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FURTHER RESULTS ON ODD LABELING OF SOME SPLITTING GRAPHS

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Abstract. Let $G = (V, E)$ be a graph with p vertices and q edges. A graph G is said to have an odd mean labeling if there exists a function $f : V(G) \to \{0, 1, 2, \ldots, 2q - 1\}$ satisfying f is $1 - 1$ and the induced map $f^* : E(G) \to \{1, 3, 5, \ldots, 2q - 1\}$ defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

is a bijection. A graph that admits an odd mean labeling is called an odd mean graph. In this paper, we have studied the odd mean labeling of some splitting graphs.

1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notations and terminology we follow [**3**].

Path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. $K_{1,m}$ is called a star and it is denoted by S_m. The bistar $B_{m,n}$ is the graph obtained from K_2 by identifying the center vertices of $K_{1,m}$ and $K_{1,n}$ at the end vertices of K_2 respectively. $B_{m,m}$ is often denoted by $B(m)$. The H-graph denoted by H_n, is the graph obtained from two copies of P_n with vertices $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even. If m number of pendant vertices are attached at each vertex of G, then the resultant graph obtained from G is the graph $G \odot mK_1$. When $m = 1$, $G \odot K_1$ is the corona of G.

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The graph obtained by attaching m pendant vertices to each vertex of a path of length $2n-1$ is denoted by $B(m)_{(n)}$. The slanting ladder SL_n is a graph obtained from two paths $u_1u_2u_3\dots u_n$ and $v_1v_2v_3\dots v_n$ by joining each u_i with $v_{i+1}, 1 \leqslant i \leqslant n-1$.

The splitting graph $S(G)$ was introduced by Sampathkumar and Walikar [**7**]. For each vertex v of a graph G, take a new vertex v' and join v' to all the vertices of G adjacent to v. The resulting graph is the splitting graph of G, denoted by $S(G)$.

The graceful labeling of graphs was first introduced by Rosa in 1961 [**1**] and R. B. Gnanajothi introduced odd graceful graphs [**2**]. The concept of mean labeling was first introduced and studied by S. Somasundaram and R. ponraj [**8**]. Further some more results on mean graphs are discussed in [**5, 6, 9, 10**]. The concept of odd mean labeling was introduced and studied by K. Manickam and M. Marudai [**4**]. Also, odd mean property for some graphs are discussed in [**11, 12**].

A graph G is said to have an odd mean labeling if there exists a function $f : V(G) \to \{0, 1, 2, \dots, 2q-1\}$ satisfying f is $1-1$ and the induced map $f^* : E(G) \to \{1, 3, 5, \dots, 2q-1\}$ defined by

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is a bijection. A graph that admits an odd mean labeling is called an odd mean graph.

For example, an odd mean labeling of SL_5 is shown in Figure 1.

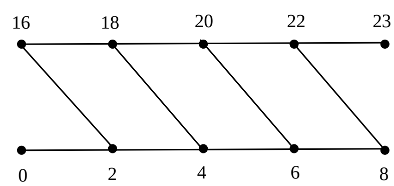

Figure 1.

In this paper, we prove that the splitting graph of H-graph, $H_n \odot K_1$ for $n \geqslant 2$ and $B(m)_{(n)}$ for $m \geqslant 1, n \geqslant 1$ are odd mean graphs.

2. Odd Mean Graphs

THEOREM 2.1. *$S'(H_n)$ is an odd mean graph.*

PROOF. Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ be the vertices of the H-graph H_n. Let $V(H_n)$ together with $u'_1, u'_2, \dots, u'_n$ and $v'_1, v'_2, \dots, v'_n$ form the vertex set of $S'(H_n)$ and the edge set of $S'(H_n)$ is $E(H_n)$ together with $\{u_iu_{i+1}, v_iv_{i+1}, u'_iu_{i+1}, v'_iv_{i+1},\ u_iu'_{i+1}, v_iv'_{i+1} : 1 \leqslant i \leqslant n-1\} \cup \Big\{u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}, u'_{\frac{n+1}{2}}v_{\frac{n+1}{2}}, u_{\frac{n+1}{2}}v'_{\frac{n+1}{2}}$ $\Big(u_{\frac{n}{2}+1}v_{\frac{n}{2}}, u'_{\frac{n}{2}+1}v_{\frac{n}{2}}, u_{\frac{n}{2}+1}v'_{\frac{n}{2}}\Big) : n \text{ } is \text{ } odd \text{ } (even)\Big\}$.

Case (i). n is odd.

Define $f : V(G) \to \{0, 1, 2, \ldots, 12n-7\}$ as follows:

$$f(u_i) = \begin{cases} 6i-2, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 6i-10, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$
$$f(u_i') = 6i-6, \quad 1 \leqslant i \leqslant n,$$
$$f(v_i) = \begin{cases} 6n+6i-10, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 6n+6i-2, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$
$$f(v_i') = 6n+6i-6, \quad 1 \leqslant i \leqslant n-1$$
$$\text{and } f(v_n') = 12n-7.$$

The induced edge labeling f^* is obtained as follows:

$$f^*(u_iu_{i+1}) = 6i-3, \quad 1 \leqslant i \leqslant n-1,$$
$$f^*(u_i'u_{i+1}) = \begin{cases} 6i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 6i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$
$$f^*(u_iu_{i+1}') = \begin{cases} 6i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 6i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$
$$f^*(v_iv_{i+1}) = 6n+6i-3, \quad 1 \leqslant i \leqslant n-1,$$
$$f^*(v_i'v_{i+1}) = \begin{cases} 6n+6i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 6n+6i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$
$$f^*(v_iv_{i+1}') = \begin{cases} 6n+6i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 6n+6i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$
$$f^*\left(u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\right) = 6n-3,$$
$$f^*\left(u'_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\right) = \begin{cases} 6n-5, & \frac{n+1}{2} \text{ is odd} \\ 6n-1, & \frac{n+1}{2} \text{ is even} \end{cases}$$
$$\text{and } f^*\left(u_{\frac{n+1}{2}}v'_{\frac{n+1}{2}}\right) = \begin{cases} 6n-1, & \frac{n+1}{2} \text{ is odd} \\ 6n-5, & \frac{n+1}{2} \text{ is even.} \end{cases}$$

Thus, f is an odd mean labeling of $S'(H_n)$. Hence, $S'(H_n)$ is an odd mean graph. For example, an odd mean labeling of $S'(H_7)$ and $S'(H_9)$ are shown in Figure 2.

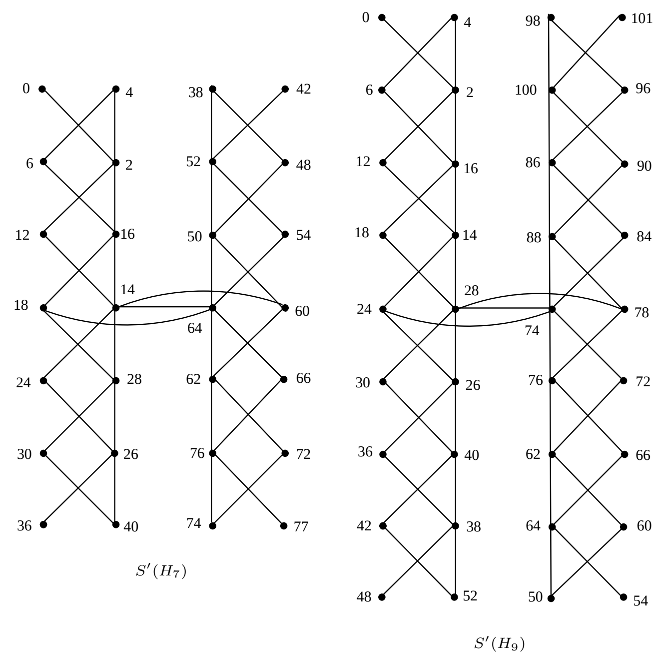

Figure 2. An odd mean labeling of $S'(H_7)$ and $S'(H_9)$.

Case (ii). n is even.
Define $f : V(G) \to \{0, 1, 2, \ldots, 12n-7\}$ as follows:

$$f(u_i) = \begin{cases} 6i-2, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 6i-10, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f(u_i') = 6i-6, \quad 1 \leqslant i \leqslant n,$$

$$f(v_i) = \begin{cases} 6n+6i-2, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 6n+6i-10, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_i') = 6n+6i-6, \quad 1 \leqslant i \leqslant n-1$$

$$\text{and } f(v_n') = 12n-7.$$

The induced edge labeling f^* is obtained as follows:

$$f^*(u_i u_{i+1}) = 6i-3, \quad 1 \leqslant i \leqslant n-1,$$

$$f^*(u_i' u_{i+1}) = \begin{cases} 6i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 6i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(u_i u_{i+1}') = \begin{cases} 6i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 6i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(v_iv_{i+1}) = 6n + 6i - 3, \quad 1 \leqslant i \leqslant n-1,$$

$$f^*(v_i'v_{i+1}) = \begin{cases} 6n+6i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 6n+6i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(v_iv_{i+1}') = \begin{cases} 6n+6i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 6n+6i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$

$$f^*\left(u_{\frac{n}{2}+1}v_{\frac{n}{2}}\right) = 6n-3,$$

$$f^*\left(u'_{\frac{n}{2}+1}v_{\frac{n}{2}}\right) = \begin{cases} 6n-1, & \frac{n}{2} \text{ is odd} \\ 6n-5, & \frac{n}{2} \text{ is even} \end{cases}$$

$$\text{and } f^*\left(u_{\frac{n}{2}+1}v'_{\frac{n}{2}}\right) = \begin{cases} 6n-5, & \frac{n}{2} \text{ is odd} \\ 6n-1, & \frac{n}{2} \text{ is even.} \end{cases}$$

Thus, f is an odd mean labeling of $S'(H_n)$. Hence, $S'(H_n)$ is an odd mean graph. For example, an odd mean labeling of $S'(H_6)$ and $S'(H_8)$ are shown in Figure 3.

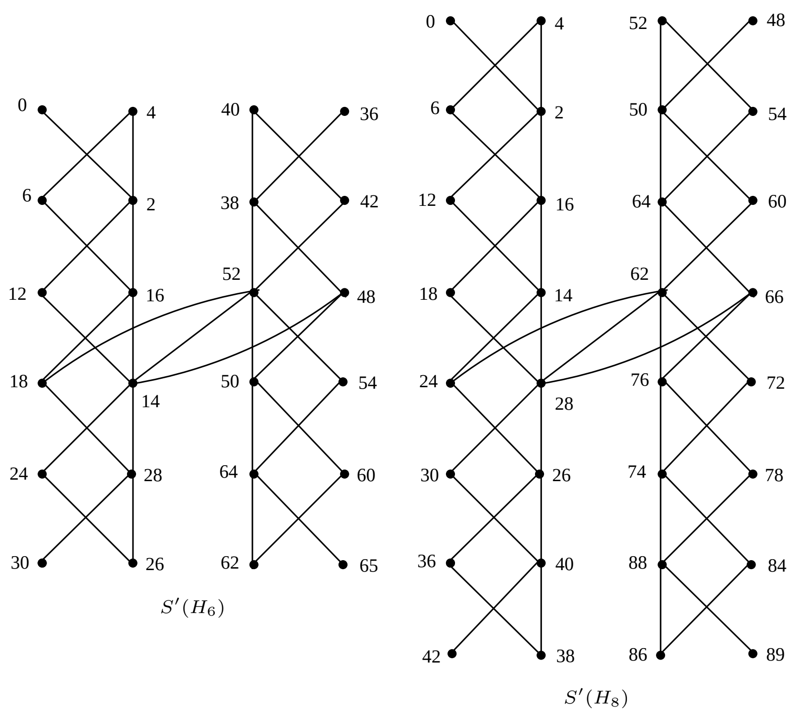

Figure 3. An odd mean labeling of $S'(H_6)$ and $S'(H_8)$.

□

THEOREM 2.2. *The splitting graph of* $H_n \odot K_1$ *is an odd mean graph for* $n \geqslant 2$.

PROOF. Let $u_i, v_i, x_i, y_i : 1 \leqslant i \leqslant n$ be the vertices of $H_n \odot K_1$. Let u'_i, v'_i, x'_i, y'_i $(1 \leqslant i \leqslant n)$ be the new vertices corresponding to $u_i, v_i, x_i, y_i (1 \leqslant i \leqslant n)$ respectively.

Then, $V(S'(H_n \odot K_1)) = V(H_n \odot K_1) \cup \{u'_i, v'_i, x'_i, y'_i : 1 \leqslant i \leqslant n\}$ and $E(S'(H_n \odot K_1)) = E(H_n \odot K_1) \cup \{u_i u'_{i+1}, u'_i u_{i+1}, v_i v'_{i+1}, v'_i v_{i+1} : 1 \leqslant i \leqslant n-1\} \cup \{u'_i x_i, u_i x'_i, v'_i y_i, v_i y'_i : 1 \leqslant i \leqslant n\} \cup \left\{ u_{\frac{n+1}{2}} v'_{\frac{n+1}{2}}, u'_{\frac{n+1}{2}} v_{\frac{n+1}{2}} \left(u_{\frac{n+1}{2}} v'_{\frac{n}{2}}, u'_{\frac{n+1}{2}} v_{\frac{n}{2}} \right) : n \textit{ is odd } (n \textit{ is even}) \right\}$.

The graph $S'(H_n \odot K_1)$ has $8n$ vertices and $12n-3$ edges.

Case (i). n is odd.

Define $f : V(S'(H_n \odot K_1)) \to \{0, 1, 2, \ldots, 2q-1 = 24n-7\}$ as follows:

$$f(u_i) = \begin{cases} 16n + 4i - 6, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 4i - 4, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even}, \end{cases}$$

$$f(x_i) = \begin{cases} 4i - 4, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 16n + 4i - 6, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even}, \end{cases}$$

$$f(u'_i) = \begin{cases} 4i - 2, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 16n + 4i - 8, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even}, \end{cases}$$

$$f(x'_i) = \begin{cases} 16n + 4i - 8, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 4i - 2, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even}, \end{cases}$$

$$f(v_i) = \begin{cases} 4n + 4i - 4, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 20n + 4i - 6, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even}, \end{cases}$$

$$f(y_i) = \begin{cases} 20n + 4i - 6, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 4n + 4i - 4, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even}, \end{cases}$$

$$f(y_n) = 24n - 7,$$

$$f(v'_i) = \begin{cases} 20n + 4i - 8, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 4n + 4i - 2, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even} \end{cases}$$

$$\text{and } f(y'_i) = \begin{cases} 4n + 4i - 2, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 20n + 4i - 8, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even}. \end{cases}$$

The induced edge labeling f^* is obtained as follows:

$$f^*(u_i u_{i+1}) = 8n + 4i - 3, \quad 1 \leqslant i \leqslant n-1,$$

$$f^*(u_i u'_{i+1}) = \begin{cases} 16n + 4i - 5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 4i - 1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even}, \end{cases}$$

$$f^*(u'_i u_{i+1}) = \begin{cases} 4i - 1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 16n + 4i - 5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even}, \end{cases}$$

$$f^*(u_i x_i) = 8n + 4i - 5, \quad 1 \leqslant i \leqslant n,$$

$$f^*(x_i u'_i) = \begin{cases} 4i - 3, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 16n + 4i - 7, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even}, \end{cases}$$

$$f^*(u_ix_i') = \begin{cases} 16n+4i-7, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 4i-3, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(v_iv_{i+1}) = 12n+4i-3, \quad 1 \leqslant i \leqslant n-1,$$

$$f^*(v_iv_{i+1}') = \begin{cases} 4n+4i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 20n+4i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(v_i'v_{i+1}) = \begin{cases} 20n+4i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 4n+4i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(v_iy_i) = 12n+4i-5, \quad 1 \leqslant i \leqslant n,$$

$$f^*(v_i'y_i) = \begin{cases} 20n+4i-7, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 4n+4i-3, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(v_iy_i') = \begin{cases} 4n+4i-3, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 20n+4i-7, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f^*\left(u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\right) = 12n-3,$$

$$f^*\left(u'_{\frac{n+1}{2}}v_{\frac{n+1}{2}}\right) = \begin{cases} 4n-1 & \text{if } \frac{n+1}{2} \text{ is odd} \\ 20n-5 & \text{if } \frac{n+1}{2} \text{ and } i \text{ is even} \end{cases}$$

$$\text{and } f^*\left(u_{\frac{n+1}{2}}v'_{\frac{n+1}{2}}\right) = \begin{cases} 20n-5 & \text{if } \frac{n+1}{2} \text{ is odd} \\ 4n-1 & \text{if } \frac{n+1}{2} \text{ and } i \text{ is even.} \end{cases}$$

Thus, f is an odd mean labeling of $S'(H_n \odot K_1)$. Hence, $S'(H_n \odot K_1))$ is an odd mean graph. For example, an odd mean labeling of $S'(H_7 \odot K_1))$ is shown in Figure 4.

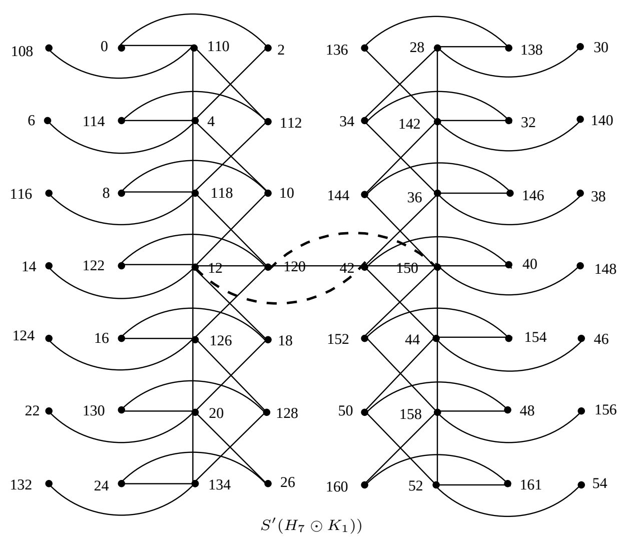

Figure 4. An odd mean labeling of $S'(H_7 \odot K_1))$

Case (ii). n is even.

Define $f : V(S'(H_n \odot K_1)) \to \{0, 1, 2, \ldots, 2q-1 = 24n-7\}$ as follows:

$$f(u_i) = \begin{cases} 4i-2, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 16n+4i-8, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f(x_i) = \begin{cases} 16n+4i-8, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 4i-2, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f(u_i') = \begin{cases} 16n+4i-6, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 4i-4, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f(x_i') = \begin{cases} 4i-4, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 16n+4i-6, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_i) = \begin{cases} 4n+4i-2, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 20n+4i-8, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f(y_i) = \begin{cases} 20n+4i-8, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 4n+4i-2, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_i') = \begin{cases} 20n+4i-6, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 4n+4i-4, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f(y_i') = \begin{cases} 4n+4i-4, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 20n+4i-6, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even} \end{cases}$$

and $f(y_n') = 24n-7$.

The induced edge labeling f^* is obtained as follows:

$$f^*(u_iu_{i+1}) = 8n+4i-3, \quad 1 \leqslant i \leqslant n-1,$$

$$f^*(u_iu_{i+1}') = \begin{cases} 4i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 16n+4i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(u_i'u_{i+1}) = \begin{cases} 16n+4i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 4i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(u_ix_i) = 8n+4i-5, \quad 1 \leqslant i \leqslant n,$$

$$f^*(x_iu_i') = \begin{cases} 16n+4i-7, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 4i-3, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(u_ix_i') = \begin{cases} 4i-3, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 16n+4i-7, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(v_iv_{i+1}) = 12n+4i-3, \quad 1 \leqslant i \leqslant n-1,$$

$$f^*(v_iv_{i+1}') = \begin{cases} 4n+4i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 20n+4i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(v_i'v_{i+1}) = \begin{cases} 20n+4i-5, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is odd} \\ 4n+4i-1, & 1 \leqslant i \leqslant n-1 \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(v_iy_i) = 12n+4i-5, \quad 1 \leqslant i \leqslant n,$$

$$f^*(v_i'y_i) = \begin{cases} 20n+4i-7, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 4n+4i-3, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(v_iy_i') = \begin{cases} 4n+4i-3, & 1 \leqslant i \leqslant n \text{ and } i \text{ is odd} \\ 20n+4i-7, & 1 \leqslant i \leqslant n \text{ and } i \text{ is even,} \end{cases}$$

$$f^*\left(u_{\frac{n}{2}+1}v_{\frac{n}{2}}\right) = 12n-3,$$

$$f^*\left(u'_{\frac{n}{2}+1}v_{\frac{n}{2}}\right) = \begin{cases} 4n-1 & \text{if } \frac{n}{2} \text{ is odd} \\ 20n-5 & \text{if } \frac{n}{2} \text{ and } i \text{ is even} \end{cases}$$

$$\text{and } f^*\left(u_{\frac{n}{2}+1}v'_{\frac{n}{2}}\right) = \begin{cases} 20n-5 & \text{if} \frac{n}{2} \text{ is odd} \\ 4n-1 & \text{if } \frac{n}{2} \text{ and } i \text{ is even.} \end{cases}$$

Thus, f is an odd mean labeling of $S'(H_n \odot K_1)$. Hence, $S'(H_n \odot K_1))$ is an odd mean graph. For example, an odd mean labeling of $S'(H_6 \odot K_1))$ is shown in Figure 5.

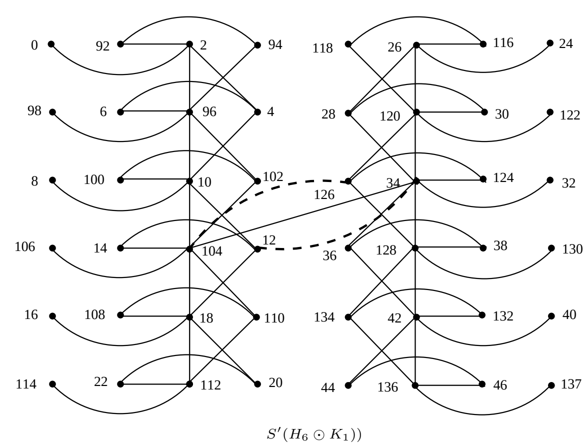

Figure 5. An odd mean labeling of $S'(H_6 \odot K_1))$.

□

THEOREM 2.3. *The graph $S'(B(m)_{(n)})$ is an odd mean graph.*

PROOF. Let $u_i, v_i, u_{ij}, v_{ij} : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m$ be the vertices of $B(m)_{(n)}$ and $u_i', v_i', u_{ij}', v_{ij}' : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m$ be the vertices corresponding to u_i, v_i, u_{ij}, v_{ij} of $B(m)_{(n)}$ which are added to obtain $S'(B(m)_{(n)})$.

Then $V(S'(B(m)_{(n)}) = \{u_i, v_i, u_{ij}, v_{ij}, u_i', v_i', u_{ij}'v_{ij}' : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m\}$ and $E(S'(B(m)_{(n)}) = \{u_iv_i, v_iu_{i+1}, u_iu_{ij}, v_iv_{ij}, u_iv_i', v_iu_i', u_iu_{ij}', v_iv_{ij}', v_i'u_{i+1}, v_iu_{i+1}', u_{ij}u_i', v_{ij}v_i' : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m\}$.

The graph $S'(B(m)_{(n)})$ has $4(m+1)n$ vertices and $6(m+1)n-3$ edges.

Define $f : V(S'(B(m)_{(n)}) \to \{0, 1, 2, 3, \ldots, 2q-1 = 12(m+1)n-7\}$ as follows:

For $1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m$,

$$\begin{aligned}
f(u_i) &= 4(m+1)i - 2(2m+1)\\
f(v_i) &= (8n+4i)(m+1) - 8\\
f(u_{ij}) &= (8n+4i-4)(m+1) + 4(j-2)\\
f(v_{ij}) &= 4(i-1)(m-1) + 8i - 2 + 4(j-1)\\
f(u'_i) &= (8n+4i-4)(m+1) - 2\\
f(v'_i) &= 4i(m+1) - 4\\
f(u'_{ij}) &= (4i-4)(m+1) + 4(j-1)\\
f(v'_{ij}) &= (8n+4i-4)(m+1) + 4(j-1) + 2 \text{ and}\\
f(v'_{nm}) &= 12(m+1)n - 7.
\end{aligned}$$

The induced edge labeling f^* is obtained as follows:

For $1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m$,

$$\begin{aligned}
f^*(u_iv_i) &= 4(n+i-1)(m+1) + 2m - 1\\
f^*(u_iu_{ij}) &= 4(n+i-1)(m+1) + 2(j-1) - 1\\
f^*(v_iv_{ij}) &= 4(n+i-1)(m+1) + 2m + 1 + 2(j-1)\\
f^*(u_iv'_i) &= 4(m+1)i - 2(m+2) + 1\\
f^*(v_iu'_i) &= (8n+4i-4)(m+1) + 2m - 3\\
f^*(u_iu'_{ij}) &= 4(m+1)i - (4m+3) + 2(j-1)\\
f^*(v_iv'_{ij}) &= (8n+4i-4)(m+1) + 2m - 1 + 2(j-1)
\end{aligned}$$

$$\begin{aligned}
f^*(u_{ij}u'_i) &= (8n+4i-4)(m+1) - 3 + 2(j-1)\\
f^*(v_{ij}v'_i) &= 4(m+1)i - (2m+1) + 2(j-1)
\end{aligned}$$

For $1 \leqslant i \leqslant n-1, 1 \leqslant j \leqslant m$,

$$\begin{aligned}
f^*(v_iu_{i+1}) &= 4(n+i-1)(m+1) + 4m + 1\\
f^*(v'_iu_{i+1}) &= 4(i-1)(m+1) + 4m + 3\\
f^*(v_iu'_{i+1}) &= (8n+4i-4)(m+1) + 4m - 1.
\end{aligned}$$

Thus, f is an odd mean labeling of $S'(B(m)_{(n)})$. Hence $S'(B(m)_{(n)})$ is an odd mean graph. For example, an odd mean labeling of $S'(B(4)_{(3)}$ is shown in Figure 6. □

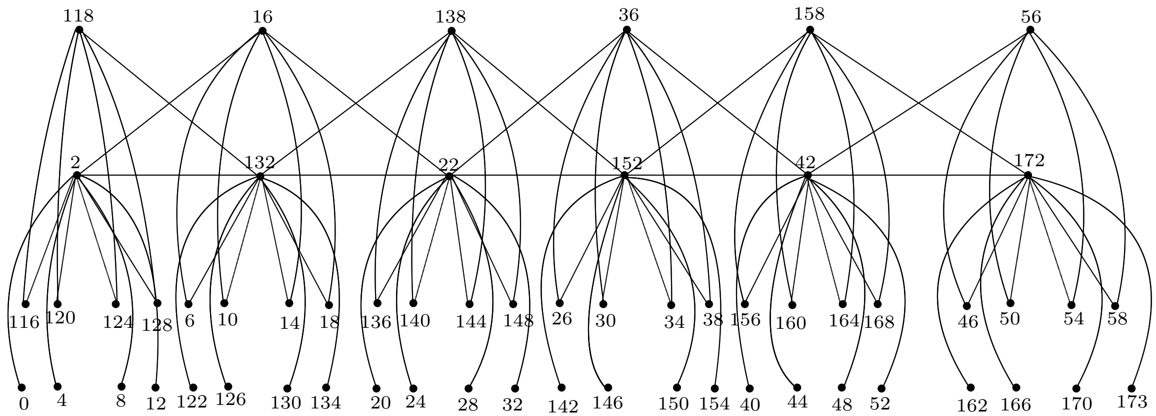

Figure 6. An odd mean labeling of $S'(B(4)_{(3)})$

COROLLARY 2.1. *$S'(B(m))$ is an odd mean graph.*

PROOF. It follows from Theorem 2.3. □

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