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COMPLEMENTARY EDGE DOMINATION IN SHADOW DISTANCE GRAPHS

Vijayachandra Kumar U. and Murali. R

Abstract. The shadow graph of connected graph G, denoted $D_2(G)$, is the graph constructed from G by taking two copies of G, say G itself and $G^{'}$ and joining each vertex u in G to the neighbors of the corresponding vertex $u^{'}$ in $G^{'}$. Let D be the set of all distances between distinct pairs of vertices in G and let D_s (called the distance set) be a subset of D. The distance graph of G, denoted by $D(G, D_s)$, is the graph having the same vertex set as that of G and two vertices u and v are adjacent in $D(G, D_s)$ whenever $d(u, v) \in D_s$. In this paper, we determine the complementary edge domination number of the shadow distance graph of the path graph, the cycle graph and the sunlet graph with specified distance sets.

1. Introduction

By a graph $G = (V, E)$ we mean a finite undirected graph without loops and multiple edges. A subset S of V is called a dominating set of G if every vertex not in S is adjacent to some vertex in S. The domination number of G denoted by $\gamma(G)$ is the minimal cardinality taken over all dominating sets of G. A subset F of E is called an edge dominating set if each edge in E is either in F or is adjacent to an edge in F. An edge dominating set F is called minimal if no proper subset of F is an edge dominating set. The

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edge domination number of G denoted by $\gamma^{'}(G)$ is the minimum cardinality taken over all edge dominating sets of G.

Let F be a minimal edge dominating set of G. If $E - F$ contains an edge dominating set, say $F^{'}$ of G, then $F^{'}$ is called a complementary edge dominating set with respect to F. The complementary edge domination number $\gamma_c^{'}(G)$ of G is the minimal complementary edge dominating set (or MCEDS) with minimum cardinality of G [**4**].

The open neighbourhood of an edge $e \in E$ denoted by $N(e)$ is the set of all edges adjacent to e in G. If $e = (u, v)$ is an edge in G, the degree of e denoted by $deg(e)$ is defined as $deg(e) = deg(u) + deg(v) - 2$. The maximum degree of an edge in G is denoted by $\triangle^{'}(G)$.

The shadow graph of G, denoted by $D_2(G)$ is the graph constructed from G by taking two copies of G, namely G itself and $G^{'}$ and by joining each vertex u in G to the neighbors of the corresponding vertex $u^{'}$ in $G^{'}$.

Let D be the set of all distances between distinct pairs of vertices in G and let D_s (called the distance set) be a subset of D. The distance graph of G denoted by $D(G, D_s)$ is the graph having the same vertex set as that of G and two vertices u and v are adjacent in $D(G, D_s)$ whenever $d(u, v) \in D_s$.

The shadow distance graph of G, denoted by $D_{sd}(G, D_s)$ [**7**] is constructed from G with the following conditions:

(1) consider two copies of G say G itself and $G^{'}$
(2) if $u \in V(G)$ (first copy) then we denote the corresponding vertex as $u^{'} \in V(G^{'})$ (second copy)
(3) the vertex set of $D_{sd}(G, D_s)$ is $V(G) \cup V(G^{'})$
(4) the edge set of $D_{sd}(G, D_s)$ is $E(G) \cup E(G^{'}) \cup E_{ds}$ where E_{ds} is the set of all edges (called the shadow distance edges) between two distinct vertices $u \in V(G)$ and $v^{'} \in V(G^{'})$ that satisfy the condition $d(u, v) \in D_s$ in G.

The n-sunlet graph denoted by S_n is the graph on $2n$ vertices obtained by attaching $n-$pendant edges to each of the vertices of the cycle graph C_n.

By P_n, C_n and S_n respectively we mean the path graph, the cycle graph, the n- sunlet graph on n vertices.

2. Preliminaries

We recall the following results related to the complementary edge domination number of a graph and edge domination number of a graph.

Theorem 2.1. ([**4**]) *$\gamma_c^{'}(C_n) = \lceil \frac{n}{3} \rceil$ for $n \geqslant 3$.*

Theorem 2.2. ([**4**]) *$\gamma_c^{'}(P_n) = \lceil \frac{n}{3} \rceil$ for $n \geqslant 3$.*

Theorem 2.3. ([**7**]) *For $n \geqslant 3$, $\gamma^{'}(D_{sd}\{P_n, \{2\}\}) = 2\lceil \frac{n-2}{2} \rceil$.*

THEOREM 2.4. ([**7**])

$$\gamma^{'}(D_{sd}\{P_n,\{3\}\}) = \begin{cases} 3, & n=4 \\ 2\lceil \frac{n-2}{2} \rceil, & n \geqslant 5 \end{cases} \tag{2.1}$$

THEOREM 2.5. ([**7**]) *For* $n \geqslant 4$,

$$\gamma^{'}(D_{sd}\{C_n,\{2\}\}) = \begin{cases} 2\lceil \frac{n-1}{3} \rceil, & \text{if } n \equiv 0 \text{ or } 2 \text{ (mod3)} \\ 2\lceil \frac{n+1}{3} \rceil, & \text{otherwise} \end{cases} \tag{2.2}$$

THEOREM 2.6. ([**7**]) *For* $n \geqslant 6$, $\gamma^{'}(D_{sd}\{C_n,\{3\}\}) = 2\lceil \frac{n-2}{2} \rceil$.

THEOREM 2.7. ([**7**]) *For* $n \geqslant 3$, $\gamma^{'}(D_{sd}\{S_n,\{2\}\}) = 2\lfloor \frac{n+1}{2} \rfloor$.

THEOREM 2.8. ([**7**]) *For* $n \geqslant 3$, $\gamma^{'}(D_{sd}\{S_n,\{3\}\}) = 2\lceil \frac{2n-1}{2} \rceil$.

THEOREM 2.9. ([**5**]) *An edge dominating set F is minimal if and only if for each edge $e \in F$, one of the following two conditions holds:*

(1) $N(e) \cap F = \phi$

(2) *there exists an edge $e_1 \in E - F$ such that $N(e_1) \cap F = \{e\}$.*

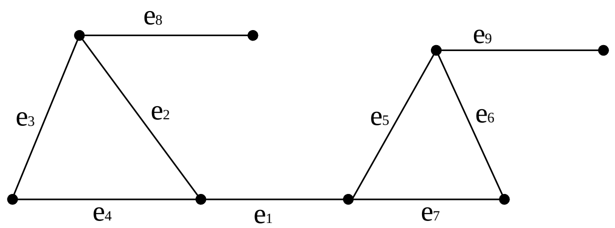

FIGURE 1. $F = \{e_2, e_6\}$ and $F^{'} = \{e_3, e_5\}$.
Hence $\gamma^{'}(G) = 2 = \gamma_c^{'}(G)$

THEOREM 2.10. ([**4**]) *Let F be a minimum edge dominating set of G. If for each $e \in F$, the induced subgraph $< N(e) >$ is a star, then $\gamma_c^{'}(G) = \gamma^{'}(G)$.*

3. The Main Results

We begin our results with the shadow distance associated with the path P_n.

THEOREM 3.1. *Let $n \geqslant 3$. Then* $\gamma_c^{'}(D_{sd}\{P_n,\{2\}\}) = 2\lfloor \frac{n}{2} \rfloor$

Proof. Consider two copies of P_n, one P_n itself and the other denoted by $P_n^{'}$. Let $v_1, v_2, ..., v_n$ be the vertices of P_n and let $v_1^{'}, v_2^{'}, ..., v_n^{'}$ be the vertices of $P_n^{'}$. Let $e_1, e_2, ...e_{n-1}$ be the edges of the first copy P_n and $e_1^{'}, e_2^{'}, ..., e_{n-1}^{'}$ be the edges of the second copy $P_n^{'}$, where $e_i = (v_i, v_{i+1})$, $e_i^{'} = (v_i^{'}, v_{i+1}^{'})$ for $i = 1, 2, ...n-1$. Let $G = (D_{sd}\{P_n, \{2\}\})$. Then $|V(G)|$ $= 2n$, $|E(G)| = 4n-6$ and $E(G) = \{e_i\} \cup \{e_i^{'}\} \cup \{e_{(j),(j+2)'}\} \cup \{e_{(k-2)',(k)}\}$ where $1 \leqslant i \leqslant n-1$, $1 \leqslant j \leqslant n-2$, $3 \leqslant k \leqslant n$.

For $n = 3$, the set $F = \{e_2, e_2^{'}\}$ is the minimal dominating set with minimum cardinality of G. Then under the hypothesis, $F^{'} = \{e_1, e_1^{'}\}$ is a MCEDS with minimum cardinality since $G \cong C_6$ it follows that $\gamma_c^{'}(G) = 2$

For $n = 4$, the set $F = \{e_2, e_2^{'}\}$ is the minimal dominating set with minimum cardinality of G. Then under the hypothesis, $F^{'} = \{e_1, e_3, e_1^{'}, e_3^{'}\}$ is a MCEDS with minimum cardinality and it follows that $\gamma_c^{'}(G) = 4$.

For $n = 5$, the set $F = \{e_2, e_4, e_2^{'}, e_4^{'}\}$ is the minimal dominating set with minimum cardinality of G. Then under the hypothesis, $F^{'} = \{e_1, e_3, e_1^{'}, e_3^{'}\}$ is a MCEDS with minimum cardinality and it follows that $\gamma_c^{'}(G) = 4$.

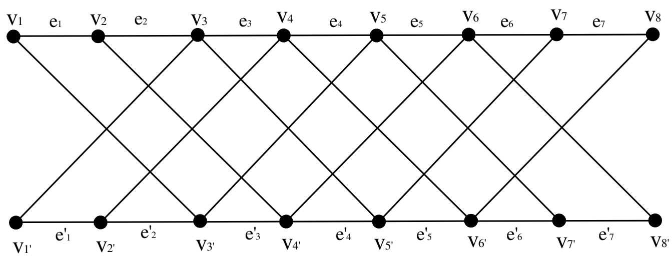

Figure 2. $F = \{e_2, e_4, e_6, e_2^{'}, e_4^{'}, e_6^{'}\}$ and $F^{'} = \{e_1, e_3, e_5, e_7, e_1^{'}, e_3^{'}, e_5^{'}, e_7^{'}\}$ for the path P_8

Let $n \geqslant 6$. Consider the set $F = \{e_2, e_2^{'}, e_4, e_4^{'}, ..., e_{2i+2}, e_{2i+2}^{'}\}$ for each i such that $0 \leqslant i \leqslant \lceil \frac{n-4}{2} \rceil$. Then, clearly, $|F| = 2\lceil \frac{n-2}{2} \rceil$ (Theorem 2.3) This set F is the minimal dominating set with minimum cardinality of G. Then under the hypothesis, the set $F^{'} = \{e_{2j-1}\} \in \{e_{2j-1}^{'}\}$, where $1 \leqslant j \leqslant \lfloor \frac{n}{2} \rfloor$ is a MCEDS with minimum cardinality since for any edge $e_i \in F^{'}$, $F^{'} - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G. But the graph G is such that there are 4 edges of degree 3, 4 edges of degree 4, 8 edges of degree 5 and $2(2n-11)$ edges of degree 6. Hence atmost $2(2n-11)$ distinct edges of G can dominate seven distinct edges including itself and each of the remaining edges can dominate less than 6 edges of G. Hence, any set containing edges less than that of $F^{'}$ cannot be a dominating set of G.

This implies that the set $F^{'}$ described above is of minimum cardinality and since $|F^{'}| = 2\lfloor\frac{n}{2}\rfloor$, it follows that $\gamma_c^{'}(D_{sd}\{P_n, \{2\}\}) = 2\lfloor\frac{n}{2}\rfloor$. □

THEOREM 3.2. *Let* $n \geqslant 5$. *Then* $\gamma_c^{'}(D_{sd}\{P_n, \{3\}\}) = 2\lfloor\frac{n}{2}\rfloor$

PROOF. The vertex set and edge set of G are as in theorem 2.10.

For $n = 5$, the set $F = \{e_2, e_4, e_2^{'}, e_4^{'}\}$ is a minimum edge dominating set of G. Then under the hypothesis, $F^{'} = \{e_1, e_3, e_1^{'}, e_3^{'}\}$ is a MCEDS with minimum cardinality. It follows that $\gamma_c^{'}(G) = 4$.

For $n = 6$, the set $F = \{e_2, e_4, e_2^{'}, e_4^{'}\}$ is a minimal edge dominating set of G. Then under the hypothesis, $F^{'}$ $=\{e_1, e_3, e_5, e_1^{'}, e_3^{'}, e_5^{'}\}$ is a MCEDS with minimum cardinality. It follows that $\gamma_c^{'}(G) = 6$.

For $n = 7$, the set $F = \{e_2, e_4, , e_6, e_2^{'}, e_4^{'}, e_6^{'}\}$ is a minimal edge dominating set of G. Then under the hypothesis, $F^{'}$ $=\{e_1, e_3, e_5, e_1^{'}, e_3^{'}, e_5^{'}\}$ is a MCEDS with minimum cardinality. It follows that $\gamma_c^{'}(G) = 6$.

For $n = 8$, the set $F = \{e_2, e_4, , e_6, e_2^{'}, e_4^{'}, e_6^{'}\}$ is a minimal edge dominating set of G. Then under the hypothesis, $F^{'}$ $=\{e_1, e_3, e_5, e_7, e_1^{'}, e_3^{'}, e_5^{'}, e_7^{'}\}$ is a MCEDS with minimum cardinality. It follows that $\gamma_c^{'}(G) = 8$.

Let $n \geqslant 9$.

Consider the set $F = \{e_2, e_2^{'}, e_4, e_4^{'}, ..., e_{2i+2}, e_{2i+2}^{'}\}$ for each i such that $1 \leqslant i \leqslant \lceil\frac{n-4}{2}\rceil$.

This set F is a minimal edge dominating set with minimum cardinality (Theorem 2.4).

Consider the set $F^{'} = \{e_{2j+2}\} \cup \{e_{2j+2}^{'}\}$, where $1 \leqslant j \leqslant \lfloor\frac{n}{2}\rfloor$

This set $F^{'}$ is a MCEDS with minimum cardinality since for any edge $e_i \in F^{'}$, $F^{'} - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G. But the graph G is such that there are 4 edges of degree 3, 8 edges of degree 4, 12 edges of degree 5 and $2(2n-16)$ edges of degree 6. Hence atmost $2(2n-16)$ distinct edges of G can dominate seven distinct edges including itself and each of the remaining edges can dominate less than 6 edges of G. Hence, any set containing the edges less that in $F^{'}$ cannot be an edge dominating set of G.

This implies that the set $F^{'}$ described above is of minimum cardinality and since $|F^{'}| = 2\ \lfloor\frac{n}{2}\rfloor$, it follows that $\gamma_c^{'}(D_{sd}\{P_n, \{3\}\}) = 2\ \lfloor\frac{n}{2}\rfloor$ □

For the cycle graph C_n, we have the following results.

THEOREM 3.3. *For* $n \geqslant 4$, *Then* $\gamma_c^{'}(D_{sd}\{C_n, \{2\}\}) = 2\ \lceil\frac{n}{3}\rceil$

PROOF. Consider two copies of C_n, one C_n itself and the other denoted by $C_n^{'}$. Let $v_1, v_2, ..., v_n$ be the vertices of C_n and $v_1^{'}, v_2^{'}, ..., v_n^{'}$ be the vertices of $C_n^{'}$. Let $e_1, e_2, ...e_n$ be the edges of the first copy C_n and $e_1^{'}, e_2^{'}, ..., e_n^{'}$ be the edges of the second copy $C_n^{'}$. where $e_i = (v_i, v_{i+1})$ and $e_i^{'} = (v_i^{'}, v_{i+1}^{'})$ for $i = 1, 2, ...n$, where computation is under modulo n. Let $G = (D_{sd}\{C_n, \{2\}\})$.

For $n = 4$, the set $F = \{e_2, e_4, e_2^{'}, e_4^{'}\}$ is the minimal dominating set with minimum cardinality of G. Then under the hypothesis $F^{'} = \{e_1, e_3, e_1^{'}, e_3^{'}\}$ is a MCEDS with minimum cardinality of G.

For $n = 5$, the set $F = \{e_2, e_5, e_2^{'}, e_5^{'}\}$ is the minimal dominating set with minimum cardinality of G. Then under the hypothesis $F^{'} = \{e_1, e_3, e_1^{'}, e_3^{'}\}$ is a MCEDS with minimum cardinality of G.

For $n = 6$, the set $F = \{e_2, e_5, e_2^{'}, e_5^{'}\}$ is the minimal dominating set with minimum cardinality of G. Then under the hypothesis $F^{'} = \{e_1, e_4, e_1^{'}, e_4^{'}\}$ is a MCEDS with minimum cardinality of G.

Let $n \geqslant 7$.

Consider the set

$$F = \begin{cases} \{e_2, e_5, ..., e_{3i+2}, e'_2, e'_5, ...e'_{3i+2}\} & if \text{ n} \equiv 0 \text{ or } 2 \text{ (mod3)} \\ \{e_2, e_5, ..., e_{3i+2}, e'_2, e'_5, ...e'_{3i+2}\} \cup \{e_n, e'_n\} & otherwise \end{cases}$$

where $0 \leqslant i \leqslant \lfloor \frac{n-2}{3} \rfloor$. Clearly, $|F|$=$2\lceil \frac{n-1}{3} \rceil$ for $n \equiv 0$ or $2(mod3)$

This set F is a minimal edge dominating set with minimum cardinality of G. Then under the hypothesis the set

$$F^{'} = \begin{cases} \{e_{3k-2}\} \cup \{e_{n-1}\} & n \equiv 1(mod3) \\ \{e_{3j-2}\} & Otherwise \end{cases}$$

where $1 \leqslant k \leqslant \lfloor \frac{n}{3} \rfloor$, $1 \leqslant j \leqslant \lceil \frac{n}{3} \rceil$ is a MCEDS with minimum cardinality since for any edge $e_i \in F^{'}$, $F^{'} - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G. Hence, any set containing edges less than that of $F^{'}$ cannot be a dominating set of G. Also G is regular of degree 4 and each edge of G is of degree 6 and an edge of G can dominate atmost seven distinct edges of G including itself.

This implies that the set $F^{'}$ described above is of minimum cardinality and since $|F^{'}| = 2 \lceil \frac{n}{3} \rceil$, it follows that $\gamma_c^{'}(D_{sd}\{C_n, \{2\}\}) = 2 \lceil \frac{n}{3} \rceil$ □

THEOREM 3.4. *Let* $n \geqslant 6$. *Then* $\gamma_c^{'}(D_{sd}\{C_n, \{3\}\}) = 2\lceil \frac{n-1}{2} \rceil$.

PROOF. The vertex set and edge set of G are as in Theorem 2.12. Let $G = \gamma_c^{'}(D_{sd}\{C_n, \{3\}\})$

For $n = 6$, the set $F = \{e_2, e_4, e_6, e_2^{'}, e_4^{'}, e_6^{'}\}$ is a minimal edge dominating set with minimum cardinality of G. Then under the hypothesis $F^{'} =$

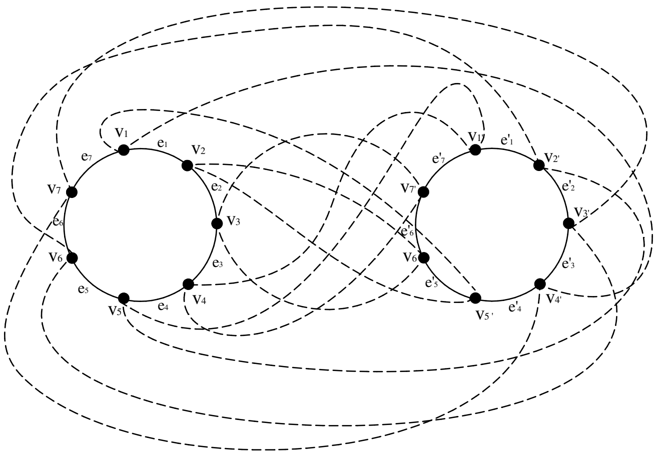

FIGURE 3. $F = \{e_2, e_5, e_7, e_2^{'}, e_5^{'}, e_7^{'}\}$ and $F^{'} = \{e_1, e_4, e_6, e_1^{'}, e_4^{'}, e_6^{'}\}$. for cycle C_7

$\{e_1, e_3, e_5, e_1^{'}, e_3^{'}, e_5^{'}\}$ is a minimal complementary edge dominating set with minimum cardinality and Hence $\gamma_c^{'}(G) = 6$.

For $n = 7$, the set $F = \{e_2, e_5, e_7, e_2^{'}, e_5^{'}, e_7^{'}\}$ is a minimal edge dominating set with minimum cardinality of G. Then under the hypothesis $F^{'} = \{e_1, e_4, e_6, e_1^{'}, e_4^{'}, e_6^{'}\}$ is a is a MCEDS with minimum cardinality of G. Hence $\gamma_c^{'}(G) = 6$.

Let $n \geqslant 8$.

Consider the set

$$F = \begin{cases} \{e_2, e_4, ..., e_{2i}, e_2^{'}, e_4^{'}, ..., e_{2i}^{'}\} \cup \{e_n, e_n^{'}\}, & 1 \leqslant i \leqslant \frac{n}{2}\, (n\, is\, even) \\ \{e_2, e_5, ..., e_{3i+2}, e_2^{'}, e_5^{'}, ..., e_{3i+2}^{'}\} \cup \{e_n, e_n^{'}\}, & 1 \leqslant j \leqslant \lceil \frac{n-1}{3} \rceil\, (n\, is\, odd) \end{cases}$$

(Theorem 2.6)

This set F is a minimal edge dominating set with minimum cardinality of G.

Consider the set $F^{'} =$

$$\begin{cases} \{e_{2j-1}\} \cup \{e_{2j-1}^{'}\}, & 1 \leqslant j \leqslant \frac{n}{2} \quad (n \quad is \quad even) \\ \{e_1\} \cup \{e_{2i+4}\} \cup \{e_1^{'}\} \cup \{e_{2i+2}^{'}\}, & 1 \leqslant i \leqslant \lfloor \frac{n}{3} \rfloor - 1 \quad (\text{n} \quad is \quad odd) \end{cases}$$

This set $F^{'}$ is a MCEDS with minimum cardinality since for any edge $e_i \in F^{'}$, $F^{'} - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G. Hence, any set containing edges less than that of $F^{'}$ cannot be a dominating set of G. Also G is regular of degree 4 and each edge of G is of degree 6 and an edge of G can dominate atmost seven distinct edges of G including itself.

This implies that the set $F^{'}$ described above is of minimum cardinality and since $|F^{'}| = 2\lceil\frac{n-2}{2}\rceil$, it follows that $\gamma_c^{'}(D_{sd}\{C_n, \{3\}\}) = 2\lceil\frac{n-2}{2}\rceil$. □

For the sunlet graph S_n, we have the following results.

THEOREM 3.5. *Let* $n \geqslant 3$. *Then* $\gamma_c^{'}(D_{sd}\{S_n, \{2\}\}) = 2\lfloor\frac{n+1}{2}\rfloor$.

PROOF. Consider two copies of S_n namely S_n itself and $S_n{}'$. In the first copy S_n, let $(v_1)_1, (v_2)_1, ..., (v_n)_1$ be the vertices of the cycle, $(v_1)_1^{'}$, $(v_2)_1^{'}$, ..., $(v_n)_1^{'}$ be the pendant vertices, let the edges of the cycle be $e_i = ((v_1)_i, (v_1)_{i+1})$, $i= 1, 2, ...n$ where computation is under modulo n and let the pendant edges be $e_{p_i} = ((v_i)_1, (v_i)_1^{'})$ where $i= 1, 2, ...n$. In the second copy, let $(v_1)_2, (v_2)_2, ..., (v_n)_2$ be the vertices of the cycle, $(v_1)_2^{'}, (v_2)_2^{'}, ..., (v_n)_2^{'}$ be the pendant vertices, let the edges of the cycle be $e_i^{'} = ((v_2)_i^{'}, (v_2)_{i+1}^{'})$, $i= 1, 2, ...n$ where computation is under modulo n and let the pendant edges be $e_{p_i}^{'} = ((v_i)_2^{'}, (v_i)_2^{'})$ where $i= 1, 2, ...n$. Let G=$(D_{sd}\{S_n, \{2\}\})$.

For $n = 3$, the set F =$\{e_1, e_{p_3}, e_1^{'}, e_{p_3}^{'}\}$ is a minimal edge dominating set with minimum cardinality of G. Then under the hypothesis $F^{'} = \{e_2, e_{p_1}, e_2^{'}, e_{p_1}^{'}\}$ is a MCEDS with minimum cardinality and hence $\gamma_c^{'}(G) = 4$.

Let $n \geqslant 4$

Consider the set F =$\{e_1, e_3, e_5......, e_{2i+1}, e_1^{'}, e_3^{'}, e_5^{'}....., e_{2i+1}^{'}\}$ Where $0 \leqslant i \leqslant \lfloor\frac{n-1}{2}\rfloor$. (Theorem 2.7)

This set F is a minimal edge dominating set with minimum cardinality of G.

Consider the set $F^{'}$ =

$$\begin{cases} \{e_{2j}\} \cup \{e_{2j}^{'}, \} & 1 \leqslant j \leqslant \frac{n}{2} \quad \text{n} \quad is \quad even \\ \{e_{p_1}, e_{p_1}^{'}\} \cup e_{2i}\} \cup \{e_{2i}^{'}\} \cup \{e_{p_1}\}, & 1 \leqslant i \leqslant \lfloor\frac{n}{2}\rfloor \quad \text{n} \quad is \quad odd \end{cases}$$

This set $F^{'}$ is a MCEDS with minimum cardinality since for any edge $e_i \in F^{'}$, $F^{'} - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G. Hence any set containing edges less than that of F cannot be a dominating set of G. Further, $\triangle^{'}(G) = 12$ which implies that an edge of G can dominate atmost 13 distinct edges including itself. But the graph G is such that there are $6n$ edges of degree 8 and $4n$ edges of degree 12. Hence atmost $4n$ distinct edges of G can dominate 13 distinct edges including itself and each of the remaining edges can dominate less than 12 edges of G. Therefore, any set containing the edges less that in $F^{'}$ can not be an edge dominating set of G.

This implies that the set $F^{'}$ described above is of minimum cardinality and since $|F^{'}| = 2\lfloor\frac{n+1}{2}\rfloor$., it follows that $\gamma_c^{'}(D_{sd}\{S_n, \{2\}\}) = 2\lfloor\frac{n+1}{2}\rfloor$. □

THEOREM 3.6. *Let* $n \geqslant 3$. *Then* $\gamma_c^{'}(D_{sd}\{S_n, \{3\}\}) = 2\lceil\frac{2n-1}{2}\rceil$

PROOF. Let G=($D_{sd}\{S_n,\{3\}\}$),The vertex set of G is as in theorem 2.14.

For n= 3, the set $F = \{e_{p_1}, e_{p_2}, e_{p_3}, e^{'}_{p_1}, e^{'}_{p_2}, e^{'}_{p_3}\}$ is a minimal edge dominating set with minimum cardinality of G . Then under the hypothesis $F^{'}$ $= \{e_2, e_3, e_{14'}, e^{'}_2, e^{'}_3, e^{'}_{1'4}\}$ is a MCEDS with minimum cardinality and hence $\gamma^{'}_c(G) = 6$.

For n= 4, the set $F = \{e_{p_1}, e_{p_2}, e_{p_3}, e_{p_4}, e^{'}_{p_1}, e^{'}_{p_2}, e^{'}_{p_3}, e^{'}_{p_4}\}$ is a minimal edge dominating set with minimum cardinality of G. Then under the hypothesis $F^{'} = \{e_1, e_3, e_{14'}, e_{68'}, e^{'}_1, e^{'}_3, e^{'}_{14'}, e^{'}_{6'8}\}$ is a MCEDS with minimum cardinality and hence $\gamma^{'}_c(G) = 8$.

For n= 5, the set $F = \{e_{p_1}, e_{p_2}, e_{p_3}, e_{p_4}, e_{p_5}, e^{'}_{p_1}, e^{'}_{p_2}, e^{'}_{p_3}, e^{'}_{p_4}, e^{'}_{p_5}\}$ is a minimal edge dominating set with minimum cardinality G.

For n= 6, the set $F = \{e_{p_1}, e_{p_2}, e_{p_3}, e_{p_4}, e_{p_5}, , e_{p_6}, e^{'}_{p_1}, e^{'}_{p_2}, e^{'}_{p_3}, e^{'}_{p_4}, e^{'}_{p_5}, e^{'}_{p_6}\}$ is a minimal edge dominating set with minimum cardinality of G.

Let $n \geqslant 7$.

Consider the set F =$\{e_{p_1}, e_{p_2}, e_{p_3}......, e_i, e^{'}_{p_1}, e^{'}_{p_2}, e^{'}_{p_3}....., e^{'}_i\}$.where $1 \leqslant i \leqslant n$. (Theorem 2.8)

This set F is a minimal edge dominating set with minimum cardinality G.

For all $n \geqslant 5$, consider the set $F^{'} = \{e_{14'}\} \cup \{e_{1'4}\} \cup F^{'}_1 \cup F^{'}_2 \cup F^{'}_3 \cup F^{'}_4$, where $F^{'}_1 = \{e_{2j-1}\}$,$F^{'}_2 = \{e^{'}_{2j-1}\}$, $F^{'}_3 = \{e_{(4k+2)(4k+4)'}\}$, $F_4 = \{e_{(4k+2)'(4k+4)}\}$, where $1 \leqslant j \leqslant \lceil \frac{n}{2} \rceil$, $1 \leqslant k \leqslant \lfloor \frac{n}{2} \rfloor - 1$

This set $F^{'}$ is a MCEDS with minimum cardinality since for any edge $e_i \in F^{'}$, $F^{'} - \{e_i\}$ is not an edge dominating set for $N(e_i)$ in G. Hence any set containing edges less than that of F cannot be a dominating set of G. Further, $\triangle^{'}(G) = 12$ which implies that an edge of G can dominate atmost 13 distinct edges including itself. But the graph G is such that there are $2n$ edges of degree 8 , $6n$ edges of degree 10 and $4n$ edges of degree 12. Hence atmost $4n$ distinct edges of G can dominate 13 distinct edges including itself and each of the remaining edges can dominate less than 12 edges of G. Therefore, any set containing the edges less that in $F^{'}$ cannot be an edge dominating set of G. This implies that the set $F^{'}$ described above is of minimum cardinality and therefore $\gamma^{'}_c(D_{sd}\{S_n, \{3\}\}) = 2\lceil \frac{2n-1}{2} \rceil$. □

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References

[1] F Harary. *Graph Theory.* Addison - Wesley Publications, 1969.
[2] S. T. Hedetniemi and R. C. Laskar. Bibliography on domination in graphs and some basic definitions of domination parameters. *Discrete Math.*, **86**(1-2)(1990), 257–277.
[3] S. R. Jayaram. Line domination in graphs. *Graphs and Combinatorics*, **3**(1)(1987), 357–363.
[4] V. R. Kulli and N. D. Soner. Complementary Edge domination in Graphs. *Indian J. pure appl. Math*, **28**(7)(1997), 917–920.
[5] V. R. Kulli. *Theory of domination in graphs*, Vishwa International Publications, 2013.
[6] S. K. Vaidya and R. M. Pandit. Edge domination in some path and cycle related graphs. *ISRN Discrete Mathematics* Volume **2014**, Article ID 975812, 5 pages
[7] U. Vijaya Chandra Kumar and R. Murali. Edge Domination in Shadow distance Graphs. *Int. J. Math. Appl.* **4**(2-D)(2016), 125-130

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Department of Mathematics, REVA University, Bengaluru, India
E-mail address: uvijaychandrakumar@reva.edu.in

Department of Mathematics, Dr. Ambedkar Institute of Technology, Bengaluru,India
E-mail address: muralir2968@gmail.com