Bull.Soc.Math. Banja Luka
ISSN 0354-5792 (p), 1986-521X (o)
Vol. 16 (2009), 43-51

SOME NOTES ON QUASI-ANTIORDERS AND COEQUALITY RELATIONS[1]

Daniel A. Romano[2] and Milovan Vinčić[3]

Abstract

It is known that each quasi-antiorder on anti-ordered set X induces coequality q on X such that X/q is an anti-ordered set. The converse of this statement also holds: Each coequality q on a set X such that X/q is anti-ordered set induces a quasi-antiorder on X. In this paper we give proofs that the families of all coequality relations q on X and the family of all quasi-antiorder relation on set X are complete lattices.

AMS Mathematics Subject Classification (2010): Primary 03F65, Secondary: 03E04

Key words and phrases: Constructive mathematics, set with apartness, coequality, anti-order and quasi-antiorder relations

1 Introduction and preliminary

This short investigation, in Bishop's constructive mathematics in the sense of well-known books [2], [4], [6] and [18] and Bogdanić, Romano and Vinčić's paper [3], Jojić and Romano's paper [6], and Romano's papers [7]-[16], is continuation of forthcoming the second author's papers [17]. Bishop's constructive mathematics is developed on Constructive Logic - logic without the Law of Excluded Middle $P \vee \neg P$. Let us note that in the Constructive Logic the 'Double Negation Law' $\neg\neg P \Longrightarrow P$ does not hold, but the following implication $P \Longrightarrow \neg\neg P$ does even in the Minimal Logic. Since the Constructive Logic is a part of the Classical Logic, these results, in the Constructive mathematics, are compatible with suitable results in the Classical mathematics. Let us recall that the following deduction principle $A \vee B, \neg B \vdash A$ is acceptable in the Constructive Logic.

Let $(X, =, \neq)$ be a set, where the relation $\neq$ is a binary relation on X, called *diversity* on X, which satisfies the following properties:

$$\neg(x \neq x), \quad x \neq y \Longrightarrow y \neq x, \quad x \neq y \wedge y = z \Longrightarrow x \neq z.$$

[1]This paper is partially supported by the Ministry of Science and Technology of the Republic of Srpska, Banja Luka, Bosnia and Herzegovina
[2]Faculty of Education, 76300 Bijeljina, 24 Sveti Sava Street, Bosnia and Herzegovina
[3]Faculty of Mechanical Engineering, 78000 Banja Luka, 75, Vojvoda Stepa Stepanović Street, Bosnia and Herzegovina

Following Heyting, if the following implication $x \neq z \Longrightarrow x \neq y \vee y \neq z$ holds, the diversity $\neq$ is called *apartness*. Let x be an element of X and A a subset of X. We write $x \bowtie A$ if and only if $(\forall a \in A)(x \neq a)$, and $A^C = \{x \in X : x \bowtie A\}$. In $X \times X$ the equality and diversity are defined by $(x, y) = (u, v) \Longleftrightarrow x = u \wedge y = v$, $(x, y) \neq (u, v) \Longleftrightarrow x \neq u \vee y \neq v$, and equality and diversity relations in power-set $\wp(X \times X)$ of $X \times X$ by

$$\alpha =_2 \beta \Longleftrightarrow (\forall (x, y) \in X \times X)((x, y) \in \alpha \Longleftrightarrow (x, y) \in \beta),$$

$$\alpha \neq_2 \beta \Longleftrightarrow$$

$$(\exists x, y \in X)((x, y) \in \alpha \wedge (x, y) \bowtie \beta) \vee (\exists x, y \in X)((x, y) \in \beta \wedge (x, y) \bowtie \alpha).$$

Let us note that the diversity relation $\neq_2$ is not an apartness relation in general case.

Example I: (1) The relation $\neg(=)$ is an apartness on the set $\mathbf{Z}$ of integers. (2) The relation q, defined on the set $\mathbf{Q}^{\mathbf{N}}$ by

$$(f, g) \in q \Longleftrightarrow (\exists k \in \mathbf{N})(\exists n \in \mathbf{N})(m \geq n \Longrightarrow |f(m) - g(m)| > k^{-1}),$$

is an apartness relation. ♦

A relation q on X is a coequality relation ([7]-[9]) on X if and only if it is consistent, symmetric and cotransitive:

$$q \subseteq \neq, \; q = q^{-1}, \; q \subseteq q * q,$$

where "$*$" is the operation of relations $\alpha \subseteq X \times X$ and $\beta \subseteq X \times X$, called filled product ([8], [9], [12]-[15]) of relations α and β, are relation on X defined by

$$(a, c) \in \beta * \alpha \Longleftrightarrow (\forall b \in X)((a, b) \in \alpha \vee (b, c) \in \beta).$$

For further study of coequality relation we suggest to read articles [8], [11], [13]-[16] (Specially, in articles [10], [12], [13] and [14], the author researches coequality relations compatible with the algebraic operations.) In article [7] and [8], problems of existence of compatible equality and coequality relations on set with apartness are discussed. In article [9], the author has proved the following: If e is an equivalence on set X, then there exists the maximal coequality relation q on X compatible with e in the following sense:

$$e \circ q \subseteq q \text{ and } q \circ e \subseteq q.$$

Opposite to the previous, if q is a coequality relation on set X, then the relation $q^C = \{(x, y) \in X \times X : (x, y) \bowtie q\}$ is an equivalence on X compatible with q ([8], [11]), and we can ([11]) construct the factor-set $X/(q^C, q) = \{aqC : a \in X\}$ with:

$$aq^C =_1 bq^C \Longleftrightarrow (a, b) \bowtie q, \;\; aq^C \neq_1 bq^C \Longleftrightarrow (a, b) \in q.$$

Also, we can ([8],[11]) construct the factor-set $X/q = \{aq : a \in X\}$: If q is a coequality relation on a set X, then X/q is a set with:

$$aq =_1 bq \Longleftrightarrow (a,b) \bowtie q, \;\; aq \neq_1 bq \Longleftrightarrow (a,b) \in q.$$

It is easily to check that $X/q \cong X/(q^C, q)$. besides, it is clear that the mapping $\pi : X \longrightarrow X/q$, defined by $\pi(x) = xq$, is a strongly extensional surjective function.

Subset $C(x) = \{y \in X : y \neq x\}$ satisfies the following implication:

$$y \in C(x) \,\wedge\, z \in X \Longrightarrow y \neq z \,\vee\, z \in C(x).$$

It is called a principal strongly extensional subset of X such that $x \bowtie C(x)$. Following this special case, for a subset A of X, we say that it is a strongly extensional subset of X if and only if the following implication

$$x \in A \wedge y \in X \Longrightarrow x \neq y \,\vee\, y \in A$$

holds.

Examples II: (1) ([7]) Let T be a set and J be a subfamily of $\wp(T)$ such that

$$\emptyset \in J, \; A \subseteq B \,\wedge\, B \in J \Longrightarrow A \in J, \; A \cap B \in J \Longrightarrow A \in J \,\vee\, B \in J.$$

If $(X_t)_{t\in T}$ is a family of sets, then the relation q on $\prod_{t\in T} X_t (\neq \emptyset)$, defined by $(f,g) \in q \Longleftrightarrow \{s \in T : (f(s) = g(s)\} \in J$, is a coequality relation on the Cartesian product $\prod_t X_t$.

(2) A ring R is a local ring if for each $r \in R$, either r or $1 - r$ is a unit, and let M be a module over R. The relation q on M, defined by $(x,y) \in q$ if there exists a homomorphism $f : M \longrightarrow R$ such that $f(x-y)$ is a unit, is a coequality relation on M.

(3) ([11]) Let T be a strongly extensional consistent subset of semigroup S, i.e. let $(\forall x, y \in S)(xy \in T \Longrightarrow x \in T \wedge y \in T)$ holds. Then, relation q on semigroup S, defined by $(a,b) \in q$ if and only if $a \neq b \,\wedge\, (a \in T \,\vee\, b \in T)$, is a coequality relation on S and compatible with semigroup operation in the following sense $(\forall x, y, a, b \in S)((xay, xby) \in q \Longrightarrow (a,b) \in q)$.

(4)Let $(R, =, \neq, +, 0, \cdot, 1)$ be a commutative ring. A subset Q of R is a coideal of R if and only if

$$0 \bowtie Q, \; -x \in Q \Longrightarrow x \in Q, \; x + y \in Q \Longrightarrow x \in Q \,\vee\, y \in Q,$$

$$xy \in Q \Longrightarrow x \in Q \,\wedge\, y \in Q.$$

Coideals of commutative ring with apartness were first defined and studied by Ruitenburg 1982 in his dissertation. After that, coideals (anti-ideals) are studied by A.S. Troelstra and D. van Dalen in their monograph [18]. This author proved, in 1988], if Q is a coideal of a ring R, then the relation q on R, defined by $(x,y) \in q \Longleftrightarrow x - y \in Q$, satisfies the following properties:

(a) q is a coequality relation on R;

(b) $(\forall x,y,u,v \in R)((x+u, y+v) \in q \Longrightarrow (x,y) \in q \vee (u,v) \in q)$;

(c) $(\forall x,y,u,v \in R)((xu, yv) \in q \Longrightarrow (x,y) \in q \vee (u,v) \in q)$.

A relation q on R, which satisfies the property (a)-(c), is called anticongruence on R ([4]) or coequality relation compatible with ring operations. If q is an anticongruence on a ring R, then the set $Q = \{x \in R : (x,0) \in q\}$ is a coideal of R. ♦

As in [12],[13], [14] and [15] a relation α on X is antiorder on X if and only if

$$\alpha \subseteq \neq,\ \alpha \subseteq \alpha * \alpha,\quad \neq \subseteq\ \alpha \cup \alpha^{-1} \text{ (linearity)}.$$

Let g be a strongly extensional mapping of anti-ordered set from $(X, =, \neq, \alpha)$ into $(Y, =, \neq, \beta)$. For g we say that it is:

(i) isotone if $(\forall a, b \in X)((a,b) \in \alpha \Longrightarrow (g(a), g(b)) \in \beta)$ holds;

(ii) reverse isotone if $(\forall a, b \in X)((g(a), g(b)) \in \beta \Longrightarrow (a,b) \in \alpha)$ holds.

A relation σ on X is a quasi-antiorder ([11]-[16]) on X if

$$\sigma\ \subseteq (\alpha\ \subseteq)\ \neq,\ \ \sigma\ \subseteq\ \sigma * \sigma.$$

It is clear that each coequality relation q on set X is a quasi-antiorder relation on X, and the apartness is a trivial anti-order relation on X. It is easy to check that if σ is a quasi-antiorder on X, then ([10]) the relation $q = \sigma \cup \sigma^{-1}$ is a coequality relation on X. The notion of quasi-antiorder is defined for first time in article [8], and the notion of anti-order relation is defined for the first time in article [10]. Those relations and their properties are investigated by Baroni in [1], Bogdanić, Jojić and Romano in [3], Jojić and Romano in [6], and van Plato in [19] also.

Examples III: Let a and b be elements of semigroup $(S, =, \neq, \cdot)$. Then ([11]), the set $C_{(a)} = \{x \in S : x \bowtie SaS\}$ is a consistent subset of S such that :
- $a \bowtie C_{(a)}$;
- $C_{(a)} \neq \emptyset \Longrightarrow 1 \in C_{(a)}$;
- Let a be an invertible element of S. Then $C_{(a)} = \emptyset$;
- $(\forall x, y \in S)(C_{(a)} \subseteq C_{(xay)})$;
- $C_{(a)} \cup C_{(b)}\ \subseteq\ C_{(ab)}$.

Let a be an arbitrary element of a semigroup S with apartness. The consistent subset $C_{(a)}$ is called a principal consistent subset of S generated by a. We introduce relation f, defined by $(a,b) \in f \Longleftrightarrow b \in C_{(a)}$. The relation f has the following properties ([11, Theorem 7]):

- f is a consistent relation ;

- $(a, b) \in f \Longrightarrow (\forall x, y \in S)((xay, b) \in f)$;

- $(a, b) \in f \Longrightarrow (\forall n \in \mathbf{N})((a^n, b) \in f)$;

- $(\forall x, y \in S)((a, xby) \in f \Longrightarrow (a, b) \in f)$;

- $(\forall x, y \in S)((a, xay) \in f)$.

We can construct the cotransitive relation $c(f) = \bigcap {}^n f$ as cotransitive fulfillment of the relation f ([8]-[11],[15]). As consequences of these assertions we have the following results. The relation $c(f)$ satisfies the following properties:

- $c(f)$ is a quasi-antiorder on S ;

- $(\forall x, y \in S)((a, xay) \bowtie c(f))$;

- $(\forall n \in \mathbf{N})((a, a^n) \bowtie c(f))$;

- $(\forall x, y \in S)((a, b) \in c(f) \Longrightarrow (xay, b) \in c(f))$;

- $(\forall n \in \mathbf{N})((a, b) \in c(f) \Longrightarrow (an, b) \in c(f))$;

- $(\forall x, y \in S)((a, xby) \in c(f) \Longrightarrow (a, b) \in c(f))$. ♦

For a given anti-ordered set $(X, =, \neq, \alpha)$ is essential to know if there exists a coequality relation q on X such that X/q is an anti-ordered set. This plays an important role for studying the structure of anti-ordered sets. The following question is natural: If $(X, =, \neq, \alpha)$ is an anti-ordered set and q a coequality relation on X, is the factor-set X/q anti-ordered set? Naturally, anti-order on X/q should be the relation Θ on X/q defined by means of the anti-order α on X such that $\Theta = \{(xq, yq) \in X/q : (x, y) \in \alpha\}$, but it is not held in general case. The following question appears: Is there coequality relation q on X for which X/q is an anti-ordered set such that the natural mapping $\pi : X \longrightarrow X/q$ is reverse isotone? The concept of quasi-antiorder relation was introduced by this author in his papers [8] and [9]-[16] (Particularly, in articles [10] and [14], the author investigated anti-ordered algebraic systems with apartness.). According to Lemma 0 in [12], if $(X, =, \neq)$ is a set and σ is a quasi-antiorder on X, then ([12, Lemma 1]) the relation q on X, defined by $q = \sigma \cup \sigma^{-1}$, is a coequality relation on X, and the set X/q is an anti-ordered set under anti-order Θ defined by $(xq, yq) \in \Theta \Longleftrightarrow (x, y) \in \sigma$. So, according to results in [12] and [13], each quasi-antiorder σ on an ordered set X under anti-order α induces an coequality relation $q =_2 \sigma \cup \sigma^{-1}$ on X such that X/q is an anti-ordered set under Θ . (For a further study of quasi-antiorders on anti-ordered set we refer to papers [12], [13] and forthcoming the author's paper [17].) In paper [14] we proved that the converse of this statement also holds. If $(X, =, \neq, \alpha)$ is an anti-ordered set and q coequality relation on X, and if there exists an order relation Θ_1 on X/q such that the $(X/q, =_1, \neq_1, \Theta_1)$ is an anti-ordered and the mapping $\pi : X \longrightarrow X/q$ is reverse isotone (so-called regular coequality), then there exists a quasi-antiorder σ on X such that $q =_2 \sigma \cup \sigma^{-1}$. So, each regular coequality q on a set $(X, =, \neq, \alpha)$

induces a quasi-antiorder on X. Besides, connections between the family of all quasi-antiorders on X, the family of coequality relations on X, and the family of all regular coequality relations q on X are given.

Lemma 1.1 *Let τ be a quasi-antiorder on set X. Then $x\tau$ (τx) is a strongly extensional subset of X, such that $x \bowtie x\tau$ $(x \bowtie \tau x)$, for each $x \in X$. Besides, the following implication $(x,z) \in \tau \Longrightarrow x\tau \cup \tau z = X$ holds for each x, z of X.*

Proof: From $\tau \subseteq \neq$ it follows $x \bowtie x\tau$. Let $yx \in \tau holds$, and let z be an arbitrary element of X. Thus, $(x,y) \in \tau$ and $(x,z) \in \tau \vee (z,y) \in \tau$. So, we have $z \in x\tau \vee y \neq z$. Therefore, $x\tau$ is a strongly extensional subset of X such that $x \bowtie x\tau$.
The proof that τx is a strongly extensional subset of X such that $x \bowtie \tau x$ is analogous. Besides, the following implication $(x,z) \in \tau \Longrightarrow x\tau \cup \tau z = X$ holds for each x, y of X. Indeed, if $(x,z) \in \tau$ and y is an arbitrary element of X, then $(\forall y \in X)((x,y) \in \tau \vee (y,z) \in \tau)$. Thus, $X = x\tau \cup \tau z$. □

Let τ be a quasi-antiorder on set X. Then for every pair (x,z) of τ there exists a pair (A_x, B_z) of strongly extensional subsets of X such that $x \bowtie A_x \wedge z \bowtie B_z$ and $X = A_x \cup B_z$ and $x \in B_z \wedge z \in A_x$.

Example IV: If A is a strongly extensional subset of X, then the relation σ on X, defined by $(x,y) \in \sigma \Longleftrightarrow x \in A \wedge x \neq y$, is a quasi-antiorder relation on X.

Proof: It is clear that σ is a consistent relation on X. Assume $(x,z) \in \sigma$ and let y be an arbitrary element of X. Then, $x \in A \wedge x \neq z$. Thus, $x \neq y \vee y \neq z$. If $x \neq y$ and $x \in A$, then $(x,y) \in \sigma$. If $y \neq z$ and $x \in A$, by strongly extensionality of A, we have $y \neq z$ and $x \in A$ and $x \neq y \vee y \in A$. In the case of $y \neq z \wedge x \in A \wedge x \neq y$ we have again $(x,y) \in \sigma$; in the case of $y \neq z$ and $x \in A$ and $y \in A$ we have $(y,z) \in \sigma$. So, the relation σ is a cotransitive relation. Therefore, relation σ is a quasi-antiorder relation on X. Further on, we have:

$$x \in A \Longrightarrow x\sigma = C(x), \;\; \neg(x \in A) \Longrightarrow x\sigma = \emptyset;$$

$$y \in A \Longrightarrow \sigma y = C(y) \cap A, \;\; y \bowtie A \Longrightarrow \sigma y = A. \;\blacklozenge$$

2 Main Results

In the following proposition we give a connection between the family $\Im(X)$ of all quasi-antiorders on set X and the family $\mathbf{q}(X)$ of all coequality relation on X

For a set $(X, =, \neq, \alpha)$ by $\Re(X, \alpha)$ we denote the family of all regular coequality relations q on X with respect to α, and by $\Im(X, \alpha)$ denotes the family of all quasi-antiorder relation on X included in α .

Let us note that families $\Im(X)$, $\Im(X, \alpha)$ and $\mathbf{q}(X)$ are complete lattices. Indeed, in the following two theorems we give proofs for those facts:

Theorem 2.1 *If $\{\tau_k\}_{k\in J}$ is a family of quasi-antiorders on a set $(X,=,\neq)$, then $\bigcup_{k\in J}\tau_k$ and $c(\bigcap_{k\in J}\tau_k)$ are quasi-antiorders in X. So, the families $\Im(X)$ and $\Im(X,\alpha)$ are complete lattices.*

Proof: (1) Let $\{\tau_k\}_{k\in J}$ is a family of quasi-antiorders on a set $(X,=,\neq)$ and let x, z be an arbitrary elements of X such that $(x,z)\in\bigcup_{k\in J}\tau_k$. Then, there exists k in J such that $(x,z)\in\tau_k$. Hence, for every $y\in X$ we have $(x,y)\in\tau_k \vee (y,z)\in\tau_k$. So, $(x,y)\in\bigcup_{k\in J}\tau_k \vee (y,z)\in\bigcup_{k\in J}\tau_k$. On the other hand, for every k in J holds $\tau_k\subseteq\neq$. From this we have $\bigcup_{k\in J}\tau_k\subseteq\neq$. So, we can put $sup\{\tau_k : k\in J\}=\bigcup_{k\in J}\tau_k$.

(2) Let $R\,(\subseteq\neq)$ be a relation on a set $(X,=,\neq)$. Then for an inhabited family of quasi-antiorders under R there exists the biggest quasi-antiorder relation under R. That relation is exactly the relation $c(R)$. In fact:
By (1), there exists the biggest quasi-antiorder relation on X under R. Let Q_R be the inhabited family of all quasi-antiorder relation on X under R. With (R) we denote the biggest quasi-antiorder relation $\bigcup Q_R$ on X under R. On the other hand, the fulfillment $c(R)=\bigcap_{n\in N}{}^nR$ of the relation R is a cotransitive relation on set X under R. Therefore, $c(R)\subseteq(R)$ holds.
We need to show that $(R)\subseteq c(R)$. Let $\tau\,(\subseteq (R)=\bigcup Q_R)$ be a quasi-antiorder relation in X under R. Firstly, we have $\tau\subseteq R={}^1R$. Assume $(x,z)\in\tau$. Then, out of $(\forall y\in X)((x,y)\in\tau \vee (y,z)\in\tau)$ we conclude that for every y in X holds $(x,y)\in R\vee(y,z)\in R$, i.e. holds $(x,z)\in R*R={}^2R$. So, we have $\tau\subseteq{}^2R$. Now, we will suppose that $\tau\subseteq{}^nR$, and suppose that $(x,z)\in\tau$. Then, $(\forall y\in X)((x,y)\in\tau\vee(y,z)\in\tau)$ implies that $(x,y)\in R\vee(y,z)\in{}^nR$ holds for every $y\in X$. Therefore, $(x,z)\in{}^{n+1}R$. So, we have $\tau\subseteq{}^{n+1}R$. Thus, by induction, we have $\tau\subseteq\bigcap{}^nR$. let us remember that τ is an arbitrary quasi-antiorder on X under R. Hence, we proved that $(R)=\bigcup Q_R\subseteq c(R)$. If $\{\tau_k\}_{k\in J}$ is a family of quasi-antiorders on a set $(X,=,\neq)$, then $c(\bigcap_{k\in J}\tau_k)$ is a quasi-antiorder in X, and we can set $inf\{\tau_k : k\in J\}=c(\bigcap_{k\in J}\tau_k)$. □

Theorem 2.2 *Let $(X,=,\neq)$ be a set with apartness. The family $\boldsymbol{q}(X)$ is a complete lattice.*

Proof: If $\{q_k : k\in\Lambda\}$ is a family of coequality relations on X, then $\bigcup q_k$ and $c(\bigcap q_k)$ are coequality relations on X such that $(\forall k\in\Lambda)(q_k\subseteq\bigcup q_k)$ and $(\forall k\in\Lambda)(c(\bigcap q_k)\subseteq q_k)$. Since $\bigcup q_k$ is the minimal extension of every q_k we can put $sup\{q_k : k\in\Lambda\}=\bigcup q_k$, and since $c(\bigcap q_k)$ is the maximal coequality relation under $\bigcap q_k(\subseteq q_k)$ we can set $inf\{q_k : k\in\Lambda\}=c(\bigcap q_k)$. □

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Received by the editors on October 13, 2009,
and revised version on December 21, 2009