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SOME REMARKS ON FUZZY GLOBALLY DISCONNECTED SPACES

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Abstract. In this paper, several characterizations of fuzzy globally disconnected spaces, are established. By means of fuzzy globally disconnectedness, conditions for fuzzy topological spaces to become fuzzy Baire spaces, are established. The conditions under which fuzzy sets become fuzzy simply open sets and fuzzy residual sets, fuzzy somewhere dense sets, fuzzy dense sets become fuzzy open sets in fuzzy globally disconnected spaces, are also obtained.

1. Introduction

In 1965, L. A. Zadeh [**25**] introduced the concept of fuzzy sets as a new approach for modeling uncertainties.The potential of fuzzy notion was realized by the researchers and has successfully been applied in all branches of Mathematics. Topology provided the most natural framework for the concepts of fuzzy sets to flourish. In 1968, C. L. Chang [**6**] introduced the concept of fuzzy topological spaces. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In 1969, the concept of globally disconnected spaces was introduced and studied by A. G. El'kin [**7**] in classical topology.The concept of fuzzy globally disconnected spaces is introduced and studied in [**22**]. In this paper several characterizations of fuzzy globally disconnected spaces, are established. By means of fuzzy globally disconnectedness of fuzzy topological spaces, conditions for fuzzy topological spaces to become fuzzy Baire spaces, are established. The conditions under which fuzzy sets become fuzzy simply open sets and fuzzy residual sets, fuzzy somewhere dense sets, fuzzy dense sets become fuzzy open sets in fuzzy globally disconnected spaces are also obtained.

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2. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I, the unit interval $[0, 1]$. A fuzzy set λ in X is a function from X into I. The null set 0 is the function from X into I which assumes only the value 0 and the whole fuzzy set 1 is the function from X into I which takes the value 1 only.

Definition 2.1. ([**6**]) Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). The interior and the closure of λ are defined respectively as follows:

(i) $int(\lambda) = \vee\{\mu/\mu \leqslant \lambda, \mu \in T\}$.
(ii) $cl(\lambda) = \wedge\{\mu/\lambda \leqslant \mu, 1 - \mu \in T\}$.

Lemma 2.1 ([**1**]). *For a fuzzy set λ of a fuzzy topological space X,*

(i) $1 - int(\lambda) = cl(1 - \lambda)$,
(ii) $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.2. ([**11**]) A fuzzy set λ in a fuzzy topological space (X, T) is called

(i) fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $cl(\lambda) = 1$, in (X, T).
(ii) fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$.That is, $intcl(\lambda) = 0$, in (X, T).

Definition 2.3. ([**11**]) A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy first category set if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.

Definition 2.4. ([**2**]) A fuzzy set λ in a fuzzy topological space (X, T) is called a

(i) fuzzy G_δ-set in (X, T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$. for $i \in I$.
(ii) fuzzy F_σ-set in (X, T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in T$. for $i \in I$.

Definition 2.5. ([**2**]) Let A be a fuzzy set of an fuzzy topological space (X, T). Then A is said to be fuzzy semi-open if there is an $U \in T$ such that $U \leqslant A \leqslant cl(U)$.

Definition 2.6. A fuzzy set λ in a fuzzy topological space (X, T) is called

(1) fuzzy pre-open if $\lambda \leqslant intcl(\lambda)$ and fuzzy pre-closed if $clint(\lambda) \leqslant \lambda$ ([**4**]).
(2) fuzzy semi-open if $\lambda \leqslant clint(\lambda)$ and fuzzy semi-closed if $intcl(\lambda) \leqslant \lambda$ ([**1**]).
(3) fuzzy β-open if $\lambda \leqslant clintcl(\lambda)$ and fuzzy β-closed if $intclint(\lambda) \leqslant \lambda$ ([**4**]).

Definition 2.7. ([**9**]) Let λ be a fuzzy set in a fuzzy topological space (X, T). The fuzzy boundary of λ is defined as $Bd(\lambda) = cl(\lambda) \wedge cl(1 - \lambda)$

Definition 2.8. A fuzzy set λ in a fuzzy topological space (X, T), is called

(i) fuzzy simply open set if $Bd(\lambda)$ is a fuzzy nowhere dense set in (X,T) ([**20**]).
(ii) fuzzy σ -nowhere dense set if λ is a fuzzy F_σ-set in (X,T) with $int(\lambda)=0$ ([**18**]).

Definition 2.9. ([**5**]) Let A be a fuzzy set of an fuzzy topological space X. Then A is said to be

(a) fuzzy feebly open if there exists fuzzy open set U in X such that $U \leqslant A \leqslant SclU$,
(b) fuzzy feebly closed if its complement is fuzzy feebly open.

Definition 2.10. ([**6**]) Let λ be a fuzzy first category set in a fuzzy topological space (X,T). Then $1-\lambda$ is called a fuzzy residual set in (X,T).

Definition 2.11. ([**23**]) Let (X,T) be a fuzzy topological space. A fuzzy set λ defined on X is said to have the property of fuzzy Baire, if $\lambda=(\mu\wedge\delta)\vee\eta$, where $\mu$is a fuzzy open set, δ is a fuzzy residual set and η is a fuzzy first category set in (X,T).

Definition 2.12. A fuzzy topological space (X,T) is called a

(i) fuzzy sub-maximal space if for each fuzzy set λ in (X,T) such that $cl(\lambda)=1$, $\lambda\in T$ in (X,T) ([**2**]).
(ii) fuzzy hyper-connected space if every non-null fuzzy open subset of (X,T) is fuzzy dense set in (X,T) ([**8**]).
(iii) fuzzy P-space if each fuzzy G_δ - set in (X,T), is a fuzzy open set in (X,T) ([**10**]).
(vi) fuzzy GID-space if for each fuzzy dense and fuzzy G_δ-set λ in (X,T), $clint(\lambda)=1$ in (X,T) ([**19**]).
(v) fuzzy globally disconnected space if each fuzzy semi-open set in (X,T) is a fuzzy open set ([**22**]).
(vi) fuzzy Baire space if $int[\vee_{i=1}^{\infty}(\lambda_i)]=0$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X,T) ([**15**]).
(vii) fuzzy strongly irresolvable space if for every dense set λ in (X,T), $clint(\lambda)$ $=1$ in (X,T) ([**13**]).
(viii) fuzzy open hereditarily irresolvable space if $int[cl(\lambda)]\neq 0$, then $int(\lambda)\neq 0$, for any non-zero fuzzy set λ in (X,T) ([**14**]).
(xi) fuzzy extremally disconnected space if $\lambda\in T$ implies $cl(\lambda)\in T$ ([**3**]).

Definition 2.13. ([**15**]) Let (X,T) be a fuzzy topological space. A fuzzy set λ defined on X is called

(i) a fuzzy somewhere dense set, if $intcl(\lambda)\neq 0$ in (X,T) ([**14**]).
(ii) a fuzzy σ-first category set if $\lambda=\vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy σ-nowhere dense sets in(X,T) ([**18**]).

Definition 2.14. ([**18**]) A fuzzy set λ in a topological space (X,T) is called a fuzzy σ-first category set if $\lambda=\vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'$ s are fuzzy σ-nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy σ-second category.

Theorem 2.1 ([**16**]). *If λ is a fuzzy dense and fuzzy G_δ-set in a fuzzy topological space (X, T),then $1 - \lambda$ is a fuzzy first category set in (X, T).*

Theorem 2.2 ([**22**]). *If λ is a fuzzy residual set in a fuzzy globally disconnected space (X, T), then λ is a fuzzy G_δ- set in (X, T).*

Theorem 2.3 ([**22**]). *If λ is a fuzzy first category set in a fuzzy globally disconnected space (X, T), then λ is a fuzzy F_σ-set in (X, T).*

Theorem 2.4 ([**20**]). *If λ is a fuzzy simply open set in a fuzzy topological space (X, T) , then $\lambda \wedge (1 - \lambda)$is a fuzzy nowhere dense set in (X, T).*

Lemma 2.2 ([**26**]). *Let A be a fuzzy set of an fuzzy topological space X. Then, $intA \leqslant SintA \leqslant A \leqslant SclA \leqslant clA$.*

Theorem 2.5 ([**22**]). *If λ is a fuzzy nowhere dense set in a fuzzy globally disconnected space (X, T),then*

(i) *$\lambda$is a fuzzy closed set in (X, T).*
(ii) *λ is a fuzzy simply open set in (X, T).*

3. Fuzzy globally disconnected space

Proposition 3.1. *If $clint(\lambda) = 1$, for a fuzzy set λ defined on X in a fuzzy globally disconnected space (X, T), then λ is a fuzzy simply open set in (X, T).*

Proof. Let λ be a fuzzy set defined on X such that $clint(\lambda) = 1$,in (X, T). Then, clearly $\lambda \leqslant clint(\lambda)$ and hence λ is a fuzzy semi-open set in (X, T). Since (X, T) is a fuzzy globally disconnected space, the fuzzy semi-open set λ is a fuzzy open set in (X, T) and hence $int(\lambda) = \lambda$ in (X, T). Now $clint(\lambda) \leqslant cl(\lambda)$, implies that $1 \leqslant cl(\lambda)$ and thus, $cl(\lambda) = 1$ in (X, T). Now

$$\begin{aligned} intcl[bd(\lambda)] &= intcl[cl(\lambda) \wedge cl(1 - \lambda)] = intcl[1 \wedge cl(1 - \lambda)] = intcl[cl(1 - \lambda)] \\ &= intcl(1 - \lambda) = 1 - clint(\lambda) = 1 - cl(\lambda) = 1 - 1 = 0. \end{aligned}$$

Hence λ is a fuzzy simply open set in (X, T). □

Proposition 3.2. *If $clint(\lambda) = 1$, for a fuzzy set λ defined on X in a fuzzy globally disconnected space (X, T), then $\lambda \wedge (1 - \lambda)$ is a fuzzy nowhere dense set in (X, T).*

Proof. Let λ be a fuzzy set defined on X such that $clint(\lambda) = 1$,in (X, T). Since (X, T) is a fuzzy globally disconnected space, by Proposition 3.1, λ is a fuzzy simply open set in (X, T). Then, by Theorem 2.4, $\lambda \wedge (1 - \lambda)$ is a fuzzy nowhere dense set in (X, T). □

Proposition 3.3. *If λ is a fuzzy semi-open and fuzzy dense set in a fuzzy globally disconnected space (X, T), then λ is a fuzzy simply open set in (X, T).*

Proof. Let λ be a fuzzy semi-open set in (X, T) such that $cl(\lambda) = 1$. Since (X, T) is a fuzzy globally disconnected space, the fuzzy semi-open set is a fuzzy open in (X, T). Now

$$intcl[bd(\lambda)] = intcl[cl(\lambda) \wedge cl(1-\lambda)] = intcl[1 \wedge cl(1-\lambda)] = intcl[cl(1-\lambda)]$$
$$= intcl(1-\lambda) = 1 - clint(\lambda) = 1 - cl(\lambda) = 1 - 1 = 0.$$

Hence λ is a fuzzy simply open set in (X, T). □

Proposition 3.4. *If λ is a fuzzy β-open set in a fuzzy globally disconnected space (X, T), then $cl(\lambda)$ is a fuzzy open set in (X, T).*

Proof. Let λ be a fuzzy β-open set in (X, T). Then, $\lambda \leqslant clintcl(\lambda)$ in (X, T). This implies that $cl(\lambda) \leqslant cl[clintcl(\lambda)]$ and then $cl(\lambda) \leqslant clintcl(\lambda)$. Hence $cl(\lambda) \leqslant clint[cl(\lambda)]$ shows that $cl(\lambda)$ is a fuzzy semi-open set in (X, T). Since (X, T) is a fuzzy globally disconnected space, $cl(\lambda)$ is a fuzzy open set in (X, T). □

Theorem 3.1 ([**23**]). *If λ is a fuzzy set with fuzzy Baire property in a fuzzy topological space (X, T), then $\lambda = [\wedge_{i=1}^{\infty}(\mu \wedge \delta_i)] \vee \eta$, where (δ_i)'s are fuzzy sets defined on X such that $cl[int(\delta_i)] = 1$, $\mu \in T$ and η is a fuzzy first category set in (X, T).*

Proposition 3.5. *If λ is a fuzzy set with fuzzy Baire property in a fuzzy globally disconnected space (X, T),then $\lambda = (\mu \wedge \delta) \vee \eta$, where μ is a fuzzy open set, δ is a fuzzy G_δ-set and η is a fuzzy F_σ-set in (X, T).*

Proof. Let λ be a fuzzy set with fuzzy Baire property in (X, T). Then, $\lambda = (\mu \wedge \delta) \vee \eta$, where μ is a fuzzy open set, δ is a fuzzy residual set and η is a fuzzy first category set in (X, T). Since (X, T) is a fuzzy globally disconnected space, by Theorems 2.2 and Theorem 2.3, the fuzzy residual set δ is a fuzzy G_δ-set and the fuzzy first category set η is a fuzzy F_σ-set in (X, T). Hence $\lambda = (\mu \wedge \delta) \vee \eta$ where $\mu \in T$, δ is a fuzzy G_δ-set and η is a fuzzy F_σ-set in (X, T). □

Proposition 3.6. *If λ is a fuzzy set with fuzzy Baire property in a fuzzy globally disconnected space (X, T), then $\lambda = [\wedge_{i=1}^{\infty}(\mu \wedge \delta_i)]$, where (δ_i)'s are fuzzy simply open sets, $\mu \in T$ and η is a fuzzy F_σ- set in (X, T).*

Proof. Let λ be a fuzzy set with fuzzy Baire property in (X, T). Then, by Theorem 3.2, $\lambda = [\wedge_{i=1}^{\infty}(\mu \wedge \delta_i)]$, where (δ_i)'s are fuzzy sets defined on X such that $cl[int(\delta_i)] = 1, \mu \in T$ and η is a fuzzy first category set in (X, T). Since (X, T) is a fuzzy globally disconnected space, by Theorems 2.3, the fuzzy first category set η is a fuzzy F_σ-set in (X, T). Also since $cl[int(\delta_i)] = 1$,in (X, T), by proposition 3.1 , (δ_i)'s are fuzzy simply open sets in (X, T). Thus, $\lambda = [\wedge_{i=1}^{\infty}(\mu \wedge \delta_i)]$, where (δ_i)'s are fuzzy simply open sets, $\mu \in T$ and η is a fuzzy F_σ- set in (X, T). □

Remark 3.1. It is to be noted that every fuzzy open set is fuzzy feebly open in a fuzzy topological space, but the converse need not be true ([**5**]) The following proportion gives a condition for a fuzzy feebly open set to be a fuzzy open set in a fuzzy topological space.

Proposition 3.7. *If λ is a fuzzy feebly open set in a fuzzy globally disconnected space (X, T), then λ is a fuzzy open set in (X, T).*

PROOF. Let λ be a fuzzy feebly open set in (X,T). Then, $\mu \leqslant \lambda \leqslant Scl(\mu)$, where $\mu \in T$. Now $Scl(\mu) \leqslant cl(\mu)$, implies that $\mu \leqslant \lambda \leqslant cl(\mu)$, in (X,T) and thus λ is a fuzzy semi-open set in (X,T). Since (X,T) is a fuzzy globally disconnected space, the fuzzy semi-open set λ is a fuzzy open in (X,T). Hence the fuzzy feebly open set λ is a fuzzy open in a fuzzy globally disconnected space (X,T). □

THEOREM 3.2 ([**21**]). *Let $\lambda \leqslant 1-\mu$, for any two fuzzy sets λ and μ defined on (X,T).*

(a) *If μ is a fuzzy dense set in (X,T), then λ is not a fuzzy open set in (X,T).*

(b) *If λ is a fuzzy dense set in (X,T), then μ is not a fuzzy open set in (X,T).*

(c) *If λ is a fuzzy open set in (X,T), then μ is not a fuzzy dense set in (X,T).*

(d) *If μ is a fuzzy open set in (X,T), then λ is not a fuzzy dense set in (X,T).*

(e) *If $clint(\mu) = 1$, then λ is a fuzzy nowhere dense set in (X,T).*

PROPOSITION 3.8. *If $\lambda \leqslant 1-\mu$, for any two fuzzy sets λ and μ defined on X such that $clint(\mu) = 1$, in a fuzzy globally disconnected space (X,T), then λ is a fuzzy closed set and μ is a fuzzy simply open set in (X,T).*

PROOF. Suppose that $\lambda \leqslant 1-\mu$, for any two fuzzy sets λ and μ defined on X. Since $clint(\mu) = 1$, in a fuzzy globally disconnected space (X,T), by proposition 3.1, μ is a fuzzy simply open set in (X,T). Now $\lambda \leqslant 1-\mu$ and $clint(\mu) = 1$, implies by Theorem 3.2(e) that, λ is a fuzzy nowhere dense set in (X,T).Then, $intcl(\lambda) = 0$, in (X,T). This implies that $intcl(\lambda) \leqslant \lambda$ in (X,T). Thus λ is a fuzzy semi-closed set in (X,T). Then $1-\lambda$ is a fuzzy semi-open set in (X,T). Since (X,T) is a fuzzy globally disconnected space, $1-\lambda$ is a fuzzy open set in (X,T) and hence λ is a fuzzy closed set in (X,T). □

4. Fuzzy globally disconnected spaces and other fuzzy topological spaces

PROPOSITION 4.1. *If $int(\vee_{i=1}^{\infty}[\lambda_i \wedge (1-\lambda_i)]) = 0$, where λ_i's are fuzzy sets defined on X such that $clint(\lambda_i) = 1$ in a fuzzy globally disconnected space (X,T), then (X,T) is a fuzzy Baire space.*

PROOF. Let (λ_i)'s ($i = 1$ to ∞) be the fuzzy sets defined on X such that $clint(\lambda_i) = 1$, in (X,T). Since (X,T) is a fuzzy globally disconnected space, by Proposition 3.2, $[\lambda_i \wedge (1-\lambda_i)]$'s are fuzzy nowhere dense sets in (X,T). By hypothesis, $int(\vee_{i}^{\infty}[\lambda_i \wedge (1-\lambda_i)]) = 0$ in (X,T). Thus, $int(\vee_{i}^{\infty}[\lambda_i \wedge (1-\lambda_i)]) = 0$, where $[\lambda_i \wedge (1-\lambda_i)]$'s are fuzzy nowhere dense sets in (X,T), implies that (X,T) is a fuzzy Baire space. □

Proposition 4.2. *If* $int(\vee_{i=1}^{\infty}[\lambda_i \wedge (1-\lambda_i)]) = 0$*, where* (λ_i)*'s* $(i = 1to\infty)$ *be fuzzy dense sets in a fuzzy strongly irresolvable and fuzzy globally disconnected space* (X,T)*, then* (X,T) *is a fuzzy Baire space.*

Proof. Let (λ_i)'s ($i = 1$ to ∞) be the fuzzy dense sets in (X,T). Since (X,T) is a fuzzy strongly irresolvable space, for the fuzzy dense sets (λ_i)'s in (X,T), we have $clint(\lambda_i) = 1$ in (X,T). Since (X,T) is fuzzy globally disconnected space, by Proposition 3.2, $[\lambda_i \wedge (1-\lambda_i)]$'s, are fuzzy nowhere dense sets in (X,T). By hypothesis, $int(\vee_{i=1}^{\infty}[\lambda_i \wedge (1-\lambda_i)]) = 0$, in (X,T). Thus, $int(\vee_{i=1}^{\infty}[\lambda_i \wedge (1-\lambda_i)]) = 0$, where $[\lambda_i \wedge (1-\lambda_i)]$'s are fuzzy nowhere dense sets in (X,T), implies that (X,T) is a fuzzy Baire space. □

Proposition 4.3. *If* λ *is a fuzzy set with fuzzy Baire property in a fuzzy globally disconnected and fuzzy P-space* (X,T)*, then* $\lambda = \gamma \vee \eta$*, where* γ *is a fuzzy open set and* η *is a fuzzy closed set in* (X,T)*.*

Proof. Let λ be a fuzzy set with fuzzy Baire property in (X,T). Since (X,T) is a fuzzy globally disconnected space, by Proposition 3.5, $\lambda = (\mu \wedge \delta) \vee \eta$, where μ is a fuzzy open set, δ is a fuzzy G_δ-set and η is a fuzzy F_σ-set in (X,T). Also since (X,T) is a fuzzy P-space, the fuzzy G_δ-set δ is a fuzzy open set and the fuzzy F_σ-set η is a fuzzy closed set in (X,T). Let $\gamma = \mu \wedge \delta$ in (X,T) and then γ is a fuzzy open set in (X,T). Thus, if λ is a fuzzy set with fuzzy Baire property in a fuzzy globally disconnected and fuzzy P-space (X,T), then $\lambda = \gamma \vee \eta$, where γ, $1-\eta \in T$. □

Theorem 4.1 ([**22**]). *If* (X,T) *is a fuzzy globally disconnected space, then* (X,T) *is a fuzzy nodec space.*

Proposition 4.4. *If a fuzzy topological space* (X,T) *is a fuzzy globally disconnected space, then* (X,T) *is a fuzzy extremally disconnected space.*

Proof. Let λ be a fuzzy open set in (X,T). Then, λ is a fuzzy β-open set in (X,T). Since (X,T) is a fuzzy globally disconnected space, by proposition 3.4, $cl(\lambda)$ is a fuzzy open set in (X,T). Hence $cl(\lambda) \in T$ for $\lambda \in T$ implies that (X,T) is a fuzzy extremally disconnected space. □

Proposition 4.5. *If a fuzzy topological space* (X,T) *is a fuzzy globally disconnected space, then* (X,T) *is a fuzzy extremally disconnected and fuzzy nodec space.*

Proof. The proof follows from the Theorem 4.1 and the Proposition 4.3. □

Theorem 4.2 ([**3**]). *For any fuzzy topological space* (X,T)*, the following are equivalent:*

(a) *X is fuzzy extremally disconnected space.*
(b) *For each fuzzy closed set* λ*,* $int(\lambda)$ *is fuzzy closed.*
(c) *For each fuzzy open set* λ*,* $cl(\lambda) + cl[1 - cl(\lambda)] = 1$*.*

(d) *For every pair of fuzzy open sets* λ *and* μ *in* X *with* $cl(\lambda)+\mu=1$, $cl(\lambda)+cl(\mu)=1$.

PROPOSITION 4.6. *If* (X,T) *is a fuzzy globally disconnected space, then for each fuzzy semi-closed set* λ *in* (X,T), $cl(1-\lambda)=1-cl(int[\lambda])$, *in* (X,T).

PROOF. Let λ be a fuzzy semi-closed set in (X,T). Then, $1-\lambda$ is a fuzzy semi-open set in (X,T). Since (X,T) is a fuzzy globally disconnected space,$1-\lambda$ is a fuzzy open set in (X,T). By Proposition 4.5, (X,T) is a fuzzy extremally disconnected space. Then, by Theorem 4.1, for the fuzzy open set $1-\lambda$, $cl(1-\lambda)+cl(1-cl[1-\lambda])=1$, in (X,T). This implies that $cl(1-\lambda)+cl(1-[1-int(\lambda])=1$ and then, $cl(1-\lambda)+cl(int[\lambda])=1$, in (X,T). Thus, $cl(1-\lambda)=1-clint(\lambda)$, in (X,T). □

PROPOSITION 4.7. *If* λ *is a fuzzy nowhere dense set in a fuzzy globally disconnected space* (X,T), *then*

(i) $clint(\lambda)=int(\lambda)$, *in* (X,T).

(ii) $cl(1-\lambda)=1-clint(\lambda)$, *in* (X,T).

PROOF. Let λ be a fuzzy nowhere dense set in (X,T). Since (X,T) is a fuzzy globally disconnected space, by Proposition 4.5, (X,T) is a fuzzy extremally disconnected and fuzzy nodec space. Since (X,T) is a fuzzy nodec space, the fuzzy nowhere dense set λ is a fuzzy closed set in (X,T). Also since (X,T) is a fuzzy extremally disconnected space, for the fuzzy closed set λ in (X,T), $int(\lambda)$ is fuzzy closed. Then, $clint(\lambda)=int(\lambda)$ in (X,T).

(ii). Since λ is a fuzzy nowhere dense set in (X,T), $intcl(\lambda)=0$, in (X,T). This implies that $intcl(\lambda)\leqslant\lambda$ and thus λ is a fuzzy semi-closed set in (X,T). Since (X,T) is a fuzzy globally disconnected space, by proposition 4.6, $cl(1-\lambda)=1-clint(\lambda)$, in (X,T). □

PROPOSITION 4.8. *If* λ *is a fuzzy dense set in a fuzzy strongly irresolvable and fuzzy globally disconnected space* (X,T), *then*

(i) λ *is a fuzzy simply open set in* (X,T).

(ii) $\lambda\wedge(1-\lambda)$ *is a fuzzy nowhere dense set in* (X,T).

(iii) $1-\lambda$ *is a fuzzy closed set in* (X,T).

PROOF. Let λ be a fuzzy dense set in (X,T). Since (X,T) is a fuzzy strongly irresolvable space, for the fuzzy dense set λ, $clint(\lambda)=1$ in (X,T). Since (X,T) is a fuzzy globally disconnected space, (i) by Proposition 3.1, λ is a fuzzy simply open set in (X,T). (ii) by Proposition 3.2, $\lambda\wedge(1-\lambda)$ is a fuzzy nowhere dense set in (X,T).

(iii) Now $clint(\lambda)=1$ in (X,T), implies that $1-clint(\lambda)=0$ in (X,T). Then, $intcl(1-\lambda)=0$ in (X,T). Thus, $1-\lambda$ is a fuzzy nowhere dense set in (X,T). By Theorem 2.5, the fuzzy nowhere dense set $1-\lambda$ is a fuzzy closed set in (X,T). □

THEOREM 4.3 ([**24**]). *If* λ *is a fuzzy* σ*-nowhere dense set in a fuzzy strongly irresolvable space* (X,T), *then* λ *is a fuzzy nowhere dense and fuzzy semi-closed set in* (X,T).

Proposition 4.9. *If λ is a fuzzy σ-nowhere dense set in a fuzzy globally disconnected and fuzzy strongly irresolvable space (X, T), then λ is a fuzzy closed set in (X, T).*

Proof. Let λ be a fuzzy σ-nowhere dense set in (X, T). Since (X, T) is a fuzzy strongly irresolvable space,λ is a fuzzy semi-closed set in (X, T). Since (X, T) is a fuzzy globally disconnected space, the fuzzy semi-closed set λ is a fuzzy closed set in (X, T). □

Proposition 4.10. *If λ is a fuzzy σ-first category set in a fuzzy globally disconnected and fuzzy strongly irresolvable space (X, T), then λ is a fuzzy F_σ-set in (X, T).*

Proof. Let λ be a fuzzy σ-first category set in (X, T). Then ,$\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T). Since (X, T) is a fuzzy globally disconnected and fuzzy strongly irresolvable space, by Proposition 4.9, the fuzzy σ-nowhere dense sets (λ_i)' s are fuzzy closed sets in (X, T). Hence $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy closed sets in (X, T), implies that λ is a fuzzy F_σ-set in (X, T). □

Theorem 4.4 ([**19**]). *Let (X, T) be a fuzzy topological space. Then, the following are equivalent :*

(i) *(X, T) is a fuzzy GID-space.*

(ii) *Each fuzzy dense and fuzzy G_δ-set in (X, T), is fuzzy semi-open in (X, T).*

Proposition 4.11. *If λ is a fuzzy dense and fuzzy G_δ-set in a fuzzy globally disconnected and fuzzy GID-space (X, T), then λ is a fuzzy open set in (X, T).*

Proof. Let λ be a fuzzy dense and fuzzy G_δ-set in (X, T). Since (X, T) is a fuzzy GID-space, by theorem 4.4, λ is a fuzzy semi-open set in (X, T). Since (X, T) is a fuzzy globally disconnected space, the fuzzy semi-open set λ is a fuzzy open set in (X, T). □

Proposition 4.12. *If λ is a fuzzy dense and fuzzy G_δ-set in a fuzzy globally disconnected and fuzzy GID-space (X, T), then $1 - \lambda$ is a fuzzy closed and fuzzy first category set in (X, T).*

Proof. Let λ be a fuzzy dense and fuzzy G_δ-set in (X, T). Since (X, T) is a fuzzy globally disconnected and fuzzy GID-space, by Proposition 4.11, λ is a fuzzy open set in (X, T) and then $1 - \lambda$ is a fuzzy closed set in (X, T). By Theorem 2.1, for the fuzzy dense and fuzzy G_δ-set λ in (X, T), $1 - \lambda$ is a fuzzy first category set in (X, T). Hence $1 - \lambda$ is a fuzzy closed and fuzzy first category set in (X, T). □

Theorem 4.5 ([**19**]). *If λ is a fuzzy σ-nowhere dense set in a fuzzy GID-space (X, T), then λ is a fuzzy nowhere dense set in (X, T).*

Proposition 4.13. *If λ is a fuzzy σ-nowhere dense set in a fuzzy globally disconnected and fuzzy GID-space (X, T), then λ is a fuzzy simply open set in (X, T).*

Proof. Let λ be a fuzzy σ-nowhere dense set in (X, T). Since (X, T) is a fuzzy GID-space, by Theorem 4.5, λ is a fuzzy nowhere dense set in (X, T). Since (X, T) is a fuzzy globally disconnected space, by Theorem 2.5, the fuzzy nowhere dense set λ is a fuzzy simply open set in (X, T). □

Theorem 4.6 ([**24**]). *If λ is a fuzzy dense set in a fuzzy open hereditarily irresolvable space (X, T), then $1 - \lambda$ is a fuzzy nowhere dense set in (X, T).*

Theorem 4.7 ([**24**]). *If a fuzzy topological space (X, T) is a fuzzy submaximal space, then (X, T) is a fuzzy open hereditarily irresolvable space.*

Remark 4.1. The converse of the theorem 4.7, need not be true. That is, a fuzzy open hereditarily irresolvable space need not be a fuzzy submaximal space.

Proposition 4.14. *If (X, T) is a fuzzy open hereditarily irresolvable and fuzzy globally disconnected space, then (X, T) is a fuzzy submaximal space.*

Proof. Let λ be a fuzzy dense set in (X, T). Since (X, T) is a fuzzy open hereditarily irresolvable space, by Theorem 4.6, $1 - \lambda$ is a fuzzy nowhere dense set in (X, T). Since (X, T) is a fuzzy globally disconnected space, by theorem 2.5, the fuzzy nowhere dense set $1 - \lambda$ is a fuzzy closed set in (X, T), and then λ is a fuzzy open set in (X, T). Thus, (X, T) is a fuzzy submaximal space. □

Theorem 4.8 ([**24**]). *If λ is a fuzzy first category set in a fuzzy open hereditarily irresolvable and fuzzy Baire space (X, T), then λ is a fuzzy nowhere dense set in (X, T).*

Proposition 4.15. *If λ is a fuzzy first category set in a fuzzy Baire, fuzzy open hereditarily irresolvable and fuzzy globally disconnected space (X, T), then λ is a fuzzy closed set in (X, T).*

Proof. Let λ be a fuzzy first category set in (X, T). Since (X, T) is a fuzzy open hereditarily irresolvable and fuzzy Baire space, by Theorem 4.8, λ is a fuzzy nowhere dense set in (X, T). Also since (X, T) is a fuzzy globally disconnected space, by Theorem 2.5, the fuzzy nowhere dense set λ is a fuzzy closed set in (X, T). □

Proposition 4.16. *If λ is a fuzzy residual set in a fuzzy Baire, fuzzy open hereditarily irresolvable and fuzzy globally disconnected space (X, T), then λ is a fuzzy open set in (X, T).*

Proof. Let λ be a fuzzy residual set in (X, T).Then, $1 - \lambda$ is a fuzzy first category set in (X, T). Since (X, T) is a fuzzy Baire, fuzzy open hereditarily irresolvable and fuzzy globally disconnected space, by Proposition 4.15, $1 - \lambda$ is a fuzzy closed set in (X, T).Hence λ is a fuzzy open set in (X, T). □

Theorem 4.9 ([**24**]). *If λ is a fuzzy somewhere dense set in a fuzzy hyperconnected and fuzzy open hereditarily irresolvable space, then $1 - \lambda$ is a fuzzy nowhere dense set in (X, T).*

Proposition 4.17. *If λ is a fuzzy somewhere dense set in a fuzzy hyperconnected,fuzzy open hereditarily irresolvable and fuzzy globally disconnected space (X,T), then λ is a fuzzy open set in (X,T).*

Proof. Letλ be a fuzzy somewhere dense set in (X,T). Since (X,T) is a fuzzy hyperconnected and fuzzy open hereditarily irresolvable space, by Theorem 4.9, $1-\lambda$ is a fuzzy nowhere dense set in (X,T). Also since (X,T) is a fuzzy globally disconnected space, by Theorem 2.5, the fuzzy nowhere dense set $1-\lambda$ is a fuzzy closed setin (X,T). Hence λ is a fuzzy open set in (X,T). □

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