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CONTRA $\mathcal{I}_{wg}$-CONTINUITY IN IDEAL SPACES

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Abstract. In this paper, the concepts of $\mathcal{I}_{wg}$-closed sets and $\mathcal{I}_{wg}$-open sets are introduced and they are used to define and investigate a new class of functions called contra $\mathcal{I}_{wg}$-continuous functions in ideal spaces. We discuss the relationships with some other related functions.

1. Introduction and Preliminaries

Throughout this paper, by a space X, we always mean a topological space (X,τ) with no separation properties assumed. Let H be a subset of X. We denote the interior, the closure and the complement of a subset H by $int(H)$, $cl(H)$ and $X \backslash H$ or H^c, respectively. The set of all open sets containing a point $x \in X$ is denoted by $\sum(x)$ ([**5**]).

Definition 1.1. ([**10**]) A subset H of a space X is said to be preopen if $H \subseteq int(cl(H))$. The complement of a preopen set is called preclosed.

Definition 1.2. ([**8**]) A space X is said to be regular if for each closed set F of X and each $x \notin F$, there exist disjoint open sets P and Q such that $x \in P$ and $F \subseteq Q$.

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Definition 1.3. ([**12**]) A space X is called locally indiscrete if every open set is closed.

Definition 1.4. ([**15**]) A space X is called Urysohn if for every pair of points $x, y \in X$, $x \neq y$ there exist $U \in \sum(x), V \in \sum(y)$ such that $cl(U) \cap cl(V) = \emptyset$.

The collection of all clopen subsets of X will be denoted by $CO(X)$. We set $CO(X, x)$=$\{V \in CO(X) | x \in V\}$ for $x \in X$ ([**11**]).

Definition 1.5. ([**13**]) A space X is said to be

(1) Ultra Hausdorff if for each pair of distinct points x and y in X there exist $U \in CO(X, x)$ and $V \in CO(X, y)$ such that $U \cap V = \emptyset$.
(2) Ultra normal if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.

Definition 1.6. ([**5**]) Let $f : (X, \tau) \to (Y, \sigma)$ be any function. Then the subset $G(f) = \{(x, f(x)) : x \in X\}$ of the product space $(X \times Y, \tau \times \sigma)$ is called the graph of f.

An ideal $\mathcal{I}$ on a space X is a non-empty collection of subsets of X which satisfies (i) $P \in \mathcal{I}$ and $Q \subseteq P \Rightarrow Q \in \mathcal{I}$ and (ii) $P \in \mathcal{I}$ and $Q \in \mathcal{I} \Rightarrow P \cup Q \in \mathcal{I}$. Given a space X with an ideal $\mathcal{I}$ on X and if $\wp$(X) is the set of all subsets of X, a set operator $(\cdot)^* : \wp(X) \to \wp(X)$, called a local function [**9**] of H with respect to τ and $\mathcal{I}$ is defined as follows: for $H \subseteq X$, $H^*(\mathcal{I}, \tau) = \{x \in X \mid U \cap H \notin \mathcal{I}$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau \mid x \in U\}$. We will make use of the basic facts about the local functions [[**7**], Theorem 2.3] without mentioning it explicitly. Kuratowski closure operator $cl^*(\cdot)$ for a topology $\tau^*(\mathcal{I}, \tau)$, called the $\star$-topology, finer than τ, is defined by $cl^*(H) = H \cup H^*(\mathcal{I}, \tau)$ [**14**]. When there is no chance for confusion, we will simply write H^* for $H^*(\mathcal{I}, \tau)$ and τ^* for $\tau^*(\mathcal{I}, \tau)$. If $\mathcal{I}$ is an ideal on X, then $(X, \tau, \mathcal{I})$ is called an ideal space. $\mathcal{N}$ is the ideal of all nowhere dense subsets in (X, τ). A subset H of an ideal space $(X, \tau, \mathcal{I})$ is called $\mathcal{I}_g$-closed [**4**] if $H^* \subseteq U$ whenever $H \subseteq U$ and U is open.

Definition 1.7. ([**6**]) A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is called $\mathcal{I}_g$-continuous if the inverse image of every closed set in Y is $\mathcal{I}_g$-closed in X.

Let us say that $w \subseteq P$ is a weak structure (briefly WS) on X iff $\emptyset \in w$. Clearly each generalized topology and each minimal structure is a WS [**2**].

Each member of w is said to be w-open and the complement of a w-open set is called w-closed.

Let w be a weak structure on X and $H \subseteq X$. We define (as in the general case) $i_w(H)$ is the union of all w-open subsets contained in H and $c_w(H)$ is the intersection of all w-closed sets containing H [**2**].

Remark 1.1 ([**1**]). If w is a WS on X, then $i_w(\emptyset) = \emptyset$ and $c_w(X) = X$.

Theorem 1.1 ([**2**]). *If w is a WS on X and $A, B \in w$ then*

(1) $i_w(A) \subseteq A \subseteq c_w(A)$,
(2) $A \subseteq B \Rightarrow i_w(A) \subseteq i_w(B)$ *and* $c_w(A) \subseteq c_w(B)$,

(3) $i_w(i_w(A)) = i_w(A)$ *and* $c_w(c_w(A)) = c_w(A)$,
(4) $i_w(X - A) = X - c_w(A)$ *and* $c_w(X - A) = X - i_w(A)$.

Definition 1.8. ([**1**]) Let w be a WS on a space X. Then $H \subseteq X$ is said to be wg-closed if $cl(H) \subseteq U$ whenever $H \subseteq U$ and U is w-open in X. The complement of a wg-closed set is called a wg-open set.

Remark 1.2 ([**1**]). For a WS w on a space X, every w-closed set is gw-closed but not conversely.

2. Properties of Contra $\mathcal{I}_{wg}$-continuity

Definition 2.1. Let w be a WS on a space X. Then X is said to be wg-normal if each pair of non-empty disjoint closed sets can be separated by disjoint wg-open sets.

Example 2.1. (1) Let

$$X = \{a, b, c\}, \tau = \{\emptyset, X, \{c\}, \{a, b\}\} \text{ and } w = \{\emptyset, X, \{a\}, \{a, b\}\}.$$

Then wg-open sets are $\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}$ and $\{a, c\}$. Clearly X is wg-normal.

(2) Let

$$X = \{a, b, c\}, \tau = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\} \text{ and } w = \{\emptyset, X, \{a, b\}, \{b, c\}, \{a, c\}\}.$$

Then wg-open sets are $\emptyset, X, \{a\}, \{a, b\}$ and $\{a, c\}$. Clearly X is not wg-normal.

Definition 2.2. Let w be a WS on a space X. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be

(1) contra wg-continuous if for each open set V in (Y, σ), $f^{-1}(V)$ is wg-closed in (X,τ).
(2) contra w-continuous if for each open set V in (Y, σ), $f^{-1}(V)$ is w-closed in (X,τ).
(3) w-continuous if for each closed set V in (Y, σ), $f^{-1}(V)$ is w-closed in (X,τ).
(4) contra continuous [**3**] if for each closed set V in (Y, σ), $f^{-1}(V)$ is open in (X, τ).

Proposition 2.1. *Every contra w-continuous function is contra wg-continuous.*

Proof. Let w be a WS on a space X. Let $f : (X, \tau) \to (Y, \sigma)$ be a contra w-continuous function and let V be any open set in Y. Then, $f^{-1}(V)$ is w-closed in X. Since every w-closed set is wg-closed, $f^{-1}(V)$ is wg-closed in X. Therefore f is contra wg-continuous. □

However, converse need not be true as seen from the following Example.

Example 2.2. Let $X = Y = \{a, b, c\}, \tau = \sigma = \{\emptyset, \{c\}, \{a, b\}, X = Y\}$ and $w = \{\emptyset, X, \{a\}, \{a, b\}\}$. Then w is a WS on a space X. Also the identity function $f : (X, \tau) \to (Y, \sigma)$ is contra wg-continuous but not contra w-continuous.

DEFINITION 2.3. Let w be a WS on a space X. A graph $G(f)$ of a function $f:(X,\tau)\to(Y,\sigma)$ is said to be contra wg-closed in $(X\times Y)$ if for each $(x,y)\in(X\times Y)\backslash G(f)$, there exist an $P\in wGO(X)$ containing x and a closed set Q of (Y,σ) containing y such that $f(P)\cap Q=\emptyset$ where $wGO(X)$ denotes the family of all wg-open sets of X.

EXAMPLE 2.3. Let $X=Y=\{a,b,c\},\tau=\sigma=\{\emptyset,\{a\},\{b\},\ \{c\},\ \{a,b\},\ \{a,c\},\ \{b,c\},\ X=Y\}$ and $w=\{\emptyset,\{a\},\{b,c\},X\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be an identity function. Then w is a WS on a space X and $G(f)$ is contra wg-closed in $X\times Y$.

DEFINITION 2.4. Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. A subset $H\subseteq X$ is said to be $\mathcal{I}_{wg}$-closed if $H^{\star}\subseteq U$ whenever $H\subseteq U$ and U is w-open in X. The complement of an $\mathcal{I}_{wg}$-closed set is called $\mathcal{I}_{wg}$-open. The family of all $\mathcal{I}_{wg}$-open sets of $(X,\tau,\mathcal{I})$ is denoted by $\mathcal{I}wGO(X)$.

DEFINITION 2.5. Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. Then $(X,\tau,\mathcal{I})$ is said to be $\mathcal{I}_{wg}$-normal if each pair of non-empty disjoint closed sets can be separated by disjoint $\mathcal{I}_{wg}$-open sets.

EXAMPLE 2.4. (1) Let $X=\{a,b,c\}$, $\tau=\{\emptyset,X,\{a\},\{b,c\}\}$, $w=\{\emptyset,X,\{b,c\}\}$ and $\mathcal{I}=\{\emptyset\}$. Then $\mathcal{I}wGO(X)=P(X)$. Clearly $(X,\tau,\mathcal{I})$ is $\mathcal{I}_{wg}$-normal.
(2) Let $X=\{a,b,c\}$, $\tau=\{\emptyset,X,\{a\},\{a,b\},\{a,c\}\}$, $w=\{\emptyset,X,\{a\},\{a,c\},\{a,b\}\}$ and $\mathcal{I}=\{\emptyset\}$. Then $\mathcal{I}_{wg}$-open sets are $\emptyset,X,\{a\},\{a,b\}and\{a,c\}$. Clearly $(X,\tau,\mathcal{I})$ is not $\mathcal{I}_{wg}$-normal.

DEFINITION 2.6. Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. A function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is said to be $\mathcal{I}_{wg}$-continuous if $f^{-1}(V)$ is $\mathcal{I}_{wg}$-closed in $(X,\tau,\mathcal{I})$ for each closed set V in (Y,σ).

DEFINITION 2.7. Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. A function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is said to be contra $\mathcal{I}_{wg}$-continuous if $f^{-1}(V)$ is $\mathcal{I}_{wg}$-closed in $(X,\tau,\mathcal{I})$ for each open set V in (Y,σ).

PROPOSITION 2.2. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. If $\tau\subseteq w$ then every $\mathcal{I}_{wg}$-closed set is $\mathcal{I}_g$-closed.*

PROOF. The result follows immediately from the given condition. □

However, converse need not be true as seen from the following Example.

EXAMPLE 2.5. Let $X=\{a,b,c\}$, $\tau=\{\emptyset,\{a\},X\}$, $w=\{\emptyset,\{a\},\{b\},X\}$ and $\mathcal{I}=\{\emptyset,\{c\}\}$. Then $\tau\subseteq w$. Also $\{b\}$ is an $\mathcal{I}_g$-closed set but not $\mathcal{I}_{wg}$-closed.

PROPOSITION 2.3. *For a WS w on an ideal space $(X,\tau,\mathcal{I})$, every wg-closed set is $\mathcal{I}_{wg}$-closed.*

PROOF. The proof follows immediately from the fact that $H^{\star}\subseteq cl(H)$. □

However, converse need not be true as seen from the following Example.

EXAMPLE 2.6. Let $X=\{a,b,c,d\}$, $\tau=\{\emptyset,\{b\},\{b,c,d\},X\}$, $w=\{\emptyset,\{a,b,c\},X\}$ and $\mathcal{I}=\{\emptyset,\{c\}\}$. Then $\{c\}$ is an $\mathcal{I}_{wg}$-closed set but not wg-closed.

PROPOSITION 2.4. *Every contra wg-continuous function is contra $\mathcal{I}_{wg}$-continuous.*

PROOF. Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a contra wg-continuous function and let V be any open set in Y. Then, $f^{-1}(V)$ is wg-closed in X. Since every wg-closed set is $\mathcal{I}_{wg}$-closed, $f^{-1}(V)$ is $\mathcal{I}_{wg}$-closed in X. Therefore f is contra $\mathcal{I}_{wg}$-continuous. □

However, converse need not be true as seen from the following Example.

EXAMPLE 2.7. Let $X = Y = \{a, b, c\}$, $\tau = \sigma = \{\emptyset, \{a\}, X = Y\}$, $\mathcal{I} = \{\emptyset, \{a\}\}$ and $w = \{\emptyset, X, \{a\}, \{c\}\}$. Then the identity function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $\mathcal{I}_{wg}$-continuous but not contra wg-continuous.

REMARK 2.1. The following two examples show that the concepts of $\mathcal{I}_{wg}$-continuity and contra $\mathcal{I}_{wg}$-continuity are independent of each other.

EXAMPLE 2.8. Let $X = Y = \{a, b, c\}$, $\tau = \sigma = \{\emptyset, \{a\}, X = Y\}$, $\mathcal{I} = \{\emptyset, \{c\}\}$ and $w = \{\emptyset, \{a\}, \{a, c\}, X\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be defined by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Since the inverse image of every open set of Y is $\mathcal{I}_{wg}$-closed in X, f is contra $\mathcal{I}_{wg}$-continuous. For the closed set $\{b, c\}$ of Y, $f^{-1}(\{b, c\}) = \{a, c\}$ is not $\mathcal{I}_{wg}$-closed in $(X, \tau, \mathcal{I})$. Therefore f is not $\mathcal{I}_{wg}$-continuous.

EXAMPLE 2.9. Let $X = Y = \{a, b, c\}$, $\tau = \sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X = Y\}$, $\mathcal{I} = \{\emptyset, \{a, c\}\}$ and $w = \{\emptyset, \{b\}, \{a, c\}, X\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be defined by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Since the inverse image of every closed set of Y is $\mathcal{I}_{wg}$-closed in X, f is $\mathcal{I}_{wg}$-continuous. For the open set $\{b\}$ of (Y, σ), $f^{-1}(\{b\}) = \{b\}$ is not $\mathcal{I}_{wg}$-closed in $(X, \tau, \mathcal{I})$. Therefore f is not contra $\mathcal{I}_{wg}$-continuous.

PROPOSITION 2.5. *If $\tau \subseteq w$, then every contra $\mathcal{I}_{wg}$-continuous function is contra $\mathcal{I}_g$-continuous.*

PROOF. The proof follows immediately from Proposition 2.2. □

However, converse need not be true as seen from the following Example.

EXAMPLE 2.10. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$, $\sigma = \{\emptyset, \{b\}, \{a, c\}, Y\}$, $w = \{\emptyset, X, \{a\}, \{a, c\}\}$ and $\mathcal{I} = \{\emptyset, \{c\}\}$. Then the identity function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $\mathcal{I}_g$-continuous but not contra $\mathcal{I}_{wg}$-continuous.

THEOREM 2.1. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a function. Then the following are equivalent:*

(1) *f is contra $\mathcal{I}_{wg}$-continuous.*
(2) *The inverse image of each closed set in Y is $\mathcal{I}_{wg}$-open in X.*
(3) *For each point x in X and each closed set Q in Y with $f(x) \in Q$, there is an $\mathcal{I}_{wg}$-open set P in X containing x such that $f(P) \subseteq Q$.*

PROOF. (1) $\Rightarrow$ (2) Let G be a closed set in Y. Then $Y - G$ is open in Y. By definition of contra $\mathcal{I}_{wg}$-continuity, $f^{-1}(Y - G)$ is $\mathcal{I}_{wg}$-closed in X. But $f^{-1}(Y - G) = X - f^{-1}(G)$. This implies $f^{-1}(G)$ is $\mathcal{I}_{wg}$-open in X.

(2) $\Rightarrow$ (3) Let $x \in X$ and Q be any closed set in Y with f(x)∈Q. By (2), $f^{-1}(Q)$ is $\mathcal{I}_{wg}$-open in X. Set $P = f^{-1}(Q)$. Then there is an $\mathcal{I}_{wg}$-open set P in X containing x such that f(P)⊆Q.

(3) $\Rightarrow$ (1) Let $x \in X$ and Q be any closed set in Y with f(x)∈Q. Then $Y - Q$ is open in Y with f(x)∈Q. By (3), there is an $\mathcal{I}_{wg}$-open set P in X containing x such that $f(P) \subseteq Q$. This implies $P = f^{-1}(Q)$. Therefore, $X - P = X - f^{-1}(Q) = f^{-1}(Y - Q)$ which is $\mathcal{I}_{wg}$-closed in X. □

THEOREM 2.2. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$ and let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \mu)$. Then the following properties hold:*

1. *If f is contra $\mathcal{I}_{wg}$-continuous and g is continuous then $g \circ f$ is contra $\mathcal{I}_{wg}$-continuous.*
2. *If f is contra $\mathcal{I}_{wg}$-continuous and g is contra continuous then $g \circ f$ is $\mathcal{I}_{wg}$-continuous.*
3. *If f is $\mathcal{I}_{wg}$-continuous and g is contra continuous then $g \circ f$ is contra $\mathcal{I}_{wg}$-continuous.*

PROOF. (1) Let V be any closed set in Z. Since g is continuous, $g^{-1}(V)$ is closed in Y. Since f is contra $\mathcal{I}_{wg}$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\mathcal{I}_{wg}$-open in X. Therefore $g \circ f$ is contra $\mathcal{I}_{wg}$-continuous.

(2) Let V be any closed set in Z. Since g is contra continuous, $g^{-1}(V)$ is open in Y. Since f is contra $\mathcal{I}_{wg}$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\mathcal{I}_{wg}$-closed in X. Therefore $g \circ f$ is $\mathcal{I}_{wg}$-continuous.

(3) Let V be any closed set in Z. Since g is contra continuous, $g^{-1}(V)$ is open in Y. Since f is $\mathcal{I}_{wg}$-continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\mathcal{I}_{wg}$-open in X. Therefore $g \circ f$ is contra $\mathcal{I}_{wg}$-continuous. □

THEOREM 2.3. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. If a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $\mathcal{I}_{wg}$-continuous and Y is regular, then f is $\mathcal{I}_{wg}$-continuous.*

PROOF. Let x be an arbitrary point of X and Q be an open set of Y containing $f(x)$. Since Y is regular, there exists $R \in \tau$ such that $f(x) \in R \subseteq cl(R) \subseteq Q$. Since f is contra $\mathcal{I}_{wg}$-continuous, by Theorem 2.1, there exists an $\mathcal{I}_{wg}$-open set P containing x such that $f(P) \subseteq cl(R)$. Thus $f(P) \subseteq cl(R) \subseteq Q$. Hence f is $\mathcal{I}_{wg}$-continuous. □

DEFINITION 2.8. Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. Then $(X, \tau, \mathcal{I})$ is said to be an $\mathcal{I}_{wg}$-space if every $\mathcal{I}_{wg}$-open set of X is w-open in X.

EXAMPLE 2.11. (1) Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $w = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a, c\}\}$. Then $\mathcal{I}_{wg}$-open sets are $\{a\}$, $\{b\}$, $\{a, b\}$, $\emptyset$ and X. Then $(X, \tau, \mathcal{I})$ is $\mathcal{I}_{wg}$-space.

(2) Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $w = \{\emptyset, \{b\}, \{a, c\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a, c\}\}$. Then $\mathcal{I}_{wg}$-open sets are $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$, $\emptyset$ and X. Then $(X, \tau, \mathcal{I})$ is not $\mathcal{I}_{wg}$-space.

THEOREM 2.4. *Let w be a WS on an $\mathcal{I}_{wg}$-space X. If a function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is contra $\mathcal{I}_{wg}$-continuous then f is contra continuous.*

PROOF. Let V be any closed set in Y. Since f is contra $\mathcal{I}_{wg}$-continuous, $f^{-1}(V)$ is $\mathcal{I}_{wg}$-open in X. Since X is an $\mathcal{I}_{wg}$-space, $f^{-1}(V)$ is open in X. Therefore f is contra continuous. □

DEFINITION 2.9. Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. Then $(X,\tau,\mathcal{I})$ is said to be $\mathcal{I}_{wg}$-T_2 space if for each pair of distinct points x and y in $(X,\tau,\mathcal{I})$, there exist an $\mathcal{I}_{wg}$-open set P containing x and an $\mathcal{I}_{wg}$-open set Q containing y such that P$\cap$Q=$\emptyset$.

EXAMPLE 2.12. (1) Let $X=\{a,b,c\}$, $\tau=\{\emptyset,\{a\},\{b,c\},X\}$, $w=\{\emptyset,\ \{b,c\},$ $X\}$ and $\mathcal{I}=\{\emptyset\}$. Then $(X,\tau,\mathcal{I})$ is $\mathcal{I}_{wg}$-T_2 space.
(2) Let $X=\{a,b,c\}$, $\tau=\{\emptyset,X,\{a\},\{b,c\}\}$, $w=\{\emptyset,X,\{a\},\{a,b\}\}$ and $\mathcal{I}=\{\emptyset\}$. Then $(X,\tau,\mathcal{I})$ is not $\mathcal{I}_{wg}$-T_2 space.

THEOREM 2.5. *If w is a WS on an ideal space $(X,\tau,\mathcal{I})$ and for each pair of distinct points x_1,x_2 in X, there exists a function f from $(X,\tau,\mathcal{I})$ into a Urysohn space Y such that $f(x_1)\neq f(x_2)$ and f is contra $\mathcal{I}_{wg}$-continuous at x_1 and x_2, then X is $\mathcal{I}_{wg}$-T_2.*

PROOF. Let x_1 and x_2 be any two distinct points in X. Then by hypothesis, there is a function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$, such that $f(x_1)\neq f(x_2)$. Let $y_i=f(x_i)$ for $i=1,2$. Then $y_1\neq y_2$. Since Y is Urysohn, there exist open neighbourhoods Q_{y_1} and Q_{y_2} of y_1 and y_2 respectively in Y such that $cl(Q_{y_1})\cap cl(Q_{y_2})=\emptyset$. Since f is contra $\mathcal{I}_{wg}$-continuous, there exists an $\mathcal{I}_{wg}$-open set P_{x_i} of x_i in X such that $f(P_{x_i})\subseteq\mathrm{cl}(Q_{y_i})$ for $i=1,2$. Hence we get $P_{x_1}\cap P_{x_2}=\emptyset$ because $cl(Q_{y_1})\cap cl(Q_{y_2})=\emptyset$. Thus X is $\mathcal{I}_{wg}$-T_2. □

COROLLARY 2.1. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. If f is a contra $\mathcal{I}_{wg}$-continuous injection of $(X,\tau,\mathcal{I})$ into a Urysohn space (Y,σ), then $(X,\tau,\mathcal{I})$ is $\mathcal{I}_{wg}$-T_2.*

PROOF. Let x_1 and x_2 be any pair of distinct points in X. Since f is contra $\mathcal{I}_{wg}$-continuous and injective, we have $f(x_1)\neq f(x_2)$. Therefore by Theorem 2.5, X is $\mathcal{I}_{wg}$-T_2. □

COROLLARY 2.2. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. If f is a contra $\mathcal{I}_{wg}$-continuous injection of $(X,\tau,\mathcal{I})$ into a Ultra Hausdorff space (Y,σ), then $(X,\tau,\mathcal{I})$ is $\mathcal{I}_{wg}$-T_2.*

PROOF. Let x_1 and x_2 be any two distinct points in X. Then since f is injective and Y is Ultra Hausdorff, $f(x_1)\neq f(x_2)$ and there exist two clopen sets V_1 and V_2 in Y such that $f(x_1)\in V_1, f(x_2)\in V_2$ and $V_1\cap V_2=\emptyset$. Then $x_i\in f^{-1}(V_i)\in\mathcal{I}w$GO(X) for $i=1,2$ and $f^{-1}(V_1)\cap f^{-1}(V_2)=\emptyset$. Thus X is $\mathcal{I}_{wg}$-T_2. □

THEOREM 2.6. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. If $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is a contra $\mathcal{I}_{wg}$-continuous, closed injection and Y is Ultra normal, then $(X,\tau,\mathcal{I})$ is $\mathcal{I}_{wg}$-normal.*

PROOF. Let G_1 and G_2 be disjoint closed subsets of X. Since f is closed and injective, $f(G_1)$ and $f(G_2)$ are disjoint closed subsets of Y. Since Y is Ultra normal, $f(G_1)$ and $f(G_2)$ are separated by disjoint clopen sets Q_1 and Q_2 respectively. Hence $G_i \subseteq f^{-1}(Q_i), f^{-1}(Q_i) \in \mathcal{I}w$GO(X) for $i = 1, 2$ and $f^{-1}(Q_1) \cap f^{-1}(Q_2) = \emptyset$. Thus X is $\mathcal{I}_{wg}$-normal. □

DEFINITION 2.10. Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. A graph $G(f)$ of a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be contra $\mathcal{I}_{wg}$-closed if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exists $P \in \mathcal{I}wGO(X)$ containing x and a closed set Q of (Y, σ) containing y such that $f(P) \cap Q = \emptyset$.

EXAMPLE 2.13. Let $X = Y = \{a, b, c\}$, $\tau = \sigma = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X = Y\}$, $w = \{\emptyset, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be an identity function. Then $G(f)$ is contra $\mathcal{I}_{wg}$-closed in $X \times Y$.

THEOREM 2.7. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $\mathcal{I}_{wg}$-continuous and (Y, σ) is Urysohn, then $G(f)$ is contra $\mathcal{I}_{wg}$-closed in $X \times Y$.*

PROOF. Let $(x, y) \in (X \times Y) \backslash G(f)$, then $f(x) \neq y$ and there exist open sets Q, R such that f(x)∈Q, y∈R and cl(Q)∩cl(R)=∅. Since f is contra $\mathcal{I}_{wg}$-continuous there exists P∈$\mathcal{I}w$GO(X) containing x such that f(P)⊆cl(Q). Since cl(Q)∩cl(R)=∅, we have f(P)∩cl(R)=∅. This shows that $G(f)$ is contra $\mathcal{I}_{wg}$-closed in $X \times Y$. □

REMARK 2.2. The following Example shows that the condition Urysohn on the space (Y, σ) in Theorem 2.7 cannot be dropped.

EXAMPLE 2.14. Let $X = Y = \{a, b, c\}$, $\tau = \sigma = \{\emptyset, \{a\}, X = Y\}$, $w = \{\emptyset, \{a, b\}, X\}$ and $\mathcal{I} = \{\emptyset, \{a\}\}$. Then Y is not a Urysohn space. Also the identity function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $\mathcal{I}_{wg}$-continuous but not contra $\mathcal{I}_{wg}$-closed.

COROLLARY 2.3. *Let w be a WS on a space X. If $f : (X, \tau) \to (Y, \sigma)$ is contra wg-continuous function and (Y, σ) is a Urysohn space, then $G(f)$ is contra-wg-closed in $X \times Y$.*

PROOF. The proof follows from the Theorem 2.7 if $\mathcal{I} = \{\emptyset\}$. □

REMARK 2.3. The following Example shows that the condition Urysohn on the space (Y, σ) in Corollary 2.3 cannot be dropped.

EXAMPLE 2.15. Let $X = Y = \{a, b, c\}$, $\tau = \sigma = \{\emptyset, \{c\}, \{a, b\}, X = Y\}$, and $w = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then Y is not a Urysohn space. Also the identity function $f : (X, \tau) \to (Y, \sigma)$ is contra wg-continuous but not contra wg-closed.

DEFINITION 2.11. Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. Then $(X, \tau, \mathcal{I})$ is said to be $\mathcal{I}_{wg}$-connected if $(X, \tau, \mathcal{I})$ cannot be expressed as the union of two disjoint non-empty $\mathcal{I}_{wg}$-open subsets of $(X, \tau, \mathcal{I})$.

EXAMPLE 2.16. (1) Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, c\}, \{a, b\}, X\}$, $w = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\mathcal{I} = \{\emptyset\}$. Then $\mathcal{I}_{wg}$-open sets are $\{a\}$, $\{a, b\}$, $\{a, c\}$, $\emptyset$ and X. Then $(X, \tau, \mathcal{I})$ is $\mathcal{I}_{wg}$-connected.

(2) Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$, $w = \{\emptyset, \{a\}, \{b, c\}, X\}$ and $\mathcal{I} = \{\emptyset\}$. Then $\mathcal{I}_{wg}$-open sets are $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, $\emptyset$ and X. Then $(X, \tau, \mathcal{I})$ is not $\mathcal{I}_{wg}$-connected.

THEOREM 2.8. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. Then a contra $\mathcal{I}_{wg}$-continuous image of a $\mathcal{I}_{wg}$-connected space is connected.*

PROOF. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a contra $\mathcal{I}_{wg}$-continuous function of an $\mathcal{I}_{wg}$-connected space $(X, \tau, \mathcal{I})$ onto a space (Y, σ). If possible, let Y be disconnected. Let M and N form a disconnection of Y. Then M and N are clopen and Y=M∪N where M∩N=∅. Since f is contra $\mathcal{I}_{wg}$-continuous, $X = f^{-1}(Y) = f^{-1}(M \cup N) = f^{-1}(M) \cup f^{-1}(N)$, where $f^{-1}(M)$ and $f^{-1}(N)$ are nonempty $\mathcal{I}_{wg}$-open sets in X. Also $f^{-1}(M) \cap f^{-1}(N) = \emptyset$. Hence X is not $\mathcal{I}_{wg}$-connected. This is a contradiction. Therefore Y is connected. □

DEFINITION 2.12. Let w be a WS on a space (X, τ). Then (X, τ) is said to be wg-connected if (X, τ) can not be expressed as the union of two disjoint non-empty wg-open subsets of (X, τ).

EXAMPLE 2.17. (1) Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{b\}, \{b, c\}, X\}$ and $w = \{\emptyset, \{b, c\}, \{a, b\}, X\}$. Then (X, τ) is wg-connected.

(2) Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{b\}, \{a, c\}, X\}$ and $w = \{\emptyset, \{a\}, \{c\}, \{a, b\}, X\}$. Then (X, τ) is not wg-connected.

COROLLARY 2.4. *Let w be a WS on a space X. Then a contra wg-continuous image of a wg-connected space is connected.*

PROOF. The proof follows from the Theorem 2.8 if $\mathcal{I}$=$\{\emptyset\}$. □

LEMMA 2.1. *For a WS w on an ideal space $(X, \tau, \mathcal{I})$, the following are equivalent:*

(1) *X is $\mathcal{I}_{wg}$-connected.*

(2) *The only subset of X which are both $\mathcal{I}_{wg}$-open and $\mathcal{I}_{wg}$-closed are the empty set $\emptyset$ and X.*

PROOF. (1) ⇒ (2). Let G be an $\mathcal{I}_{wg}$-open and $\mathcal{I}_{wg}$-closed subset of X. Then $X - G$ is both $\mathcal{I}_{wg}$-open and $\mathcal{I}_{wg}$-closed. Since X is $\mathcal{I}_{wg}$-connected, X can be expressed as union of two disjoint non-empty $\mathcal{I}_{wg}$-open sets X and $X - G$, which implies $X - G$ is empty.

(2) ⇒ (1). Suppose $X = P \cup Q$ where P and Q are disjoint non-empty $\mathcal{I}_{wg}$-open subsets of X. Then P is both $\mathcal{I}_{wg}$-open and $\mathcal{I}_{wg}$-closed. By assumption either P=∅ or X which contradicts the assumption that P and Q are disjoint nonempty $\mathcal{I}_{wg}$-open subsets of X. Therefore X is $\mathcal{I}_{wg}$-connected. □

DEFINITION 2.13. Let w be a WS on a space X. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then f is called w-preclosed if $f(V)$ is preclosed in Y for each w-closed set V of X.

THEOREM 2.9. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a surjective w-preclosed contra $\mathcal{I}_{wg}$-continuous function. If X is an $\mathcal{I}_{wg}$-space, then Y is locally indiscrete.*

PROOF. Suppose that Q is open in Y. Since f is contra $\mathcal{I}_{wg}$-continuous, $f^{-1}(Q) = P$ is $\mathcal{I}_{wg}$-closed in X. Since X is an $\mathcal{I}_{wg}$-space, P is w-closed in X. Since f is w-preclosed, then Q is preclosed in Y. Now we have $cl(Q) = cl(int(Q)) \subseteq Q$. This means that Q is closed and hence Y is locally indiscrete. $\square$

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