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ON THE TOTAL IRREGULARITY STRENGTH OF SOME GRAPHS

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ABSTRACT. *A totally irregular total k-labeling* $f : V \cup E \to \{1, 2, 3, \ldots, k\}$ is a labeling of vertices and edges of G in such a way that for any two different vertices x and y their vertex-weights $wt_h(x) \neq wt_h(y)$ where the vertex-weight $wt_h(x) = h(x) + \sum_{xy \in E} h(xz)$ and also for every two different edges xy and $x'y'$ of G their edge-weights $wt_h(xy) = h(x) + h(xy) + h(y)$ and $wt_h(x'y') = h(x') + h(x'y') + h(y')$ are distinct. A total irregularity strength of graph G, denoted by $ts(G)$ is defined as the minimum k for which a graph G has a totally irregular total k-labeling. In this paper, we investigate double fan, double triangular snake, joint-wheel and $P_m + \overline{K_m}$ whose *total irregularity strength* equals to the lower bound.

1. Introduction

Let G be a finite, simple and undirected graph with the vertex set V and edge set E. A labeling of a graph G is a mapping that carries a set of graph elements into a set of numbers (usually to positive or non-negative integer). If the domain of mapping is a vertex set, or an edge set or a union of vertex and edge set, then the labeling is called *vertex labeling or edge labeling or total labeling* respectively. Bača et al. [**3**] introduced an edge irregular total labeling and a vertex irregular total labeling. They determined the total edge irregular strength (tes) and total vertex irregular strength(tvs) of some certain graphs. Also, they obtained the exact values of the tes of path, cycle, star, wheel and friendship graph. Ivančo and Jendroľ [**5**]

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proved that

$$tes(G) \geqslant max\left\{ \left\lceil \frac{(|E(G)|+2)}{3} \right\rceil, \left\lceil \frac{(\Delta(G)+1)}{2} \right\rceil \right\}. \tag{1.1}$$

We found [**6, 7, 8, 11, 12**] the total edge irregularity strength of closed helm graph CH_n and flower graph Fl_n, the disjoint union of wheel graphs, double wheel graphs, armed crown graph, splitting graph, tadpole graph. In [**4**] Hungund, Akka determined Total edge irregularity strength of triangular snake and double triangular snake.

$$tes(G_p^{'}) = 2p+1, p \geqslant 1. \tag{1.2}$$

Nurdin et al. [**16**] determined the lower bound of tvs for any graph G.

Theorem 1.1 ([**16**]). *Let G be a connected graph having n_i vertices of degree i $(i = \delta, \delta+1, \delta+2, ..., \Delta)$ where δ and Δ are the minimum and maximum degree of G, respectively. Then*

$$tvs(G) \geqslant max\left\{ \left\lceil \frac{\delta+n_\delta}{\delta+1} \right\rceil, \left\lceil \frac{\delta+n_\delta+n_{\delta+1}}{\delta+2} \right\rceil, \ldots, \left\lceil \frac{\delta+\sum\limits_{i=\delta}^{\Delta}(n_i)}{\Delta+1} \right\rceil \right\}. \tag{1.3}$$

Ahmad et al. [**1**] found the total vertex irregularity strength of helm graph H_n and flower graph Fl_n. We found [**9**] the total vertex irregularity strength of corona product of some graphs. Combining the ideas of vertex irregular total k-labeling and edge irregular total k-labeling, Marzuki et al. [**15**] introduced another irregular total k-labeling called the *totally irregular total k-labeling.* A labeling $h : V(G) \cup E(G) \to \{1, 2, 3, \ldots, k\}$ to be a *totally irregular totalk-labeling* of the graph G if for every two different vertices x and y the vertex-weights $wt_h(x) \neq wt_h(y)$ where the vertex-weight $wt_h(x) = h(x) + \sum\limits_{xz \in E} h(xz)$ and also for every two different edges xy and $x'y'$ of G the edge-weights $wt_h(xy) = h(x) + h(xy) + h(y)$ and $wt_h(x'y') = h(x') + h(x'y') + h(y')$ are distinct. The *total irregularity strength* $ts(G)$ is defined as the minimum k for which a graph G has a totally irregular total k-labeling. For the total irregularity strength of a graph G, they observed that

$$ts(G) \geqslant max\left\{tes(G), tvs(G)\right\}. \tag{1.4}$$

They also determined the total irregularity strength of cycles and paths.

Ramdani and Salman [**17**] obtained the total irregularity strength of some Cartesian product graphs. Ramdani et al. [**18**] determined the total irregularity strength of gear graph $G_n, n \geqslant 3$, fungus graph $Fg_n, n \geqslant 3$ and disjoint union of star $mS_n, n, m \geqslant 2$. Ali Ahmad et al. [**2**] obtained the total irregularity strength of generalized Petersen graph. Also, we found [**13, 14**] the total irregularity strength of wheel related graphs and disjoint union of crossed prism and necklace graphs. We use the following definitions in the subsequent section.

Definition 1.1. The graph $P_n + 2K_1$ is called a double fan DF_n.

DEFINITION 1.2. A double triangular snake DT_p is a graph formed by two triangular snake having a common path, that is a double triangular snake with p blocks is obtained from a path $v_0, v_1, ..., v_p$ by joining v_i and v_{i+1} to two new vertices v_{p+1+i} and v_{2p+1+i} for $i = 0, 1, .., p-1$.

DEFINITION 1.3. A Joint-wheel graph WH_n consists of two disjoint copies of wheel which are joined by an edge between two rim vertices. WH_n has $2n+2$ vertices and $4n+1$ edges, where n is the number of rim vertices in one copy of the wheel graph.

2. Main Results

In this section,we determine the total irregularity strength of double fan graph DF_n for $n \geqslant 3$, double triangular snake DT_p for $p \geqslant 3$, joint-wheel graph WH_n for $n \geqslant 3$ and $P_m + \overline{K_m}, m \geqslant 3$ whose total irregularity strength equals to the lower bound. In addition, we show that these graphs admit totally irregular total k-labeling. Further we determine the exact value of their ts.

THEOREM 2.1. *Let $n \geqslant 3$ and DF_n be a double fan graph with n+2 vertices and 3n-1 edges. Then $ts(DF_n) = n+1$.*

PROOF. Since $|V(DF_n)| = n+2$ and $|E(DF_n)| = 3n-1$. The vertex set $V(DF_n) = \{v_i, a, b : 1 \leqslant i \leqslant n\}$ and edge set $E(DF_n) = \{av_i, bv_i : 1 \leqslant i \leqslant n-1\} \cup \{v_iv_{i+1} : 1 \leqslant i \leqslant n\}$, by (1.1), (1.3) and (1.4) we have $ts(DF_n) \geqslant n+1$. For the reverse inequality, we define a total labeling $f : V \cup E \to \{1, 2, 3, \dots, n+1\}$ by considering the following two cases.

Case(i): $n = 5$
$f(b) = 6; f(a) = 1; f(av_1) = 1; f(av_2) = 2; f(av_3) = 3; f(av_4) = 4; f(av_5) = 5; f(bv_1) = f(bv_2) = f(bv_3) = f(bv_4) = 5; f(bv_5) = 6; f(v_1v_2) = f(v_2v_3) = f(v_3v_4) = f(v_4v_5) = 1.$

Case(ii): $n \geqslant 3; n \neq 5$
$f(v_i) = i, 1 \leqslant i \leqslant n; f(a) = 1; f(b) = n+1; f(v_iv_{i+1}) = 1,\ 1 \leqslant i \leqslant n-1; f(bv_i) = n,\ 1 \leqslant i \leqslant n; f(av_i) = i,\ 1 \leqslant i \leqslant n.$

We see that all the vertex and edge labels are at most $n+1$. Since

$$wt(av_i) = 2i+1,\ 1 \leqslant i \leqslant n; wt(v_iv_{i+1}) = 2i+2,\ 1 \leqslant i \leqslant n-1;$$
$$wt(bv_i) = 2n+1+i,\ 1 \leqslant i \leqslant n,$$

the weights of the edges under the labeling f are $\{3, 4, 5, \dots, 2n+1, 2n+2, 2n+3, \dots, 3n+1\}$. Thus the edge weights are pair-wise distinct.

The vertex weights are

$$wt(v_i) = \begin{cases} n+3, & if\ i = 1 \\ n+2i+2, & if\ 2 \leqslant i \leqslant n-1 \\ 3n+1, & if\ i = n; \end{cases}$$

$$wt(a) = \frac{n^2+n+2}{2};$$

$$wt(b) = n^2 + n + 1.$$

That is $\{n+3, n+6, n+8, ..., 3n, 3n+1, \frac{n^2+n+2}{2}, n^2+n+1\}$. Hence all the vertex weights are distinct .This labeling construction shows that $ts(DF_n) \leqslant n+1$. Combining this with the lower bound, we conclude that $ts(DF_n) = n+1$. This completes the proof. □

Figure 1 shows a totally irregular total labeling of double fan graph DF_8.

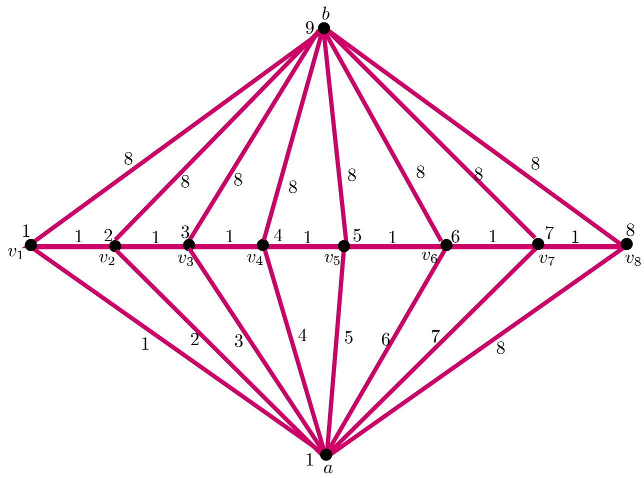

Figure1. $ts(DF_8) = 9$.

THEOREM 2.2. *Let $p \geqslant 2$ and DT_p be a double triangular graph. Then*

$$ts(DT_p) = 2p + 1.$$

PROOF. Let $V(DT_p) = \{v_i : 0 \leqslant i \leqslant p\} \cup \{w_i, w'_i : 0 \leqslant i \leqslant p-1\}$ and $E(DT_p) = \{v_iv_{i+1} : 0 \leqslant i \leqslant p\} \cup \{v_iw_i, w_iv_{i+1}, w'_iv_i, w'_iv_{i+1} : 0 \leqslant i \leqslant p-1\}$ by (1.1), (1.3) and (1.4) we have $ts(G) \geqslant 2p+1$. For the reverse inequality, we define a total labeling $f : V \cup E \to \{1, 2, 3, \ldots, 2p+1\}$ by considering the following two cases.

Case(i): p is odd.

$f(v_i) = i+1,\ 0 \leqslant i \leqslant p$; $f(w_i) = i+2,\ 0 \leqslant i \leqslant p-1$; $f(v_iw_i) = 1,\ 0 \leqslant i \leqslant p-1$; $f(w_iv_{i+1}) = 1,\ 0 \leqslant i \leqslant p-1$; $f(w'_iv_{i+1}) = 2p+1,\ 0 \leqslant i \leqslant p-1$; $f(w'_iv_i) = 2p+1,\ 0 \leqslant i \leqslant p-1$; $f(w'_i) = p+2+i,\ 0 \leqslant i \leqslant p-1$; $f(v_iv_{i+1}) = 2p+1-i,\ 0 \leqslant i \leqslant p-1$.

We see that all the vertex and edge labels are at most $2p+1$. The edge weights are

$wt(v_iw_i) = 4+2i,\ 1 \leqslant i \leqslant p-1$; $wt(w_iv_{i+1}) = 5+2i,\ 0 \leqslant i \leqslant p-1$; $wt(w'_iv_{i+1}) = 3p+5+2i,\ 0 \leqslant i \leqslant p-1$; $wt(w'_iv_i) = 3p+4+2i,\ 0 \leqslant i \leqslant p-1$; $wt(v_iv_{i+1}) = 2p+4+i,\ 0 \leqslant i \leqslant p-1$,

the weights of the edges under the labeling f are $\{4, 6, 8, \ldots, 2p+2, 5, 7, \ldots, 2p+3, 3p+5, 3p+7, \ldots, 5p+3, 3p+4, 3p+6, 3p+8, \ldots, 5p+2, 2p+4, 2p+5, \ldots, 3p+2, 3p+3\}$. Thus the edge weights are pair-wise distinct.

The vertex weights are

$$wt(w_i) = 4 + i, 0 \leqslant i \leqslant p - 1;$$

$$wt(v_i) = \begin{cases} 4p + 4, & if\ i = 0 \\ 4p + 5, & if\ i = p \\ 8p + 8 - i, & if\ 1 \leqslant i \leqslant p - 1; \end{cases}$$

$$wt(w_i') = 5p + 4 + i.$$

That is $\{4, 5, 6, ..., p+3, 4p+4, 4p+5, 8p+7, 8p+6, ..., 7p+9\}$. Hence all the vertex weights are distinct.

Case(ii): p is even.

$f(v_i) = i + 1,\ 0 \leqslant i \leqslant p$; $f(w_i) = 1,\ 0 \leqslant i \leqslant p - 1$; $f(w_i') = 2p + 1,\ 0 \leqslant i \leqslant p - 1$; $f(v_i w_i) = i + 1,\ 0 \leqslant i \leqslant p - 1$; $f(w_i v_{i+1}) = i + 1,\ 0 \leqslant i \leqslant p - 1$; $f(v_i v_{i+1}) = 2p - i,\ 0 \leqslant i \leqslant p-1$; $f(w_i' v_{i+1}) = p+1+i,\ 0 \leqslant i \leqslant p-1$; $f(w_i' v_i) = p+1+i,\ 0 \leqslant i \leqslant p-1$.

We see that all the vertex and edge labels are at most $2p + 1$. Since

$wt(v_i w_i) = 3 + 2i,\ 1 \leqslant i \leqslant p - 1$; $wt(w_i v_{i+1}) = 4 + 2i,\ 0 \leqslant i \leqslant p - 1$; $wt(v_i v_{i+1}) = 2p + 3 + i,\ 0 \leqslant i \leqslant p - 1$; $wt(w_i' v_i) = 3p + 3 + 2i,\ 0 \leqslant i \leqslant p - 1$; $wt(w_i' v_{i+1}) = 3p + 4 + 2i,\ 0 \leqslant i \leqslant p - 1$,

the weights of the edges under the labeling f are $\{3, 5, \ldots, 2p + 1, 4, 6, 8, \ldots, 2p + 2, 2p+3, 2p+4, \ldots, 3p+2, 3p+3, 3p+5, \ldots, 5p+1, 3p+4, 3p+6, 3p+8, \ldots, 5p+2\}$. Thus the edge weights are pair-wise distinct.

The vertex weights are

$$wt(w_i) = 3 + 2i, 0 \leqslant i \leqslant p - 1;$$

$$wt(v_i) = \begin{cases} 3p + 3, & if\ i = 0 \\ 5p + 2, & if\ i = p \\ 6p + 4 + 3i, & if\ 1 \leqslant i \leqslant p - 1; \end{cases}$$

$$wt(w_i') = 4p + 3 + 2i; 0 \leqslant i \leqslant p - 1.$$

That is $\{3, 5, 7, \ldots, 2p+1, 3p+3, 5p+2, 4p+3, 4p+5, 4p+7, \ldots, 6p+1, 6p+4, 6p+7, \ldots, 9p+1\}$. Hence all the vertex weights are distinct. This labeling construction shows that $ts(DT_p) \leqslant 2p + 1$.

Combining this with the lower bound, we conclude that $ts(DT_p) = 2p+1$. This completes the proof. □

Figure 2 shows a totally irregular total labeling of double triangular DT_6.

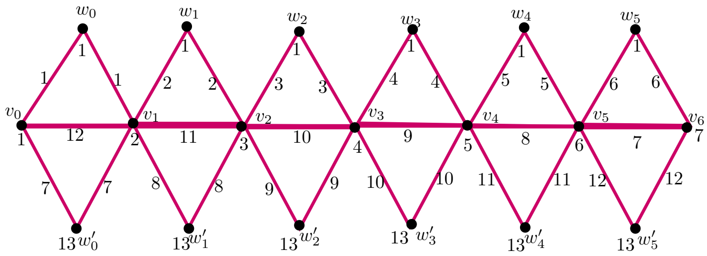

Figure 2. $ts(DT_6) = 13$.

THEOREM 2.3. *Let $n \geqslant 3$ and WH_n be a joint-wheel graph. Then*

$$ts(WH_n) = \left\lceil \frac{4n+3}{3} \right\rceil.$$

PROOF. Let $V(WH_n) = \{v_i, u_i, c_1, c_2 : 1 \leqslant i \leqslant n\}$ and

$$E(WH_n) = \{v_iv_{i+1}, u_iu_{i+1}, c_1v_i, c_2u_i, v_nu_n : 1 \leqslant i \leqslant n\}$$

with indices taken modulo n by (1.1), (1.3) and (1.4) we have $ts(WH_n) \geqslant \left\lceil \frac{4n+3}{3} \right\rceil$. Let $k = \left\lceil \frac{4n+3}{3} \right\rceil$. For the reverse inequality, we define a total labeling f as follows: $f(v_i) = i,\ 1 \leqslant i \leqslant n$; $f(c_1) = 1$, $f(c_1v_i) = 1,\ 1 \leqslant i \leqslant n$; $f(v_iv_{i+1}) = n+1-i,\ 1 \leqslant i \leqslant n-1$; $f(v_nv_1) = n+1$; $f(u_i) = k+1-i,\ 1 \leqslant i \leqslant n$; $f(u_iu_{i+1}) = 3n+2-2k+i,\ 1 \leqslant i \leqslant n$; $f(c_2) = k$, $f(c_2u_i) = k,\ 1 \leqslant i \leqslant n$; $f(v_nu_n) = 2n+2-k$.

We see that all the vertex and edge labels are at most k.

We have

$wt(c_1v_i) = 2+i,\ 1 \leqslant i \leqslant n$; $wt(v_iv_{i+1}) = n+2+i,\ 1 \leqslant i \leqslant n$; $wt(v_nu_n) = 2n+3$;

$$wt(u_iu_{i+1}) = \begin{cases} 3n+3-i, & if\ 1 \leqslant i \leqslant n-1 \\ 3n+3, & if\ i = n; \end{cases} \quad wt(c_2u_i) = 3k+1-i,\ 1 \leqslant i \leqslant n,$$

the weights of the edges under the labeling f are $\{3, 4, \dots, n+2, n+3, n+4, \dots, 2n+2, 2n+3, 3n+2, 3n+1, 3n, \dots, 2n+4, 3n+3, 3k, 3k-1, \dots, 3k+1-n\}$. Thus the edge weights are pair-wise distinct.

The vertex weights are

$$wt(c_1) = n+1,\ 1 \leqslant i \leqslant n;\ wt(v_i) = \begin{cases} 2n+4-i, & if\ 1 \leqslant i \leqslant n-1 \\ 4n+6-k, & if\ i = n; \end{cases} \quad wt(u_i) =$$

$$\begin{cases} 7n+5-2k, & if\ i = 1 \\ 6n+4-2k+i, & if\ 2 \leqslant i \leqslant n-1 \\ 9n+6-3k, & if\ i = n; \end{cases} \quad wt(c_2) = k(1+n).$$

That is $\{n+1, 2n+3, 2n+2, 2n+1, \dots, n+6, n+5, 4n+6-k, 7n+5-2k, \dots, 6n+6-2k, 6n+7-2k, \dots, 7n+2-2k, 7n+3-2k, 9n+6-3k, k(1+n)\}$.

Hence all the vertex weights are distinct. This labeling construction shows that $ts(WH_n) \leqslant k$. Combining this with the lower bound, we conclude that $ts(WH_n) = k$. This completes the proof. Figure 3 shows a totally irregular total labeling of joint-wheel graph WH_6. □

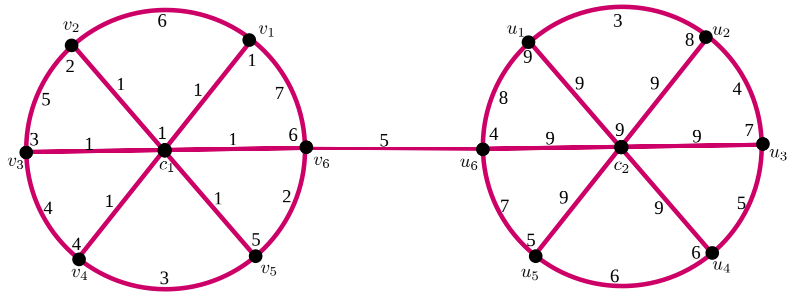

Figure 3. $ts(WH_6) = 9$.

THEOREM 2.4. *Let* $m \geqslant 3$. *Then* $ts(P_m + \overline{k_m}) = m + 1$.

PROOF. Let $V(P_m + \overline{k_m}) = \{v_i, u_i : 1 \leqslant i \leqslant m\}$ and

$$E(P_m + \overline{k_m}) = \{v_1u_i, v_mu_i, 1 \leqslant i \leqslant m\} \cup \{v_iv_{i+1} : 1 \leqslant i \leqslant m-1\}$$

by (1.1), (1.3) and (1.4) we have $ts(P_m + \overline{k_m}) \geqslant m+1$. For the reverse inequality, we define a total labeling $f : V \cup E \to \{1, 2, 3, \ldots, m+1\}$ by considering the following two cases.

Case(i): m is odd.

$f(v_i) = \begin{cases} 1, & if\ i = 1; \\ m+1, & if\ 2 \leqslant i \leqslant m. \end{cases}$ $f(u_i) = i,\ 1 \leqslant i \leqslant m$; $f(v_1u_i) = 1,\ 1 \leqslant i \leqslant m$; $f(v_mu_i) = 1,\ 1 \leqslant i \leqslant m$; $f(v_iv_{i+1}) = \begin{cases} m+1, & if\ i = 1 \\ i, & if\ 2 \leqslant i \leqslant m-1. \end{cases}$

We see that all the vertex and edge labels are at most $m+1$. We have $wt(v_1u_i) = 2+i,\ 1 \leqslant i \leqslant m$; $wt(v_mu_i) = m+2+i,\ 1 \leqslant i \leqslant m$; $wt(v_iv_{i+1}) = \begin{cases} 2m+3, & if\ i = 1 \\ 2m+2+i, & if\ 2 \leqslant i \leqslant m-1. \end{cases}$

the weights of the edges under the labeling f are $\{3, 4, \ldots, m+2, m+3, m+4, \ldots, 2m+2, 2m+3, 2m+4, 2m+5, \ldots, 3m+1\}$. Thus the edge weights are pair-wise distinct.

The vertex weights are

$wt(u_i) = 2+i,\ 1 \leqslant i \leqslant m$; $wt(v_i) = \begin{cases} 2m+2, & if\ i = 1 \\ 3m, & if\ i = m; \end{cases}$

That is $\{3, 4, \ldots, m+2, 2m+2, 3m\}$. Hence all the vertex weights are distinct.

Case(ii): m is even.

$f(v_i) = \begin{cases} 1, & if\ i = 1 \\ m+1, & if\ 2 \leqslant i \leqslant m; \end{cases}$ $f(u_i) = 1,\ 1 \leqslant i \leqslant m$; $f(v_1u_i) = f(v_mu_i) = i,\ 1 \leqslant i \leqslant m$; $f(v_iv_{i+1}) = \begin{cases} m+1, & if\ i = 1, 2 \\ m+3-i, & if\ 3 \leqslant i \leqslant m-1. \end{cases}$

We see that all the vertex and edge labels are at most $m+1$.

We have

$wt(v_1u_i) = 2+i,\ 1 \leqslant i \leqslant m;\ wt(v_mu_i) = m+2+i,\ 1 \leqslant i \leqslant m;\ wt(v_iv_{i+1}) = \begin{cases} 2m+3, & if\ i=1 \\ 3m+3, & if\ i=2 \\ 3m+5-i, & if\ 3 \leqslant i \leqslant m-1; \end{cases}$

The weights of the edges under the labeling f are $\{3, 4, \ldots, m+2, m+3, m+4, \ldots, 2m+2, 2m+3, 3m+3, 3m+2, 3m+1, 3m, \ldots, 2m+6\}$. Thus the edge weights are pair-wise distinct.

The vertex weights are

$$wt(u_i) = 2i+1,\ 1 \leqslant i \leqslant m;\ wt(v_i) = \begin{cases} \frac{m^2+3m+4}{2}, & if\ i=1 \\ \frac{m^2+3m+10}{2}, & if\ i=m. \end{cases}$$

That is $\{3, 5, \ldots 2m+1, \frac{m^2+3m+4}{2}, \frac{m^2+3m+10}{2}\}$. Hence all the vertex weights are distinct. This labeling construction shows that $ts(P_m + \overline{k_m}) \leqslant m+1$.

Combining this with the lower bound, we conclude that $ts(P_m + \overline{k_m}) = m+1$. This completes the proof. $\square$

Figure 4 shows a totally irregular total labeling of $P_5 + \overline{k_5}$.

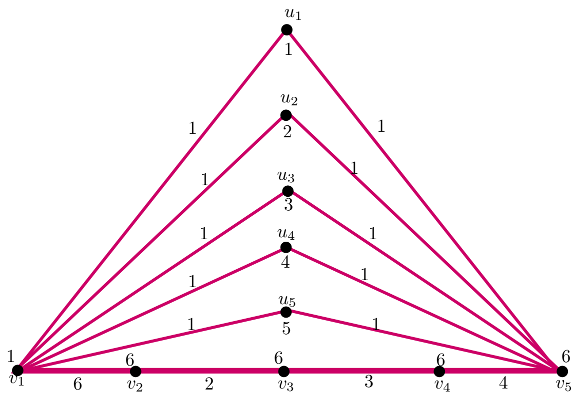

Figure 4. $ts(P_5 + \overline{k_5}) = 6$.

References

[1] A. Ahmad and K. M. Awan, I. Javaid and Slamin. Total vertex irregularity strength of wheel related graphs. *Australas. J. Combin.,* **51** (2011), 147–156.

[2] A. Ahmad, M. Ibrahim and M. K. Siddiqui. On the total trregularity strength of generalized Petersen graph. *Math. Reports,* **18**(68)(2)(2016), 197–204.

[3] M. Bača, S. Jendroĺ, M. Miller and J. Ryan. On irregular total labellings. *Discrete Math.,* **307**(11-12)(2007), 1378–1388.

[4] N. S. Hungund and D. G. Akka. Total irregularity strength of triangular snake and double triangular snake. *International Refereed Research Journal,* **3**(2011), 67-69.

[5] Ivančo and S. Jendroĺ. Total edge irregularity strength of trees. *Discussiones Math. Graph Theory,* **26**(3)(2006), 449–456.

[6] P. Jeyanthi and A. Sudha. Total edge irregularity strength of wheel related graphs. *J. Graph Labeling,* **2**(1)(2016), 45–57.
[7] P. Jeyanthi and A. Sudha. Total edge irregularity strength of disjoint union of Wheel graphs. *El. Notes Dis. Math.,* **48**(2015), 175–182.
[8] P. Jeyanthi and A. Sudha. Total edge trregularity strength of disjoint union of double Wheel graphs. *Proyecciones J. Math.,* **35**(3)(2016), 251–262.
[9] P. Jeyanthi and A. Sudha. Total vertex irregularity strength of corona product of some graphs. *J. Algorithms and Computation,* **48**(1)(2016), 127–140.
[10] P. Jeyanthi and A. Sudha. Total vertex irregularity strength of some graphs. *Palestine J. Math.,* **7**(2)(2018), 725–733.
[11] P. Jeyanthi and A. Sudha. Total edge irregularity strength of some families of graphs. *Utilitas Mathematica,* **109**(2018), 139–153.
[12] P. Jeyanthi and A. Sudha. Some results on edge irregular total labeling. *Bull. Int. Math. Virtual Inst.,* **9**(1)(2019), 73–91.
[13] P. Jeyanthi and A. Sudha. On the total irregularity strength of wheel related graphs. *Utilitas Mathematica,* to appear.
[14] P. Jeyanthi and A. Sudha. The total irregularity strength of disjoint union of crossed prism and necklace graphs. *Utilitas Mathematica,* to appear
[15] C. C. Marzuki, A. N. M. Salman and M. Miller. On the total irregularity strength on cycles and paths. *Far East J. Math. Sci.,* **82**(1)(2013), 1–21.
[16] Nurdin, E. T. Baskoro, A. N. M. Salman and N. N. Gaos. On the total vertex irregularity strength of trees. *Discrete. Math.,* **310**(21)(2010), 3043–3048.
[17] R. Ramdani and A. N. M. Salman. On the total irregularity strength of some Cartesian product graphs. *AKCE Int. J. Graphs Comb.*, **10**(2)(2013), 199–209.
[18] R. Ramdani, A. N. M. Salman, H. Assiyatun, A. Semaničová-Fenňovčková and M. Bača. Total irregularity strength of three families of graphs. *Math. Comput. Sci.,* **9**(2)(2015), 229–237.

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