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A NEW FORM OF SOME NANO SETS

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Abstract. In this paper, we made an attempt to notions of nano $t^{\#}$-set, nano $\mathcal{B}^{\#}$-set, nano t_α-set, nano $\mathcal{B}_\alpha$-set and strong nano $\mathcal{B}$-set are introduce and investigate.

1. Introduction and Preliminaries

M. Lellis Thivagar et al [**2, 3**] introduced the concept of nanotopological spaces which was defined in terms of approximations and boundary region of a subset of a universe U using an equivalence relation on it.

In this paper, we introduce and investigate the new notion called nano semi-open sets which is weaker than nano α-open sets and stronger than nano β-open sets. Also we introduce and investigate some new nano generalized classes of $\mathcal{N}$.

Throughout this paper $(U, \mathcal{N})$ (or U) represent nano topological spaces(briefly NT's) on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space $(U, \mathcal{N})$, $n\text{-}cl(H)$ and $n\text{-}int(H)$ denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 1.1. ([**4**]) Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(1) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

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(2) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X \neq \phi\}$.

(3) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Proposition 1.1 ([**2**]). *If (U, R) is an approximation space and $X, Y \subseteq U$, then*

(1) $L_R(X) \subseteq X \subseteq U_R(X)$;
(2) $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$;
(3) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
(4) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
(5) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
(6) $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
(7) $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;
(8) $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$;
(9) $U_R U_R(X) = L_R U_R(X) = U_R(X)$;
(10) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 1.2. ([**2**]) Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(1) U and $\phi \in \tau_R(X)$.
(2) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
(3) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

This means that $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X and $(U, \tau_R(X))$ as a nano topological space. The elements of $\tau_R(X)$ are called nano-open sets (briefly n-open sets).

In the rest of the paper, we denote a nano topological space by $(U, \mathcal{N})$, where $\mathcal{N} = \tau_R(X)$. The nano-interior and nano-closure of a subset A of U are denoted by $n\text{-}int(A)$ and $n\text{-}cl(A)$, respectively.

Definition 1.3. ([**2**]) A subset H of a nano topological space $(U, \mathcal{N})$ is called

(1) nano α-open set (briefly, $n\alpha$-open set) if $H \subseteq n\text{-}int(n\text{-}cl(n\text{-}int(H)))$.
(2) nano semi open set if $H \subseteq n\text{-}cl(n\text{-}int(H))$.
(3) nano pre open set if $H \subseteq n\text{-}int(n\text{-}cl(H))$.
(4) nano b-open set (briefly, nb-open set) if

$$H \subseteq n\text{-}cl(n\text{-}int(H)) \cup n\text{-}int(n\text{-}cl(H)).$$

(5) nano β-open set (briefly, $n\beta$-open set) if $H \subseteq n\text{-}cl(n\text{-}int(n\text{-}cl(H)))$.

The complements of the above mentioned sets are called their respective nano closed sets.

Definition 1.4. ([**1**]) A subset H of a space $(U, \mathcal{N})$ is called a

(1) nano t-set (briefly, nt-set) if $n\text{-}int(H) = n\text{-}int(n\text{-}cl(H))$;
(2) nano $\mathcal{B}$-set (briefly, $n\mathcal{B}$-set) if $H = P \cap Q$, where P is n-open and Q is nt-set.

2. On nano $t^{\#}$-set and nano $\mathcal{B}^{\#}$-set

DEFINITION 2.1. A subset H of a space $(U, \mathcal{N})$ is called a

(1) nano $t^{\#}$-set (briefly, $nt^{\#}$-set) if $n\text{-}int(H) = n\text{-}cl(n\text{-}int(H))$,
(2) nano $\mathcal{B}^{\#}$-set (briefly, $n\mathcal{B}^{\#}$-set) if $H = P \cap Q$, where P is n-open and Q is $nt^{\#}$-set.

REMARK 2.1. (1) In U with the nano topology $\mathcal{N}$, a subset H with $n\text{-}int(H) = \phi$ is $nt^{\#}$-set if and only if $n\text{-}int(H) = \phi = n\text{-}int(H)$.

(2) In U with the topology $\mathcal{N}$, there is no proper subset H, with $n\text{-}int(H) \neq \phi$ which is $nt^{\#}$-set. (or) The only subset in U, with nonempty interior, which is $nt^{\#}$-set is U itself.

PROOF. (1). Proof is direct from the definition.

(2). If H is a proper $nt^{\#}$-set in U with $n\text{-}int(H) \neq \phi$, then $n\text{-}int(H) = n\text{-}cl(n\text{-}int(H)) = n\text{-}cl(n\text{-}int(H))$. Thus $n\text{-}int(H)$ is a proper n-clopen subset in U contradicting the connectedness of U. Hence the result. □

EXAMPLE 2.1. Let $U = \{m_1, m_2, m_3, m_4\}$ with

$$U/R = \{\{m_1\}, \{m_2\}, \{m_3, m_4\}\} \text{ and } X = \{m_2, m_4\}.$$

Then the nanotopology $\mathcal{N} = \{\phi, \{m_2\}, \{m_3, m_4\}, \{m_2, m_3, m_4\}, U\}$. Then the set $\{m_3\}$ is $nt^{\#}$-set and $n\mathcal{B}^{\#}$-set.

REMARK 2.2. In a space $(U, \mathcal{N})$,

(1) each n-open set is $n\mathcal{B}^{\#}$-set.
(2) each $nt^{\#}$-set is $n\mathcal{B}^{\#}$-set.

EXAMPLE 2.2. In Example 2.1, then

(1) the set $\{m_3\}$ is $n\mathcal{B}^{\#}$-set but not n-open.
(2) the set $\{m_2\}$ is $n\mathcal{B}^{\#}$-set but not $nt^{\#}$-set.

PROPOSITION 2.1. *If P and Q are $nt^{\#}$-sets of a space $(U, \mathcal{N})$, then $P \cap Q$ is $nt^{\#}$-set.*

PROOF. Let P and Q be $nt^{\#}$-sets. Then we have $n\text{-}int(P \cap Q) \subset n\text{-}int(P \cap Q) \subset n\text{-}cl(n\text{-}int(P \cap Q)) = n\text{-}cl(n\text{-}int(P) \cap n\text{-}int(Q)) \subset n\text{-}cl(n\text{-}int(P)) \cap n\text{-}cl(n\text{-}int(Q)) = n\text{-}int(P) \cap n\text{-}int(Q) = n\text{-}int(P \cap Q)$. Thus $n\text{-}int(P \cap Q) = n\text{-}cl(n\text{-}int(P \cap Q))$ and hence $P \cap Q$ is $nt^{\#}$-set. □

REMARK 2.3. The diagram true for any subset of a space $(U, \mathcal{N})$:

$$\begin{array}{ccccc} & & n\alpha\textbf{-open} & \longrightarrow & \textbf{nano pre-open} \\ n\textbf{-open} & \nearrow & \downarrow & & \downarrow \\ & \searrow & \textbf{nano semi-open} & \longrightarrow & nb\textbf{-open} \\ & & & & \downarrow \\ & & & & n\beta\textbf{-open} \end{array}$$

In this diagram, none of the implications are reversible.

THEOREM 2.1. *For a subset H of a space $(U, \mathcal{N})$, the following are equivalent:*

(1) *H is n-open,*
(2) *H is nano semi-open and $n\mathcal{B}^{\#}$-set.*

PROOF. (1)⇒(2): (2) follows by Remark 2.3 and (1) of Remark 2.2.

(2)⇒(1): Given H is nano semi-open and $n\mathcal{B}^{\#}$-set. So $H = P \cap Q$ where P is n-open and $n\text{-}int(Q) = n\text{-}cl(n\text{-}int(Q))$. Then $H \subset P = n\text{-}int(P)$. Also H is nano semi-open implies $H \subset n\text{-}cl(n\text{-}int(H)) \subset n\text{-}cl(n\text{-}int(Q)) = n\text{-}int(Q)$ by assumption. Thus $H \subset n\text{-}int(P) \cap n\text{-}int(Q) = n\text{-}int(P \cap Q) = n\text{-}int(H)$ and hence H is n-open. □

REMARK 2.4. The familial concepts of nano semi-open and the familial concepts $n\mathcal{B}^{\#}$-set are independent.

EXAMPLE 2.3. In Example 2.1, then

(1) the set $\{m_1, m_2\}$ is nano semi-open but not $n\mathcal{B}^{\#}$-set.
(2) the set $\{m_3\}$ is $n\mathcal{B}^{\#}$-set but not nano semi-open.

3. On nano t_α-set and nano $\mathcal{B}_\alpha$-set

DEFINITION 3.1. A subset H of a space $(U, \mathcal{N})$ is called a

(1) nano t_α-set (briefly, nt_α-set) if $n\text{-}int(H) = n\text{-}cl(n\text{-}int(n\text{-}cl(H)))$.
(2) nano $\mathcal{B}_\alpha$-set (briefly, $n\mathcal{B}_\alpha$-set) if $H = P \cap Q$, where P is n-open and Q is nt_α-set.

REMARK 3.1. In a space $(U, \mathcal{N})$,

(1) each n-open set is $n\mathcal{B}_\alpha$-set.
(2) each nt_α-set is $n\mathcal{B}_\alpha$-set.

EXAMPLE 3.1. In Example 2.1, then

(1) the set $\{m_1\}$ is $n\mathcal{B}_\alpha$-set but not n-open.
(2) the set $\{m_3, m_4\}$ is $n\mathcal{B}_\alpha$-set but not nt_α-set.

PROPOSITION 3.1. *If P and Q are nt_α-sets of a space $(U, \mathcal{N})$, then $P \cap Q$ is nt_α-set.*

PROOF. Let P and Q be nt_α-sets. Then we have $n\text{-}int(P \cap Q) \subset n\text{-}int(P \cap Q) \subset n\text{-}int(n\text{-}cl(P \cap Q)) \subset n\text{-}cl(n\text{-}int(n\text{-}cl(P \cap Q))) \subset n\text{-}cl(n\text{-}int(n\text{-}cl(P))) \cap n\text{-}cl(n\text{-}int(n\text{-}cl(Q))) = n\text{-}int(P) \cap n\text{-}int(Q) = n\text{-}int(P \cap Q)$. Then $n\text{-}int(P \cap Q) = n\text{-}cl(n\text{-}int(n\text{-}cl(P \cap Q)))$ and hence $P \cap Q$ is nt_α-set. □

REMARK 3.2. In a nanotopological space the notions of $n\beta$-open and being a $n\mathcal{B}_\alpha$-set are independent.

EXAMPLE 3.2. In Example 2.1, then

(1) the set $\{m_4\}$ is $n\beta$-open set but not $n\mathcal{B}_\alpha$-set.
(2) the set $\{m_1\}$ is $n\mathcal{B}_\alpha$-set but not $n\beta$-open.

THEOREM 3.1. *For a subset H of a space $(U,\mathcal{N})$, the following are equivalent:*

(1) *H is n-open,*
(2) *H is $n\beta$-open and $n\mathcal{B}_\alpha$-set.*

PROOF. (1) ⇒ (2): follows by Remark 2.3 and (1) of Remark 3.1.

(2) ⇒ (1): Given H is $n\beta$-open and $n\mathcal{B}_\alpha$-set. So $H = P \cap Q$ where P is n-open and Q is nt_α-set. Then $H \subset P = n\text{-}int(P)$. Also H is $n\beta$-open implies $H \subset n\text{-}cl(n\text{-}int(n\text{-}cl(H))) \subset n\text{-}cl(n\text{-}int(n\text{-}cl(Q))) = n\text{-}int(Q)$ since Q is nt_α-set. Thus $H \subset n\text{-}int(P) \cap n\text{-}int(Q) = n\text{-}int(P \cap Q) = n\text{-}int(H)$ and hence H is n-open. □

4. Properties of nano strong $\mathcal{B}$-set

DEFINITION 4.1. A subset H of a space $(U,\mathcal{N})$ is called a nano strong $\mathcal{B}$-set (briefly, strong $n\mathcal{B}$-set) if $H = P \cap Q$, where P is n-open and Q is a nt-set and $n\text{-}int(n\text{-}cl(H)) = n\text{-}cl(n\text{-}int(H))$.

PROPOSITION 4.1. *In a space $(U,\mathcal{N})$,*

(1) *each n-open set is strong $n\mathcal{B}$-set.*
(2) *each nt-set H with $n\text{-}int(n\text{-}cl(H)) = n\text{-}cl(n\text{-}int(H))$ is strong-$n\mathcal{B}$-set.*

PROOF. Proof follows directly from the Definition of a strong $n\mathcal{B}$-set. □

REMARK 4.1. The converses of (1) in Proposition 4.1 is need not to be true as shown in the next Example.

EXAMPLE 4.1. Let $U = \{m_1, m_2, m_3, m_4\}$ with

$$U/R = \{\{m_1\}, \{m_3\}, \{m_2, m_4\}\} \text{ and } X = \{m_1, m_2\}.$$

Then the nanotopology $\mathcal{N} = \{\phi, \{m_1\}, \{m_2, m_4\}, \{m_1, m_2, m_4\}, U\}$. Then the set $\{m_3\}$ is strong $n\mathcal{B}$-set but not n-open.

PROPOSITION 4.2. *In a space $(U,\mathcal{N})$, each strong $n\mathcal{B}$-set is $n\mathcal{B}$-set.*

PROOF. Proof follows from the fact that an nt-set H with $n\text{-}int(n\text{-}cl(H)) = n\text{-}cl(n\text{-}int(H))$ is an nt-set. □

THEOREM 4.1. *For a subset H of a space $(U,\mathcal{N})$, the following are equivalent:*

(1) *H is n-open,*
(2) *H is nb-open and strong $n\mathcal{B}$-set.*

PROOF. (1) ⇒ (2): (2) follows by Remark 2.3 and Proposition 4.1.

(2) ⇒ (1): Given H is nb-open and strong $n\mathcal{B}$-set. Since H is a strong $n\mathcal{B}$-set, $H = P \cap Q$ where U is n-open and V is nt-set with $n\text{-}int(n\text{-}cl(V)) = n\text{-}cl(n\text{-}int(V))$. Then $H \subset U = n\text{-}int(U)$. Also H is nb-open implies $H \subset n\text{-}int(n\text{-}cl(H)) \cup n\text{-}cl(n\text{-}int(H)) \subset n\text{-}int(n\text{-}cl(V)) \cup n\text{-}cl(n\text{-}int(V)) = n\text{-}int(V)$ by assumption. Thus $H \subset n\text{-}int(U) \cap n\text{-}int(V) = n\text{-}int(P \cap Q) = n\text{-}int(H)$ and hence H is n-open. □

REMARK 4.2. The familial concepts of nb-open and the familial concepts strong $n\mathcal{B}$-set are independent.

EXAMPLE 4.2. (1) In Example 4.1, then the set $\{m_2\}$ is nb-open but not strong $n\mathcal{B}$-set.

(2) In Example 2.1, then the set $\{m_3\}$ is strong $n\mathcal{B}$-set but not nb-open.

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