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SHEAF REPRESENTATION OF STONE ALMOST DISTRIBUTIVE LATTICES

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Abstract. A Stone ADL L is a pseudo-complemented ADL in which $x^\star \vee x^{\star\star} = 0^\star$ for all $x \in L$. If (S, Π, X) is a sheaf of dense ADLs over a Boolean space such that there is a maximal global section and for any global section f, support of f is open, then it is proved that the set $\Gamma(X, S)$ of all global sections is a Stone ADL. Conversely, it is proved that every Stone ADL is isomorphic to the ADL of global sections of a suitable sheaf of dense ADLs over a Boolean space.

1. Introduction

After the Booles axiomatization of the two valued propositional calculus into a Boolean algebra, many generalizations of the Boolean algebras have come into being. The concept of an Almost Distributive Lattice (ADL) was introduced by Swamy and Rao as a common abstraction of almost all the lattice theoretic and ring theoretic generalizations of a Boolean algebra like p-rings, regular rings, bi-regular rings, associate rings, p_1-rings, triple systems, Baire rings and m-domain rings. An almost distributive lattice(ADL) $(L, \vee, \wedge, 0)$ is an algebra of type $(2, 2, 0)$ which satisfies all the axioms of a distributive lattice with 0 except the commutativity of the binary operations and, in this case, the commutativity of either of the binary operation is equivalent to that of the other. A unary operation $x \mapsto x^\star$ on an ADL L is called a pseudo-complementation if it satisfies the property $x \wedge y = 0 \iff x^\star \wedge y = x^\star$ for all elements x and y of L.

In one of our earlier papers entitled "Pseudo-complementation on Almost Distributive Lattices" [**7**], we have proved that the pseudo-complementations is one-to-one correspondence with the maximal elements and certain other fundamental

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results on ADLs with pseudo-complementation. In another paper entitled "Stone Almost Distributive Lattices" [**8**], we have introduced the notion of a Stone ADL as an ADL with pseudo-complementation in which $x^\star \vee x^{\star\star}$ is maximal for each element x of L and proved several results regarding the Stone ADLs. In particular, we have proved that the set of pseudo-complements of elements of a Stone ADL forms a Boolean algebra and hence a Boolean ring. With this motivation, we have considered the prime spectrum of the Boolean algebra of pseudo-complements of a Stone ADL which forms a compact, Hausdroff and totally disconnected topological space and it is well known that such spaces are called Boolean space. With this as the base space, we have explored the possibilities of constructing a sheaf of ADLs whose ADL of continues sections is isomorphic to the given Stone ADL. If successful, the elements of a Stone ADL can be identified with continuous sections of a sheaf of ADLs (probably satisfying certain stronger properties) over a Boolean space. This facilitates us to use stronger properties of the stalks and those of the Boolean space to interpret the earlier results in a more convenient form and to predict new results on Stone ADLs. Later, we can extend this to a more general classes of ADLs.

2. Preliminaries

Firstly, we recall certain definitions and properties of ADLs from [**1, 2, 3, 6, 7**] that are required in the text.

Definition 2.1. An algebra $(L, \vee, \wedge, 0)$ of type (2,2,0) is called an ADL if, for any $x, y, z \in L$,

(1) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
(2) $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$
(3) $(x \vee y) \wedge x = x$
(4) $(x \vee y) \wedge y = y$
(5) $x \vee (x \wedge y) = x$
(6) $0 \wedge x = 0$

Definition 2.2. Let A be a non-empty set and $a_0 \in A$. For any $a, b \in A$, define

$$a \vee b = \begin{cases} a & \text{if } a \neq a_0 \\ b & \text{if } a = a_0 \end{cases} \quad \textit{and} \quad a \wedge b = \begin{cases} b & \text{if } a \neq a_0 \\ a_0 & \text{if } a = a_0 \end{cases}$$

Then $(L, \vee, \wedge, a_0)$ is an ADL and this ADL is called discrete ADL.

Lemma 2.1. *Let L be an ADL. Then we have the following:*

(1) $a \vee b = a \Leftrightarrow a \wedge b = b$
(2) $a \vee b = b \Leftrightarrow a \wedge b = a$.
(3) *If* $a \leqslant b$ *is defined when* $a = a \wedge b$, *then* $\leqslant$ *is a partial ordering on* L *w.r.t. which 0 is the least element.*
(4) $a \wedge b = b \wedge a$ *whenever* $a \leqslant b$.
(5) $\wedge$ *is associative in* L.
(6) $a \wedge b \wedge c = b \wedge a \wedge c$.

(7) $(a \vee b) \wedge c = (b \vee a) \wedge c$.
(8) $a \wedge b = 0 \Leftrightarrow b \wedge a = 0$.
(9) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$.
(10) *if* $a \leqslant c, b \leqslant c$ *for some* $c \in L$, *then* $a \wedge b = b \wedge a$ *and* $a \vee b = b \vee a$.

Definition 2.3. A homomorphism between ADLs L and L' is a mapping $f : L \to L'$ satisfying the following:
(1) $f(a \vee b) = f(a) \vee f(b)$
(2) $f(a \wedge b) = f(a) \wedge f(b)$
(3) $f(0) = 0$.

A nonempty subset I of L is called an ideal of L if $x \vee y \in I$ and $x \wedge a \in I$ whenever $x, y \in I$ and $a \in L$. For any $A \subseteq L$, the ideal generated by A is $(A] = \{(\bigvee_{i=1}^{n} a_i) \wedge x \mid a_i \in A, x \in L, n \in Z^+\}$. If $A = \{a\}$,then we write (a] for (A] and this is called a principal ideal generated by a. The set of all principal ideals of L is a distributive lattice and it is denoted by $P\mathfrak{I}(L)$. A proper ideal P of L is called prime if for any $x, y \in L, x \wedge y \in P$ then $x \in P$ or $y \in P$. For any $x, y \in L$ with $x \leqslant y, [x, y] = \{t \in L \mid x \leqslant t \leqslant y\}$ is a bounded distributive lattice with respect to the operations induced from those on L. An element m is maximal in $(L, \leqslant)$ if and only if $m \wedge x = x$ for all $x \in L$. For any $A \subseteq L, A^\star = \{x \in L \mid a \wedge x = 0 \ \forall \ a \in A\}$ is an ideal of L. We write $[a]^\star$ for $\{a\}^\star$. We don't know, so far , whether $\vee$ is associative in an ADL or not. In this paper L denotes an ADL in which $\vee$ is associative.

Lemma 2.2. *Let L be an ADL and I is an ideal of L. Then, for any* $a, b \in L$, *we have the following:*
(1) $(a] = \{a \wedge x \mid x \in L\}$.
(2) $a \in (b] \Leftrightarrow b \wedge a = a$.
(3) $a \wedge b \in I \Leftrightarrow b \wedge a \in I$.
(4) $(a] \cap (b] = (a \wedge b] = (b \wedge a]$
(5) $(a] \vee (b] = (a \vee b] = (b \vee a]$
(6) $(a] = L \Longleftrightarrow a$ *is maximal.*

Lemma 2.3. *Let L be an ADL and* $x, y \in L$. *Then the following statements hold:*
(1) $[x \vee y]^\star = [x]^\star \cap [y]^\star$
(2) $[x \wedge y]^\star = [y \wedge x]^\star$
(3) $[x]^{\star\star\star} = [x]^\star$
(4) $x \leqslant y \Rightarrow [y]^\star \subseteq [x]^\star$
(5) $[x \wedge y]^{\star\star} = [x]^{\star\star} \cap [y]^{\star\star}$

Definition 2.4. Let L be an ADL . A unary operation $\star$ on L is called a pseudo-complementation on L if, for any $x, y \in L$,
(1) $x \wedge y = 0 \Leftrightarrow x^\star \wedge y = y$
(2) $(x \vee y)^\star = x^\star \wedge y^\star$

It can be easily seen that the above conditions (1) and (2) of a pseudocomplementation are independent of each other.

LEMMA 2.4. *Let L be an ADL with pseudo-complementation $*$. Then, for any $a, b \in L$, we have the following:*

(1) $0^\star$ *is maximal*
(2) *if a is maximal, then* $a^\star = 0$
(3) $0^{\star\star} = 0$
(4) $a^\star \wedge a = 0$
(5) $a^{\star\star} \wedge a = a$
(6) $a^\star = a^{\star\star\star}$
(7) $a^\star = 0 \Leftrightarrow a^{\star\star}$ *is maximal*
(8) $a^\star \leqslant 0^\star$
(9) $a^\star \wedge b^\star = b^\star \wedge a^\star$
(10) $a \leqslant b \Rightarrow b^\star \leqslant a^\star$
(11) $a^\star \leqslant (a \wedge b)^\star$ *and* $b^\star \leqslant (a \wedge b)^\star$
(12) $a^\star \leqslant b^\star \Leftrightarrow b^{\star\star} \leqslant a^{\star\star}$
(13) $a = 0 \Leftrightarrow a^{\star\star} = 0$
(14) $(a \wedge b)^{\star\star} = a^{\star\star} \wedge b^{\star\star}$
(15) $(a \wedge b)^\star = (b \wedge a)^\star$
(16) $(a \vee b)^\star = (b \vee a)^\star$

THEOREM 2.1. *If $\star$ and $\perp$ are pseudo-complementations on an ADL L, then $a^\star \mapsto a^\perp$ is an isomorphism of $L^\star = \{a^\star | a \in L\}$ onto $L^\perp = \{a^\perp | a \in L\}$.*

DEFINITION 2.5. An ADL L is called a $\star - ADL$ if, to each $x \in L, [x]^{\star\star} = [x']^\star$ for some $x' \in L$.

DEFINITION 2.6. Let L be an ADL and $\star$ a pseudo-complementation on L. Then L is called a Stone ADL if ,for any $x \in L, x^\star \vee x^{\star\star} = 0^\star$

THEOREM 2.2. *Let L be a $\star - ADL$ with a maximal element m. Then L is a Stone ADL if and only if $[x \wedge y]^\star = [x]^\star \vee [y]^\star$ for all $x, y \in L$.*

THEOREM 2.3. *An ADL L is stone ADL if and only if $P\mathfrak{I}(L)$ of principal ideals of L is a stone lattice.*

THEOREM 2.4. *Let L be an ADL with maximal element m . Then L is a Stone ADL if and only if for any $x \in L, [x]^\star \vee [x]^{\star\star} = L$.*

THEOREM 2.5. *If L is a Stone ADL with respect to a pseudo-complementation $\star$, then $L^\star$ is a Boolean algebra.*

DEFINITION 2.7. A non trivial ADL L is called dense if $a \wedge b \neq 0$ for all $a \neq 0$ and $b \neq 0$.

THEOREM 2.6. [*Chinese remainder theorem*] *Let A be an Algebra and L be a sub lattice of the structure lattice $Con(A)$. Then L is a distributive permutable sublattice of $Con(A)$if and only if, for any $n \in Z^+, \theta_1, \theta_2, ..., \theta_n \in L$ and $a_1, a_2, ..., a_n \in A$ with $(a_i, a_j) \in \theta_i \circ \theta_j$ for $i \neq j, 1 \leqslant i, j \leqslant n$, there exists $a \in A$ such that $(a, a_i) \in \theta_i$ for $1 \leqslant i \leqslant n$.*

3. Sheaf Representation of Stone ADLs

DEFINITION 3.1. A triple (S, Π, X) is called sheaf of algebras of (the same) type τ over X if S and X are topological spaces and $\Pi : S \to X$ is a local homeomorphism of S onto X satisfying the following:

(1) For each $p \in X, \Pi^{-1}(p) = S_p$ in an algebra of type τ; S_p is called the stalk at p.
(2) If f is an n-ary operation in τ and $n > 0$, the map $(s_1, s_2, ..., s_n) \mapsto f(s_1, s_2, ..., s_n)$ is a continuous mapping of $S^{(n)}$ into S, where $S^{(n)} = \{(s_1, s_2, ..., s_n) \in S^n | \Pi(s_1) = \Pi(s_2) = ... = \Pi(s_n)\}$
(3) If f_0 is a nullary operation in τ, then the map $p \mapsto f_0^p$ is a continuous map of X into S, where f_0^p is the element in the stalk S_p corresponding to f_0.

If (S, Π, X) is a sheaf of algebras of type τ over X and $Y \subseteq X$, then a continuous map $\sigma : Y \to S$ is called a section on Y if $\Pi \circ \sigma$ is the identity map on Y. Sections on the whole space X are called global sections . The set of all global sections will be denoted by $\Gamma(X, S)$. It can be easily verified that $\Gamma(X, S)$ is an algebra of type τ under the pointwise operations. Let us recall that a topological space is called a Boolean space if it is compact, Hausdroff and totally disconnected. Now, we first prove the following.

THEOREM 3.1. *Let (S, Π, X) be a sheaf of dense ADLs over a Boolean space X such that there is a maximal global section and that , for any global section f , support $|f| = \{p \in X | f(p) \neq 0\}$ is open. Then $\Gamma(X, S)$ is a Stone ADL.*

PROOF. Clearly $\Gamma(X, S)$ is an ADL under the pointwise operations. It is well known that, for any global sections f and g, the set

$$< f, g >= \{p \in X | f(p) \neq f(g)\}$$

is a closed set in X and by hypothesis $< f, 0 > (= |f|)$ is open also. Thus $|f|$ is a clopen set in X for all global sections f. Let g_0 be a maximal global section; that is, g_0 is a maximal element in the ADL $\Gamma(X, S)$. Now, for each $f \in \Gamma(X, S)$, define $f^\star : X \to S$ by

$$f^\star(p) = \begin{cases} 0 & \text{if } f(p) \neq 0. \\ g_0(p) & \text{if } f(p) = 0 \end{cases}$$

Since g_0 is continuous and $|f|$ is clopen, it follows that $f^\star$ is continuous and $f^\star \in \Gamma(X, S)$. Since each stalk is dense , it can be easily seen that

$$f \wedge g = 0 \Leftrightarrow f^\star \wedge g = g \;\text{ and }\; (f \vee g)^\star = f^\star \wedge g^\star$$

for any global sections f and g. Therefore $f \mapsto f^\star$ is a pseudo-complementation on $\Gamma(X, S)$. Also,

$$f^\star \vee f^{\star\star} = g_0 (= 0^\star)$$

for any $f \in \Gamma(X, S)$. Thus $\Gamma(X, S)$ is a Stone ADL. □

Next, we shall prove a converse of the above theorem , in the sense that , every Stone ADL is isomorphic to $\Gamma(X, S)$ for a suitable sheaf (S, Π, X) of dense ADLs satisfying the properties in the hypothesis of the above theorem. First we shall recall the following.

Definition 3.2. For any element a in an ADL L, define

$$\theta_a = \{(x, y)|a \wedge x = a \wedge y\}.$$

Then θ_a is a congruence relation on L.

Lemma 3.1. *Let L be a Stone ADL. Then $\theta_a \cap \theta_b = \theta_{a\vee b}$ and $\theta_a \circ \theta_b = \theta_{a\wedge b}$ for any $a, b \in L^{\star\star}$ and hence $\{\theta_a|a \in L^{\star\star}\}$ is a permutable and distributive sublattice of the lattice $Con(L)$ of congruence relations on L.*

Proof. For any $a, b \in L^{\star\star}$ and $x, y \in L$, we have $a \wedge x = a \wedge y$ implies that $a \wedge b \wedge x = b \wedge a \wedge x = b \wedge a \wedge y = a \wedge b \wedge y$ and $b \wedge a \wedge x = b \wedge a \wedge y$ and hence $\theta_a, \theta_b \subseteq \theta_{a\wedge b}$, so that $\theta_a \circ \theta_b \subseteq \theta_{a\wedge b}$. On the other hand, let $x, y \in \theta_{a\wedge b}$ so that $a\wedge b\wedge x = a\wedge b\wedge y$. Put $z = (b\wedge x)\vee b^\star \wedge y$. Then $a\wedge z = (a\wedge b\wedge x)\vee(a\wedge b^\star \wedge y) = (a\wedge b\wedge y)\vee(a\wedge b^\star\wedge y) = (b\wedge a\wedge y)\vee(b^\star\wedge a\wedge y) = (b\vee b^\star)\wedge a\wedge y = 0^\star\wedge a\wedge y = a\wedge y$ and $b\wedge z = (b \wedge b \wedge x) \vee (b \wedge b^\star \wedge y) = b \wedge x$ and hence $(x, z) \in \theta_b$ and $(z, y) \in \theta_a$ so that $(x, y) \in \theta_a \circ \theta_b$. Thus $\theta_a \circ \theta_b = \theta_{a\wedge b}$. It is easy to see that $\theta_{a\vee b} = \theta_a \cap \theta_b$. □

Let us recall that, for any Boolean algebra B, $Spec(B)$ denotes the space of all prime ideals of B together with the hull-kernel topology for which $\{X_a|a \in B\}$ is a base , where $X_a = \{p \in Spec(B)|a \notin P\}$ and that $Spec(B)$ is a Boolean space.

Lemma 3.2. *Let L be a Stone ADL and P be a prime ideal of the Boolean algebra $L^{\star\star} = \{x^{\star\star}|x \in L\}$. Let*

$$\theta_P = \{(x, y) \in L \times L|a \wedge x = a \wedge y \ for\ some\ a \in L^{\star\star} - P\}.$$

Then θ_P is a congruence relation on L and L/θ_P is a dense ADL.

Proof. Clearly, θ_P is a congruence relation on L. Also, if $a \in L^{\star\star} - P$, then $(a, 0) \notin \theta_P$ and hence L/θ_P is a nontrivial ADL. Let $x, y \in L$ such that $x/\theta_P \wedge y/\theta_P = 0/\theta_P$. Then $(x \wedge y, 0) \in \theta_P$ and hence $a \wedge x \wedge y = 0$ for some $a \in L^{\star\star} - P$. Now, $a \wedge x^{\star\star} \wedge y^{\star\star} = 0 \in P$ and hence $x^{\star\star} \in P$ or $y^{\star\star} \in P$ and therefore $x^\star \notin P$ or $y^\star \notin P$ so that $(x, 0) \in \theta_P$ or $(y, 0) \in \theta_P$ (since $x^\star \wedge x = 0$ and $x^\star \in L^{\star\star}$). Therefore $x/\theta_P = 0/\theta_P$ or $y/\theta_P = 0/\theta_P$. Thus L/θ_P is a dense ADL. □

It can be easily verified that if L is a Stone ADL and P is a minimal prime ideal of L, then $P^{\star\star} = \{x^{\star\star}|x \in P\}$ is a prime ideal of $L^{\star\star}$ and $P \mapsto P^{\star\star}$ is a *homeomorphism* of the space of minimal prime ideals of L onto $Spec(L^{\star\star})$.

Lemma 3.3. *Let L be a Stone ADL and a be an element in the Boolean algebra $L^{\star\star}$. Then*

$$\theta_a = \bigcap\{\theta_P|P \in Spec(L^{\star\star}) \ and\ a \notin P\}$$

Proof. By the definition of θ_a and θ_P, it is clear that $\theta_a \subseteq \theta_P$ for all $P \in Spec(L^{\star\star})$ such that $a \notin P$. Conversely, suppose that $(x, y) \notin \theta_a$. Then $a\wedge x \neq a\wedge y$. Put

$$I = \{b \in L^{\star\star}|b \wedge x = b \wedge y\}$$

Then clearly I is an ideal of $L^{\star\star}$ not containing a and hence there exists a prime ideal P of $L^{\star\star}$ such that $I \subseteq P$ and $a \notin P$. Now, $(x, y) \notin \theta_P, P \in Spec(L^{\star\star})$ and $a \notin P$. □

Corollary 3.1. *If L is a Stone ADL, then* $\bigcap\{\theta_P | P \in Spec(L^{\star\star})\} = \triangle$*, the diagonal relation on L.*

Proof. Consider $0^\star$, which is a maximal element in L. Then $0^\star \wedge x = x$, so that $\theta_{0^\star} = \triangle$. Also, $0^\star$ is the largest element in $L^{\star\star}$ and hence $0^\star$ is not in any prime ideal of $L^{\star\star}$. □

Theorem 3.2. *Any Stone ADL is isomorphic to* $\Gamma(X, S)$ *for a suitable sheaf* (S, Π, X) *of dense ADLs over a Boolean space X.*

Proof. Let L be an *ADL* and X the Boolean space $Spec(L^{\star\star})$. Let S be the disjoint union of L/θ_P's, $P \in X$. For each x in L, define $\hat{x} : X \to S$ by $\hat{x}(P) = x/\theta_P$. Let S be equipped with the largest topology with respect to which each $\hat{x}, x \in L$, is continuous. Define $\Pi : S \to X$ by $\Pi(s) = P$ for all $s \in L/\theta_P$. Then we shall prove that (S, Π, X) is a sheaf of dense *ADLs* over the Boolean space X and that $L \cong \Gamma(X, S)$. First we shall prove that

$$\{\hat{x}(X_a) | x \in L \text{ and } a \in L^{\star\star}\}$$

is a base for the largest topology on S with respect to which each $\hat{x}, x \in L$, is continuous.

For any $x, y \in L$ and $a, b \in L^{\star\star}$, we have

$$\begin{aligned} s \in \hat{x}(X_a) \cap \hat{y}(X_b) &\Rightarrow s = x/\theta_P = y/\theta_Q,\ P \in X_a, Q \in X_b \\ &\Rightarrow P = Q \in X_a \cap X_b \text{ and } c \wedge x = c \wedge y \text{ for some } c \in L^{\star\star} - P. \\ &\Rightarrow P \in X_{a \wedge b \wedge c}, \text{ and } s = \hat{x}(P) = \hat{y}(P), \\ &\Rightarrow s \in \hat{x}(X_{a \wedge b \wedge c}) \subseteq \hat{x}(X_a) \cap \hat{y}(X_b) \end{aligned}$$

Therefore the class mentioned is a base for a topology τ on S. Also,

$$\begin{aligned} P \in \hat{y}^{-1}(\hat{x}(X_a)) &\Rightarrow x/\theta_P = y/\theta_P \text{ and } P \in X_a \\ &\Rightarrow b \wedge x = b \wedge y \text{ for some } b \in L^{\star\star} - P. \\ &\Rightarrow P \in X_{b \wedge a} \subseteq \hat{y}^{-1}(\hat{x}(X_a)) \end{aligned}$$

Therefore each $\hat{x}, x \in L$ is continuous with respect to the topology τ and it can be easily proved that this topology τ is the largest one on S with respect to which each $\hat{x}$ is continuous.

Any element of S is of the form $x/\theta_P (= \hat{x}(P))$ for some $x \in L$ and $P \in X$ and, clearly, for any open set W in X, the restriction of Π to the open set $\hat{x}(W)$ is a homeomorphism of $\hat{x}(W)$ onto W. Therefore Π is a local homeomorphism of S onto S. The continuity of the ADL operations follows from the facts that each θ_P is a congruence on L and, for any $x, y \in L, \{P \in X | (x, y) \in \theta_P\}$ is an open subset of X. Also, for each $P \in X$ by Lemma 3.2, L/θ_P is a dense ADL. Thus (S, Π, X) is a sheaf of dense *ADLs* over the Boolean space X. Finally, we shall prove that $x \mapsto \hat{x}$ is an isomorphism of L onto the ADL $\Gamma(X, S)$ of all global sections. For any $x, y \in L, \hat{x} = \hat{y}$ implies that $x/\theta_P = y/\theta_P$ for all $P \in X$ and hence, by Corollary 3.1, $x = y$. Therefore $x \mapsto \hat{x}$ is an injection. Since the operations on $\Gamma(X, S)$

are pointwise, $x \mapsto \hat{x}$ is a homomorphism of $ADLs$. Lastly, let $\sigma \in \Gamma(X, S)$. For each $P \in X, \sigma(P) = x_P(P)$ for some $x_P \in L$ and there exists $a_P \in L^{\star\star}$ such that $\sigma/X_{a_P} = \hat{x}_P/X_{a_p}$ (since $< \sigma, x_p >$ is a closed set in X and X_as, $a \in L^{\star\star}$, form a base for open sets). By using the compactness of X and the fact that X is a Boolean space, there exist $a_1, a_2, ..., a_n \in L^{\star\star}$ and $x_1, x_2, ..., x_n \in L$ such that

$$a_1 \wedge a_2 \wedge .... \wedge a_n = 0,\ a_1 \vee a_2 \vee .... \vee a_n = 0^\star \text{ and}$$
$$\sigma/X_{a_i} = \hat{x}_i/X_{a_i}, \text{ for all } 1 \leqslant i \leqslant n.$$

Now, since $X_{a \wedge b} = X_a \cap X_b$ for all $a, b \in L^{\star\star}$, it follows that,

$$\hat{x}_i/X_{a_i \wedge a_j} = \sigma/X_{a_i \wedge a_j} = \hat{x}_j/X_{a_i \wedge a_j}$$

for all $1 \leqslant i, j \leqslant n$ and hence by lemma 3.2, we have

$$(x_i, x_j) \in \theta_{a_i \wedge a_j} \text{ for all } 1 \leqslant i, j \leqslant n.$$

Now, by Lemma 3.1, $\theta_{a_1}, \theta_{a_2}, \theta_{a_3}, ..., \theta_{a_n}$ are members of a permutable and distributive sublattice of the congruence lattice $Con(L)$ and

$$(x_i, x_j) \in \theta_{a_i} \circ \theta_{a_j} \text{ for all } 1 \leqslant i, j \leqslant n$$

Therefore, by Theorem 2.6 , there exists $x \in L$ such that $(x, x_i) \in \theta_{a_i}$ for all $1 \leqslant i \leqslant n$. Then

$$\hat{x}/X_{a_i} = \hat{x}_i/X_{a_i} = \sigma/X_{a_i} \text{ for all } i$$

and hence $\hat{x} = \sigma$. Then $x \mapsto \hat{x}$ is an isomorphism of L onto $\Gamma(X, S)$. □

NOTE. Note that in the above sheaf (S, Π, X), the support of each global section is open, since $\hat{x}(P) = 0$ if and only if $x^{\star\star} \in P$, for any $x \in L$ and $P \in X$, so that $|\hat{x}| = X_{x^{\star\star}}$.

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