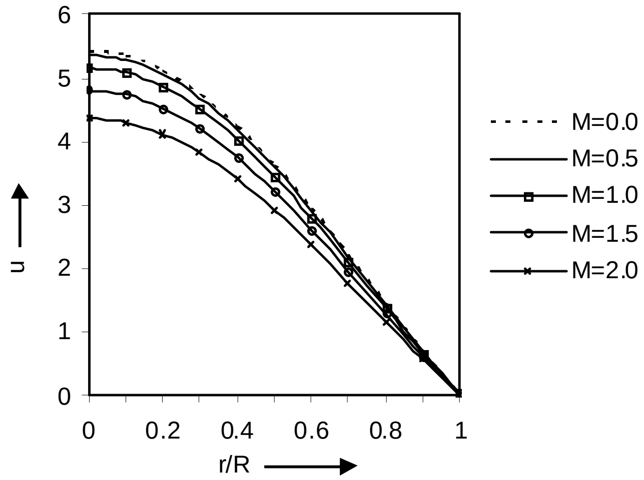

Fig. 2(a). Variation of velocity profiles for aorta artery against r/R with □=0.0, t=0.0.

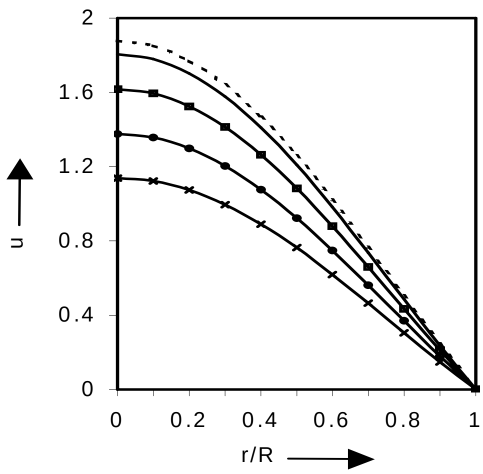

Fig. 2(b). Variation of velocity profiles for femorat artery against r/R with ϕ=0, t=0.

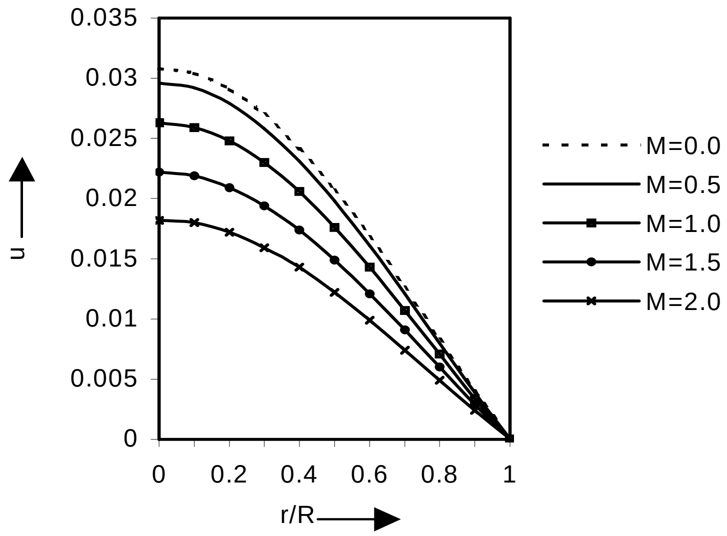

Fig. 2(c). Variation of velocity profiles for coronary artery against r/R with ϕ=0.0, t=0.0.

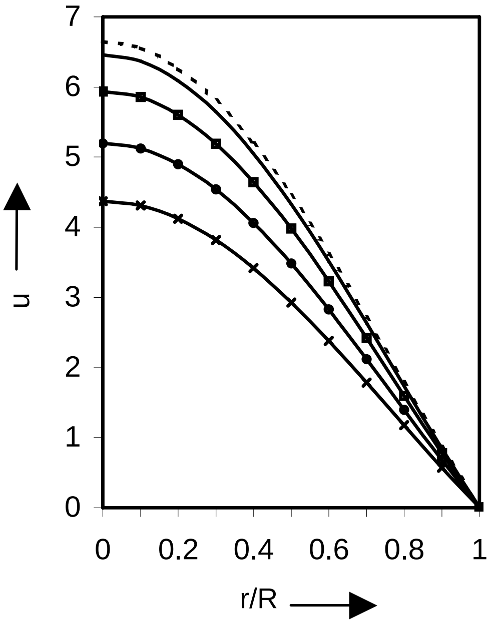

Fig. 3(a). Variation of velocity profiles for aorta artery against r/R with ϕ=0.0, t=45.

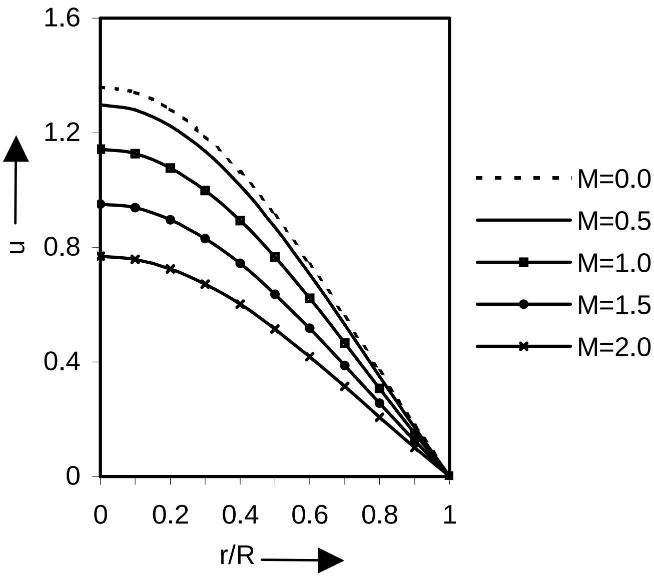

Fig. 3(b). Variation of velocity profiles for femorat artery against r/R with ϕ=0, t=45.

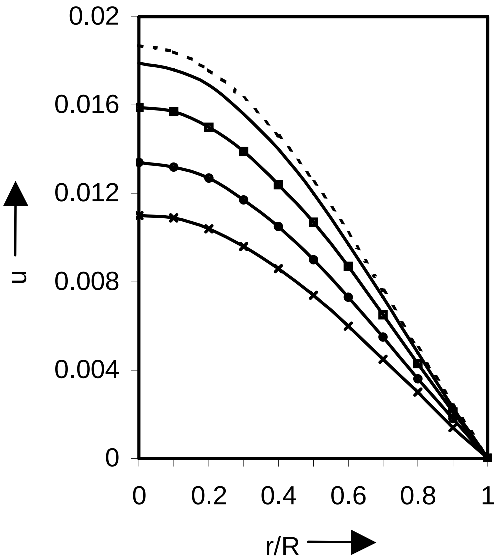

Fig. 3(c). Variation of velocity profiles for coronary artery against r/R with ϕ=0, t=45.

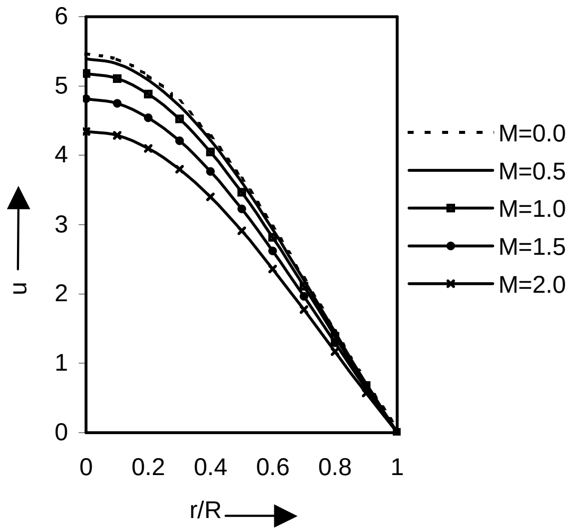

Fig. 4(a). Variation of velocity profiles for aorta artery against r/R with ϕ=45, t=0.0.

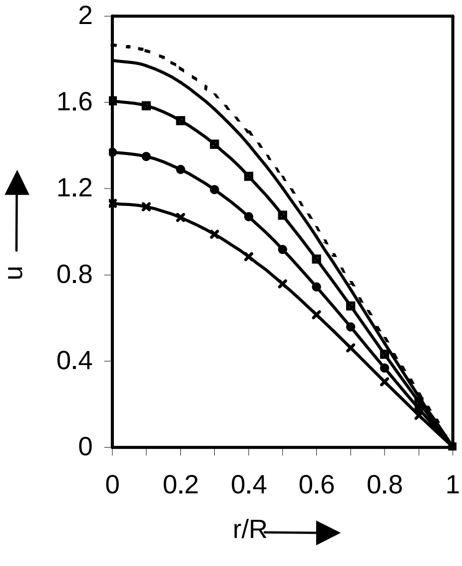

Fig. 4(b). Variation of velocity profiles for femorat artery against r/R with ϕ=45, t=0.0

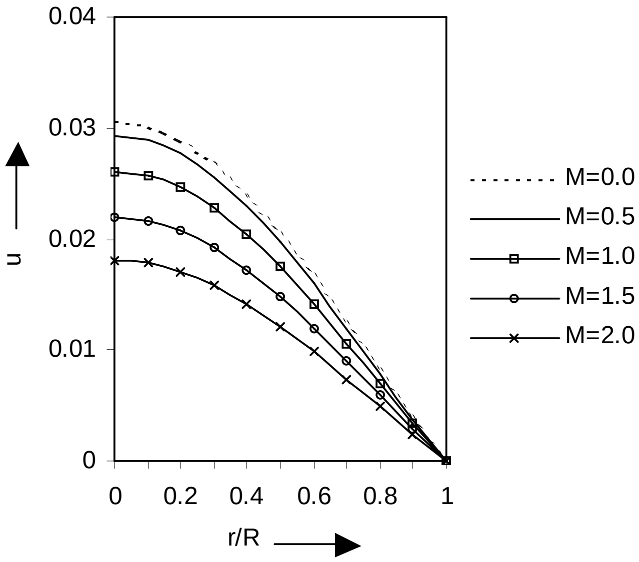

Fig. 4(c). Variation of velocity profiles for coronary artery against r/R with ϕ=45, t=0.0.

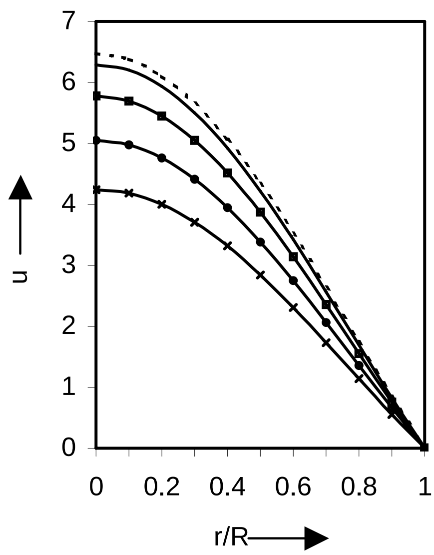

Fig. 5(a). Variation of velocity profiles for aorta artery against r/R with ϕ=90, t=45.

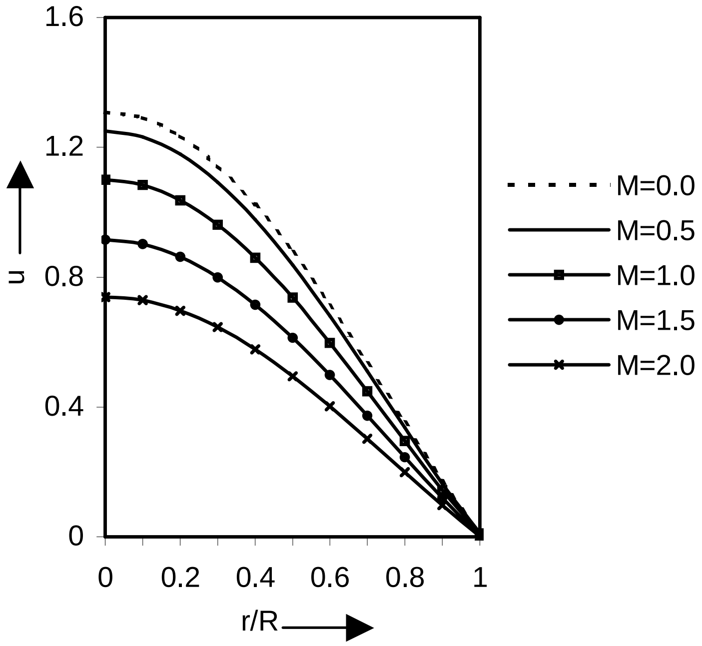

Fig. 5(b). Variation of velocity profiles for femorat artery against r/R with ϕ=90, t=45.

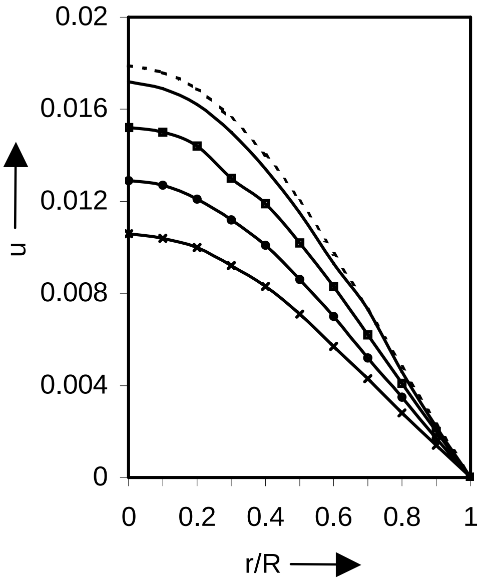

Fig. 5(c). Variation of velocity profiles for coronary artery against with ϕ=90, t=45.

Variation

of volumetric flow rate Q has been studied and shown in figures 6(a) to 6(c) for different values of Hartmann number (M). For fixed value of ϕ, it is observed that increase in M decreases the maximum value of the flow rate Q and the oscillatory nature of the curves with time is nearly same for different values of M. Figures 7(a) to 7(c) show that flow rate Q decreases with increase in the Hartmann number (M) at the

particular time for different values of phase angle and it decreases more rapidly with decreasing the radius of the artery.

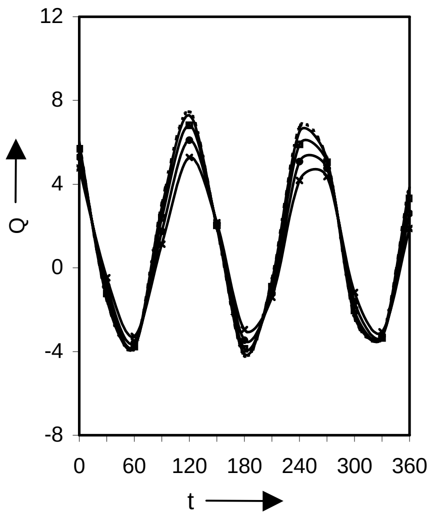

Fig. 6(a). Variation of flow rate for aorta artery against t when ϕ=45.

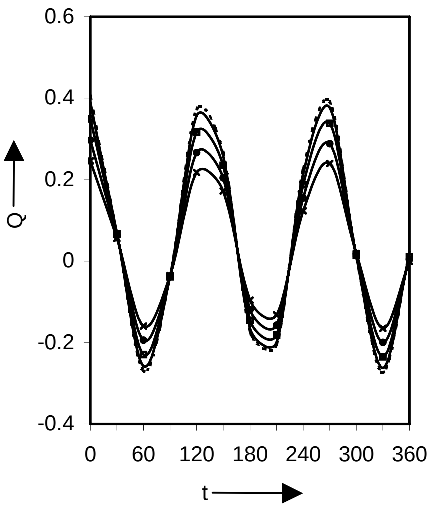

Fig. 6(b). Variation of flow rate for femorat artery against t when ϕ=45.

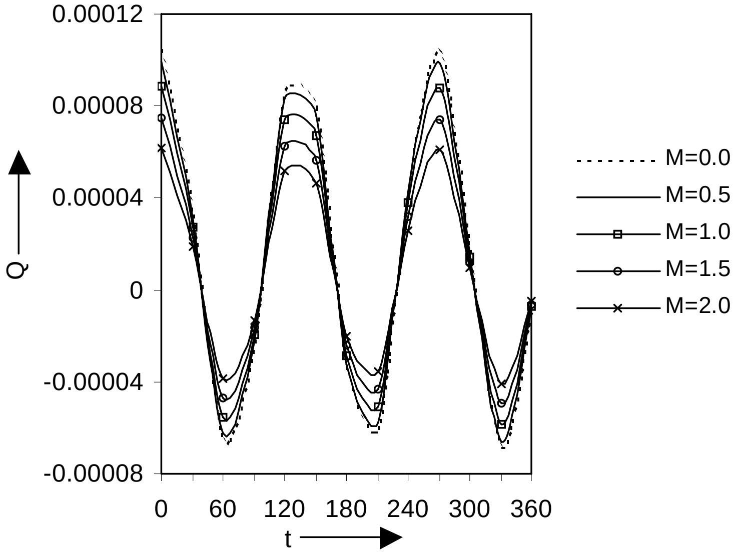

Fig. 6(c). Variation of flow rate for coronary artery against t when ϕ=45.

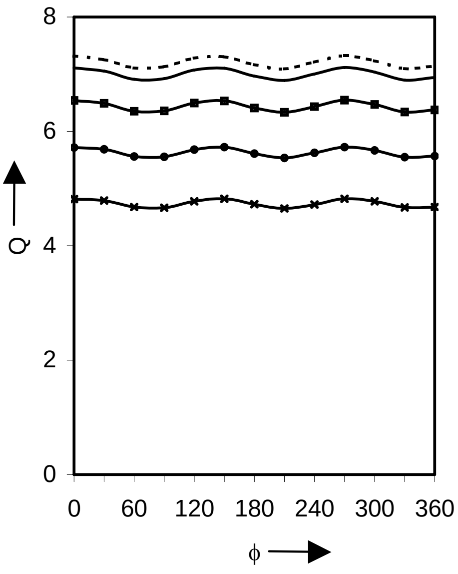

Fig. 7(a). Variation of flow rate for aorta artery against ϕ when t=45.

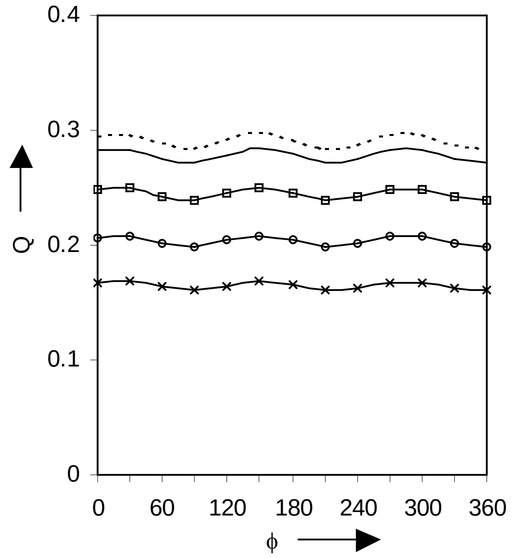

Fig. 7(b). Variation of flow rate for femorat artery against ϕ when t=45.

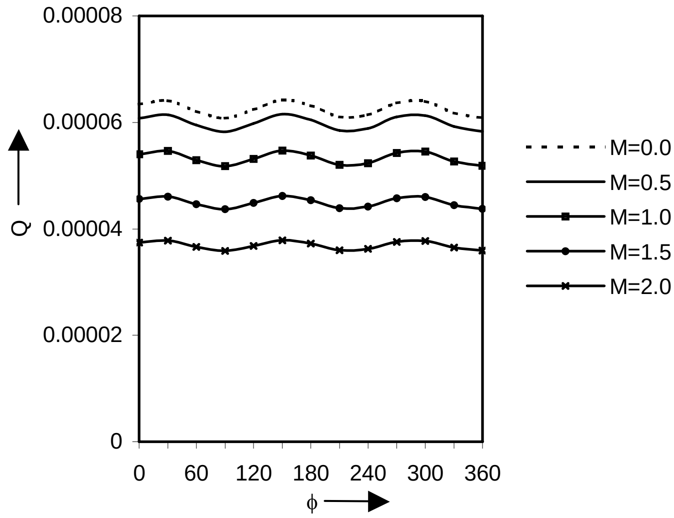

Fig. 7(c). Variation of flow rate for coronary artery against ϕ when t=45.

Figures 8(a) to 9(c) indicates the effect of Hartmann number on wall shear stress τ_w. For fixed value of ϕ, it is found from figures 8(a) to 8(c) that the maximum value of the wall shear stress decreases with increase in M whereas from 9(a) to 9(c), it is observed that for fixed value of t, the maximum value of τ_w increases with increase in M. In both cases, wall shear stress decreases with increasing the radius of the artery.

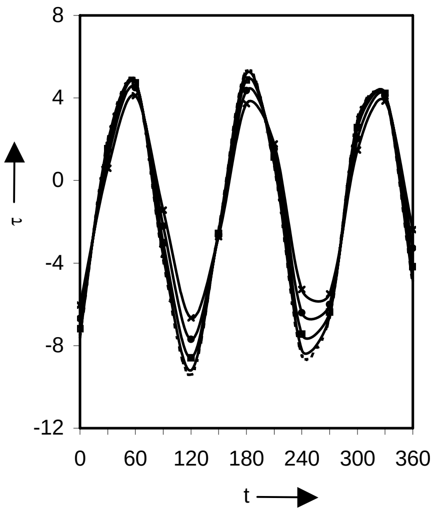

Fig. 8(a). Variation of wall shear stress for aorta artery against t when ϕ=45.

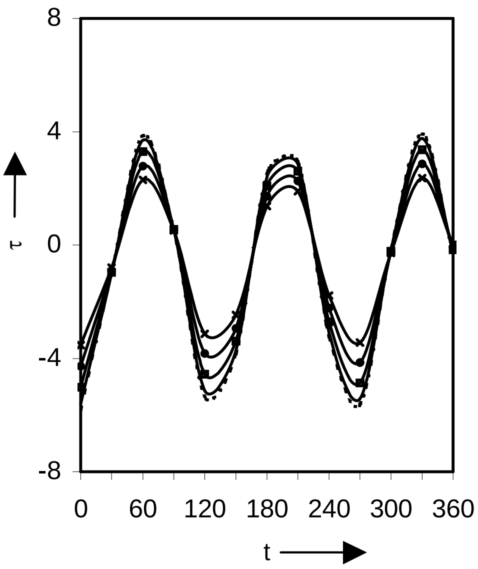

Fig. 8(b). Variation of wall shear stress for femorat artery against t when ϕ=45.

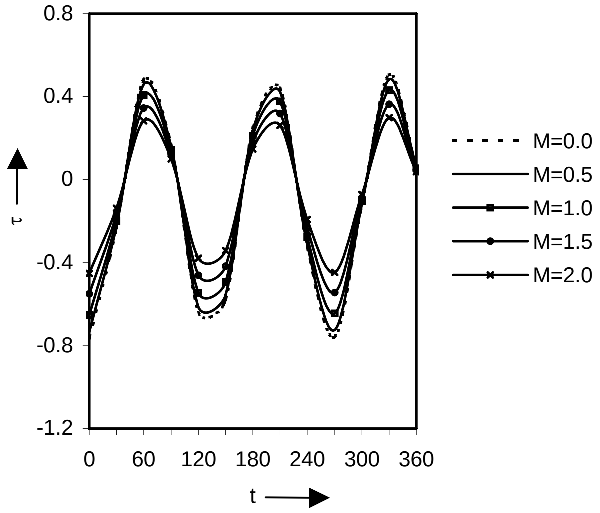

Fig. 8(c). Variation of wall shear stress for coronary artery against t when ϕ=45.

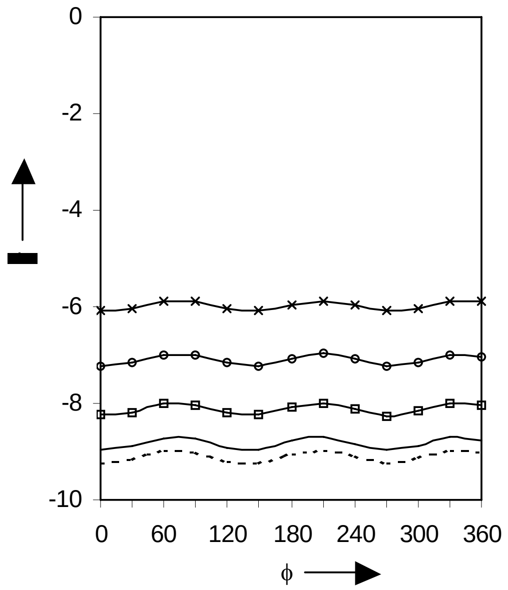

Fig. 9(a). variation of wall shear stress for aorta artery against ϕ when t=45.

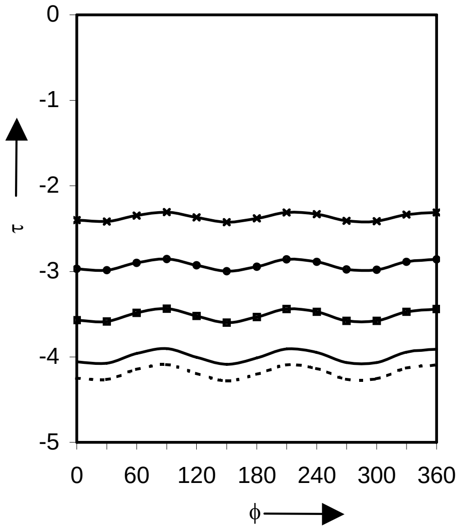

Fig. 9(b). Variation of wall shear stress for femorat artery against ϕ when t=45.

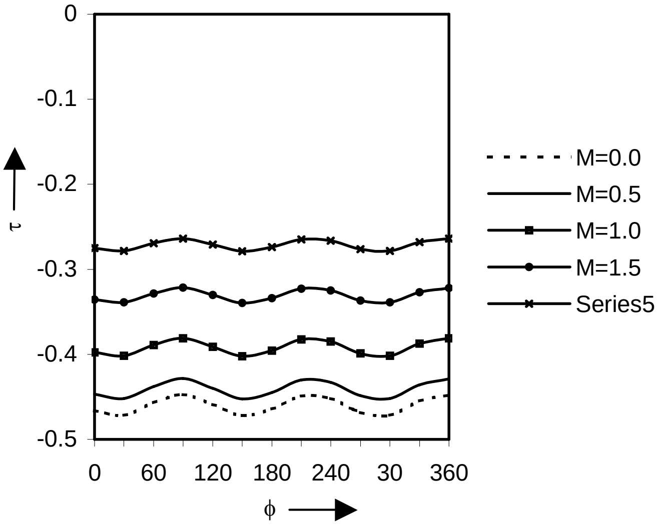

Fig. 9(c). Variation of wall shear stress for coronary artery against ϕ when t=45.

DIS CUSSIONS

In the present study, we considered the effect of transverse uniform magnetic field on pulsatile flow of blood through a stenosed porous medium with body acceleration. Using physiological data, the following observations have been made.

The velocity and maximum value of volumetric flow rate decreases with increase in Hartmann number (M) and for a particular value of phase angle (ϕ), the maximum value of wall shear stress (τ_w) increases with increase in M but the effect is reverse for a fixed value of t. A comparison of the present investigation with Bhardwaj and Kanodia[16] and previous investigations show that our results are more interesting due to the presence of magnetic field. Most of them neglect this magnetic effect on the flow variables like instantaneous volume flow rate, wall shear etc but due to presence of magnetic field these variables changes sharply in smaller arteries. It is clear from the previous numerical results that the transverse magnetic field effects largely on the axial flow velocity of blood. So, by taking appropriate values of magnetic field parameter (M) we may regulate the axial flow velocity.Thus, in case of magnetotherapy, by maintaining a proper magnetic field, the influence of magnetic instruments on blood flow velocity may be regulated.

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