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NEW GENERALIZED CLOSED SETS IN IDEAL NANOTOPOLOGICAL SPACES

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Abstract. We have introduce $\mathcal{L}$-nI_g-closed subsets, $\mathcal{S}$-nI_g-closed subsets, $\mathcal{R}$-nI_g-closed subsets and $nI^\star$-$\mathcal{O}$-sets in this paper. Also we have discussed their properties related to other subsets.

1. Introduction

The concept of nanotopology was introduced by Lellis Thivagar et al [**9**]. which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. Also nano closed sets, nano-interior and nano-closure of a subset were defined.

The Concept of an ideal nanotopological space and some of its properties were introduced by Parimala et al(2017).

In this paper we have introduce $nI^\star$-$\mathcal{O}$-sets, $\mathcal{L}$-nI_g-closed subsets, $\mathcal{S}$-nI_g-closed subsets and $\mathcal{R}$-nI_g-closed subsets. Also we have discussed some special properties of $\mathcal{L}$-nI_g, $\mathcal{S}$-nI_g and $\mathcal{R}$-nI_g-closed subsets.

2. Preliminaries

An ideal I [**12**] on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions.

(1) $A \in I$ and $B \subset A$ imply $B \in I$ and
(2) $A \in I$ and $B \in I$ imply $A \cup B \in I$.

Given a topological space (X, τ) with an ideal I on X. If $\wp(X)$ is the family of all subsets of X, a set operator $(.)^\star : \wp(X) \to \wp(X)$, called a local function of A with respect to τ and I is defined as follows: for $A \subset X$, $A^\star(I, \tau) = \{x \in X : U \cap A \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$ [**2**]. The closure operator defined

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by $cl^{\star}(A) = A \cup A^{\star}(I, \tau)$ [**11**] is a Kuratowski closure operator which generates a topology $\tau^{\star}(I, \tau)$ called the $\star$-topology finer than τ. The topological space together with an ideal on X is called an ideal topological space or an ideal space denoted by (X, τ, I). We will simply write $A^{\star}$ for $A^{\star}(I, \tau)$ and $\tau^{\star}$ for $\tau^{\star}(I, \tau)$.

DEFINITION 2.1. ([**8**]) Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.

(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

DEFINITION 2.2. ([**9**]) Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(1) U and $\phi \in \tau_R(X)$,

(2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$,

(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus $\tau_R(X)$ is a topology on U called the nanotopology with respect to X and $(U, \tau_R(X))$ is called the nanotopological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets). The complement of a n-open set is called n-closed. In the rest of the paper, we denote a nanotopological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $n\text{-}int(A)$ and $n\text{-}cl(A)$, respectively.

A nanotopological space $(U, \mathcal{N})$ with an ideal I on U is called [**5**] an ideal nanotopological space and is denoted by $(U, \mathcal{N}, I)$. $G_n(x) = \{G_n \mid x \in G_n, G_n \in \mathcal{N}\}$, denotes [**5**] the family of nano open sets containing x.

In future an ideal nanotopological space $(U, \mathcal{N}, I)$ will be simply called a space.

DEFINITION 2.3. ([**5**]) Let $(U, \mathcal{N}, I)$ be a space with an ideal I on U. Let $(.)^{\star}_n$ be a set operator from $\wp(U)$ to $\wp(U)$ ($\wp(U)$ is the set of all subsets of U). For a subset $A \subseteq U$, $A^{\star}_n(I, \mathcal{N}) = \{x \in U : G_n \cap A \notin I$, for every $G_n \in G_n(x)\}$ is called the nano local function (briefly, n-local function) of A with respect to I and $\mathcal{N}$. We will simply write $A^{\star}_n$ for $A^{\star}_n(I, \mathcal{N})$.

THEOREM 2.1 ([**5**]). *Let $(U, \mathcal{N}, I)$ be a space and A and B be subsets of U. Then*

(1) $A \subseteq B \Rightarrow A_n^\star \subseteq B_n^\star$,
(2) $A_n^\star = n\text{-}cl(A_n^\star) \subseteq n\text{-}cl(A)$ *($A_n^\star$ is a n-closed subset of $n\text{-}cl(A)$)*,
(3) $(A_n^\star)_n^\star \subseteq A_n^\star$,
(4) $(A \cup B)_n^\star = A_n^\star \cup B_n^\star$,
(5) $V \in \mathcal{N} \Rightarrow V \cap A_n^\star = V \cap (V \cap A)_n^\star \subseteq (V \cap A)_n^\star$,
(6) $J \in I \Rightarrow (A \cup J)_n^\star = A_n^\star = (A - J)_n^\star$.

THEOREM 2.2 ([**5**]). *Let $(U, \mathcal{N}, I)$ be a space with an ideal I and A be a subset of U. If $A \subseteq A_n^\star$, then $A_n^\star = n\text{-}cl(A_n^\star) = n\text{-}cl(A)$.*

DEFINITION 2.4. ([**5**]) Let $(U, \mathcal{N}, I)$ be a space. The set operator $n\text{-}cl^\star$ called a nano $\star$-closure is defined by $n\text{-}cl^\star(A) = A \cup A_n^\star$ for $A \subseteq U$.

It can be easily observed that $n\text{-}cl^\star(A) \subseteq n\text{-}cl(A)$.

THEOREM 2.3 ([**7**]). *In a space $(U, \mathcal{N}, I)$, if A and B are subsets of U, then the following results are true for the set operator $n\text{-}cl^\star$.*

(1) $A \subseteq n\text{-}cl^\star(A)$,
(2) $n\text{-}cl^\star(\phi) = \phi$ *and* $n\text{-}cl^\star(U) = U$,
(3) *If* $A \subset B$, *then* $n\text{-}cl^\star(A) \subseteq n\text{-}cl^\star(B)$,
(4) $n\text{-}cl^\star(A) \cup n\text{-}cl^\star(B) = n\text{-}cl^\star(A \cup B)$.
(5) $n\text{-}cl^\star(n\text{-}cl^\star(A)) = n\text{-}cl^\star(A)$.

DEFINITION 2.5. ([**6**]) A subset A of a space $(U, \mathcal{N}, I)$ is called $n\star$-dense in itself (resp. $n\star$-perfect and $n\star$-closed) if $A \subseteq A_n^\star$ (resp. $A = A_n^\star$ and $A_n^\star \subseteq A$).

DEFINITION 2.6. A subset A of a nanotopological space $(U, \mathcal{N})$ is called nano nowhere dense (briefly, n-nowhere dense) [**10**] if $n\text{-}int(n\text{-}cl(A)) = \phi$.

DEFINITION 2.7. A subset A of a space $(U, \mathcal{N}, I)$ is called

(1) nano g-closed (briefly, ng-closed) [**1**] if $n\text{-}cl(A) \subseteq B$, whenever $A \subseteq B$ and B is n-open. The complement of a ng-closed set is said to be ng-open.
(2) nano I_g-closed (briefly, nI_g-closed) [**6**] if $A_n^\star \subseteq B$ whenever $A \subseteq B$ and B is n-open. The complement of a nI_g-closed set is said to be nI_g-open.
(3) nano pre*-I-closed (briefly, pre*-nI-closed) [**3**] if $n\text{-}cl^\star(n\text{-}int(A)) \subseteq A$. The complement of a pre*-nI-closed set is said to be pre*-nI-open.
(4) nano $\mathcal{Q}$-I-closed (briefly, $\mathcal{Q}$-nI-closed) [**4**] if $A = n\text{-}cl^\star(n\text{-}int(A))$. The complement of a $\mathcal{Q}$-nI-closed set is said to be $\mathcal{Q}$-nI-open.

THEOREM 2.4 ([**6**]). *In a space $(U, \mathcal{N}, I)$, each $n\star$-closed set is nI_g-closed.*

3. New generalized closed subsets of $(U, \mathcal{N}, I)$

DEFINITION 3.1. A subset A of a space $(U, \mathcal{N}, I)$, is called

(1) lightly nano I_g-closed (briefly $\mathcal{L}$-nI_g-closed) if $(n\text{-}int(A))_n^\star \subseteq B$ whenever $A \subseteq B$ and B is n-open. The complement of a $\mathcal{L}$-nI_g-closed set is said to be $\mathcal{L}$-nI_g-open.

(2) softly nano I_g-closed (briefly $\mathcal{S}$-nI_g-closed) if $(n\text{-}int(A))_n^\star \subseteq B$ whenever $A \subseteq B$ and B is ng-open. The complement of a $\mathcal{S}$-nI_g-closed set is said to be $\mathcal{S}$-nI_g-open.

(3) robustly nano I_g-closed (briefly $\mathcal{R}$-nI_g-closed) if $A^\star_n \subseteq B$ whenever $A \subseteq B$ and B is ng-open. The complement of a $\mathcal{R}$-nI_g-closed set is said to be $\mathcal{R}$-nI_g-open.

THEOREM 3.1. *In a space $(U, \mathcal{N}, I)$ the following results hold for a subset A of U. A is $\mathcal{R}$-nI_g-closed $\Rightarrow$ A is $\mathcal{S}$-nI_g-closed $\Rightarrow$ A is $\mathcal{L}$-nI_g-closed.*

PROOF. A is $\mathcal{R}$-nI_g-closed $\Rightarrow$ A is $\mathcal{S}$-nI_g-closed.

Let A be $\mathcal{R}$-nI_g-closed and $A \subseteq B$ where B is ng-open. Since A is $\mathcal{R}$-nI_g-closed $A^\star_n \subseteq B$. But $(n\text{-int(A)})^\star_n \subseteq A^\star_n$. Thus $(n\text{-int(A)})^\star_n \subseteq B$ which proves that A is $\mathcal{S}$-nI_g-closed.

A is $\mathcal{S}$-nI_g-closed $\Rightarrow$ A is $\mathcal{L}$-nI_g-closed.

Let A be $\mathcal{S}$-nI_g-closed and $A \subseteq G$ where G is n-open. Since A is $\mathcal{S}$-nI_g-closed and G is ng-open being n-open, $(n\text{-int(A)})^\star_n \subseteq G$. This proves that A is $\mathcal{L}$-nI_g-closed. $\square$

REMARK 3.1. None of the implications in 3.1 is reversible as seen from the following Example.

EXAMPLE 3.1. Consider

$U = \{e_1, e_2, e_3, e_4, e_5\}$, $U/R = \{\{e_1\}, \{e_2, e_3\}, \{e_4, e_5\}\}$ and $X = \{e_1, e_2\}$.

Then $\mathcal{N} = \{\phi, \{e_1\}, \{e_2, e_3\}, \{e_1, e_2, e_3\}, U\}$. Let $I = \{\phi, \{e_1\}\}$. In this space $(U, \mathcal{N}, I)$,

(1) $\mathcal{L}$-nI_g-closed $\nRightarrow$ $\mathcal{S}$-nI_g-closed. $A = \{e_2, e_3, e_4\}$ is $\mathcal{L}$-nI_g-closed for U is the only n-open set containing A. Also A is ng-open and $A \subseteq A$ whereas

$(n\text{-}int(A))^\star_n = \{e_2, e_3\}^\star_n = \{e_2, e_3, e_4, e_5\} \nsubseteq A$,

which proves that A is not $\mathcal{S}$-nI_g-closed.

(2) $\mathcal{S}$-nI_g-closed $\nRightarrow$ $\mathcal{R}$-nI_g-closed. $B = \{e_2, e_4\}$ is $\mathcal{S}$-nI_g-closed for $(n\text{-}int(B))^\star_n = \phi^\star_n = \phi$. Also B is ng-open and $B \subseteq B$ whereas $B^\star_n = \{e_2, e_3, e_4, e_5\} \nsubseteq B$, which verifies that B is not $\mathcal{R}$-nI_g-closed.

THEOREM 3.2. *In a space $(U, \mathcal{N}, I)$, the following results hold for a subset A of U .*

(1) *A is $\mathcal{R}$-nI_g-closed $\Rightarrow$ A is nI_g-closed.*
(2) *A is nI_g-closed $\Rightarrow$ A is $\mathcal{L}$-nI_g-closed.*
(3) *A is n-closed $\Rightarrow$ A is $\mathcal{R}$-nI_g-closed.*

PROOF. (1) let $A \subseteq B$ where B is n-open. Since B is ng-open and A is $\mathcal{R}$-nI_g-closed, $A^\star_n \subseteq B$. This proves that A is nI_g-closed.

(2) Let $A \subseteq B$ where B is n-open. Since A is nI_g-closed, $A^\star_n \subseteq B$. But $(n\text{-int(A)})^\star_n \subseteq A^\star_n \subseteq B$, which shows that A is $\mathcal{L}$-nI_g-closed.

(3) Since A is n-closed, A is $n\star$-closed and $A^\star_n \subseteq A$. Let $A \subseteq B$ where B is ng-open. then $A^\star_n \subseteq A \subseteq B$ which proves that A is $\mathcal{R}$-nI_g-closed. $\square$

REMARK 3.2. From Theorem 2.4, 3.1 and 3.2 the results are given in a diagram.

$$\begin{array}{ccccc} n\text{-closed} & \longrightarrow & \mathcal{R}\text{-}nI_g\text{-closed} & \longrightarrow & \mathcal{S}\text{-}nI_g\text{-closed} \\ \downarrow & & \downarrow & & \downarrow \\ n\star\text{-closed} & \longrightarrow & nI_g\text{-closed} & \longrightarrow & \mathcal{L}\text{-}nI_g\text{-closed}. \end{array}$$

Here $A \longrightarrow B$ means A implies B.

Remark 3.3. None of the implications is reversible as seen from the following Example.

Example 3.2. In Example 3.1,

(1) $n\star$-closed $\nrightarrow$ n-closed. $B = \{e_1\}$ is $n\star$-closed for $B^\star_n = \{e_1\}^\star_n = \phi \subseteq B$. But B is not n-closed.

(2) $\mathcal{R}$-nI_g-closed $\nrightarrow$ n-closed. $C = \{e_1\}$ is $\mathcal{R}$-nI_g-closed for if $C \subseteq K$ where K is ng-open then $C^\star_n = \{e_1\}^\star_n = \phi \subseteq K$. But C is not n-closed.

(3) $\mathcal{L}$-nI_g-closed $\nrightarrow$ nI_g-closed. $D = \{e_2\}$ is $\mathcal{L}$-nI_g-closed for $(n\text{-}int(D))^\star_n = \phi^\star_n = \phi$. $D = \{e_2\} \subseteq \{e_2, e_3\}$ is n-open. But $D^\star_n = \{e_2\}^\star_n = \{e_2, e_3, e_4, e_5\} \nsubseteq \{e_2, e_3\}$ which proves that D is not nI_g-closed.

(4) nI_g-closed $\nrightarrow$ $n\star$-closed. $E = \{e_5\}$ is nI_g-closed for U is the only n-open set containing E. But $E^\star_n = \{e_5\}^\star_n = \{e_4, e_5\} \nsubseteq E$. Thus E is not $n\star$-closed.

(5) nI_g-closed $\nrightarrow$ $\mathcal{R}$-nI_g-closed. $F = \{e_5\}$ is nI_g-closed . But F is ng-open and $F \subseteq F$ whereas $F^\star_n = \{e_5\}^\star_n = \{e_4, e_5\} \nsubseteq F$ which proves that F is not $\mathcal{R}$-nI_g-closed.

Thus Examples 3.1 and 3.2 verify Remark 3.3.

Definition 3.2. A subset B of a space $(U, \mathcal{N}, I)$ is called a nano $I^\star$-$\mathcal{O}$-set (briefly $nI^\star$-$\mathcal{O}$-set) if $A = P \cup Q$ where P is n-closed and Q is $pre^\star$-nI-open.

Theorem 3.3. *In a space $(U, \mathcal{N}, I)$ a subset A of U is*

(1) $pre^\star$-nI-open $\Rightarrow$ A is a $nI^\star$-$\mathcal{O}$-set.
(2) n-closed $\Rightarrow$ A is a $nI^\star$-$\mathcal{O}$-set.

Proof. (1) $A = A \cup \phi$ where A is $pre^\star$-nI-open and ϕ is n-closed. Hence A is a $nI^\star$-$\mathcal{O}$-set.

(2) $A = \phi \cup A$ where ϕ is $pre^\star$-nI-open and A is n-closed. Hence A is a $nI^\star$-$\mathcal{O}$-set. □

Remark 3.4. The converse of Theorem 3.3 is not true as shown in the following Example.

Example 3.3. Let $U = \{a, b, c, d\}$ with $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ and $X = \{c, d\}$. Then $\mathcal{N} = \{\phi, \{d\}, \{a, c\}, \{a, c, d\}, U\}$. Let the ideal be $I = \{\phi, \{c\}\}$. In $(U, \mathcal{N}, I)$, $\{a, b\}$ is a $nI^\star$-$\mathcal{O}$-set for $\{a, b\} = \{a\} \cup \{b\}$ where $\{a\}$ is $pre^\star$-nI-open and $\{b\}$ is n-closed. But $\{a, b\}$ is neither $pre^\star$-nI-open or n-closed.

Theorem 3.4. *In a space $(U, \mathcal{N}, I)$, the following properties are equivalent for a n-open subset A of U.*

(1) *A is $n\star$-closed.*
(2) *A is $\mathcal{Q}$-nI-closed.*
(3) *A is $\mathcal{L}$-nI_g-closed.*

Proof. (1) $\Rightarrow$ (2): Since A is $n\star$-closed and n-open,
$A = n\text{-}cl^\star(A) = n\text{-}cl^\star(n\text{-}int(A))$.
Hence A is $\mathcal{Q}$-nI-closed.

(2) $\Rightarrow$ (3): Since A is $\mathcal{Q}$-nI-closed, $A = n\text{-}cl^\star(n\text{-}int(A)) = n\text{-}cl^\star(A)$. Thus A is $n\star$-closed and hence $A_n^\star \subseteq A$. If $A \subseteq B$ where B is n-open, then $(n\text{-}int(A))_n^\star \subseteq A_n^\star \subseteq A \subseteq B$, which proves that A is $\mathcal{L}$-nI_g-closed.

(3) $\Rightarrow$ (1): $A \subseteq A$ where A is n-open. Since A is $\mathcal{L}$-nI_g-closed, $(n\text{-}int(A))_n^\star \subseteq A$. This implies $A_n^\star \subseteq A$ for A is n-open. Hence A is $n\star$-closed. □

THEOREM 3.5. *In a space $(U, \mathcal{N}, I)$,*

(1) *a n-nowhere dense subset of U is $\mathcal{S}$-nI_g-closed.*
(2) *a n-nowhere dense subset of U is $\mathcal{L}$-nI_g-closed.*

PROOF. (1) Let A be a n-nowhere dense subset of U. Then $n\text{-}int(n\text{-}cl(A)) = \phi$ and $n\text{-}int(A) \subseteq n\text{-}int(n\text{-}cl(A)) = \phi \Rightarrow n\text{-}int(A) = \phi$. If $A \subseteq B$ where B is ng-open then $(n\text{-}int(A))_n^\star = \phi_n^\star = \phi \subseteq B$. Thus A is $\mathcal{S}$-nI_g-closed.

(2) If A is n-nowhere dense, by (1) A is $\mathcal{S}$-nI_g-closed, which implies A is $\mathcal{L}$-nI_g-closed. □

REMARK 3.5. The converses of (1) and (2) in Theorem 3.5 are not true as shown in the following Example.

EXAMPLE 3.4. (1) In (2) of Example 3.1, $B = \{e_2, e_4\}$ is $\mathcal{S}$-nI_g-closed but not n-nowhere dense for $n\text{-}int(n\text{-}cl(B)) = n\text{-}int(\{e_2, e_3, e_4, e_5\}) = \{e_2, e_3\} \neq \phi$.

(2) By (1) B is $\mathcal{L}$-nI_g-closed but not n-nowhere dense.

THEOREM 3.6. *In a space $(U, \mathcal{N}, I)$ the family of $\mathcal{R}$-nI_g-closed subsets and the family of n-nowhere dense subsets are independent.*

EXAMPLE 3.5. In Example 3.1,

(1) $A = \{e_1\}$ is $\mathcal{R}$-nI_g-closed for $A_n^\star = \{e_1\}_n^\star = \phi$. But A is not n-nowhere dense for $n\text{-}int(n\text{-}cl(A)) = n\text{-}int(\{e_1, e_4, e_5\}) = \{e_1\} \neq \phi$.
(2) $B = \{e_5\}$ is n-nowhere dense for $n\text{-}int(n\text{-}cl(B)) = n\text{-}int(\{e_4, e_5\}) = \phi$. But $B \subseteq \{e_2, e_5\}$ which is ng-open and $B_n^\star = \{e_4, e_5\} \nsubseteq \{e_2, e_5\}$. Hence B is not $\mathcal{R}$-nI_g-closed.

4. Some more properties of $\mathcal{L}$-nI_g, $\mathcal{S}$-nI_g and $\mathcal{R}$-nI_g-closed subsets

Already we have given some properties of these sets related to other subsets in section 3. Here we discuss some more properties of these sets.

THEOREM 4.1. *In a space $(U, \mathcal{N}, I)$ a subset A of U is $\mathcal{R}$-nI_g-closed $\iff$ $n\text{-}cl^\star(A) \subseteq G$ whenever $A \subseteq G$ and G is ng-open.*

PROOF. Necessary part. Let $A \subseteq G$ where G is ng-open. A is $\mathcal{R}$-nI_g-closed implies $A_n^\star \subseteq G$. Thus $A \subseteq G$ and $A_n^\star \subseteq G$ imply $n\text{-}cl^\star(A) = A \cup A_n^\star \subseteq G$. This proves the necessary part.

Sufficient part. If $A \subseteq G$ where G is ng-open, then by assumption $n\text{-}cl^\star(A) \subseteq G$. Thus $A \cup A_n^\star \subseteq G$ which implies $A_n^\star \subseteq G$. Hence A is $\mathcal{R}$-nI_g-closed which proves the sufficiency part. □

THEOREM 4.2. *A subset A of U in a space $(U, \mathcal{N}, I)$ is $\mathcal{S}$-nI_g-closed $\iff$ $n\text{-}cl^\star(n\text{-}int(A)) \subseteq G$ whenever $A \subseteq G$ and G is ng-open.*

PROOF. Similar to the proof of Theorem 4.1. □

THEOREM 4.3. *Let A be a subset of U in a space $(U, \mathcal{N}, I)$. Then A is $\mathcal{L}$-nI_g-closed $\iff$ n-$cl^\star(n\text{-}int(A)) \subseteq G$ whenever $A \subseteq G$ and G is n-open.*

PROOF. Similar to the proof of Theorem 4.1. □

THEOREM 4.4. *If a subset A of U in a space $(U, \mathcal{N}, I)$ is $\mathcal{R}$-nI_g-closed and B is a subset such that $A \subseteq B \subseteq A_n^\star$, then B is $\mathcal{R}$-nI_g-closed.*

PROOF. Let G be a subset of U such that $B \subseteq G$. Then $A \subseteq B \subseteq A_n^\star$ implies $A \subseteq G$. Since by assumption A is $\mathcal{R}$-nI_g-closed, $A_n^\star \subseteq G$. But $A \subseteq B \subseteq A_n^\star$ implies $B_n^\star \subseteq ((A_n^\star))_n^\star \subseteq A_n^\star$. Thus $A_n^\star \subseteq G$ implies $B_n^\star \subseteq G$ which proves that B is $\mathcal{R}$-nI_g-closed. □

THEOREM 4.5. *If A and B are subsets of U in a space $(U, \mathcal{N}, I)$ such that A is $\mathcal{L}$-nI_g-closed and $A \subseteq B \subseteq (n\text{-}int(A))_n^\star$, then B is $\mathcal{L}$-nI_g-closed.*

PROOF. Let $B \subseteq H$ where H is n-open. By assumption $A \subseteq B \subseteq (n\text{-}int(A))_n^\star$ implies $A \subseteq H$. Since A is $\mathcal{L}$-nI_g-closed $(n\text{-}int(A))_n^\star \subseteq H$. Again
$A \subseteq B \subseteq (n\text{-}int(A))_n^\star$ implies $B_n^\star \subseteq ((n\text{-}int(A))_n^\star)_n^\star \subseteq (n\text{-}int(A))_n^\star \subseteq H$.
Hence $(n\text{-}int(B))_n^\star \subseteq B_n^\star \subseteq H$ which proves that B is $\mathcal{L}$-nI_g-closed. □

THEOREM 4.6. *If A and B are subsets of U in a space $(U, \mathcal{N}, I)$ such that A is $\mathcal{S}$-nI_g-closed and $A \subseteq B \subseteq (n\text{-}int(A))_n^\star$, then B is $\mathcal{S}$-nI_g-closed.*

PROOF. Similar to the proof of Theorem 4.5. □

THEOREM 4.7. *The union of two $\mathcal{R}$-nI_g-closed subsets of U in a space $(U, \mathcal{N}, I)$ is $\mathcal{R}$-nI_g-closed.*

PROOF. Let A and B be $\mathcal{R}$-nI_g-closed in $(U, \mathcal{N}, I)$. If $A \cup B \subseteq G$ where G is ng-open, then $A \subseteq G$ and $B \subseteq G$. Since A and B are $\mathcal{R}$-nI_g-closed, $A_n^\star \subseteq G$ and $B_n^\star \subseteq G$. Hence $(A \cup B)_n^\star = A_n^\star \cup B_n^\star \subseteq G$ which verifies that $A \cup B$ is $\mathcal{R}$-nI_g-closed. □

REMARK 4.1. In a space $(U, \mathcal{N}, I)$, the union of two $\mathcal{L}$-nI_g-closed subsets of U is not generally $\mathcal{L}$-nI_g-closed as seen in the following Example.

EXAMPLE 4.1. In Example 3.1, $A = \{e_2\}$ is $\mathcal{L}$-nI_g-closed for if K is any n-open subset such that $A \subseteq K$, then $(n\text{-}int(A))_n^\star = \phi_n^\star = \phi \subseteq K$. Similarly $B = \{e_3\}$ is also $\mathcal{L}$-nI_g-closed. $A \cup B = \{e_2, e_3\}$is n-open and $A \cup B \subseteq \{e_2, e_3\}$. But $(n\text{-}int(A \cup B))_n^\star = \{e_2, e_3\}_n^\star = \{e_2, e_3, e_4, e_5\} \nsubseteq \{e_2, e_3\}$. This verifies that $A \cup B$ is not $\mathcal{L}$-nI_g-closed.

REMARK 4.2. In a space $(U, \mathcal{N}, I)$, the union of two $\mathcal{S}$-nI_g-closed subsets of U is not in general $\mathcal{S}$-nI_g-closed as illustrated in the following Example.

EXAMPLE 4.2. In Example 4.1, $A = \{e_2\}$ and $B = \{e_3\}$ are $\mathcal{S}$-nI_g-closed for $(n\text{-}int(A))_n^\star = \phi = (n\text{-}int(B))_n^\star$. Also $A \cup B = \{e_2, e_3\}$ is ng-open being n-open. But $(n\text{-}int(A))_n^\star = \{e_2, e_3\}_n^\star = \{e_2, e_3, e_4, e_5\} \nsubseteq \{e_2, e_3\}$ which proves that $A \cup B$ is not $\mathcal{S}$-nI_g-closed.

REMARK 4.3. In a space $(U, \mathcal{N}, I)$, the intersection of two $\mathcal{L}$-nI_g-closed subsets of U is not generally $\mathcal{L}$-nI_g-closed as shown in the following Example.

EXAMPLE 4.3. In Example 4.2, $A = \{e_2, e_3, e_4\}$ and $B = \{e_2, e_3, e_5\}$ are $\mathcal{L}$-nI_g-closed for $U$is the only n-open set containing A as well as B. $A \cap B = \{e_2, e_3\}$ and $A \cap B \subseteq \{e_2, e_3\}$ where $\{e_2, e_3\}$ is n-open. But $(n\text{-}int(A \cap B))^\star_n = \{e_2, e_3\}^\star_n = \{e_2, e_3, e_4, e_5\} \nsubseteq \{e_2, e_3\}$. Hence $A \cap B$ is not $\mathcal{L}$-nI_g-closed.

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