BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Vol. 9(2019), 287-300
DOI: 10.7251/BIMVI1902287M

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

A STUDY OF QUASI-INTERIOR IDEALS OF SEMIRING

Murali Krishna Rao Marapureddy

Abstract. In this paper, as a further generalization of ideals, we introduce the notion of quasi-interior ideal as a generalization of quasi ideal, interior ideal,left(right) ideal, ideal ofsemiring, study the properties of quasi-interior ideals of semiring and characterize the quasi-interior simple semiring ,regular semiring using quasi-interior ideal ideals ofsemiring..

1. Introduction

The notion of a semiring was introduced by Vandiver [**19**] in 1934, but semirings had appeared in earlier studies on the theory of ideals of rings. Semiring is a generalization of ring but also of a generalization of distributive lattice.Semirings are structually similar to semigroups than to semirings. A universal algebra $(S,+,\cdot)$ is called a semiring if and only if $(S,+)$, $(S,\cdot)$ are semigroups which are connected by distributive laws, *i.e.*, $a(b+c) = ab+ac,\ (a+b)c = ac+bc,$ for all $a,b,c \in S$. The theory of rings and theory of semigroups have considerable impact on the development of theory of semirings. Semirings play an important role in studying matrices and determinants. Semirings are useful in the areas of theoretical computer science as well as in the solution of graph theory, optimization theory, in particular for studying automata,coding theory and formal languages. Semiring theory has many applications in other branches of mathematics.

Ideals play an important role in advance studies and uses of algebraic structures. Generalization of ideals in algebraic structures is necessary for further study of algebraic structures. Many mathematicians proved important results and charecterizations of algebraic structures by using the concept and the properties of generalization of ideals in algebraic structures. Henriksen [**3**] and Shabir et al. [**17**]

2010 *Mathematics Subject Classification.* 16Y60; 06Y99.
Key words and phrases. quasi-interior ideal,bi-interior ideal, bi-quasi ideal,bi-ideal,quasi ideal, interior ideal,regularsemiring, bi-quasi-interior simple semiring.

studied ideals in semirings. We know that the notion of a one sided ideal of any algebraic structure is a generalization of notion of an ideal. The quasi ideals are generalization of left ideal and right ideal whereas the bi-ideals are generalization of quasi ideals.

In 1952 the concept of bi-ideals was introduced by Good and Hughes [**2**] for semigroups. The notion of bi-ideals in rings and semirings were introduced by Lajos and Szasz [**10**]. Bi-ideal is a special case of (m-n) ideal. In 1956, Steinfeld [**17**] first introduced the notion of quasi ideals for semigroups and then for rings. Iseki [**4, 5, 6**] introduced the concept of quasi ideal for a semiring. Quasi ideals, bi-ideals in $\Gamma-$semirings studied by Jagtap and Pawar [**8**]. Murali Krishna Rao [**12, 13, 14, 15**] introduced the notion of left (right) bi-quasi ideal of semiring, semiring, $\Gamma-$semigroup and studied the properties of left bi-quasi ideals and characterized the left bi-quasi simple semiring and regular semiring using left bi-quasi ideals of semirings. Murali Krishna Rao [**15**] introduced the notion of bi-interior ideal as a generalization of quasi ideal, bi-ideal and interior ideal of semigroup and studied the properties of bi-interior ideals of semigroup, simple semigroup and regular semigroup. Murali Krishna Rao [**16**] introduced the notion of bi quasi-interior ideal of semiring as a generalization of ideal, left ideal, right ideal, bi-ideal, quasi ideal, bi-quasi ideal, bi-interior ideal and interior ideal of semiring and studied some of their properties and characterized the bi-quasi-interior simple semiring, regular semiring using bi-quasi-interior ideals of semiring.

In this paper, we introduce the notion of quasi-interior ideals as a generalization of quasi ideal, interior ideal, left(right) ideal and ideal of semiring and study the properties of quasi-interior ideals of semiring.

2. Preliminaries.

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. ([**1**]) A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively will be called semiring provided

(i) addition is a commutative operation.
(ii) multiplication distributes over addition both from the left and from the right.
(iii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$.

Example 2.1. Let M be the set of all natural numbers. Then $(M,\ max,\ min)$ is a semiring.

Example 2.2. Let M be the additive semigroup of all $m \times n$ matrices over the set of non negative rational numbers .Then with respect to usual matrix multiplication M is a semiring.

Definition 2.2. A semiring M is called a division semiring if for each non-zero element of M has multiplication inverse.

DEFINITION 2.3. A non-empty subset A of a semiring M is called

(i) a subsemiring of M if $(A, +)$ is a subsemigroup of $(M, +)$ and $AA \subseteq A$.
(ii) a quasi ideal of M if A is a subsemiring of M and $AM \cap MA \subseteq A$.
(iii) a bi-ideal of M if A is a subsemiring of M and $AMA \subseteq A$.
(iv) an interior ideal of M if A is a subsemiring of M and $MAM \subseteq A$.
(v) a left (right) ideal of M if A is a subsemiring of M and $MA \subseteq A\,(AM \subseteq A)$.
(vi) an ideal if A is a subsemiring of $M, AM \subseteq A$ and $MA \subseteq A$.
(vii) a $k-$ideal if A is a subsemiring of $M, AM \subseteq A$, $MA \subseteq A$ and $x \in M,\ x + y \in A, y \in A$ then $x \in A$.

DEFINITION 2.4. ([**16**]) A non-empty subset B of a semiring M is said to be bi-quasi-interior ideal of M if B is a subsemiring of M and $BMBMB \subseteq B$.

Every bi-quasi-interior ideal of a semiring M need not be bi-ideal,quasi-ideal, interior ideal bi-interior ideal. and bi-quasi ideals of semiring M.

DEFINITION 2.5. ([**15**]) A non-empty subset B of a semiring M is said to be bi-interior ideal of M if B is a subsemiring of M and $MBM \cap BMB \subseteq B$.

DEFINITION 2.6. ([**11**]) Let M be a semiring. A non-empty subset L of M is said to be left bi-quasi ideal (right bi-quasi ideal) of M if L is a subsemigroup of $(M, +)$ and $ML \cap LML \subseteq L$ $(LM \cap LML \subseteq L)$.

DEFINITION 2.7. ([**11**]) Let M be a semiring. L is said to be bi-quasi ideal of M if it is both a left bi-quasi ideal and a right bi-quasi ideal of M.

DEFINITION 2.8. ([**13**]) A semiring M is called a left bi-quasi simple semiring if M has no left bi-quasi ideal other than M itself.

3. Quasi-interior ideals of semirings

In this section we introduce the notion of quasi-interior ideal as a generalization of bi-ideal, quasi-ideal and interior ideal of semiring and study the properties of quasi-interior ideal of a semiring.

DEFINITION 3.1. A non-empty subset B of a semiring M is said to be left quasi-interior ideal of M if B is a subsemiring of M and $MBMB \subseteq B$.

DEFINITION 3.2. A non-empty subset B of a semiring M is said to be right quasi-interior ideal of M if B is a subsemiring of M and $BMBM \subseteq B$.

DEFINITION 3.3. A non-empty subset B of a semiring M is said to be quasi-interior ideal of M if B is a subsemiring of M and $B$is left and right quasi-interior ideal of M.

REMARK 3.1. Every quasi-interior ideal of a semiring M need not be quasi-ideal, interior ideal, bi-interior ideal. and bi-quasi ideal of semiring M.

EXAMPLE 3.1.

(i) Let Q be the set of all rational numbers, $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a, b, c, d \in Q \right\}$ be the multiplicative and the additive semigroups of M with respect to usual matrix multiplication and usual addition of matrices.Then M is a semiring

(a) If $R = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \mid 0 \neq a, 0 \neq b \in Q \right\}$ then R is a quasi ideal of a semiring M and R is neither a left ideal nor a right ideal.

(b) If $S = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \mid 0 \neq a \in Q \right\}$ then S is a bi-ideal of a semiring M.

(ii) If $M = \left\{ \begin{pmatrix} a & b \\ c & 0 \end{pmatrix} \mid a, b, c \in Q \right\}$ then M is a semiring with respect to usual addition of matrices and ternary operation is defined as usual matrix multiplication and $A = \left\{ \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} \mid 0 \neq a, 0 \neq b \in Q \right\}$. Then A is not a bi-ideal of the semiring M and A is left quasi interior ideal of the semiring M .

In the following theorem, we mention some important properties and we omit the proofs since proofs are straight forward.

Theorem 3.1. *Let M be a semring. Then the following are hold.*

(1) *Every left ideal is a left quasi-interior ideal of M.*
(2) *Every right ideal is a right quasi-interior ideal of M.*
(3) *Every quasi ideal is a quasi-interior ideal of M.*
(4) *Every ideal is a quasi-interior ideal of M.*
(5) *Intersection of a right ideal and a left ideal of M is a quasi-interior ideal of M.*
(6) *If L is a left ideal and R is a right ideal of a semiring M then $B = RL$ is a quasi-interior ideal of M.*
(7) *If B is a quasi-interior ideal and T is a subsemiring of M then $B \cap T$ is a quasi-interior ideal of a semiring M.*
(8) *Let M be asemiring and B be a subsemiring of M.If $MMMB \subseteq B$ then B is a left quasi-interior ideal of M.*
(9) *Let M be asemiring and B be a subsemiring of M.If $MMMB \subseteq B$ and$BMMM \subseteq B$ then B is a quasi-interior ideal of M.*
(10) *Intersection of a right quasi-interior ideal and a left quasi-interior ideal of M is a quasi-interior ideal of M.*
(11) *If L is a left ideal and R is a right ideal of a semiring M then $B = R \cap L$ is a quasi-interior ideal of M.*

Theorem 3.2. *If B be a left quasi-interior ideal of a semiring M, then B is a left bi-quasi ideal of M.*

Proof. Suppose B is a left quasi-interior ideal of the semiring M. Then $MBMB \subseteq B$. We have $BMB \subseteq MBMB$. Therefore $MB \cap BMB \subseteq BMB \subseteq MBMB \subseteq B$. Hence B is a left bi-quasi ideal of M. □

Corollary 3.1. *If B be a right quasi-interior ideal of a semiring M, then B is a right bi-quasi ideal of M.*

COROLLARY 3.2. *If B be a quasi-interior ideal of a semiring M, then B is a bi-quasi ideal of M.*

THEOREM 3.3. *If B be a left quasi-interior ideal of a semiring M, then B is a bi–interior ideal of M.*

PROOF. Suppose B is the left quasi-interior ideal of a semiring M. Then $MBMB \subseteq B$. We have $MBM \cap BMB \subseteq BMB \subseteq MBMB \subseteq B$. Hence B is a bi–interior ideal of M. □

COROLLARY 3.3. *If B be a right quasi-interior ideal of a semiring M, then B is a bi–interior ideal of M.*

COROLLARY 3.4. *If B be a quasi-interior ideal of a semiring M, then B is a bi–interior ideal of M.*

THEOREM 3.4. *Every left quasi-interior ideal of a semiring M is a bi-ideal of a semiring M.*

PROOF. Let B be a left quasi-interior ideal of the semiring M. Then $BMB \subseteq MBMB \subseteq B$. Therefore $BMB \subseteq B$. Hence every left quasi-interior ideal of a semiring M is a bi-ideal of the semiring M. □

COROLLARY 3.5. *Every right quasi-interior ideal of a semiring M is a bi-ideal of a semiring M.*

COROLLARY 3.6. *Every quasi-interior ideal of a semiring M is a bi-ideal of a semiring M.*

THEOREM 3.5. *Every left quasi-interior ideal of a semiring M is a bi-quasi interior ideal of a semiring M.*

PROOF. Let B be a left quasi-interior ideal of the semiring M. Then $MBMB \subseteq B$. Therefore $BMBMB \subseteq MBMB \subseteq B$. This completes the proof. □

COROLLARY 3.7. *Every right quasi-interior ideal of a semiring M is a bi-quasi interior ideal of a semiring M.*

COROLLARY 3.8. *Every quasi-interior ideal of a semiring M is a bi-quasi interior ideal of a semiring M.*

THEOREM 3.6. *Every interior ideal of a semiring M is a left quasi-interior ideal of M.*

PROOF. Let I be an interior ideal of the semiring M. Then $MIMI \subseteq MIM \subseteq I$. Hence I is a left quasi-interior ideal of the semiring M. □

THEOREM 3.7. *Let M be asemiring and B be a subsemiring of M. B is a quasi-interior ideal of M if and only if there exmist left ideals L and R such that $RL \subseteq B \subseteq R \cap L$.*

Proof. Suppose B is a quasi-interior ideal of the semiring$M$. Then $MBMB \subseteq B$. Let $R = MB$ and $L = MB$. Then L and R are left ideals of M. Therefore $RL \subseteq B \subseteq R \cap L$.

Conversely suppose that there exmist L and R are left ideals of M such that $RL \subseteq B \subseteq R \cap L$ .Then

$$MBMB \subseteq M(R \cap L)M(R \cap L) \subseteq M(R)M(L) \subseteq RL \subseteq B.$$

Hence B is a left quasi-interior ideal of a semiring M. □

Corollary 3.9. *Let M be a semiring and B be a subsemiring of M. B is a right quasi-interior ideal of M if and only if there exmist right ideals L and R such that $RL \subseteq B \subseteq R \cap L$.*

Theorem 3.8. *The intersection of a left quasi-interior ideal B of a semiring M and a right ideal A of M is always a left quasi-interior ideal of M.*

Proof. Suppose $C = B \cap A$.

$$MCMC \subseteq MBMB \subseteq B$$
$$MCMC \subseteq MAMA \subseteq A \text{ since } A \text{ is a left ideal of } M$$

Therefore $MCMC \subseteq B \cap A = C$.

Hence the intersection of a left quasi-interior ideal o B of the semiring M and a left ideal A of M is always a left quasi-interior ideal of M. □

Corollary 3.10. *The intersection of a right quasi-interior ideal B of a semiring M and a right ideal A of M is always a right quasi-interior ideal of M.*

Corollary 3.11. *The intersection of a quasi-interior ideal B of a semiring M and an ideal A of M is always a quasi-interior ideal of M.*

Theorem 3.9. *Let A and C be left quasi-interior ideals of a semiring M, $B = AC$ and B is additively subsemigroup of M. If $CC = C$ then B is a left quasi-interior ideal of M.*

Proof. Let A and C be left quasi-interior ideals of the semiring M and $B = AC$. Then $BB = ACAC = ACCCAC \subseteq ACMCMC \subseteq AC = B$. Therefore $B = AC$ is a subsemiring of M

$$\begin{aligned} MBMB &= MACMAC \\ &\subseteq MAMAC \subseteq AC = B. \end{aligned}$$

Hence B is a left quasi-interior ideal of M. □

Corollary 3.12. *Let A and C be quasi-interior ideal ideals of a semiring M, $B = CA$ and B is additively subsemigroup of M. If $CC = C$ then B is a quasi-interior ideal of M.*

Theorem 3.10. *Let A and C be subsemirings of M and $B = AC$. If A is the left ideal then B is a quasi-interior ideal of M.*

Proof. Let A and C be subsemirings of M and $B = AC$. Suppose A is the left ideal of M. Then $BB = ACAC \subseteq AC = B$.

$$\begin{aligned} MBMB &= MACMAC \\ &\subseteq AC = B. \end{aligned}$$

Hence B is a left quasi-interior ideal of M. □

Corollary 3.13. *Let A and C be subsemirings of a semiring M and $B = AC$and B is additively subsemigroup of M. If C is a right ideal then B is a right quasi-interior ideal of M.*

Theorem 3.11. *Let M be a semiring and T be a non-empty subset of M. If subsemiring B of M containing $MTMT$ and $B \subseteq T$ then is a left quasi-interior ideal of a semiring M.*

Proof. Let B be a subsemiring of M containing $MTMT$. Then

$$\begin{aligned} MBMB &\subseteq MTMT \\ &\subseteq B. \end{aligned}$$

Therefore $MBMB \subseteq B$. Hence B is a left quasi-interior ideal of M. □

Theorem 3.12. *B is a left quasi-interior ideal of a semiring M if and only if B is a left ideal of some ideal ofsemiring$M$.*

Proof. Suppose B is a left ideal of some ideal R of the semiring$M$.Then $RB \subseteq B, MR \subseteq R$.Hence $MBMB \subseteq MRMB \subseteq RMB \subseteq RB \subseteq B$. Therefore B is a left quasi-interior ideal of a semiring M.

Conversely suppose that B is a left quasi-interior ideal of the semiring M .Then $MBMB \subseteq B$. Therefore B is a left ideal of ideal MBM of the semiring$M$. □

Corollary 3.14. *B is a right quasi-interior ideal of a semiring M if and only if B is a right ideal of some ideal of semiring$M$.*

Corollary 3.15. *B is a quasi-interior ideal of a semiring M if and only if B is an ideal of some ideal of semiring$M$.*

Theorem 3.13. *If B is a left quasi-interior ideal of a semiring M, T is a subsemiring of M and $T \subseteq B$ then BT is a left quasi-interior ideal of M.*

Proof. $BTBT \subseteq BT$. Obviously, BT is a subsemigroup of $(M, +)$. Hence BT is a subsemiring of M. We have

$$MBTMBT \subseteq MBMBT \subseteq BT.$$

Hence BT is a left quasi-interior ideal of the semiring M. □

Theorem 3.14. *Let B be a left quasi-interior ideal of a semiring M and I be an interior ideal of M. Then $B \cap I$ is a left quasi-interior ideal of M.*

PROOF. Suppose B is a bi-ideal of M and I is an interior ideal of M. Obviously $B \cap I$ is subsemiring of M. Then

$$M(B \cap I)M(BI) \subseteq MBMB \subseteq B$$
$$M(B \cap I)M(BI) \subseteq MIMI \subseteq I$$

Therefore $M(B \cap I)M(BI) \subseteq B \cap I$. Hence $B \cap I$ is a left quasi-interior ideal of M. □

THEOREM 3.15. *Let M be a semiring and T be a subsemiring of M. Then every subsemiring of T containing $TMTMT$ is a left quasi-interior ideal of M.*

PROOF. Let C be a subsemiring of T containing $TMTMT$. Then

$$MCMC \subseteq MTMT \subseteq C.$$

Hence C is a quasi-interior ideal of the semiring M. □

THEOREM 3.16. *The intersection of $\{B_\lambda \mid \lambda \in A\}$ left quasi-interior ideals of a semiring M is a left quasi-interior ideal of M.*

PROOF. Let $B = \bigcap\limits_{\lambda \in A} B_\lambda$. Then B is a subsemiring of M. Since B_λ is a left quasi-interior ideal of M, we have $MB_\lambda MB_\lambda \subseteq B_\lambda$, for all $\lambda \in A$ and $M \cap B_\lambda M \cap B_\lambda) \subseteq \cap B_\lambda$. Thus $MBMB \subseteq B$. Hence B is a left quasi-interior ideal of M. □

THEOREM 3.17. *Let B be a left quasi-interior ideal of a semiring M,$e \in B$ and e be idempotent. Then eB is a left quasi-interior ideal of M.*

PROOF. Let B be a left quasi-interior ideal of the semiring M. Suppose $x \in B \cap eM$. Then $x \in B$ and $x = ey, \in, y \in M$.

$$\begin{aligned} x &= ey \\ &= eey \\ &= e(ey) \\ &= ex \in eB. \end{aligned}$$

Therefore

$$\begin{aligned} &B \cap eM \subseteq eB \\ &eB \subseteq B \text{ and } eB \subseteq eM \\ \Rightarrow\, &eB \subseteq B \cap eM \\ \Rightarrow\, &eB = B \cap eM. \end{aligned}$$

Hence eB is a left quasi-interior ideal of M. □

COROLLARY 3.16. *Let M be a semiring M and e be idempotent. Then eM and Me are left quasi-interior ideal and right quasi-interior ideal of M respectively.*

THEOREM 3.18. *Let M be a semiring. If $M = Ma$, for all $a \in M$. Then every left quasi-interior ideal of M is a quasi ideal of M.*

Proof. Let B be a left quasi-interior ideal of the semiring M and $a \in B$. Then

$$\begin{aligned}
&MBMB \subseteq B\\
\Rightarrow &Ma \subseteq MB,\\
\Rightarrow &M \subseteq MB \subseteq M\\
\Rightarrow &MB = M\\
\Rightarrow &BM = BMB \subseteq MBMB \subseteq B\\
\Rightarrow &MB \cap BM \subseteq M \cap BM \subseteq BM \subseteq B.
\end{aligned}$$

Therefore B is a quasi ideal of M. Hence the theorem. □

4. Left quasi-interior simple semiring and regular semiring

We introduce the notion of left quasi-interior simple semiring and characterize the quasi-interior simple semiring using left quasi-interior ideals of semiring and study the properties of minimal left quasi-interior ideals of semiring and quasi-interior ideals of regular semiring.

Definition 4.1. A semiring M is a left (right) simple semiring if M has no proper left (right) ideals of M.

Definition 4.2. A semiring M is said to be simple semiring l if M has no proper ideals of M.

Definition 4.3. A semiring M is said to be left quasi-interior simple semiring if M has no left quasi-interior ideal other than M itself.

Theorem 4.1. *If M is a division semiring, then M is a left quasi-interior simple semiring.*

Proof. Let B be a proper left quasi-interior ideal of the division semiring M, $x \in M$ and $0 \neq a \in B$. Since M is a divisionsemiring, there exmist $b \in M, \in$ such that $ab = 1$. Then there exmist $\beta \in$ such that $abx = x = xab$. Therefore $x \in BM$ and $M \subseteq BM$. We have $BM \subseteq M$. Hence $M = BM$. Similarly we can prove $MB = M$.

$$\begin{aligned}
M &= MB = MBB = MBBB\\
&\subseteq MBMB \subseteq B\\
M &\subseteq B
\end{aligned}$$

Therefore $M = B$.

Hence division semiring M has no proper left-quasi-interior ideals. □

Corollary 4.1. *If M is a division semiring then M is a right quasi-interior simple semiring.*

Corollary 4.2. *If M is a division semiring then M is a quasi-interior simple semiring.*

Theorem 4.2. *Let M be a simple semiring . Every left quasi-interior ideal is a left ideal of M.*

Proof. Let M be the simple semiring and B be a left quasi-interior ideal of M. Then $MBMB \subseteq B$ and MBM is an ideal of M. Since M is a simple semiring , we have $MBM = M$. Hence $MBMB \subseteq B$ and $MB \subseteq B$. Hence the theorem. $\square$

Corollary 4.3. *Let M be a simple semiring . Every right quasi-interior ideal is a right ideal of M.*

Theorem 4.3. *Let M be a semiring. M is a left quasi-interior simple semiring if and only if $< a >= M$, for all $a \in M$ and where $< a >$ is the smallest left quasi-interior ideal generated by a.*

Proof. Let M be a semiring. Suppose M is a left quasi-interior simple semiring,$a \in M$ and $B = Ma$. Then B is a left ideal of M. Therefore, by Theorem 3.5, B is a left quasi-interior ideal of M. Therefore $B = M$. Hence $Ma = M$, for all $a \in M$. Thus $Ma \subseteq < a > \subseteq M$ and $M \subseteq < a > \subseteq M$. Therefore $M =< a >$.

Suppose $< a >$ is the smallest left quasi-interior ideal of M generated by a and $< a >= M$ and A is the left quasi-interior ideal and $a \in A$. Then $< a > \subseteq A \subseteq M$ and $M \subseteq A \subseteq M$. Therefore $A = M$. Hence M is a left quasi-interior ideal simple semiring. $\square$

Theorem 4.4. *Let M be a semiring. Then M is a left quasi-interior simple $-$semriring if and only if $MaMa = M$, for all $a \in M$.*

Proof. Suppose M is a left-quasi interior simple semiring and $a \in M$. Then $MaMa$ is a quasi-interior ideal of M. Hence $MaMa = M$, for all $a \in M$.

Conversely suppose that $MaMa = M$, for all $a \in M$. Let B be a left quasi-interior ideal of the semiring M and $a \in B$. $M = MaMa \subseteq MBMB \subseteq B$. Therefore $M = B$. Hence M is a left quasi-interior simple semiring. $\square$

Corollary 4.4. *Let M be a semiring. Then M is a right quasi-interior simple semriring if and only if $aMaM = M$, for all $a \in M$.*

Corollary 4.5. *Let M be a semiring. Then M is a quasi-interior simple semiring if and only if $aMaM = M$ and $MaMa = M$,for all $a \in M$.*

Theorem 4.5. *If semiring M is a left simple semiring, then every left quasi-interior ideal of M is a right ideal of M.*

Proof. Let B be a left quasi-interior ideal of the left simple semiring M. Then MB is a left ideal of M and $MB \subseteq M$. Therefore $MB = M$. Then $BM = BMB \subseteq MBMB \subseteq B$ and $BM \subseteq B$. Hence every left quasi-interior ideal is a right ideal of M. $\square$

Corollary 4.6. *If semiring M is a right simple semiring, then every right quasi-interior ideal of M is a left ideal of M.*

Corollary 4.7. *Every quasi-interior ideal of a left and right simple semiring M is an ideal of M.*

Theorem 4.6. *Let M be a semiring and B be a left quasi-interior ideal of M. Then B is a minimal left quasi-interior ideal of M if and only if B is a left quasi-interior simple subsemiring of M.*

PROOF. Let B be a minimal left quasi-interior ideal of the semiring M and C be a left quasi-interior ideal of B. Then $BCBC \subseteq C$. and $BCBC$ is a left quasi-interior ideal of M. Since C is a quasi-interior ideal of B, we have $BCBC = B$ and $B = BCBC \subseteq C$. Thus $B = C$.

Conversely suppose that B is a left quasi-interior simple subsemiring of M. Let C be a left quasi-interior ideal of M and $C \subseteq B$. We have $BCBC \subseteq MCMC \subseteq MBMB \subseteq B$ amd $B = C$ since B is a left quasi-interior simple semiring. Hence B is a minimal left quasi-interior ideal of M. □

THEOREM 4.7. *Let M be a semiring and $B = LL$, where L is a minimal left ideal of M . Then B is a minimal left quasi-interior ideal of M.*

PROOF. Obviously $B = LL$ is a left quasi-interior ideal of M. Let A be a left quasi-interior ideal of M such that $A \subseteq B$. We have MA is a left ideal of M. Then $MA \subseteq MB = MLL \subseteq L$ since L is a left ideal of M.. Therefore $MA = L$ Hence $B = MAMA \subseteq A$. Therefore $A = B$. Hence B is a minimal left quasi-interior ideal of M. □

COROLLARY 4.8. *Let M be a semiring and $B = RR$, where R is a minimal right ideal of M . Then B is a minimal right quasi-interior ideal of M.*

We characterize regular semiring using left quasi-interior ideals of semiring.

THEOREM 4.8. *Let M be a regular semiring. Then every left quasi-interior ideal of M is a left ideal of M.*

PROOF. Let B be a left quasi-interior ideal of M. Then $MBMB \subseteq B$. Thus $MB \subseteq MBMMB \subseteq MBMB \subseteq B$ since M is regular. Hence the theorem. □

COROLLARY 4.9. *Let M be a regular semiring. Then every right quasi-interior ideal is a right ideal of M.*

COROLLARY 4.10. *Let M be a regular semiring. Then every quasi-interior ideal is an ideal of M.*

THEOREM 4.9. *M is regular $-$semiring if and only if $AB = A \cap B$ for any right ideal A and left ideal B of $-$semiring M.*

THEOREM 4.10. *Let M be semiring Then B is a quasi-interior ideal of M if and only if $BMBM = B$ and $MBMB = B$ for all quasi-interior ideals B of M.*

PROOF. Suppose M is the regular semiring, B is a bi-interior ideal of M and $x \in B$. Then $MBMB \subseteq B$ and there exmist $y \in M,\ ,\in$ such that $x = xyx\alpha yx \in MBMB$. Therefore $x \in MBMB$. Hence $MBMB = B$. Similarly we can prove $BMBM = B$.

Conversely suppose that $BMBM = B$ and $MBMB = B$ for all quasi-interior ideals B of M. Let $B = R \cap L$, and $C = RL$, where R is a right ideal and L is a left ideal of M. Then $BandC$ are quasi-interior ideals of M. Therefore

$$(R \cap L)M(R \cap L)M = R \cap L$$

and

$$\begin{aligned}R\cap L &= (R\cap L)M(R\cap L)M\\ &\subseteq RMLM\\ &\subseteq RLM\\ R\cap L &= (R\cap L)M(R\cap L)M\\ &\subseteq RLMRLM\\ &\subseteq RL\\ &\subseteq R\cap L \text{ (since } RL\subseteq L \text{ and } RL\subseteq R).\end{aligned}$$

Therefore $R\cap L = RL$. Hence M is a regular semiring. □

THEOREM 4.11. *Let B be left quasi-interior ideal of a regular semiring M. If B is a left quasi-interior ideal of M and B is regular subsemiring of M then any left quasi-interior ideal of B is a left quasi-interior ideal of M.*

PROOF. Let A be a left quasi-interior ideal of the regular subsemiring B of M. Then by Theorem 3.47, $ABAB = A$. We have $BMBM = B$ and $A\subseteq B$. Thus

$$\begin{aligned}AMAM \subseteq BMBM &= B\\ &\Rightarrow ABAB = A\subseteq B\\ &\Rightarrow A = ABAB\subseteq AMAM\\ &\Rightarrow AMAM = A, \text{ since M is a regular semiring}\end{aligned}$$

Hence A is a left quasi-interior ideal of M. □

THEOREM 4.12. *M is a regular semiring if and only if $AB = A\cap B$ for any right ideal A and left ideal B of a semiring M.*

THEOREM 4.13. *Let B be subsemiring of a regular semiring M. If B can be represented as $B = RL$, where R is a right ideal and L is a left ideal of M then B is a left quasi-interior ideal of M.*

PROOF. Suppose $B = RL$, where R is right ideal of M and L is a left ideal of M. Then $BMBM = RLMBM \subseteq R \subseteq RL = B$. Hence B is a left quasi-interior ideal of the regular semiring M.

Conversely suppose that B is a bi-quasi-interior ideal of the regular semiring M. We have $BMBM = B$. Let $R = BM$ and $L = MB$. Then $R = BM$ is a right ideal of M and $L = MB$ is a left ideal of M.

$$\begin{aligned}&BM\cap MB\subseteq BMBM = B\\ \Rightarrow\ &BM\cap MB\subseteq B\\ \Rightarrow\ &R\cap L\subseteq B.\end{aligned}$$

We have $B\subseteq BM = R$ and

$$\begin{aligned}&B\subseteq MB = L\\ \Rightarrow\ &B\subseteq R\cap L\\ \Rightarrow\ &B = R\cap L = RL\end{aligned}$$

since M is a regular semiring.

Hence B can be represented as RL, where R is the right ideal and L is the left ideal of M. Hence the theorem. □

The following theorem is a necessary and sufficient condition for semiring M to be regular using left quasi-interior ideals.

THEOREM 4.14. *M is a regular semrиring if and only if $B \cap I \cap L \subseteq BIL$, for any left quasi-interior ideal B, ideal I and left idealL of M.*

PROOF. Suppose M is the regular semiring, B, I and L are left quasi-interior ideal, ideal and left ideal of M respectively.
Let $a \in B \cap I \cap L$. Then $a \in aMa$, since M is regular.

$$a \in aMa \subseteq aMaMa \subseteq BIL$$

Hence $B \cap I \cap L \subseteq BIL$.

Conversely suppose that $B \cap I \cap L \subseteq BIL$, for any left quasi-interior ideal B, ideal I and left ideal L of M. Let R be a right ideal and L be left ideal of M. Then by assumption,

$$R \cap L = R \cap M \cap L \subseteq RML \subseteq RL$$

We have $RL \subseteq R,\ RL \subseteq L$. Therefore $RL \subseteq R \cap L$. Hence $R \cap L = RL$. Thus M is a regular semiring. □

5. Conclusion

As a further generalization of ideals ,we introduced the notion of quasi-interior ideal of semiring as a generalization of ideal,left ideal, right ideal, bi-ideal,quasi ideal and interior ideal of semiring and studied some of their properties. We introduced the notion of quasi-interior simple semiring and characterized the quasi-interior simple semiring,regular semiring using left quasi-interior ideals of semiring.We proved every bi- quasi ideal of semiring and bi-interior ideal of semiring are quasi-interior ideals and studied some of the properties of quasi-interior ideals of semiring. In continuity of this paper, we study prime quasi-interior ideals, maximal and minimal quasi-interior ideals of semiring.

References

[1] P. J. Allen. A fundamental theorem of homomorphism for semirings. *Proc. Am. Math. Sco.*, **21**(2)(1969), 412–416.

[2] R. A. Good and D. R. Hughes. Associated groups for a semigroup. *Bull. Am. Math. Soc.*, **58**(6)(1952), 624–625.

[3] M. Henriksen. Ideals in semirings with commutative addition. *Notice Am. Math. Soc.*, **6**(1958), 321.

[4] K. Iseki. Quasi-ideals in semirings without zero. *Proc. Japan Acad.*, **34**(2)(1958), 79–84.

[5] K. Iseki. Ideal theory of semiring. *Proc. Japan Acad.*, **32**(8)(1956), 554–559.

[6] K. Iseki. Ideal in semirings. *Proc. Japan Acad.*, **34**(1)(1958), 29–31.

[7] K. Izuka. On the Jacobson radical of a semiring. *Tohoku Math. J.*, **11**(2)(1959), 409–421.

[8] R. D. Jagatap and Y. S. Pawar. Quasi-ideals and minimal quasi-ideals in Γsemirings. *Novi Sad J. Math.*, **39**(2)(2009), 79–87.

[9] S. Lajos. On the bi-ideals in semigroups. *Proc. Japan Acad.*, **45**(8)(1969), 710–712.

[10] S. Lajos and F. A. Szasz. On the bi-ideals in associative ring. *Proc. Japan Acad*, **46**(6)(1970), 505–507.
[11] M. M. K. Rao. Left bi-quasi ideals of semirings. *Bull. Int. Math. Virtual Inst*, 8(1)(2018), 45–53.
[12] M. M. K. Rao. Bi-quasi-ideals and fuzzy bi-quasiideals of $\Gamma-$semigroups. *Bull. Int. Math. Virtual Inst.*, **7**(2)(2017), 231–242.
[13] M. M. K. Rao and B. Venkateswarlu. Regular $\Gamma-$incline and field $\Gamma-$semiring. *Novi Sad J. Math.*, **45**(2)(2015), 155–171.
[14] M. M. K. Rao, B. Venkateswarlu and N. Rafi. Left Bi-quasi-ideals of semirings. *Asia Pacific J. Math.*, **4**(2)(2017), 144-153. ISSN 2357-2205.
[15] M. M. K. Rao. Bi -interior ideals of semigroups. *Discuss. Math. Gen. Algebra Appl*, **38**(1)(2018), 69-70.
[16] M. M. K. Rao. A study of bi-quasi interior ideal as a new generalization of ideal of generalization of semirings, *Bull. Int. Math. Virtual Inst.*, **8**(3)(2018), 519–535.
[17] A. M. Shabir and A. Batod. A note on quasi ideal in semirings. *Southeast Asian Bull. Math.*, **27**(5)(2004), 923–928.
[18] O. Steinfeld. Uher die quasi ideals, Von halbgruppend. *Publ. Math., Debrecen*, **4**(1956), 26-275.
[19] H. S. Vandiver. Note on a simple type of algebra in which cancellation law of addition does not hold. *Bull. Am. Math. Soc.*, **40**(12)(1934), 914–920.

Received by editors 09.07.2018; Revised version 18.12.2018; Available online 31.12.2018.

Department of Mathematics, GITAM University, VISAKHAPATNAM, INDIA
E-mail address: mmarapureddy@gmail.comserver