SOME RESULTS ON
ODD HARMONIOUS LABELING OF GRAPHS

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Abstract. A graph $G(p,q)$ is said to be odd harmonious if there exists an injection $f : V(G) \to \{0, 1, 2, \ldots, 2q - 1\}$ such that the induced function $f^* : E(G) \to \{1, 3, \ldots, 2q - 1\}$ defined by $f^*(uv) = f(u) + f(v)$ is a bijection. In this paper we prove that the $m$-mirror graph $M_m(G)$, $m$-shadow graph of $C_{bn}$, $m$-splitting graph of $K_2, n$ $(r, s)$ and $\langle C_n : K_2, m : C_r \rangle$, pyramid graph $PY_n$ are odd harmonious graphs.

1. Introduction

Throughout this paper by a graph we mean a finite, simple and undirected one. For standard terminology and notation we follow Harary [3]. A graph $G = (V, E)$ with $p$ vertices and $q$ edges is called a $(p, q)$ – graph. The graph labeling is an assignment of integers to the set of vertices or edges or both, subject to certain conditions. An extensive survey of various graph labeling problems is available in [1]. Labeled graphs serves as useful mathematical models for many applications such as coding theory, including the design of good radar type codes, synch-set codes, missile guidance codes and convolution codes with optimal autocorrelation properties. They facilitate the optimal nonstandard encoding of integers. Graham and Sloane [2] introduced harmonious labeling during their study of modular versions of additive bases problems stemming from error correcting codes. A graph $G$ is said to be harmonious if there exists an injection $f : V(G) \to \mathbb{Z}_q$ such that the induced function $f^* : E(G) \to \mathbb{Z}_q$ defined by $f^*(uv) = (f(u) + f(v)) \pmod{q}$ is a bijection and $f$ is called harmonious labeling of $G$. The concept of odd harmonious labeling was due to Liang and Bai [13]. A graph $G$ is said to be odd harmonious if there exists an injection $f : V(G) \to \{0, 1, 2, \ldots, 2q - 1\}$ such that the induced function $f^* : E(G) \to \{1, 3, \ldots, 2q - 1\}$ defined by $f^*(uv) = f(u) + f(v)$ is a bijection. If $f(V(G)) = \{0, 1, 2, \ldots, q\}$ then $f$ is called as strongly odd harmonious.

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labeling and G is called as strongly odd harmonious graph. The odd harmoniousness of graph is useful for the solution of undetermined equations. Several results have been published on odd harmonious labeling. An interested reader can refer to [4, 5, 6, 7, 8, 9, 10, 11, 12].

We use the following definitions in the subsequent section.

**Definition 1.1.** The m-shadow graph \( D_m(G) \) of a connected graph G is constructed by taking m-copies of G, say \( G_1, G_2, G_3, ..., G_m \), then join each vertex \( u \) in \( G_i \) to the neighbors of the corresponding vertex \( v \) in \( G_j \), \( 1 \leq j \leq m \).

**Definition 1.2.** The m-splitting graph \( Spl_m(G) \) of a graph G is obtained by adding to each vertex \( v \) of G new m vertices, say \( v^1, v^2, ..., v^m \) such that \( v^i, 1 \leq i \leq m \) is adjacent to every vertex that is adjacent to \( v \) in G.

Hence, the 2-shadow graph is the known shadow graph \( D_2(G) \) and the 1-splitting graph is the known splitting graph.

**Definition 1.3.** Let G be any graph with p vertices and q edges. Let \( G_1, G_2, ..., G_m \) be m-copies of the graph G. The m-mirror graph \( M_m(G) \) is defined as the disjoint union of \( G_1, G_2, ..., G_m \) together with additional edges joining each vertex of \( G_i \) to its corresponding vertex in \( G_{i+1} \), \( 1 \leq i \leq m - 1 \). Hence \( M_m(G) \) has mp vertices and \( mp + (m - 1)p \) edges.

**Definition 1.4.** The wheel \( W_n \) is obtained by joining a new vertex with every vertices of n-cycle \( C_n \) by an edge.

**Definition 1.5.** Gear graph \( W_n \) is obtained from a wheel graph \( W_n \) by subdividing each edge of the n-cycle by a vertex. The vertex and the edge set of the gear graph \( W_n \) are \( V(W_n) = \{ v_i / i \in [0, 2n] \} \), \( E(W_n) = \{ v_0v_{2i-1} / i \in [1, n] \} \cup \{ v_ivi-1 / i \in [2, 2n] \} \cup \{ v_1v_{2n} \} \).

**Definition 1.6.** A pyramid graph \( PY_n \) is a graph with the vertex set

\[ V(PY_n) = \{ v_11, v_21, v_22, v_31, v_32, v_33, ..., v_n1, v_n2, ..., v_nn \} \]

and the edge set \( E(PY_n) = \{ v_i j v(i+1)j / 1 \leq i \leq n - 1 \ and \ 1 \leq j \leq n - 1 \} \cup \{ v_i v(i+1)(i+1)/1 \leq i \leq n - 1 \} \cup \{ v_i j v(i+1)(j+1)/2 \leq i \leq n - 1 \ and \ 1 \leq j \leq n - 2 \} \).

**Definition 1.7.** Let G be a graph on p vertices \( v_1, v_2, ..., v_p \) and \( H_1, H_2, ..., H_p \) be p graphs isomorphic to a graph H with n vertices. The corona graph \( G \circ H \) is obtained by joining each vertex \( v_i \) of G with every vertices of \( H_i \) for \( 1 \leq i \leq p \) and \( 1 \leq j \leq n \). The comb graph is \( P_n \circ K_1 \).

**Definition 1.8.** ([3]) The graph \( K_{2,n}(r, s) \) is obtained from \( K_{2,n}, (n \geq 2) \) by adding \( r \) and \( s \) (\( r, s \geq 1 \)) pendant edges to the two vertices of degree \( n \).

**Definition 1.9.** Consider the bipartite graph \( K_{2,m} \) with the vertex set \( V = V_1 \cup V_2 \) where \( V_1 = \{ u, v \} \) and \( V_2 = \{ v_1, v_2, ..., v_m \} \). The graph \( G = (C_n : K_{2,m} : C_r) \) is obtained by identifying the vertex u of \( V_1 \) with a vertex of \( C_n \) and the other vertex \( v \) of \( V_1 \) with a vertex of \( C_r \).
Definition 1.10. The graph \( W_{m,n} \) is obtained from the gear graph \( W_n \) as follows: Join the vertices \( v_i \) and \( v_{i+2} \) with the new vertices \( v_{i+1} \) for \( 1 \leq j \leq m \) and \( 2 \leq i \leq n-2 \) and join \( v_n \) and \( v_2 \) with \( v_1 \) for \( 1 \leq j \leq m \).

2. Main Results

In this section we prove that the \( m \)-mirror graph \( M_m(G) \), \( m \)-shadow graph of \( C_{bn} \), \( m \)-splitting graph of \( K_{2,n} \) and \( W_{m,n} \), \( \langle C_n : K_{2,m} : G_r \rangle \), pyramid graph \( PY_n \) are odd harmonious graphs.

Theorem 2.1. If \( G(p,q) \) is a strongly odd harmonious tree, then \( M_m(G) \) is an odd harmonious graph.

Proof. Let \( f \) be a strongly odd harmonious labeling of \( G \). Then \( f(V(G)) = \{0,1,2,\ldots,q\} \) and \( f^*(E(G)) = \{1,3,\ldots,2q-1\} \). Define a new labeling \( g : V(M_m(G)) \to \{0,1,2,\ldots,2q-1\} \) as \( g(u) = f(u) + (i-1)(2q+1) \) if \( u \in G_i \), \( 1 \leq i \leq m \). The induced edge labels of the edges between the graphs \( G_i \) and \( G_{i+1} \) is

\[
\begin{align*}
= \{2f(u) + (i-1)(2q+1) + i(2q+1)/u \in G\} & \text{ for } 1 \leq i \leq m - 1. \\
= \{2f(u) + (2q+1)(2i-1)/u \in G\} & \text{ for } 1 \leq i \leq m - 1. \\
= \{(2q+1)(2i-1), 2(2q+1)(2i-1), 4(2q+1)(2i-1), \ldots, 2q+(2q+1)(2i-1)\} & \text{ for } 1 \leq i \leq m - 1.
\end{align*}
\]
The induced edge labelings are \{+\} and join \(v f \Rightarrow v f\).

Hence the induced edge labels of \(M_n(G)\) is \(\{1, 3, \ldots, 2q - 1\} \cup \{2q + 1, 2q + 3, \ldots, 4q + 1\} \cup \{4q + 3, 4q + 5, \ldots, 6q + 1\} \cup \{6q + 3, 6q + 5, \ldots, 8q + 3\} \ldots \cup \{4mq - 6q + 2m - 3, \ldots, 4mq - 4q + 2m - 3\} \cup \{4mq - 4q + 2m - 1, \ldots, 4mq - 2q + 2m - 3\} = \{1, 3, 5, \ldots, 4mq - 2q + 2m - 3\} .\) Hence \(M_n(G)\) is odd harmonious.

An odd harmonious labeling of \(M_3(P_3 \odot K_1)\) is shown in Figure 1.

Figure 1: An odd harmonious labeling of \(M_3(P_3 \odot K_1)\)

**Theorem 2.2.** The graph \(W_{m,n}\) for \(n \equiv 0 \pmod{4}, m \geq 1\) is an odd harmonious graph.

**Proof.** Let \(V(G) = \{v_0, v_1, \ldots, v_n\} \cup \{v_{2i-1}^j \mid 1 \leq i \leq \frac{n}{2}, 1 \leq j \leq m\} .\) Join the vertices \(v_i\) and \(v_{i+2}\) with the vertices \(v_{i+1}^j\) for \(1 \leq j \leq m\) and \(2 \leq i \leq n - 2\) and join \(v_n\) and \(v_2\) with \(v_1^j\) for \(1 \leq j \leq m\). Let \(G = W_{m,n} .\) Then \(|V(G)| = n + \frac{mn}{2} + 1\) and \(|E(G)| = n(m + 1) + \frac{n}{2} .\) We define a labeling \(f : V(G) \to \{0, 1, 2, \ldots, 2(n(m + 1) + \frac{n}{2}) - 1\} .\) as follows:

- \(f(v_0) = 0; f(v_i) = i, 1 \leq i \leq n\) and if \(i\) is odd;
- \(f(v_i) = (i - 1) + n + 1\) and \(1 \leq i \leq \frac{n}{2} - 2\) and if \(i\) is even;
- \(f(v_1) = (i - 1) + n + 3\) and \(\frac{n}{2} \leq i \leq n - 2\) and if \(i\) is even;
- \(f(v_n) = n; f(v_{2i-1}^j) = 2n + 2i - 1, 1 \leq i \leq \frac{n}{2}\) and \(1 \leq j \leq m\).

The induced edge labelings are:

- \(f^*(v_0v_1) = i, 1 \leq i \leq n\) and if \(i\) is odd; \(f^*(v_nv_1) = n + 1;\)
- \(f^*(v_iv_{i+1}) = 2i + n + 1, 1 \leq i \leq \frac{n}{2} - 2;\)
- \(f^*(v_nv_{i+1}) = 2i + n + 3, \frac{n}{2} - 1 \leq i \leq n - 2;\)
- \(f^*(v_{n-1}v_n) = 2n - 1;\)
- \(f^*(v_{2i-1}^jv_i) = 2n + 2i + n - 1, 1 \leq i \leq \frac{n}{2} - 2\) and if \(i\) is even , \(1 \leq j \leq m;\)
- \(f^*(v_{2i-1}^jv_i) = 2n + 2i + n + 1, \frac{n}{2} \leq i \leq n - 2\) and if \(i\) is even , \(1 \leq j \leq m;\)
- \(f^*(v_{2i-1}^jv_i) = 2n + 2i + n - 1;\)
- \(f^*(v_{2i+1}^jv_i) = 2n + 2i + n + 1, 1 \leq i \leq \frac{n}{2} - 2\) , if \(i\) is even and \(1 \leq j \leq m;\)
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For $1 \leq i \leq n - 2$, if $i$ is even and $1 \leq j \leq m$;

$$f^*(v_i v_{i+1}) = 2nj + 2i + n + 3, \quad \frac{n}{2} \leq i \leq n - 2,$$

$$f^*(v_n v_1) = 2nj + n + 1.$$ In view of the above defined labeling pattern, the graph $W_{m,n}$ is an odd harmonious graph.

An odd harmonious labeling of $W_{2,8}$ is shown in Figure 2.

An odd harmonious labeling of $W_{2,8}$ is shown in Figure 2.

Figure 2: An odd harmonious labeling of $W_{2,8}$.

**Theorem 2.3.** The graph $D_m(C_{bn})$ is odd harmonious for all $m \geq 2$ and $n \geq 1$.

**Proof.** Let $v_0^0, v_1^0, \cdots, v_n^0$ and $u_0^0, u_1^0, \cdots, u_n^0$ be the vertices of the graph $C_{bn}$ and $v_1^j, v_2^j, \cdots, v_n^j, u_1^j, u_2^j, \cdots, u_n^j$ be the vertices of the $j$th copy of $C_{bn}$ where $1 \leq j \leq m - 1$. Let $G = D_m(C_{bn})$. Then $|V(G)| = 2mn$ and $|E(G)| = m^3(2n - 1)$. We define a labeling $f: V(G) \to \{0, 1, 2, \cdots, 2m^2(2n - 1) - 1\}$ as follows:

- $f(v_1^1) = 4mj$, $0 \leq j \leq m - 1$;
- $f(v_i^j) = (i - 3)m^2 + 2m(2m + j - 1)$, $i = 3, 5, \cdots, n(\text{odd})$ and $0 \leq j \leq m - 1$;
- $f(v_i^j) = (i - 2)m^2 + 2m + 1 + 2j$, $i = 2, 4, \cdots, n(\text{even})$ and $0 \leq j \leq m - 1$;
- $f(u_1^j) = 1 + 2j$, $0 \leq j \leq m - 1$;
- $f(u_i^j) = 2m^2n + (i - 5)m^2 + 2m + 1 + 2j$, $i = 3, 5, \cdots, n(\text{odd})$ and $0 \leq j \leq m - 1$;
- $f(u_i^j) = 2m^2n + (i - 2)m^2 - 2m + 2mj$, $i = 2, 4, \cdots, n(\text{even})$ and $0 \leq j \leq m - 1$.

The induced edge labels are

$$f^*(v_1^j u_1^k) = 4mj + 1 + 2k$$ for $0 \leq j, k \leq m - 1$. 

\[ \boxed{\text{Figure 2: An odd harmonious labeling of } W_{2,8}.} \]
f^r(v_i^j) = 4mj + 2m + 1 + 2k, 0 \leq j, k \leq m - 1;
\] 
\[ f^r(v_i^{j+1}) = 2m + 1 + 2j + 4mk, 0 \leq j, k \leq m - 1;
\] 
\[ f^r(v_i^{j+k}) = 2(i - 2)m^2 + 2m + 1 + 2j + 2m(2m + k - 1), i = 2, 4, \cdots, (n - 1)(\text{even}), 0 \leq j, k \leq m - 1;
\] 
\[ f^r(v_i^{j+k-1}) = (i - 2)m^2 + 2m + 1 + 2j + (i - 4)m^2 + 2m(2m + k - 1)
\]
\[ i = 4, 6, \cdots, n(\text{even}), 0 \leq j, k \leq m - 1;
\] 
\[ f^r(v_i^{j+k+1}) = (i - 3)m^2 + 2m(2m + j - 1) + (i - 1)m^2 + 2m + 1 + 2k
\]
\[ i = 3, 5, \cdots, (n - 1)(\text{odd}), 0 \leq j, k \leq m - 1;
\] 
\[ f^r(v_i^{j+k-1}) = 2(i - 3)m^2 + 2m + 1 + 2k + 2m(2m + j - 1), i = 3, 5, \cdots, n(\text{odd}), 0 \leq j, k \leq m - 1;
\] 
\[ f^r(v_i^{j+k}) = (i - 3)m^2 + 2m(2m + j - 1) + 2m^2 + (i - 5)m^2 + 2m + 1 + 2k
\]
\[ i = 3, 5, \cdots, n(\text{odd}), 0 \leq j, k \leq m - 1;
\] 
\[ f^r(v_i^{j+k+1}) = 2(i - 2)m^2 + 2m + 1 + 2j + 2m^2 + 2m + 2mk, i = 2, 4, \cdots, n(\text{even})
\]
\[ 0 \leq j, k \leq m - 1.
\]

In view of the above defined labeling pattern, the graph \( D_m(C_b) \) is an odd harmonious graph.

An odd harmonious labeling of \( D_3(C_{b2}) \) is shown in Figure 3.

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**Theorem 2.4.** The graph \( Spl_m[K_{2,n}(r,s)] \) is an odd harmonious graph for all \( n, r, s \geq 1 \).

**Proof.** Let \( v_0, v_0, w_1, w_2, \cdots, w_r, v_1, v_2, \cdots, v_n, u_1, u_2, \cdots, u_s \) be the vertices of the graph \( K_{2,n}(r,s) \) and suppose \( v_0^j, v_0^k, w_1^j, w_2^j, \cdots, w_r^j, v_1^j, v_2^j, \cdots, v_n^j, u_1^j, u_2^j, \cdots, u_s^j \), \( 1 \leq j \leq m \) be the vertices of the \( j \)th copy of \( K_{2,n}(r,s) \). Let \( G = Spl_m[K_{2,n}(r,s)] \).

Then \( |V(G)| = (m + 1)(n + r + s + 2) \) and \( |E(G)| = (2m + 1)(2n + r + s) - 1 \). We define a labeling \( f : V(G) \to \{0, 1, 2, \cdots, 2(2m + 1)(2n + r + s) - 1\} \) as follows:

\[ f(v_0^j) = 0; \quad f(w_i) = 2i - 1, 1 \leq i \leq r; \quad f(v_i) = 2(i + r) - 1, 1 \leq i \leq n;
\]
\[ f(u_i) = 2n; \quad f(w_i) = 2(r + n + i) - 1, 1 \leq i \leq s;
\]
\[ f(w_i) = (2j - 1)(4n + 2r + 2s) + 2i - 1, 1 \leq i \leq r \text{ and } 1 \leq j \leq m;
\]
\[ f(v_0^j) = (2j - 1)(4n + 2r + 2s) + 2r, 1 \leq j \leq m;
\]

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**Figure 3:** An odd harmonious labeling of \( D_3(C_{b2}) \).
In view of the above defined labeling pattern, the graph $Spl_1$ is odd harmonious.

An odd harmonious labeling of $Spl_1$ is shown in Figure 4.

Figure 4: An odd harmonious labeling of $Spl_1$.

**Theorem 2.5.** The pyramid graph $PY_n$ is an odd harmonious graph for all $n \geq 2$.

**Proof.** The graph $PY_n$ with $|V(G)| = n(n + 1)$ and $|E(G)| = n(n - 1)$. We define a labeling $f : V(G) \rightarrow \{0, 1, 2, \cdots, 2n(n - 1) - 1\}$ as follows: $f(v_{ij}) = (i - 2)i + (2j - 1)$, $1 \leq i \leq n$ and $1 \leq j \leq i$. The induced edge labels are
Theorem 2.6. The graph $G = (C_n : K_{2,m} : C_r)$ is an odd harmonious graph for $n, r \equiv 0 \pmod{4}$ and $m \geq 2$.

Proof. Let $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_r$ be the vertices of the cycles $C_n$ and $C_r$ respectively. Consider the bipartite graph $K_{2,m}$ with vertex set $V = V_1 \cup V_2$ where $V_1 = \{u, v\}$ and $V_2 = \{w_1, w_2, \ldots, w_m\}$. Then $G = (C_n : K_{2,m} : C_r)$ is obtained by identifying the vertex $u$ of $V_1$ with the vertex $v_2$ of $C_n$ and the other vertex $v$ of $V_1$ with the vertex $u_1$ of $C_r$. Then $|V(G)| = 2n + m$ and $|E(G)| = 2(m + n)$. We define a labeling $f : V(G) \to \{0, 1, 2, \ldots, 4(m + n) - 1\}$ as follows:

Case(i): When $n = r$

$$f(v_i) = i - 1, \quad 1 \leq i \leq \frac{n}{2};$$
$$f(v_i) = \begin{cases} 
  i + 1 & \text{if } i \text{ is odd} \\
  i - 1 & \text{if } i \text{ is even}
\end{cases}, \quad \frac{n}{2} + 1 \leq i \leq n;$$
$$f(w_i) = n + \frac{n}{2} + 2i, \quad 1 \leq i \leq m;$$
$$f(u_i) = 2m + \frac{n}{2} + i - 2, \quad i = 1, 3, 5, \ldots, \frac{n}{2} - 1;$$
$$f(u_i) = 2m + \frac{n}{2} + i, \quad i = \frac{n}{2} + 1, \frac{n}{2} + 3, \ldots, n - 1;$$
$$f(u_i) = \frac{3n}{2} + 2m + i, \quad i = 2, 4, 6, \ldots, n.$$

The induced edge labels are

$$f^*(v_iv_{i+1}) = 2i - 1, \quad 1 \leq i \leq \frac{n}{2} - 1; \quad f^*(v_iw_i) = 2i + 1, \quad \frac{n}{2} \leq i \leq n - 1;$$
$$f^*(v_2w_i) = 2(n + i) - 1, \quad 1 \leq i \leq m; \quad f^*(u_1w_i) = 2(m + n + i) - 1, \quad 1 \leq i \leq m;$$

In view of the above defined labeling pattern, the graph $PY_n$ is an odd harmonious graph.

An odd harmonious labeling of $PY_4$ is shown in Figure 5.

Figure 5: An odd harmonious labeling of $PY_4$. 
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\[ f^*(u_iu_{i+1}) = 2(2m + n + i) - 1, \ 1 \leq i \leq \frac{n}{2} - 1; \]
\[ f^*(u_{2i}u_{2i+1}) = 3n + 4m + 1; \ f^*(u_iu_{i+1}) = 2(2m + n + i) + 1, \ \frac{n}{2} + 1 \leq i \leq n - 1. \]

**Case (ii):** When \( n \neq r. \)

Without loss of generality assume that \( n < r. \)

\[ f(v_i) = i - 1, \ 1 \leq i \leq \frac{n}{2}; \ f(v_i) = \begin{cases} i + 1 & \text{if } i \text{ is odd,} \\ i - 1 & \text{if } i \text{ is even} \end{cases}, \ \frac{n}{2} + 1 \leq i \leq n; \]
\[ f(w_i) = n + \frac{n}{2} + 2i, \ 1 \leq i \leq m; \ f(u_i) = 2m + \frac{n}{2} + i - 2, \ i = 1, 3, 5, \ldots, \frac{r}{2} - 1; \]
\[ f(u_i) = 2m + \frac{n}{2} + i, \ i = \frac{r}{2} + 1, \frac{r}{2} + 3, \ldots, r - 1; \]
\[ f(u_i) = \frac{3n}{2} + 2m + i, \ i = 2, 4, 6, \ldots, r. \]

The induced edge labels are

\[ f^*(v_iv_{i+1}) = 2i - 1, \ 1 \leq i \leq \frac{n}{2} - 1; \ f^*(v_iv_{i+1}) = 2i + 1, \ \frac{n}{2} \leq i \leq n - 1; \]
\[ f^*(v_2w_i) = 2(n + i) - 1, \ 1 \leq i \leq m; \ f^*(u_1w_i) = 2(m + n + i) - 1, \ 1 \leq i \leq m; \]
\[ f^*(u_iu_{i+1}) = 2(2m + n + i) - 1, \ 1 \leq i \leq \frac{r}{2} - 1; \ f^*(u_{2i}u_{2i+1}) = 2n + 4m + r + 1; \]
\[ f^*(u_iu_{i+1}) = 2(2m + n + i) + 1, \ \frac{r}{2} + 1 \leq i \leq r - 1. \]

In view of the above defined labeling pattern, the graph \( \langle C_n : K_{2,m} : C_r \rangle \) is odd harmonious. \( \Box \)

An odd harmonious labeling of \( \langle C_8 : K_{2,4} : C_8 \rangle \) and \( \langle C_8 : K_{2,4} : C_4 \rangle \) are shown in Figures 6 and 7.

![Figure 6: An odd harmonious labeling of \( C_8 : K_{2,4} : C_8 \)](image-url)
Figure 7: An odd harmonious labeling of $\langle C_8 : K_{2,4} : C_4 \rangle$

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