BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Vol. 9(2019), 231-244
DOI: 10.7251/BIMVI1902231P

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

$\mathcal{I}_{w\hat{g}}$-NORMAL AND $\mathcal{I}_{w\hat{g}}$-REGULAR SPACES

Prabakaran Thiruvalagu, Sangeethasubha Vanangamudi, Seenivasagan Narayanasamy, and Ravi Ochadevar

Abstract. $\mathcal{I}_{w\hat{g}}$-normal and $\mathcal{I}_{w\hat{g}}$-regular spaces are introduced and various characterizations and properties are given. Characterizations of normal, mildly normal, $w\hat{g}$-normal and regular spaces are also given.

1. Introduction and Preliminaries

Throughout this paper, by a space X, we always mean a topological space (X, τ) with no separation properties assumed. Let H be a subset of X. We denote the interior, the closure and the complement of a set H by int(H), cl(H) and X\H or H^c, respectively.

An ideal $\mathcal{I}$ on a space X is a non-empty collection of subsets of X which satisfies
(i) P$\in\mathcal{I}$ and Q$\subseteq$P$\Rightarrow$Q$\in\mathcal{I}$ and
(ii) P$\in\mathcal{I}$ and Q$\in\mathcal{I}\Rightarrow$P$\cup$Q$\in\mathcal{I}$.

Given a space X with an ideal $\mathcal{I}$ on X and if $\wp$(X) is the set of all subsets of X, a set operator $(.)^*$: $\wp$(X)$\rightarrow\wp$(X), called a local function [**8**] of H with respect to τ and $\mathcal{I}$ is defined as follows: for H$\subseteq$X,

$$H^*(\mathcal{I},\tau) = \{x \in X \mid U \cap H \notin \mathcal{I} \text{ for every} U \in \tau(x)\}$$

where τ(x)={U$\in\tau$|x$\in$U}. We will make use of the basic facts about the local functions [[**7**], Theorem 2.3] without mentioning it explicitly. A Kuratowski closure operator cl*(.) for a topology $\tau^*(\mathcal{I},\tau)$, called the $\star$-topology, finer than τ is defined by cl*(H)=H$\cup$H$^*(\mathcal{I},\tau)$ [**30**]. When there is no chance for confusion, we will simply write H* for H$^*(\mathcal{I},\tau)$ and τ^* for $\tau^*(\mathcal{I},\tau)$. If $\mathcal{I}$ is an ideal on X, then (X,τ,$\mathcal{I}$) is called an ideal space. $\mathcal{N}$ is the ideal of all nowhere dense subsets in (X,τ). A subset H

2010 *Mathematics Subject Classification.* 54D10, 54D15.

Key words and phrases. $\mathcal{I}_{w\hat{g}}$-closed set, $\mathcal{I}_{w\hat{g}}$-open set, completely codense ideal, $w\hat{g}$-closed set, $w\hat{g}$-open set, $w\hat{g}$-normal space, mildly normal space, regular space.

of an ideal space (X,τ,$\mathcal{I}$) is called $\star$-closed [**7**] (resp. $\star$-dense in itself [**6**]) if $H^*\subseteq H$ (resp. $H\subseteq H^*$). A subset H of an ideal space (X,τ,$\mathcal{I}$) is called $\mathcal{I}_g$-closed [**4**] if $H^*\subseteq U$ whenever $H\subseteq U$ and U is open.

$int^*(H)$ will denote the interior of H in (X,τ^*). A subset A of a space (X, τ) is said to be regular open [**29**] if A=int(cl(A)) and A is said to be regular closed [**29**] if A=cl(int(A)). A subset H of a space X is called an α-open [**19**] (resp. semi-open [**9**], preopen [**14**]) set if H$\subseteq$int(cl(int(H))) (resp. H$\subseteq$cl(int(H)), H$\subseteq$int(cl(H))). The complement of a semi-open set is called semi-closed. The family of all α-open sets in (X,τ), denoted by τ^α, is a topology on X finer than τ. The complement of an α-open set is called α-closed. The interior of a subset H in (X,τ^α) is denoted by $int_\alpha(H)$. The closure of a subset H in (X,τ^α) is denoted by $cl_\alpha(H)$. A subset H of a space X is said to be g-closed [**10**] if cl(H)$\subseteq$U whenever H$\subseteq$U and U is open. H is said to be g-open if X$-$H is g-closed. The family of all semi-open sets of X is denoted by SO(X).

An ideal $\mathcal{I}$ is said to be codense [**5**] or τ-boundary [**18**] if $\tau\cap\mathcal{I}=\{\emptyset\}$. $\mathcal{I}$ is said to be completely codense [**5**] if PO(X)$\cap\mathcal{I}=\{\emptyset\}$, where PO(X) is the family of all preopen sets in (X,τ). Every completely codense ideal is codense but not converse by [**5**]. The following Lemmas will be useful in the sequel.

LEMMA 1.1. *Let (X,τ,$\mathcal{I}$) be an ideal space and H$\subseteq$X. If H$\subseteq H^*$, then $H^*=cl(H^*)=cl(H)=cl^*(H)$* ([**24**], Theorem 5).

LEMMA 1.2. *Let (X,τ,$\mathcal{I}$) be an ideal space. Then $\mathcal{I}$ is codense if and only if G $\subseteq G^*$ for every semi-open set G in X* ([**24**], Theorem 3).

LEMMA 1.3. *Let (X,τ,$\mathcal{I}$) be an ideal space. If $\mathcal{I}$ is completely codense, then $\tau^*\subseteq\tau^\alpha$* ([**24**], Theorem 6).

Recall that (X,τ,$\mathcal{I}$) is a $T_{\mathcal{I}}$ ideal space [**4**] if every $\mathcal{I}_g$-closed set is $\star$-closed.

LEMMA 1.4. *If (X,τ,$\mathcal{I}$) is a $T_{\mathcal{I}}$ ideal space and H is an $\mathcal{I}_g$-closed set, then H is a $\star$-closed set* ([**16**], Corollary 2.2).

LEMMA 1.5. *Every g-closed set is $\mathcal{I}_g$-closed but not conversely* ([**4**], Theorem 2.1).

LEMMA 1.6 ([**7**]). *Let (X,τ,$\mathcal{I}$) be an ideal space and let M and N be two subsets on X. Then*

(1) $M\subseteq N\Rightarrow M^*\subseteq N^*$.
(2) $M^*=cl(M^*)\subseteq cl(M)$ (M^* *is a closed subset of cl(M)).*
(3) $(M^*)^*\subseteq M^*$.
(4) $(M\cup N)^*=M^*\cup N^*$.
(5) $M^*-N^*=(M-N)^*-N^*\subseteq(M-N)^*$.

Let us say that $w\subseteq P$ is a weak structure (briefly WS) on X iff $\emptyset\in w$. Clearly each generalized topology and each minimal structure is a WS [**3**].

Each member of w is said to be w-open and the complement of a w-open set is called w-closed.

Let w be a weak structure on X and H⊆X. We define (as in the general case) i_w(H) is the union of all w-open subsets contained in H and c_w(H) is the intersection of all w-closed sets containing H [**3**].

Let w be a WS on a space X and H⊆X. Then H$\in\sigma(w)$ [resp. H$\in\alpha(w)$, H$\in\pi(w)$] if H$\subseteq c_w(i_w$(H)) [resp. H$\subseteq i_w(c_w(i_w$(H))), H$\subseteq i_w(c_w$(H))] [**3**].

Let w be a WS on a space X. Then H⊆X is called a $\hat{g}w$-closed set if c_w(H)⊆U whenever H⊆U∈SO(X). The complement of a $\hat{g}w$-closed set is called $\hat{g}w$-open [**25**].

Let w be a WS on a space X. Then H⊆X is said to be $w\hat{g}$-closed [**26**] if cl(H)⊆U whenever H⊆U and U$\in\sigma(w)$. A subset H of a space X is said to be rαg-closed [**21**] if cl_α(H)⊆U whenever H⊆U and U is regular open. H is said to be $w\hat{g}$-open (resp. rαg-open) if X−H is $w\hat{g}$-closed (resp. rαg-closed).

Let w be a WS on an ideal space (X,τ,$\mathcal{I}$). Then H⊆X is called $\mathcal{I}_{w\hat{g}}$-closed if H*⊆U whenever H⊆U and U$\in\sigma(w)$ [**26**] . In [**26**], every $*$-closed and hence every closed set is $\mathcal{I}_{w\hat{g}}$-closed. A subset H of an ideal space (X,τ,$\mathcal{I}$) is said to be $\mathcal{I}_{w\hat{g}}$-open [**26**] if X−H is $\mathcal{I}_{w\hat{g}}$-closed. In this paper, we define $\mathcal{I}_{w\hat{g}}$-normal, ${}_{w\hat{g}}\mathcal{I}$-normal and $\mathcal{I}_{w\hat{g}}$-regular spaces using $\mathcal{I}_{w\hat{g}}$-open sets and give characterizations and properties of such spaces. Also, characterizations of normal, mildly normal, $w\hat{g}$-normal and regular spaces are given.

Remark 1.1. ([**1**]) If w is a WS on X, then $i_w(\emptyset) = \emptyset$ and $c_w(X) = X$.

Theorem 1.1 ([**3**]). *If w is a WS on X and $A,B \in w$ then*

(1) $i_w(A)\subseteq A\subseteq c_w(A)$,

(2) $A\subseteq B\Rightarrow i_w(A)\subseteq i_w(B)$ *and* $c_w(A)\subseteq c_w(B)$,

(3) $i_w(i_w(A))=i_w(A)$ *and* $c_w(c_w(A))=c_w(A)$,

(4) $i_w(X - A)=X - c_w(A)$ *and* $c_w(X - A)=X - i_w(A)$.

Lemma 1.7 ([**1**]). *If w is a WS on X, then*

(1) $x\in i_w(A)$ *if and only if there is a w-open set $G\subseteq A$ such that $x\in G$,*

(2) $x\in c_w(A)$ *if and only if $G\cap A\neq\emptyset$ whenever $x\in G\in w$,*

(3) *If $A\in w$, then $A=i_w(A)$ and if A is w-closed then $A=c_w(A)$.*

Lemma 1.8 ([**26**], Theorem 2.47). *Let (X,τ,$\mathcal{I}$) be an ideal space where $\mathcal{I}$ is completely codense. Then the following are equivalent.*

(1) *X is normal.*

(2) *For disjoint closed sets M and N, there exist disjoint $\mathcal{I}_{w\hat{g}}$-open sets P and Q such that M⊆P, N⊆Q.*

(3) *For a closed set M and an open set Q containing M, there exists an $\mathcal{I}_{w\hat{g}}$-open set P such that M⊆P⊆cl*(P)⊆Q.*

Definition 1.1. ([**26**]) An ideal space (X,τ,$\mathcal{I}$) is said to have the property B if $P^c\in\tau$ and $Q^c\in\sigma(w)$ then X\(P∩Q)$\in\sigma(w)$.

Lemma 1.9. *If (X,τ,$\mathcal{I}$) is an ideal space and H⊆X. If $\mathcal{I}$={∅}, then H is $\mathcal{I}_{w\hat{g}}$-closed if and only if H is $w\hat{g}$-closed* ([**26**], Corollary 2.28).

LEMMA 1.10 ([**26**], Theorem 2.15). *If $(X,\tau,\mathcal{I})$ is an ideal space with property B and $H\subseteq X$, then the following are equivalent.*

1. *H is $\mathcal{I}_{w\hat{g}}$-closed.*
2. *$cl^*(H)\subseteq U$ whenever $H\subseteq U$ and $U\in\sigma(w)$.*

LEMMA 1.11 ([**26**], Theorem 2.42). *Let $(X,\tau,\mathcal{I})$ be an ideal space with property B and $H\subseteq X$. Then H is $\mathcal{I}_{w\hat{g}}$-open if and only if $F\subseteq int^*(H)$ whenever $F^c\in\sigma(w)$ and $F\subseteq H$.*

LEMMA 1.12 ([**26**], Theorem 2.46). *Let $(X,\tau,\mathcal{I})$ be an ideal space. Then every subset of X is $\mathcal{I}_{w\hat{g}}$-closed if and only if every subset of $\sigma(w)$ is $*$-closed.*

PROPOSITION 1.1 ([**26**]). *If $H\in\tau$ then $H\in\sigma(w)$.*

2. $\mathcal{I}_{w\hat{g}}$-normal spaces

Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. Then $(X,\tau,\mathcal{I})$ is said to be an $\mathcal{I}_{w\hat{g}}$-normal space if for every pair of disjoint closed sets M and N, there exist disjoint $\mathcal{I}_{w\hat{g}}$-open sets P and Q such that M$\subseteq$P and N$\subseteq$Q. Since every open set is an $\mathcal{I}_{w\hat{g}}$-open set, every normal space is $\mathcal{I}_{w\hat{g}}$-normal. The following Example 2.1 shows that an $\mathcal{I}_{w\hat{g}}$-normal space is not necessarily a normal space. Theorem 2.1 below gives characterizations of $\mathcal{I}_{w\hat{g}}$-normal spaces. Theorem 2.2 below shows that the two concepts coincide for completely codense ideal spaces.

EXAMPLE 2.1. Let $X = \{a, b, c\}$, τ={$\emptyset$, {b}, {a, b}, {b, c}, X}, w={$\emptyset$, {c}} and $\mathcal{I}$={$\emptyset$, {b}}. Then $\emptyset^*=\emptyset$, $(\{a, b\})^*=\{a\}$, $(\{b, c\})^*=\{c\}$, $(\{b\})^*=\emptyset$ and $X^*=\{a, c\}$. Here every subset of $\sigma(w)$ is $*$-closed and so, by Lemma 1.12, every subset of X is $\mathcal{I}_{w\hat{g}}$-closed and hence every subset of X is $\mathcal{I}_{w\hat{g}}$-open. This implies that $(X,\tau,\mathcal{I})$ is $\mathcal{I}_{w\hat{g}}$-normal. Now {a} and {c} are disjoint closed subsets of X which are not separated by disjoint open sets and so (X,τ) is not normal.

THEOREM 2.1. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. Then the following are equivalent.*

1. *X is $\mathcal{I}_{w\hat{g}}$-normal.*
2. *For every pair of disjoint closed sets M and N, there exist disjoint $\mathcal{I}_{w\hat{g}}$-open sets P and Q such that $M\subseteq P$ and $N\subseteq Q$.*
3. *For every closed set M and an open set Q containing M, there exists an $\mathcal{I}_{w\hat{g}}$-open set P such that $M\subseteq P\subseteq cl^*(P)\subseteq Q$.*

PROOF. (1) $\Rightarrow$ (2). The proof follows from the definition of $\mathcal{I}_{w\hat{g}}$-normal spaces.

(2) $\Rightarrow$ (3). Let M be a closed set and Q be an open set containing M. Since M and X$-$Q are disjoint closed sets, there exist disjoint $\mathcal{I}_{w\hat{g}}$-open sets P and R such that M$\subseteq$P and X$-$Q$\subseteq$R. Again, P$\cap$R=$\emptyset$ implies that P$\cap$int*(R)=$\emptyset$. Also P$\subseteq$X$-$R$\Rightarrow$cl*(P)$\subseteq$cl*(X$-$R)=X$-$int*(R). Since Q$\in\sigma(w)$ and R is $\mathcal{I}_{w\hat{g}}$-open, X$-$Q $\subseteq$R implies that X$-$Q$\subseteq$int*(R) and so X$-$int*(R)$\subseteq$Q. Thus, we have M$\subseteq$P$\subseteq$cl*(P) $\subseteq$X$-$int*(R)$\subseteq$Q which proves (3).

(3) $\Rightarrow$ (1). Let M and N be two disjoint closed subsets of X. By hypothesis, there exists an $\mathcal{I}_{w\hat{g}}$-open set P such that M$\subseteq$P$\subseteq$cl*(P)$\subseteq$X$-$N. If R=X$-$cl*(P), then

P and R are the required disjoint $\mathcal{I}_{w\hat{g}}$-open sets containing M and N respectively. So, (X,τ,$\mathcal{I}$) is $\mathcal{I}_{w\hat{g}}$-normal. □

THEOREM 2.2. *Let (X,τ,$\mathcal{I}$) be an ideal space where $\mathcal{I}$ is completely codense and w a WS on (X,τ,$\mathcal{I}$). If (X,τ,$\mathcal{I}$) is $\mathcal{I}_{w\hat{g}}$-normal, then it is a normal space.*

PROOF. Suppose that $\mathcal{I}$ is completely codense. By Theorem 2.1, (X,τ,$\mathcal{I}$) is $\mathcal{I}_{w\hat{g}}$-normal if and only if for each pair of disjoint closed sets M and N, there exist disjoint $\mathcal{I}_{w\hat{g}}$-open sets P and Q such that M⊆P and N⊆Q if and only if X is normal, by Lemma 1.8. □

THEOREM 2.3. *Let w be a WS on an ideal space (X,τ,$\mathcal{I}$) which is $\mathcal{I}_{w\hat{g}}$-normal space. If G is closed and H is a $w\hat{g}$-closed set such that H∩G=∅, then there exist disjoint $\mathcal{I}_{w\hat{g}}$-open sets P and Q such that H⊆P and G⊆Q.*

PROOF. Since H∩G=∅, H⊆X−G where X−G∈τ⊆$\sigma(w)$. Therefore, by hypothesis, cl(H)⊆ X−G. Since cl(H)∩G=∅ and X is $\mathcal{I}_{w\hat{g}}$-normal, there exist disjoint $\mathcal{I}_{w\hat{g}}$-open sets P and Q such that H⊆cl(H)⊆P and G⊆Q. □

DEFINITION 2.1. Let w be a WS on a space X. A subset H of a space X is said to be αgs$w$-closed if cl_α(H)⊆U whenever H⊆U and U∈$\sigma(w)$. The complement of an αgs$w$-closed set is called αgs$w$-open.

PROPOSITION 2.1. *For a WS w on a space X, every closed subset is αgsw-closed.*

PROOF. Let H be a closed set such that H⊆U and U∈$\sigma(w)$. Then cl(H)=H and cl_α(H)⊆cl(H)=H⊆U. Hence H is αgs$w$-closed. □

EXAMPLE 2.2. Let X={a, b, c}, τ={∅, {c}, X} and w={∅, {a}, {a, b}}. Then the αgs$w$-closed sets are {a}, {b}, {a, b}, {b, c}, ∅, X and the closed sets are ∅, X, {a, b}. It is clear that {b} is αgs$w$-closed set but it is not closed.

The following Corollaries 2.1 and 2.2 give properties of normal spaces. If $\mathcal{I}$={∅} in Theorem 2.1, then we have the following Corollary 2.8, the proof of which follows from Theorem 2.2 and Lemma 1.9, since {∅} is a completely codense ideal. If $\mathcal{I}$=$\mathcal{N}$ in Theorem 2.3, then we have the Corollary 2.2 below, since $\tau^*(\mathcal{N})=\tau^\alpha$ and $\mathcal{I}_{w\hat{g}}$-open sets coincide with αgs$w$-open sets.

COROLLARY 2.1. *Let (X,τ) be a normal space and w a WS on X. If G is a closed set and H is a $w\hat{g}$-closed set disjoint from G, then there exist disjoint $w\hat{g}$-open sets P and Q such that H⊆P and G⊆Q.*

COROLLARY 2.2. *Let (X,τ,$\mathcal{I}$) be a normal ideal space where $\mathcal{I}$=$\mathcal{N}$ and w a WS on (X,τ,$\mathcal{I}$). If G is a closed set and H is a $w\hat{g}$-closed set disjoint from G, then there exist disjoint αgsw-open sets P and Q such that H⊆P and G⊆Q.*

LEMMA 2.1. *If w is a WS on an ideal space (X,τ,$\mathcal{I}$) and H⊆X, then the following hold. If $\mathcal{I}$=$\mathcal{N}$, then H is $\mathcal{I}_{w\hat{g}}$-closed if and only if H is αgsw-closed.*

PROOF. It follows from cl*(H)=cl_α(H) for any subset H of X. □

THEOREM 2.4. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$ which is $\mathcal{I}_{w\hat{g}}$-normal. Then the following hold.*

(1) *For every closed set M and every $w\hat{g}$-open set N containing M, there exists an $\mathcal{I}_{w\hat{g}}$-open set P such that M⊆int*(P)⊆P⊆N.*
(2) *For every $w\hat{g}$-closed set M and every open set N containing M, there exists an $\mathcal{I}_{w\hat{g}}$-closed set P such that M⊆P⊆cl*(P)⊆N.*

PROOF. (1) Let M be a closed set and N be a $w\hat{g}$-open set containing M. Then M∩(X−N)=∅, where M is closed and X−N is $w\hat{g}$-closed. By Theorem 2.3, there exist disjoint $\mathcal{I}_{w\hat{g}}$-open sets P and Q such that M⊆P and X−N⊆Q. Since P∩Q=∅, we have P⊆X−Q. By Lemma 1.14, M⊆int*(P). Therefore, M⊆int*(P)⊆P⊆X−Q⊆N. This proves (1).

(2) Let M be a $w\hat{g}$-closed set and N be an open set containing M. Then X−N is a closed set contained in the $w\hat{g}$-open set X−M. By (1), there exists an $\mathcal{I}_{w\hat{g}}$-open set Q such that X−N⊆int*(Q)⊆Q⊆X−M. Therefore, M⊆X−Q⊆cl*(X−Q)⊆N. If P=X−Q, then M⊆P⊆cl*(P)⊆N and so P is the required $\mathcal{I}_{w\hat{g}}$-closed set. □

The following Corollaries 2.3 and 2.4 give some properties of normal spaces. If $\mathcal{I}$={∅} in Theorem 2.3, then we have the following Corollary 2.4. If $\mathcal{I}=\mathcal{N}$ in Theorem 2.4, then we have the Corollary 2.4 below.

COROLLARY 2.3. *Let (X,τ) be a normal space and w a WS on X. Then the following hold.*

(1) *For every closed set M and every $w\hat{g}$-open set N containing M, there exists a $w\hat{g}$-open set P such that M⊆int(P)⊆P⊆N.*
(2) *For every $w\hat{g}$-closed set M and every open set N containing M, there exists a $w\hat{g}$-closed set P such that M⊆P⊆cl(P)⊆N.*

COROLLARY 2.4. *Let (X,τ) be a normal space and w a WS on X. Then the following hold.*

(1) *For every closed set M and every $w\hat{g}$-open set N containing M, there exists an αgsw-open set P such that M⊆int_α(P)⊆P⊆N.*
(2) *For every $w\hat{g}$-closed set M and every open set N containing M, there exists an αgsw-closed set P such that M⊆P⊆cl_α(P)⊆N.*

Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. Then $(X,\tau,\mathcal{I})$ is said to be ${}_{w\hat{g}}\mathcal{I}$-normal if for each pair of disjoint $\mathcal{I}_{w\hat{g}}$-closed sets M and N, there exist disjoint open sets P and Q in X such that M⊆P and N⊆Q. Since every closed set is $\mathcal{I}_{w\hat{g}}$-closed, every ${}_{w\hat{g}}\mathcal{I}$-normal space is normal. But a normal space need not be ${}_{w\hat{g}}\mathcal{I}$-normal as the following Example 2.3 shows. Theorems 2.5 and 2.6 below give characterizations of ${}_{w\hat{g}}\mathcal{I}$-normal spaces.

EXAMPLE 2.3. Let X={a, b, c}, τ={∅, {a}, {b, c}, X}, w={∅, {b, c}, X} and $\mathcal{I}$={∅, {a}}. Every subset of $\sigma(w)$ is ∗-closed and so every subset of X is $\mathcal{I}_{w\hat{g}}$-closed. Now A={a, b} and B={c} are disjoint $\mathcal{I}_{w\hat{g}}$-closed sets, but they are not separated by disjoint open sets. So $(X,\tau,\mathcal{I})$ is not ${}_{w\hat{g}}\mathcal{I}$-normal. But $(X,\tau,\mathcal{I})$ is normal.

Theorem 2.5. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. Then the following are equivalent.*

(1) *X is $_{w\hat{g}}\mathcal{I}$-normal.*

(2) *For every $\mathcal{I}_{w\hat{g}}$-closed set M and every $\mathcal{I}_{w\hat{g}}$-open set N containing M, there exists an open set P of X such that M⊆P⊆cl(P)⊆N.*

Proof. (1) ⇒ (2). Let M be an $\mathcal{I}_{w\hat{g}}$-closed set and N be an $\mathcal{I}_{w\hat{g}}$-open set containing M. Since M and X−N are disjoint $\mathcal{I}_{w\hat{g}}$-closed sets, there exist disjoint open sets P and Q such that M⊆P and X−N⊆Q. Now P∩Q=∅ implies that cl(P)⊆X−Q. Therefore, M⊆P⊆cl(P)⊆X−Q⊆N. This proves (2).

(2) ⇒ (1). Suppose M and N are disjoint $\mathcal{I}_{w\hat{g}}$-closed sets, then the $\mathcal{I}_{w\hat{g}}$-closed set M is contained in the $\mathcal{I}_{w\hat{g}}$-open set X−N. By hypothesis, there exists an open set P of X such that M⊆P⊆cl(P)⊆X−N. If Q=X−cl(P), then P and Q are disjoint open sets containing M and N respectively. Therefore, $(X,\tau,\mathcal{I})$ is $_{w\hat{g}}\mathcal{I}$-normal. □

If $\mathcal{I}=\{\emptyset\}$, then $_{w\hat{g}}\mathcal{I}$-normal spaces coincide with $w\hat{g}$-normal spaces and so if we take $\mathcal{I}=\{\emptyset\}$, in Theorem 2.5, then we have the following characterization for $w\hat{g}$-normal spaces.

Corollary 2.5. *Let w be a WS on a space X. Then the following are equivalent.*

(1) *X is $w\hat{g}$-normal.*

(2) *For every $w\hat{g}$-closed set M and every $w\hat{g}$-open set N containing M, there exists an open set P of X such that M⊆P⊆cl(P)⊆N.*

Theorem 2.6. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. Then the following are equivalent.*

(1) *X is $_{w\hat{g}}\mathcal{I}$-normal.*

(2) *For each pair of disjoint $\mathcal{I}_{w\hat{g}}$-closed subsets M and N of X, there exists an open set P of X containing M such that cl(P)∩N=∅.*

(3) *For each pair of disjoint $\mathcal{I}_{w\hat{g}}$-closed subsets M and N of X, there exist an open set P containing M and an open set Q containing N such that cl(P)∩cl(Q)=∅.*

Proof. (1) ⇒ (2). Suppose that M and N are disjoint $\mathcal{I}_{w\hat{g}}$-closed subsets of X. Then the $\mathcal{I}_{w\hat{g}}$-closed set M is contained in the $\mathcal{I}_{w\hat{g}}$-open set X−N. By Theorem 2.15, there exists an open set P of X such that M⊆P⊆cl(P)⊆X−N. Therefore, P is the required open set containing M such that cl(P)∩N=∅.

(2) ⇒ (3). Let M and N be two disjoint $\mathcal{I}_{w\hat{g}}$-closed subsets of X. By hypothesis, there exists an open set P of X containing M such that cl(P)∩N=∅. Also, cl(P) and N are disjoint $\mathcal{I}_{w\hat{g}}$-closed sets of X. By hypothesis, there exists an open set Q of X containing N such that cl(P)∩cl(Q)=∅.

(3) ⇒ (1). The proof is clear. □

If $\mathcal{I}=\{\emptyset\}$, in Theorem 2.6, then we have the following characterizations for $w\hat{g}$-normal spaces.

COROLLARY 2.6. *Let (X,τ) be a space and w a WS on X. Then the following are equivalent.*

(1) *X is $w\hat{g}$-normal.*
(2) *For each pair of disjoint $w\hat{g}$-closed subsets M and N of X, there exists an open set P of X containing M such that cl(P)∩N=∅.*
(3) *For each pair of disjoint $w\hat{g}$-closed subsets M and N of X, there exist an open set P containing M and an open set Q containing N such that cl(P)∩cl(Q)=∅.*

THEOREM 2.7. *Let w be a WS on an ideal space (X,τ,$\mathcal{I}$) which is an ${}_{w\hat{g}}\mathcal{I}$-normal space. If M and N are disjoint $\mathcal{I}_{w\hat{g}}$-closed subsets of X, then there exist disjoint open sets P and Q such that cl*(M)⊆P and cl*(N)⊆Q.*

PROOF. Suppose that M and N are disjoint $\mathcal{I}_{w\hat{g}}$-closed sets. By Theorem 2.6(3), there exist an open set P containing M and an open set Q containing N such that cl(P)∩cl(Q)=∅. Since M is $\mathcal{I}_{w\hat{g}}$-closed, M⊆P implies that cl*(M)⊆P. Similarly cl*(N)⊆Q. □

If $\mathcal{I}$={∅}, in Theorem 2.7, then we have the following property of disjoint $w\hat{g}$-closed sets in $w\hat{g}$-normal spaces.

COROLLARY 2.7. *Let w be a WS on a space X which is a $w\hat{g}$-normal space. If M and N are disjoint $w\hat{g}$-closed subsets of X, then there exist disjoint open sets P and N such that cl(M)⊆P and cl(N)⊆Q.*

THEOREM 2.8. *Let w be a WS on (X,τ,$\mathcal{I}$) which is an ${}_{w\hat{g}}\mathcal{I}$-normal space. If M is an $\mathcal{I}_{w\hat{g}}$-closed set and N is an $\mathcal{I}_{w\hat{g}}$-open set containing M, then there exists an open set P such that M⊆cl*(M)⊆P⊆int*(N)⊆N.*

PROOF. Suppose M is an $\mathcal{I}_{w\hat{g}}$-closed set and N is an $\mathcal{I}_{w\hat{g}}$-open set containing M. Since M and X−N are disjoint $\mathcal{I}_{w\hat{g}}$-closed sets, by Theorem 2.7, there exist disjoint open sets P and Q such that cl*(M)⊆P and cl*(X−N)⊆Q. Now, X−int*(N)=cl*(X−N)⊆Q implies that X−Q⊆int*(N). Again, P∩Q=∅ implies P⊆X−Q and so M⊆cl*(M) ⊆P⊆X−Q⊆ int*(N)⊆N. □

If $\mathcal{I}$={∅}, in Theorem 2.7, then we have the following Corollary 2.8.

COROLLARY 2.8. *Let w be a WS on a space X which is a $w\hat{g}$-normal space. If M is a $w\hat{g}$-closed set and N is a $w\hat{g}$-open set containing M, then there exists an open set P such that M⊆cl(M)⊆P⊆int(N)⊆N.*

The following Theorem 2.9 gives a characterization of normal spaces in terms of $w\hat{g}$-open sets which follows from Lemma 1.8 if $\mathcal{I}$={∅}.

THEOREM 2.9. *Let (X,τ) be a space and w a WS on X. Then the following are equivalent.*

(1) *X is normal.*
(2) *For any disjoint closed sets M and N, there exist disjoint $w\hat{g}$-open sets P and Q such that M⊆P and N⊆Q.*

(3) *For any closed set M and an open set Q containing M, there exists a $w\hat{g}$-open set P such that M⊆P⊆cl(P)⊆Q.*

The rest of the section is devoted to the study of mildly normal spaces in terms of $\mathcal{I}_{w\hat{g}}$-open sets, $\mathcal{I}_g$-open sets and $\mathcal{I}_{rg}$-open sets. A space (X,τ) is said to be a mildly normal space [**28**] if disjoint regular closed sets are separated by disjoint open sets. A subset H of a space (X,τ) is said to be αg-closed [**13**] if cl_α(H)⊆U whenever H⊆U and U is open. A subset H of a space (X,τ) is said to rg-closed [**22**] if cl(H)⊆U whenever H⊆U and U is regular open in X. The complements of the above closed sets are called their respective open sets.

A subset H of an ideal space (X,τ,$\mathcal{I}$) is said to be a regular generalized closed set with respect to an ideal $\mathcal{I}$ ($\mathcal{I}_{rg}$-closed) [**17**] if H*⊆U whenever H⊆U and U is regular open. H is called $\mathcal{I}_g$-open (resp. $\mathcal{I}_{rg}$-open) if X−H is $\mathcal{I}_g$-closed (resp. $\mathcal{I}_{rg}$-closed). Clearly, every $\mathcal{I}_{w\hat{g}}$-closed set is $\mathcal{I}_g$-closed and every $\mathcal{I}_g$-closed set is $\mathcal{I}_{rg}$-closed but the separate converses are not true. Theorem 2.10 below gives characterizations of mildly normal spaces. Corollary 2.9 below gives characterizations of mildly normal spaces in terms of αgs$w$-open, αg-open and rαg-open sets. Corollary 2.10 below gives characterizations of mildly normal spaces in terms of $w\hat{g}$-open, g-open and rg-open sets. The following Lemma 2.2 is essential to prove Theorem 2.10.

LEMMA 2.2 ([**17**]). *Let (X,τ,$\mathcal{I}$) be an ideal space. A subset H⊆X is $\mathcal{I}_{rg}$-open if and only if F⊆int*(H) whenever F is regular closed and F⊆H.*

THEOREM 2.10. *Let (X,τ,$\mathcal{I}$) be an ideal space where $\mathcal{I}$ is completely codense and w a WS on (X,τ,$\mathcal{I}$). Then the following are equivalent.*

(1) *X is mildly normal.*

(2) *For disjoint regular closed sets M and N, there exist disjoint $\mathcal{I}_{w\hat{g}}$-open sets P and Q such that M⊆P and N⊆Q.*

(3) *For disjoint regular closed sets M and N, there exist disjoint $\mathcal{I}_g$-open sets P and Q such that M⊆P and N⊆Q.*

(4) *For disjoint regular closed sets M and N, there exist disjoint $\mathcal{I}_{rg}$-open sets P and Q such that M⊆P and N⊆Q.*

(5) *For a regular closed set M and a regular open set N containing M, there exists an $\mathcal{I}_{rg}$-open set P of X such that M⊆P⊆cl*(P)⊆N.*

(6) *For a regular closed set M and a regular open set N containing M, there exists an ∗-open set S of X such that M⊆S⊆cl*(S)⊆N.*

(7) *For disjoint regular closed sets M and N, there exist disjoint ∗-open sets P and Q such that M⊆P and N⊆Q.*

PROOF. (1) ⇒ (2). Suppose that M and N are disjoint regular closed sets. Since X is mildly normal, there exist disjoint open sets P and Q such that M⊆P and N⊆Q. But every open set is an $\mathcal{I}_{w\hat{g}}$-open set. This proves (2).

(2) ⇒ (3). The proof follows from the fact that every $\mathcal{I}_{w\hat{g}}$-open set is an $\mathcal{I}_g$-open set.

(3) ⇒ (4). The proof follows from the fact that every $\mathcal{I}_g$-open set is an $\mathcal{I}_{rg}$-open set.

(4) ⇒ (5). Suppose M is a regular closed and N is a regular open set containing M. Then M and X−N are disjoint regular closed sets. By hypothesis, there exist disjoint $\mathcal{I}_{rg}$-open sets P and Q such that M⊆P and X−N⊆Q. Since X−N is regular closed and Q is $\mathcal{I}_{rg}$-open, by Lemma 2.2, X−N⊆int*(Q) and so X−int*(Q)⊆N. Again, P∩Q=∅ implies that P∩int*(Q)=∅ and so cl*(P)⊆X−int*(Q)⊆N. Hence P is the required $\mathcal{I}_{rg}$-open set such that M⊆P⊆cl*(P)⊆N.

(5) ⇒ (6). Let M be a regular closed set and N be a regular open set containing M. Then there exists an $\mathcal{I}_{rg}$-open set P of X such that M⊆P⊆cl*(P)⊆N. By Lemma 2.2, M⊆int*(P). If S=int*(P), then S is an ∗-open set and M⊆S⊆cl*(S)⊆cl*(P)⊆N. Therefore, M⊆S⊆cl*(S)⊆N.

(6) ⇒ (7). Let M and N be disjoint regular closed subsets of X. Then X−N is a regular open set containing M. By hypothesis, there exists an ∗-open set P of X such that M⊆P⊆cl*(P)⊆X−N. If Q=X−cl*(P), then P and Q are disjoint ∗-open sets of X such that M⊆P and N⊆Q.

(7) ⇒ (1). Let M and N be disjoint regular closed sets of X. Then there exist disjoint ∗-open sets P and Q such that M⊆P and N⊆Q. Since $\mathcal{I}$ is completely codense, by Lemma 1.3, $\tau^*\subseteq\tau^\alpha$ and so P, Q∈τ^α. Hence M⊆P⊆int(cl(int(P)))=S and N⊆Q⊆int(cl(int(Q)))=T. S and T are the required disjoint open sets containing M and N respectively. This proves (1). □

If $\mathcal{I}$=$\mathcal{N}$, in the above Theorem 2.10, then $\mathcal{I}_{rg}$-closed sets coincide with rαg-closed sets and so we have the following Corollary 2.9.

Corollary 2.9. *Let w be a WS on a space X. Then the following are equivalent.*

(1) *X is mildly normal.*
(2) *For disjoint regular closed sets M and N, there exist disjoint αgsw-open sets P and Q such that M⊆P and N⊆Q.*
(3) *For disjoint regular closed sets M and N, there exist disjoint αg-open sets P and Q such that M⊆P and N⊆Q.*
(4) *For disjoint regular closed sets M and N, there exist disjoint rαg-open sets P and Q such that M⊆P and N⊆Q.*
(5) *For a regular closed set M and a regular open set N containing M, there exists an rαg-open set P of X such that M⊆P⊆cl_α(P)⊆N.*
(6) *For a regular closed set M and a regular open set N containing M, there exists an α-open set S of X such that M⊆S⊆cl_α(S)⊆N.*
(7) *For disjoint regular closed sets M and N, there exist disjoint α-open sets P and Q such that M⊆P and N⊆Q.*

If $\mathcal{I}$={∅} in the above Theorem 2.10, we get the following Corollary 2.10.

Corollary 2.10. *Let w be a WS on a space X. Then the following are equivalent.*

(1) *X is mildly normal.*

(2) *For disjoint regular closed sets M and N, there exist disjoint $w\hat{g}$-open sets P and Q such that M⊆P and N⊆Q.*

(3) *For disjoint regular closed sets M and N, there exist disjoint g-open sets P and Q such that M⊆P and N⊆Q.*

(4) *For disjoint regular closed sets M and N, there exist disjoint rg-open sets P and Q such that M⊆P and N⊆Q.*

(5) *For a regular closed set M and a regular open set N containing M, there exists an rg-open set P of X such that M⊆P⊆cl(P)⊆N.*

(6) *For a regular closed set M and a regular open set N containing M, there exists an open set S of X such that M⊆S⊆cl(S)⊆N.*

(7) *For disjoint regular closed sets M and N, there exist disjoint open sets P and Q such that M⊆P and N⊆Q.*

3. $\mathcal{I}_{w\hat{g}}$-regular spaces

Let w be a WS on (X,τ,$\mathcal{I}$). Then (X,τ,$\mathcal{I}$) is said to be an $\mathcal{I}_{w\hat{g}}$-regular space if for each pair consisting of a point x and a closed set N not containing x, there exist disjoint $\mathcal{I}_{w\hat{g}}$-open sets P and Q such that x∈P and N⊆Q. Every regular space is $\mathcal{I}_{w\hat{g}}$-regular, since every open set is $\mathcal{I}_{w\hat{g}}$-open. The following Example 3.1 shows that an $\mathcal{I}_{w\hat{g}}$-regular space need not be regular. Theorem 3.2 gives a characterization of $\mathcal{I}_{w\hat{g}}$-regular spaces.

Example 3.1. Consider the Example 2.1. Then ∅*=∅, ({b})*=∅, ({a, b})*={a}, ({b, c})*={c} and X*={a, c}. Since every subset of $\sigma(w)$ is $*$-closed, every subset of X is $\mathcal{I}_{w\hat{g}}$-closed and so every subset of X is $\mathcal{I}_{w\hat{g}}$-open. This implies that (X,τ,$\mathcal{I}$) is $\mathcal{I}_{w\hat{g}}$-regular. Now, {c} is a closed set not containing a∈X, {c} and a are not separated by disjoint open sets. So (X,τ,$\mathcal{I}$) is not regular.

Theorem 3.1. *Let w be a WS on an ideal space (X,τ,$\mathcal{I}$). Then the following are equivalent.*

(1) *X is $\mathcal{I}_{w\hat{g}}$-regular.*

(2) *For every closed set N not containing x∈X, there exist disjoint $\mathcal{I}_{w\hat{g}}$-open sets P and Q such that x∈P and N⊆Q.*

(3) *For every open set Q containing x∈X, there exists an $\mathcal{I}_{w\hat{g}}$-open set P of X such that x∈P⊆cl*(P)⊆Q.*

Proof. (1) and (2) are equivalent by the definition.

(2) ⇒ (3). Let Q be an open subset such that x∈Q. Then X−Q is a closed set not containing x. Therefore, there exist disjoint $\mathcal{I}_{w\hat{g}}$-open sets P and W such that x∈P and X−Q⊆W. Now, X−Q⊆W implies that X−Q⊆int*(W) and so X−int*(W)⊆Q. Again, P∩W=∅ implies that P∩int*(W)=∅ and so cl*(P)⊆X−int*(W). Therefore, x∈P⊆cl*(P)⊆Q. This proves (3).

(3) ⇒ (1). Let N be a closed set not containing x. By hypothesis, there exists an $\mathcal{I}_{w\hat{g}}$-open set P such that x∈P⊆cl*(P)⊆X−N. If W=X−cl*(P), then P and W are disjoint $\mathcal{I}_{w\hat{g}}$-open sets such that x∈P and N⊆W. This proves (1). □

THEOREM 3.2. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. If $(X,\tau,\mathcal{I})$ is an $\mathcal{I}_{w\hat{g}}$-regular, T_1-space where $\mathcal{I}$ is completely codense, then X is regular.*

PROOF. Let N be a closed set not containing x∈X. By Theorem 3.1, there exists an $\mathcal{I}_{w\hat{g}}$-open set P of X such that x∈P⊆cl*(P)⊆X−N. Since X is a T_1-space, {x} is closed and $\{x\}^c \in \sigma(w)$ and so {x}⊆int*(P), by Lemma 1.11. Since $\mathcal{I}$ is completely codense, $\tau^* \subseteq \tau^\alpha$ and so int*(P) and X−cl*(P) are α-open sets. Now, x∈int*(P) ⊆int(cl(int(int*(P))))=G and N⊆X−cl*(P)⊆int(cl(int(X−cl*(P))))=H. Then G and H are disjoint open sets containing x and N respectively. Therefore, X is regular. □

If $\mathcal{I}=\mathcal{N}$ in Theorem 3.1, then we have the following Corollary 3.1 which gives characterizations of regular spaces, the proof of which follows from Theorem 3.2.

COROLLARY 3.1. *If w is a WS on a space X which is a T_1-space, then the following are equivalent.*

(1) *X is regular.*
(2) *For every closed set N not containing x∈X, there exist disjoint αgsw-open sets P and Q such that x∈P and N⊆Q.*
(3) *For every open set Q containing x∈X, there exists an αgsw-open set P of X such that x∈P⊆cl_α(P)⊆Q.*

If $\mathcal{I}=\{\emptyset\}$ in Theorem 3.1, then we have the following Corollary 3.2 which gives characterizations of regular spaces.

COROLLARY 3.2. *If w is a WS on a space X which is a T_1-space, then the following are equivalent.*

(1) *X is regular.*
(2) *For every closed set N not containing x∈X, there exist disjoint $w\hat{g}$-open sets P and Q such that x∈P and N⊆Q.*
(3) *For every open set Q containing x∈X, there exists a $w\hat{g}$-open set P of X such that x∈P⊆cl(P)⊆Q.*

THEOREM 3.3. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. If every subset of $\sigma(w)$ is $*$-closed, then $(X,\tau,\mathcal{I})$ is $\mathcal{I}_{w\hat{g}}$-regular.*

PROOF. Suppose every subset of $\sigma(w)$ is $*$-closed. Then by Lemma 1.12, every subset of X is $\mathcal{I}_{w\hat{g}}$-closed and hence every subset of X is $\mathcal{I}_{w\hat{g}}$-open. If N is a closed set not containing x, then {x} and N are the required disjoint $\mathcal{I}_{w\hat{g}}$-open sets containing x and N respectively. Therefore, $(X,\tau,\mathcal{I})$ is $\mathcal{I}_{w\hat{g}}$-regular. □

The following Example 3.2 shows that the reverse direction of the above Theorem 3.2 is not true.

EXAMPLE 3.2. Consider the real line $\mathcal{R}$ with the usual topology. Let $\mathcal{I}=\{\emptyset\}$. Then $\mathcal{R}$ is regular and hence $\mathcal{I}_{w\hat{g}}$-regular. Let H=(0,1) be an open set and hence H∈σ(w). Then cl*(H)=cl(H)=[0,1]≠H and consequently H is not $\star$-closed. Thus the subset H of $\sigma(w)$ is not $\star$-closed.

References

[1] A. Al-Omari and T. Noiri. A unified theory of generalized closed sets in weak structures. *Acta Math. Hungar.*, **135**(1-2)(2012), 174–183.

[2] J. Antony Rex Rodrigo, O. Ravi and A. Naliniramalatha. $\hat{g}$-closed sets in ideal topological spaces. *Methods of Functional analysis and Topology*, **17**(3)(2011), 274–280.

[3] Á. Császár. Weak Structures. *Acta Math. Hungar.*, **131**(1-2)(2011), 193–195.

[4] J. Dontchev, M. Ganster and T. Noiri. Unified operation approach of generalized closed sets via topological ideals. *Math. Japonica.*, **49**(1999), 395–401.

[5] J. Dontchev, M. Ganster and D. Rose. Ideal resolvability. *Topology and its Applications*, **93**(1999), 1–16.

[6] E. Hayashi. Topologies defined by local properties. *Math. Ann.*, **156**(1964), 205–215.

[7] D. Janković and T. R. Hamlett. New topologies from old via ideals. *Amer. Math. Monthly*, **97**(4)(1990), 295–310.

[8] K. Kuratowski. *Topology, Vol. I*, New York: Academic Press, 1966.

[9] N. Levine. Semi-open sets and semi-continuity in topological spaces. *Amer. Math. Monthly*, **70**(1963), 36–41.

[10] N. Levine. Generalized closed sets in topology. *Rend. Circ. Mat. Palermo*, (2), 19(1970), 89–96.

[11] S. N. Maheshwari and R. Prasad, Some new separation axioms, Ann. Soc. Sci. Bruxelles, 89(1975), 395-402.

[12] H. Maki, R. Devi and K. Balachandran. Generalized α-closed sets in topology. *Bull. Fukuoka Univ. Ed. III*, **42**(1993), 13–21.

[13] H. Maki, R. Devi and K. Balachandran. Associated topologies of generalized α-closed sets and α-generalized closed sets. *Mem. Fac. Sci. Kochi Univ. Math.*, **15**(1994), 51–63.

[14] A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb. On precontinuous and weak precontinuous mappings. *Proc. Math. Phys. Soc. Egypt*, **53**(1982), 47–53.

[15] B. M. Munshi. Separation Axioms. *Acta Ciencia Indica*, **12**(1986), 140–144.

[16] M. Navaneethakrishnan and J. Paulraj Joseph. g-closed sets in ideal topological spaces. *Acta Math. Hungar.*, **119**(4)(2008), 365–371.

[17] M. Navaneethakrishnan, J. Paulraj Joseph and D. Sivaraj. $\mathcal{I}_g$-normal and $\mathcal{I}_g$-regular spaces. *Acta Math Hungar.*, **125**(4)(2009), 327–340.

[18] R. L. Newcomb. *Topologies which are compact modulo an ideal*, Ph.D. Dissertation, University of California, Santa Barbara, California, 1967.

[19] O. Njåstad. On some classes of nearly open sets. *Pacific J. Math.*, **15**(3)(1965), 961–970.

[20] T. Noiri and V. Popa. On g-regular spaces and some functions. *Mem. Fac. Sci. Kochi Univ. Math.*, **20**(1999), 67–74.

[21] T. Noiri. Almost αg-closed functions and separation axioms. *Acta Math. Hungar.*, **82**(3)(1999), 193–205.

[22] N. Palaniappan and K. Chandrasekra Rao. Regular generalized closed sets. *Kyungpook Math. J.*, **33**(2)(1993), 211–219.

[23] O. Ravi, S. Ganesan and S. Chandrasekar. Almost αgs-closed functions and separation axioms. Bull. Math. Anal. App., **3**(1)(2011), 165–177.

[24] V. Renuka Devi, D. Sivaraj and T. Tamizh Chelvam. Codense and completely codense ideals. *Acta Math. Hungar.*, **108**(3)(2005), 197–205.

[25] N. Seenivasagan, O. Ravi, T. Prabakaran, and V. Sangeethasubha. $\hat{g}w$-closed sets in weak structures. *International Journal of Mathematics Trends and Technology*, **58**(3)(2018), 164–175.

[26] N. Seenivasagan, O. Ravi, V. Sangeethasubha and T. Prabakaran. $w\hat{g}$-closed sets in ideal topological spaces, communicated.

[27] M. Sheik John. *A study on generalizations of closed sets and continuous maps in topological and bitopological spaces.* Ph. D Thesis, Bharathiar University, Coimbatore, 2002.

[28] M. K. Singal and A. R. Singal. Mildly normal spaces. *Kyungpook Math. J.*, **13**(1973), 27–31.

[29] R. Staum. The algebra of bounded continuous functions into a nonarchimedean field. *Pacific J. Math.*, **50**(1974), 169–185.

[30] R. Vaidyanathaswamy. *Set topology*. Chelsea Publishing Company, New York, 1946.

Received by editors 09.05.2018; Revised version 28.11.2018; Available online 10.12.2018.

Latha Mathavan Engineering College, Department of Mathematics, Madurai, Tamil Nadu, India

E-mail address: prabhakaran2611@gmail.com

Research Scholar, Department of Mathematics, Bharathidasan University, Trichy, Tamil Nadu, India

E-mail address: sangeethasubha11@gmail.com

Assistant Professor, Department of Mathematics, Govt Arts College for Women, Nilakottai, Tamil Nadu, India

E-mail address: vasagan2000@yahoo.co.in

Controller of Examinations, Madurai Kamaraj University, Madurai, Tamil Nadu, India

E-mail address: siingam@yahoo.com