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A NOTE ON THE GENERATION OF PYTHAGOREAN TRIPLES

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Abstract. We give a new method (up to our knowledge) for generating some primitive pythagorean triples. Our method is based on the construction of a new primitive pythagore-an triple from a predetermined primitive pythagorean triple. We generalize this idea and we construct a new pythagorean triple from a predetermined pythagorean n-tuple.

1. Introduction

A pythagorean triple is a triple of positive integers (a, b, c) such that

$$a^2 + b^2 = c^2.$$

Obviously, if k is a positive integer and (a, b, c) is a pythagorean triple, then (ka, kb, kc) is a pythagorean triple also. A pythagorean triple is called primitive, if the greatest common divisor of integers a, b, c is equal to 1. It is known, from ancient times, that there are infinitely many pythagorean triples.

Euclid ([**6**]) gave formula for generating all primitive pythagorean triples. The formula states that integers

$$a = m^2 - n^2,\ b = 2mn,\ c = m^2 + n^2 \tag{Eq:1}$$

form a primitive pythagorean triple, if and only if m and n are relatively prime positive integers with conditions $m > n$, where m and n are opposite parities.

The Euclid formula has more symmetric variant [**6**]:

$$a = mn,\ b = \frac{m^2 - n^2}{2},\ c = \frac{m^2 + n^2}{2}; \tag{Eq:2}$$

where m and n are odd, relatively prime positive integers such that $m > n$.

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I want to thank my professor Duško Jojić. I am grateful to my beloved aunt Grozda Stanić.

There are three important families [**6**] of pythagorean triples: Plato's family, Pythagora's family and Fermat's family.

There are many methods that generate infinitely many primitive pythagorean triples but not necessarily all of them ([**6, 7**]). Some important methods are the Stifel method and the Ozanam method.

It is important to note that, in 1934, Berggren [**2**] showed that all primitive pythagorean triples may be produced by using matrix multiplication and three Berggren's matrices. The same matrices were rediscovered later by Barning [**1**] and Hall [**3**]. For more information on this topic, we refer readers to make insight in [**5**] and [**4, 7**].

In this paper we give a new method (up to our knowledge) for generating some primitive pythagorean triples. Our method is based on the construction of a new primitive pythagorean triple from a predetermined primitive pythagorean triple. The first main result of this paper is the following theorem.

THEOREM 1.1. *Let $(a,\, b,\, c)$ be a primitive pythagorean triple. Then*

$$(ab,\, (a+b)c,\, c^2+ab)$$

is a primitive pythagorean triple also.

As a useful consequence of the previous theorem, we have the following corollary.

COROLLARY 1.1. *Let $(a,\, b,\, c)$ be a primitive pythagorean triple. Then*

$$(ab, |a-b|c, c^2-ab)$$

is a primitive pythagorean triple, too.

Our second main result in this article requires a new notion as a generalization of the term 'pythagorean triple' given in the following definition.

DEFINITION 1.1. A pythagorean n-tuple is an n-tuple of n positive integers $(a_1, a_2, \ldots, a_n)$ such that $\sum_{k=1}^{n-1} {a_k}^2 = {a_n}^2$ where n is a natural number greater than 2.

In the following theorem, as our second main result of this paper, generalizing our idea exposed in Theorem 1.1 we show that we can construct a new pythagorean triple from a predetermined pythagorean n-tuple.

THEOREM 1.2. *Let $(a_1, a_2, \ldots, a_n)$ be a pythagorean n-tuple. Let denote*

$$A_{n-1} = \sum_{1\leqslant i<j\leqslant n-1} a_i \cdot a_j \quad \textit{and} \quad B_{n-1} = \sum_{k=1}^{n-1} a_k.$$

Then

$$(A_{n-1},\, B_{n-1}\cdot a_n,\, a_n^2 + A_{n-1})$$

is a pythagorean triple.

2. The Proof of the Theorem 1.1

PROOF. Firstly, we need to show that if (a, b, c) is a pythagorean triple, then $(ab, (a+b)c, c^2+ab)$ is a pythagorean triple. Because (a, b, c) is a pythagorean triple, we know that

$$a^2 + b^2 = c^2 \tag{2.1}$$

Then we have

$$\begin{aligned}(ab)^2 + ((a+b)c)^2 &= a^2b^2 + (a+b)^2c^2 \\ &= a^2b^2 + (a^2 + 2ab + b^2)c^2 \\ &= a^2b^2 + (2ab + c^2)c^2 \qquad \text{(by the equation (2.1))} \\ &= (ab)^2 + 2(ab)(c^2) + (c^2)^2 \\ &= (ab + c^2)^2.\end{aligned}$$

Hence, we proved that

$$(ab)^2 + ((a+b)c)^2 = (c^2 + ab)^2. \tag{2.2}$$

Secondly, we need to show that $(ab, (a+b)c, c^2+ab)$ is a primitive triple, if (a, b, c) is a primitive triple. Let p be a prime number such that p divides both ab and $c^2 + ab$. Then p must divide their difference which is c^2. Since p is a prime, p must divide c. Similarly, if p divides ab, then p must divide a or b.

There are two cases.

Case 1. If p divides both a and c, then it follows, from the equation (2.1), that p must divide b. This is a contradiction, because $(a,\ b,\ c)$ is a primitive triple.

Case 2. If p divides both b and c, then p must divide a by the equation (2.1). Again a contradiction, because $(a,\ b,\ c)$ is a primitive triple.

We conclude that the greatest common divisor of numbers ab and c^2+ab must be equal to 1. Therefore, we proved that $(ab, (a+b)c, c^2+ab)$ is a primitive triple.

This completes the proof of the Theorem 1.1. □

The proof of the Corollary 1.1 is very similar to the proof of the Theorem 1.1.

3. The Method

As an immediate consequence of the Theorem 1.1, we have the following corollary:

COROLLARY 3.1. *There are infinitely many primitive pythagorean triples.*

PROOF. Let (a, b, c) be a primitive pythagorean triple. Let us define a map φ, as follows:

$$\varphi((a, b, c)) = (ab, (a+b)c, c^2 + ab). \tag{3.1}$$

By the Theorem 1.1, we know that $\varphi((a, b, c))$ is a primitive pythagorean triple. Obviously, because (a, b, c) is a triple of positive integers, it must be $c^2 + ab > c$.

From the function $\varphi^2((a, b, c)) = \varphi(ab, (a+b)c, c^2+ab)$, we obtain another pythagorean triple $(abc(a+b), (ab+(a+b)c)(c^2+ab), (c^2+ab)^2+abc(a+b))$.

Using the functions φ^2, φ^3, ... , we obtain a sequence of primitive pythagorean triples whose third coordinate is increasing.

So, we conclude if (a, b, c) is a primitive pythagorean triple, then there are infinitely many primitive pythagorean triples.

Of course, we know that one primitive pythagorean triple is (3,4,5). Hence, there are infinitely many primitive pythagorean triples. This completes the proof of the Corollary 3.1. □

EXAMPLE 3.1. If we start from the triple $(3, 4, 5)$, by using the function φ, we obtain following triples:

$$\begin{aligned} \varphi((3,4,5)) &= (12,35,37) \\ \varphi^2((3,4,5)) &= (420,1739,1789) \\ \varphi^3((3,4,5)) &= (730380,3862451,3930901) \end{aligned}$$

REMARK 3.1. We can consider the function ϑ, as follows:

$$\vartheta((a,b,c)) = (ab, |a-b|c, c^2-ab). \tag{3.2}$$

Let (a, b, c) be a primitive pythagorean triple. By the Corollary 1.1, we know that $\vartheta((a, b, c))$ is a primitive pythagorean triple. From inequalities $c > a$ and $c-1 \geqslant b$, we obtain that $c(c-1) > ab$. The last inequality is equivalent to $c^2 - ab > c$.

Using the functions ϑ^2, ϑ^3, ... , we obtain a sequence of primitive pythagorean triples whose third coordinate is increasing.

So, the function ϑ gives another proof of the Corollary 3.1.

EXAMPLE 3.2. If we start from the triple $(3, 4, 5)$, by using the function ϑ, we obtain following triples:

$$\begin{aligned} \vartheta((3,4,5)) &= (12,5,13) \\ \vartheta^2((3,4,5)) &= (60,91,109) \\ \vartheta^3((3,4,5)) &= (5460,3379,6421) \end{aligned}$$

4. The Proof of the Theorem 1.2

PROOF. After square the number B_{n-1} and use the equality $\sum_{k=1}^{n-1} a_k^2 = a_n^2$, we obtain

$$\begin{aligned} B_{n-1}^2 &= \sum_{k=1}^{n-1} a_k^2 + 2A_{n-1} \\ &= a_n^2 + 2A_{n-1}. \end{aligned}$$

So, we get

$$B_{n-1}^2 = a_n^2 + 2A_{n-1}. \tag{4.1}$$

Then we have

$$\begin{aligned}
A_{n-1}^2 + (B_{n-1} \cdot a_n)^2 &= A_{n-1}^2 + B_{n-1}^2 \cdot a_n^2 \\
&= A_{n-1}^2 + (a_n^2 + 2A_{n-1}) \cdot a_n^2 \qquad \text{(by the equation (4.1))} \\
&= A_{n-1}^2 + (a_n^2)^2 + 2A_{n-1} \cdot a_n^2 \\
&= (A_{n-1} + a_n^2)^2.
\end{aligned}$$

From the last equation above, we conclude that

$$A_{n-1}^2 + (B_{n-1} \cdot a_n)^2 = (A_{n-1} + a_n^2)^2. \tag{4.2}$$

The equation (4.2) proves the Theorem 1.2. □

5. Conclusions

In this part of our article, we will show two new ways of constructing primitive pythagorean triples from a predetermined pythagorean triple using the φ (see the proof the Corollary 3.1) and ϑ (Remark 3.1) functions. Both algorithms are expressed as consequences of previous claims.

Let (X, Y, Z) be a primitive pythagorean triple where X is even and Y, Z are odd natural numbers.

Corollary 5.1. *A primitive pythagorean triple (X, Y, Z) can be generated by the function φ if and only if $Z - 3X$ is a square of an integer.*

Corollary 5.2. *A primitive pythagorean triple (X, Y, Z) can be generated by the function ϑ if and only if $Z + 3X$ is a square of an integer.*

We give a proof of the Corollary 5.1. The proof of the Corollary 5.2 is similar to the proof of the Corollary 5.1.

Proof. Let a, b, and c be positive integers such that $\varphi((a, b, c)) = (X, Y, Z)$. Let us find a, b and c. We have three equations with three unknown variables a, b and c (we treat X, Y and Z as known variables).

$$ab = X \tag{5.1}$$

$$(a + b)c = Y \tag{5.2}$$

$$c^2 + ab = Z. \tag{5.3}$$

From equations (5.1) and (5.3) we obtain that $c^2 = Z - X$ and thus

$$c = \sqrt{Z - X}. \tag{5.4}$$

This is correct, because $(Z - X) \cdot (Z + X) = Y^2$ and the numbers $Z - X$, $Z + X$ are relatively prime numbers and both odd. Then, from equations (5.2) and (5.4), we have

$$a + b = \sqrt{Z + X}. \tag{5.5}$$

On the other hand, we have

$$\begin{aligned}(a-b)^2 &= (a+b)^2 - 4ab \\ &= (Z+X) - 4X \qquad \text{(by equations (5.1) and (5.5))} \\ &= Z - 3X.\end{aligned}$$

We obtain that

$$(a-b)^2 = Z - 3X \tag{5.6}$$

So, if a primitive pythagorean triple (X, Y, Z) can be generated by the function φ, then $Z - 3X$ must be a square of an integer.

Now, let $Z - 3X$ be a square of an integer. By the equation (5.6), we obtain that

$$|a-b| = \sqrt{Z-3X}. \tag{5.7}$$

Now, the system of equations (5.5) and (5.7) must have a solution in natural numbers, because numbers $\sqrt{Z+X}$ and $\sqrt{Z-3X}$ have same parity (they are both odd).

It is easy to show that:

$$a = \frac{\sqrt{Z+X} + \sqrt{Z-3X}}{2}$$
$$b = \frac{\sqrt{Z+X} - \sqrt{Z-3X}}{2}$$

or symmetrically

$$b = \frac{\sqrt{Z+X} + \sqrt{Z-3X}}{2}$$
$$a = \frac{\sqrt{Z+X} - \sqrt{Z-3X}}{2}$$

It is readily verified that $\varphi((a, b, c)) = \varphi((b, a, c)) = (X, Y, Z)$, and that (a, b, c) forms a pythagorean triple. Clearly, a and b have opposite parity.

Moreover, since (X, Y, Z) is a primitive pythagorean triple, it follows, from the definition of φ, that (a, b, c) also is a primitive pythagorean triple.

This proves the Corollary 5.1. □

From the proof of Theorem 1.1 follows

Corollary 5.3. *Let a, b, and c be complex numbers, with condition $c \neq 0$. Then $a^2 + b^2 = c^2$ if and only if $(ab)^2 + ((a+b)c)^2 = (c^2 + ab)^2$.*

6. An Application of Corollaries 5.1 and 5.2

In this section, we use Corollary 5.1 and Corollary 5.2 in order to describe all primitive pythagorean triple which can be generated by our functions φ and ϑ.

We need help of the following lemma:

LEMMA 6.1. *Let m,n, and t be odd, relatively prime natural numbers; with condition $m > n$. Then all solutions of a diophantine equation $m^2 + n^2 = 2t^2$ are:*

$$m = \frac{u^2 + 2uv - v^2}{2} \tag{6.1}$$

$$n = \frac{|u^2 - 2uv - v^2|}{2} \tag{6.2}$$

$$t = \frac{u^2 + v^2}{2}; \tag{6.3}$$

where u and v are odd, relatively prime natural numbers such that $u > v$.

As a direct consequence of the Corollary 5.1, we have the following lemma:

LEMMA 6.2. *Let (X, Y, Z) be a primitive pythagorean triple, and let X be even. Let m and n be odd, relatively prime natural numbers from the Euclid formula (Eq:2) $(m > n)$; which determine X, Y, and Z. Then triple (X, Y, Z) can be generated from the function φ, iff there exists an odd natural number t such that*

$$2n^2 = m^2 + t^2 \tag{6.4}$$

PROOF. Let us assume that a triple (X, Y, Z) can be generated from the function φ.

By the Corollary 5.1, we conclude that there exists an odd natural number t such that $Z - 3X = t^2$. If we use the Euclid formula (Eq:2), then the above formula gradually becomes, as follows:

$$\begin{aligned} Z - 3X &= t^2 \\ \frac{m^2 + n^2}{2} - 3 \cdot \frac{m^2 - n^2}{2} &= t^2 \\ \frac{(m^2 + n^2) - 3(m^2 - n^2)}{2} &= t^2 \\ \frac{4n^2 - 2m^2}{2} &= t^2 \\ 2n^2 - m^2 &= t^2 \end{aligned}$$

The last equation above proves the formula (6.4).

Clearly, if the equation (6.4) holds, then it must be $Z - 3X = t^2$. This completes the proof of this lemma. □

Similarly, as a direct consequence of the Corollary 5.2, we have the following lemma:

LEMMA 6.3. *Let (X, Y, Z) be a primitive pythagorean triple, and let X be even. Let m and n be odd, relatively prime natural numbers from the Euclid formula (Eq:2) $(m > n)$; which determine X, Y, and Z. Then triple (X, Y, Z) can be generated from the function ϑ, iff there exists an odd natural number t such that*

$$2m^2 = n^2 + t^2 \tag{6.5}$$

The proof of the Lemma 6.3 is similar to the proof of the Lemma 6.2.

Now, we can present main results of this section.

Proposition 6.1. *Let (X, Y, Z) be a primitive pythagorean triple, and let X be even. Then a triple (X, Y, Z) can be generated from the function φ iff there exist odd, relatively prime natural numbers u and v $(u > v)$, such that:*

$$X = \frac{uv(u^2 - v^2)}{2} \tag{6.6}$$

$$Y = \frac{(u^2 + 2uv - v^2)(u^2 + v^2)}{4} \tag{6.7}$$

$$Z = \frac{(u^2 + 2uv - v^2)^2 + (u^2 + v^2)^2}{8} \tag{6.8}$$

Proof. By the Euclid formula (Eq:2), we know that there exist odd, relatively prime natural numbers m and n $(m > n)$, such that

$$X = \frac{m^2 - n^2}{2} \tag{6.9}$$

$$Y = mn \tag{6.10}$$

$$Z = \frac{m^2 + n^2}{2}. \tag{6.11}$$

By the Lemma 6.2, it follows that the equation (6.4) must hold. In other words, we have $2n^2 = m^2 + t^2$. Since $m > n$, from the Equation (6.4), it follows that $n > t$. So, it must be $m > t$.

By the Lemma 6.1, we easily obtain that

$$n = \frac{u^2 + v^2}{2} \qquad \text{(by the Eq. (6.3))} \tag{6.12}$$

$$m = \frac{u^2 + 2uv - v^2}{2} \qquad \text{(by the Eq. (6.1));} \tag{6.13}$$

where $u > v$.

Now, Eqns. (6.12), (6.13), (6.9), (6.10), and (6.11) prove Eqns. (6.6), (6.7), and (6.8). This completes the proof of this proposition. □

The other main result is the following proposition:

Proposition 6.2. *Let (X, Y, Z) be a primitive pythagorean triple, and let X be even. Then a triple (X, Y, Z) can be generated from the function ϑ iff there exist odd, relatively prime natural numbers u and v $(u > v)$, such that:*

$$X = \frac{uv(u^2 - v^2)}{2} \tag{6.14}$$

$$Y = \frac{|u^2 - 2uv - v^2| \cdot (u^2 + v^2)}{4} \tag{6.15}$$

$$Z = \frac{(u^2 - 2uv - v^2)^2 + (u^2 + v^2)^2}{8} \tag{6.16}$$

The proof of the Proposition 6.2 is similar to the proof of the Proposition 6.1.

7. Connection with Plato's family

In this section, we will investigate the connection between our method and Plato's family of primitive pythagorean triples.

PROPOSITION 7.1. *There are no primitive pythagorean triples (X, Y, Z) from Plato's family (X is even) which can be generated from the function φ.*

PROOF. Because a triple (X, Y, Z) belongs to the Plato's family, it follows that $n = 1$ in formulas (6.9), (6.10) and (6.11).

On the other hand, a triple (X, Y, Z) can be generated from the function φ. By the formula (6.12), it follows that

$$\frac{u^2 + v^2}{2} = 1. \tag{7.1}$$

Because $u > v$, it must be $\frac{u^2+v^2}{2} > v^2 \geqslant 1$. So, $\frac{u^2+v^2}{2} > 1$. We got a contradiction. This proves this proposition. □

PROPOSITION 7.2. *Let $(\frac{m^2-1}{2}, m, \frac{m^2+1}{2})$ be a primitive pythagorean triple from Plato's family; where m is odd natural number. This triple can be generated from the function ϑ, iff there exist natural numbers r and s, so that a triple (r, s, m) belongs to the Fermat family.*

PROOF. Let us assume that this triple can be generated from the function ϑ. By the Lemma 6.3, it follows:

$$\begin{aligned} n^2 + t^2 &= 2m^2 \\ 1 + t^2 &= 2m^2 \qquad\qquad (\text{ because n} = 1) \\ \frac{1+t^2}{2} &= m^2 \\ (\frac{t+1}{2})^2 + (\frac{t-1}{2})^2 &= m^2. \end{aligned}$$

We conclude that a triple $(\frac{t-1}{2}, \frac{t+1}{2}, m)$ belongs to the Fermat family.

Clearly, the reverse direction holds. This completes the proof of this proposition. □

So, there are infinitely many Plato's primitive pythagorean triples which can be generated from the function ϑ.

8. Connection with Pythagora's family

In this section, we will investigate the connection between our method and Pythagora's family of primitive pythagorean triples.

PROPOSITION 8.1. *The only Pythagora's primitive triple which can be generated from the function φ is a triple $(12, 35, 37)$.*

Proof. From the Example 3.1, we know that $\varphi((3,4,5)) = (12,35,37)$.

Let us prove that there is no other Pythagora's triple which can be generated from the function φ.

Let (X,Y,Z) be a primitive Pythagora's triple (X is even) which can be generated from the function φ.

Because (X,Y,Z) is a primitive Pythagora's triple, it follows that, in formulas (6.9), (6.10) and (6.11), it must be

$$m - n = 2. \tag{8.1}$$

Since a triple (X,Y,Z) can be generated from the function φ, the Equations (6.12) and (6.13) are true.

From Equations (8.1), (6.12) and (6.13), it follows that

$$v(u-v) = 2. \tag{8.2}$$

The Equation (8.2) is possible, iff $v = 1$ and $u = 3$.

By Equations (6.6), (6.7) and (6.8), we obtain a triple $(12,35,37)$. □

By using the same idea, we can conclude:

Proposition 8.2. *There are no Pythagora's primitive triples which can be generated from the function ϑ.*

9. Connection with Fermat's family

In this section, we will investigate the connection between our method and Fermat's family of primitive pythagorean triples.

Proposition 9.1. *There are no Fermat's primitive triples which can be generated from the function φ.*

Proof. Let (X,Y,Z) be a primitive pythagorean triple (X is even) which belongs to Fermat's family.

By the definition of Fermat's family, it must be

$$|X - Y| = 1. \tag{9.1}$$

Since a triple (X,Y,Z) can be generated from the function φ, equations (6.6) and (6.7) hold.

By equations (6.6) and (6.7), the equation (9.1) gradually becomes:

$$\begin{aligned}
|X - Y| &= 1 \\
\left|\frac{uv(u^2-v^2)}{2} - \frac{(u^2+2uv-v^2)(u^2+v^2)}{4}\right| &= 1 \\
|2uv(u^2-v^2) - (u^2+2uv-v^2)(u^2+v^2)| &= 4 \\
|2uv(u^2-v^2) - 2uv(u^2+v^2) - (u^2-v^2)(u^2+v^2)| &= 4 \\
|-4uv^3 + v^4 - u^4| &= 4 \\
|u^4 - v^4 + 4uv^3| &= 4
\end{aligned}$$

Since $u > v >= 1$, it follows that $u^4 - v^4 > 0$. Then the last equation above becomes

$$u^4 - v^4 + 4uv^3 = 4. \tag{9.2}$$

Clearly, we have following inequalities:

$$u^4 - v^4 + 4uv^3 > 4uv^3 >= 4 * 3 * 1^3 = 12$$
$$u^4 - v^4 + 4uv^3 > 12 \tag{9.3}$$

We got a contradiction. This completes the proof of this proposition. □

10. Final Conclusion

From Corollary 5.1 and Corollary 5.2, it can be shown:

PROPOSITION 10.1. *There is no primitive pythagorean triple which can be generated from functions φ and ϑ simultaneously.*

In other words, there are not exist natural numbers X (X is even),Y,Z, and T such that (X, Y, Z) and $(3X, T, Z)$ are both primitive pythagorean triples. This can be proven by using method of infinite descent.

REMARK 10.1. During the preparation of this paper, we found a paper [**8**] in which the same method is described. However, we present several new facts, such as the Theorem 1.2, Corollaries 5.1) and 5.2 and all their applications except the Proposition 10.1.

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References

[1] F. J. M. Barning. Over Pythagorese en bijna-Pythagorese driehoeken en een generatieproces met behulp van unimodulaire matrices. *Math. Centrum, Amsterdam, Dept. Pure Math.*, (1963), ZW-011.

[2] B. Berggren. Pytagoreiska trianglar. *Tidskrift för Elementär Matematik, Fysik och Kemi*, **17**(1934), 129–139.

[3] A. Hall. Genealogy of Pythagorean Triads. *The Mathematical Gazette*, **54**(1970), 377–379.

[4] J. Roberts. *Elementary Number Theory: A Problem Oriented Approach.* MIT Press, Cambridge, MA, 1977.

[5] http://www.math.uconn.edu/~kconrad/blurbs/linmultialg/descentPythag.pdf

[6] https://en.wikipedia.org/wiki/Pythagorean_triple

[7] http://mathworld.wolfram.com/PythagoreanTriple.html

[8] https://arxiv.org/pdf/math/0701554.pdf

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