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ON FUZZY B* SETS

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ABSTRACT. In this paper, the concepts of fuzzy B^* sets in a fuzzy topological spaces are introduced and studied. Several characterizations of fuzzy B^* sets are established.

1. Introdution

In order to deal with uncertainties, the idea of fuzzy sets, fuzzy set operations was introduced by L. A. Zadeh [**16**] in 1965. In 1968 C. L. Chang [**4**], applied basic concepts of general topology to fuzzy sets and introduced fuzzy topology. The paper of Chang paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. In the recent years, there has been a growing trend among many fuzzy topologists to introduce and study different forms of fuzzy sets. D. K. Ganguly and Chandrani Mitra [**5**] introduced the concept of B* set in classical topology. The purpose of this paper is to introduce and study fuzzy B* sets in fuzzy topological spaces. Several characterizations of fuzzy B* sets are established. The condition under which fuzzy perfectly disconnected spaces become fuzzy weakly Baire spaces and fuzzy Baire spaces are also obtained by means of fuzzy B* sets.

2. Preliminaries

In order to make the exposition self-contained, some basic notions and results used in the sequel are given. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). Let X be a non-empty set and I, the unit interval [0,1]. A fuzzy set λ in X is a function from X into I. The null

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set 0 is the function from X into I which assumes only the value 0 and the whole fuzzy set 1 is the function from X into I which takes 1 only.

Definition 2.1. ([**4**]) Let (X,T) be a fuzzy topological space and λ be any fuzzy set in (X,T). The interior $int(\lambda)$ and the closure $cl(\lambda)$ are defined respectively as follows:

(i). $int(\lambda) = \vee\{\mu/\mu \leqslant \lambda, \mu \in T\}$.

(ii). $cl(\lambda) = \wedge\{\mu/\lambda \leqslant \mu, 1-\mu \in T\}$.

Lemma 2.1 ([**1**]). *For a fuzzy set λ of a fuzzy topological space X,*

(i). $1 - int(\lambda) = cl(1-\lambda)$,

(ii). $1 - cl(\lambda) = int(1-\lambda)$.

Definition 2.2. ([**10**]) A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy dense if there exists no fuzzy closed set μ in (X,T) such that $\lambda < \mu < 1$.

Definition 2.3. ([**10**]) A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X,T) such that $\mu < cl(\lambda)$. That is., $int[cl(\lambda)] = 0$, in (X,T).

Definition 2.4. ([**8**]) Let (X,T) be a fuzzy topological space.A fuzzy set λ defined on X is called a fuzzy somewhere dense set, if $intcl(\lambda) \neq 0$ in (X,T).

Definition 2.5. ([**8**]) A fuzzy set λ in a fuzzy topological space (X,T) is called a fuzzy first category set if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X,T). Any other fuzzy set in (X,T) is said to be of fuzzy second category.

Definition 2.6. ([**9**]) Let λ be a fuzzy first category set in a fuzzy topological space (X,T). Then $1-\lambda$ is called a fuzzy residual set in (X,T).

Definition 2.7. ([**7**]) Let λ be a fuzzy topological space (X,T). A fuzzy set λ in (X,T) is called a fuzzy residual set if $\lambda = \bigwedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are such that $clint(\lambda_i) = 1$ in (X,T).

Definition 2.8. A fuzzy set λ in a fuzzy topological space (X,T) is called a

(i). fuzzy G_δ-set in (X,T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$. ([**2**])

(ii). fuzzy F_σ-set in (X,T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1-\lambda_i \in T$. ([**2**])

(iii). fuzzy regular open set in (X,T), if $intcl(\lambda) = \lambda$ and fuzzy regular closed set in (X,T), if $clint(\lambda) = \lambda$. ([**1**])

(iv). fuzzy pre - open if $\lambda \leqslant intcl(\lambda)$ and fuzzy pre - closed if $clint(\lambda) \leqslant \lambda$. ([**3**])

(v). fuzzy semi G_δ-set in (X,T) if $\lambda = \bigwedge_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy semi open sets in (X,T). ([**7**])

(vi). fuzzy faintly open set if either $\lambda = 0$ or $int(\lambda) \neq 0$ in (X,T). ([**15**])

Definition 2.9. ([**7**]) Let (X,T) be a fuzzy topological space. A fuzzy set λ defined on X is said to have the property of fuzzy Baire, if $\lambda = (\mu \wedge \delta) \vee \eta$, where μ is a fuzzy open set, δ is a fuzzy residual set and η is a fuzzy first category set in (X,T).

Definition 2.10. ([**7**]) Let (X, T) be a fuzzy topological space. A fuzzy set λ defined on X is called a fuzzy Baire set, if $\lambda = (\mu \wedge \delta)$, where μ is a fuzzy open set and δ is a fuzzy residual set in (X, T).

Definition 2.11. A fuzzy topological space (X, T) is called a

(i). fuzzy globally disconnected space if each fuzzy semi-open set in (X, T) is a fuzzy open set. ([**13**])

(ii). fuzzy perfectly disconnected space if for any two non zero fuzzy sets λ and μ defined on X with $\lambda \leqslant 1 - \mu$, $cl(\lambda) \leqslant 1 - cl(\mu)$, in (X, T). ([**12**])

Definition 2.12. A fuzzy topological space (X, T) is called a

(i). fuzzy submaximal space if for each fuzzy set λ in (X, T) such that $cl(\lambda) = 1$, then $\lambda \in T$ in (X, T). ([**2**])

(ii). fuzzy hyperconnected space if every non-null fuzzy open subset of (X, T) is fuzzy dense set in (X, T). ([**6**])

Definition 2.13. ([**8**]) A fuzzy topological space (X, T) is called a fuzzy open hereditarily irresolvable space if $int[cl(\lambda)] \neq 0$, then $int(\lambda) \neq 0$, for any non-zero fuzzy set λ in (X, T).

Definition 2.14. ([**10**]) A fuzzy topological space (X, T) is called a fuzzy first category space if $\vee_{\alpha=1}^{\infty}\{\lambda_\alpha\} = 1_X$, where $\{\lambda_\alpha\}$'s are fuzzy nowhere dense sets in (X,T).A fuzzy topological space which is not of fuzzy first category, is said to be of fuzzy second category.

Definition 2.15. Let (X, T) be a fuzzy topological space. Then (X, T) is called a

(i). fuzzy Baire space if $int[\vee_{i=1}^{\infty}(\lambda_i)] = 0$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X, T). ([**9**])

(ii). fuzzy weakly Baire space if $int\{\bigvee_{i=1}^{\infty}(\mu_i)\} = 0$, where $(\mu_i) = cl(\lambda_i) \wedge [1 - (\lambda_i)]$and (λ_i)'s are fuzzy regular open sets in (X, T). ([**7**])

Definition 2.16. ([**11**]) A fuzzy topological space (X, T) is called a fuzzy resolvable space if there exists a fuzzy dense set λ in (X, T) such that $cl(1-\lambda) = 1$. Otherwise (X, T) is called fuzzy irresolvable.

Theorem 2.1 ([**7**]). *If (X, T) is a fuzzy weakly Baire and fuzzy open hereditarily irresolvable space, then (X, T) is a fuzzy Baire space.*

Theorem 2.2 ([**7**]). *If the fuzzy topological $(X, T)$is a fuzzy Baire space, then (X, T) is a fuzzy second category space.*

Theorem 2.3 ([**15**]). *A fuzzy topological space (X, T) is a fuzzy irresolvable space if and only if every fuzzy dense set is a fuzzy faintly open set in (X, T).*

Theorem 2.4 ([**14**]). *If λ is a fuzzy somewhere dense set in a fuzzy topological space (X, T), then there exits a fuzzy regular closed set η in (X, T) such that $\eta \leqslant cl(\lambda)$.*

Theorem 2.5 ([**14**]). *If λ is a fuzzy somewhere dense set in a fuzzy perfectly disconnected space (X, T), then $cl(\lambda)$ is a fuzzy pre-closed set in (X, T).*

3. Fuzzy B* set

Definition 3.1. Let (X,T) be a fuzzy topological space. A fuzzy set λ defined on X is called a fuzzy B* set, if λ is a fuzzy set with fuzzy Baire property in (X,T) such that $intcl(\lambda) \neq 0$ in (X,T).

Example 3.1. Let $X = \{a,b,c\}$. Then the fuzzy sets α, β, γ, λ and η are defined on X as follows:

$\alpha : X \to [0,1]$ defined as $\alpha(a) = 0.5; \alpha(b) = 0.4; \alpha(c) = 0.6$.

$\beta : X \to [0,1]$ defined as $\beta(a) = 0.6; \beta(b) = 0.5; \beta(c) = 0.7$.

$\gamma : X \to [0,1]$ defined as $\gamma(a) = 0.6; \gamma(b) = 0.6; \gamma(c) = 0.7$.

$\lambda : X \to [0,1]$ defined as $\lambda(a) = 0.5; \lambda(b) = 0.5; \lambda(c) = 0.6$.

$\eta : X \to [0,1]$ defined as $\eta(a) = 0.5; \eta(b) = 0.6; \eta(c) = 0.6$.

Then, $T = \{0, \alpha, \beta, \gamma, 1\}$ is a fuzzy topology on X. On computation, one can see that $cl(\alpha) = 1$; $cl(\beta) = 1$; $cl(\gamma) = 1$; $cl(\lambda) = 1$ and $cl(\eta) = 1$. Also, $cl(1-\lambda) = 1-\alpha$; $cl(1-\beta) = 1-\alpha$, $int(1-\alpha) = int(1-\beta) = int(1-\gamma) = 0$; $int(1-\eta) = 0$; $int(1-\lambda) = 0$, $int(\eta) = \alpha$; $int(\lambda) = \alpha$. Now, $cl[int(\alpha)] = cl(\alpha) = 1$; $cl[int(\beta)] = cl(\beta) = 1$; $cl[int(\gamma)] = cl(\gamma) = 1$; $cl[int(\lambda)] = cl(\alpha) = 1$; $cl[int(\eta)] = cl(\alpha) = 1$. Then $\alpha \wedge \beta \wedge \gamma \wedge \lambda \wedge \eta = \alpha$ and thus α is a fuzzy residual set in (X,T). Also $1-\alpha$ is a fuzzy first category set in (X,T). Now $(\alpha \wedge \alpha) \vee (1-\alpha) = \eta$; $(\beta \wedge \alpha) \vee (1-\alpha) = \eta$; $(\gamma \wedge \alpha) \vee (1-\alpha) = \eta$. Hence η is a fuzzy set with fuzzy Baire property in (X,T). Also $intcl(\eta) \neq 0$,in (X,T), and hence η is the fuzzy B* set in (X,T).

Proposition 3.1. *If λ is a fuzzy B* set in a fuzzy topological space (X,T), then $1-\lambda = (\alpha \vee \beta) \wedge \gamma$, where α is a fuzzy closed set, γ is a fuzzy residual set and β is a fuzzy first category set in (X,T) and $int(1-\lambda)$ is not a fuzzy dense set in (X,T).*

Proof. Let λ be a fuzzy B* set in (X,T). Then

$$1-\lambda = 1-[(\mu \wedge \delta) \vee \eta] = [1-(\mu \wedge \delta)] \wedge (1-\eta) = [(1-\mu) \vee (1-\delta)] \wedge (1-\eta),$$

when $\mu \in T$, η is a fuzzy first category set and δ is a fuzzy residual set in (X,T). Since μ is a fuzzy open set, $1-\mu$ is a fuzzy closed set in (X,T). Also since δ is a fuzzy residual set, $1-\delta$ is a fuzzy first category set and η is a fuzzy first category set, implies that $1-\eta$ is a fuzzy residual set in (X,T). Let $\alpha = 1-\mu$, $\beta = 1-\delta$ and $\gamma = 1-\eta$. Hence $1-\lambda = (\alpha \vee \beta) \wedge \gamma$, where α is a fuzzy closed in (X,T) and β is a fuzzy first category set and γ is a fuzzy residual set in (X,T). Since λ is a fuzzy B* set in (X,T), $intcl(\lambda) \neq 0$ in (X,T). Then $1-intcl(\lambda) \neq 1$ in (X,T) and then $clint(1-\lambda) \neq 1$. Thus $int(1-\lambda)$ is not a fuzzy dense set in (X,T). ☐

Proposition 3.2. *If λ is a fuzzy B* set in a fuzzy topological space (X,T), then $int(\lambda) \neq 0$ in (X,T).*

Proof. Let λ be a fuzzy B* set in (X,T). Then $\lambda = (\mu \wedge \delta) \vee \eta$, where μ is a fuzzy open set, δ is a fuzzy residual set and η is a fuzzy first category set in (X,T). Now,

$$int(\lambda) = int[(\mu \wedge \delta) \vee \eta] \geqslant [int(\mu \wedge \delta)] \vee int(\eta) = \{int(\mu) \wedge int(\delta)\} \vee int(\eta) = [\mu \wedge int(\delta)] \vee int(\eta).$$

Since μ, int(δ) are fuzzy open sets in (X,T), $\mu \wedge int(\delta)$ is also a fuzzy open set and hence $[\mu \wedge int(\delta)] \vee int(\eta)$ is a non zero fuzzy open set in (X,T) and thus $int(\lambda) \neq 0$, in (X,T) $\square$

REMARK 3.1. In view of the proposition 3.2 and Definition 2.8, it is clear that fuzzy B* sets are fuzzy faintly open set in fuzzy topological spaces.

PROPOSITION 3.3. *If λ is a fuzzy B*set in a fuzzy topological space (X,T), then there exists a fuzzy Baire set σ in (X,T) such that $\sigma \leqslant \lambda$, in (X,T).*

PROOF. Let λ be a fuzzy B*set in (X,T). Then, $\lambda = (\mu \wedge \delta) \vee \eta$, where μ is a fuzzy open set, δ is a fuzzy residual set and η is a fuzzy first category set in (X,T). Let $\sigma = \mu \wedge \delta$. Since μ is a fuzzy open set and δ is a fuzzy residual set in (X,T), σ is a fuzzy Baire set in (X,T).Thus, $\lambda = \sigma \vee \eta$ in (X,T). This implies that $\sigma \leqslant \lambda$, in (X,T). $\square$

PROPOSITION 3.4. *If λ is a fuzzy B* set in a fuzzy topological space (X,T), then $\lambda = [\wedge_{i=1}^{\infty}(\mu \wedge \delta_i)] \vee \eta$ where (δ_i)'s are fuzzy sets defined on X such that $cl[int(\delta_i)] = 1$, $\mu \in T$ and η is a fuzzy first category set in (X,T).*

PROOF. Let λ be a fuzzy B* set in (X,T). Then, $\lambda = (\mu \wedge \delta) \vee \eta$, where μ is a fuzzy open set δ is a fuzzy residual set and η is a fuzzy first category set in (X,T). Since δ is a fuzzy residual set in (X,T), $\delta = \wedge_{i=1}^{\infty}(\delta_i)$, where (δ_i)'s are fuzzy sets defined on X such that $cl[int(\delta_i)] = 1$, in (X,T). Then

$$\mu \wedge \delta = \mu \wedge [\wedge_{i=1}^{\infty}(\delta_i)] = \wedge_{i=1}^{\infty}(\mu \wedge \delta_i)$$

and hence $\lambda = [\wedge_{i=1}^{\infty}(\mu \wedge \delta_i)] \vee \eta$, where (δ_i)'s are fuzzy sets defined on X such that $cl[int(\delta_i)] = 1$, $\mu \in T$ and η is a fuzzy first category set in (X,T). $\square$

PROPOSITION 3.5. *If λ is a fuzzy B* set in a fuzzy topological space (X,T), then there exist a fuzzy regular closed set η in (X,T) such that $\eta \leqslant cl(\lambda)$.*

PROOF. Let λ be a fuzzy B* set in (X,T). Then λ is a fuzzy set defined on X with fuzzy Baire property such that $intcl(\lambda) \neq 0$ in (X,T). Now $intcl(\lambda) \neq 0$ in (X,T) implies that λ is a fuzzy somewhere dense set in (X,T). Then by theorem 2.4, there exists a fuzzy regular closed set η in (X,T) such that $\eta \leqslant cl(\lambda)$. $\square$

PROPOSITION 3.6. *If λ is a fuzzy B* set in a fuzzy perfectly disconnected space (X,T), then $cl(\lambda)$ is a fuzzy pre-closed set in (X,T).*

PROOF. Let λ be a fuzzy B* set in (X,T). Then λ is a fuzzy set defined on X with fuzzy Baire property such that $intcl(\lambda) \neq 0$ in (X,T). Now $intcl(\lambda) \neq 0$ in (X,T) implies that λ is a fuzzy somewhere dense set in (X,T). Since (X,T) is a fuzzy perfectly disconnected space, by theorem.2.5, $cl(\lambda)$ is a fuzzy pre-closed set in (X,T). $\square$

PROPOSITION 3.7. *If λ is a fuzzy B* set in a fuzzy hyperconnected space (X,T), then $(1-\lambda)$ is a fuzzy nowhere dense set in (X,T).*

Proof. Let λ be a fuzzy B* set in (X,T). Then, by proposition 3.2, $int(\lambda) \neq 0$ in (X,T). Since (X,T) is a fuzzy hyperconnected space $cl[int(\lambda)] = 1$, for the fuzzy open set $int(\lambda)$ in (X,T). This implies that $1 - clint(\lambda) = 0$ in (X,T), and thus $intcl(1-\lambda) = 0$. Therefore $(1-\lambda)$ is a fuzzy nowhere dense set in (X,T). □

Theorem 3.1 ([**7**]). *If λ is a fuzzy residual set in a fuzzy topological space (X,T), then λ is a fuzzy semi - G_δ set in (X,T).*

Proposition 3.8. *If λ is a fuzzy B* set in a fuzzy topological space (X,T), then $\lambda = (\mu \wedge \delta) \vee \eta$, where μ is a fuzzy open set, η is a fuzzy first category set and δ is a fuzzy semi - G_δ set in (X,T).*

Proof. Let λ be a fuzzy B* set in (X,T). Then, $\lambda = (\mu \wedge \delta) \vee \eta$, where $\mu \in T$, η is fuzzy first category set in (X,T) and δ is a fuzzy residual set in (X,T). By theorem 3.1, the fuzzy residual set δ is a fuzzy semi - G_δ set in (X,T) and hence $\lambda = (\mu \wedge \delta) \vee \eta$, where μ is a fuzzy open set, η is a fuzzy first category set and δ is a fuzzy semi - G_δ set in (X,T). □

Theorem 3.2 ([**7**]). *If λ is a fuzzy Baire set in a fuzzy globally disconnected space (X,T), then $\lambda = \mu \wedge \delta$, where μ is a fuzzy open set and δ is a fuzzy G_δ - set in (X,T).*

Proposition 3.9. *If λ is a fuzzy B* set in a fuzzy globally disconnected space (X,T), then $\mu \wedge \delta \leqslant \lambda$,where μ is a fuzzy open set and σ is a fuzzy G_δ - set in (X,T).*

Proof. Let λ be a fuzzy B* set in (X,T). Then, by proposition 3.3, there exists a fuzzy Baire set σ in (X,T), such that $\sigma \leqslant \lambda$, in (X,T). Since (X,T) is a fuzzy globally disconnected space, by theorem 3.2, for the fuzzy Baire set σ in (X,T), $\sigma = \mu \wedge \delta$, where $\mu \in T$ and δ is a fuzzy G_δ - set in (X,T). Thus $\mu \wedge \delta \leqslant \lambda$, where μ is a fuzzy open set and δ is a fuzzy G_δ - set in (X,T). □

Theorem 3.3 ([**7**]). *If λ is a fuzzy Baire set in a fuzzy submaximal space (X,T), then $\lambda = \mu \wedge \sigma$, where μ is a fuzzy open set and δ is a fuzzy G_δ - set in (X,T).*

Proposition 3.10. *If λ is a fuzzy B* set in a fuzzy submaximal space (X,T), then $\mu \wedge \delta \leqslant \lambda$, where μ is a fuzzy open set and δ is a fuzzy G_δ - set in (X,T).*

Proof. Let λ be a fuzzy B* set in (X,T). Then, by proposition 3.3, there exists a fuzzy Baire set σ in (X,T) such that $\sigma \leqslant \lambda$, in (X,T). Since (X,T) is a fuzzy sub-maximal space, by theorem 3.3, for the fuzzy Baire set σ in (X,T), $\sigma = \mu \wedge \delta$, where μ is a fuzzy open set and δ is a fuzzy G_δ - set in (X,T). Thus $\mu \wedge \delta \leqslant \lambda$, where $\mu \in T$ and δ is a fuzzy G_δ - set in (X,T). □

Theorem 3.4 ([**7**]). *If a fuzzy topological space (X,T) is fuzzy second category (but not a fuzzy Baire) and fuzzy hyperconnected space and λ is a fuzzy residual set in (X,T), then*

(i) λ is a fuzzy nowhere dense set in (X,T).

(ii) $(1-\lambda)$ is a fuzzy dense set in (X,T).

Proposition 3.11. *If λ is a fuzzy B^* set in a fuzzy second category (but not a fuzzy Baire) and fuzzy hyperconnected space (X,T), then $(1-\lambda)$ is not a fuzzy dense set in (X,T).*

Proof. Let λ be a fuzzy B* set in (X,T). Then, $\lambda = (\mu \wedge \delta) \vee \eta$, where μ is a fuzzy open set, δ is a fuzzy residual set and η is a fuzzy first category set in (X,T) and $intcl(\lambda) \neq 0$, in (X,T). Since δ is a fuzzy residual set in the fuzzy second category but not fuzzy Baire space and fuzzy hyperconnected space, δ is a fuzzy nowhere dense set and hence $intcl(\delta) = 0$. But $in(\delta) \leqslant intcl(\delta)$ implies that $int(\delta) = 0$ in (X,T). Now

$$int(\lambda) = int[(\mu \wedge \delta) \vee \eta] \geqslant [int(\mu \wedge \delta)] \vee int(\eta) = [int(\mu) \wedge int(\delta)] \vee int(\eta) = [\mu \wedge 0] \vee in(\eta) = 0 \vee int(\eta) = int(\eta).$$

Then $int(\lambda) \geqslant int(\eta)$ in (X,T) implies that $1 - int(\lambda) \leqslant 1 - int(\eta)$ and thus $cl(1-\lambda) \leqslant cl(1-\eta)$. Since η is a fuzzy first category set in (X,T), $1-\eta$ is a fuzzy residual set in (X,T). Since (X,T) is not fuzzy Baire space $cl(1-\eta) \neq 1$ in (X,T). Thus $cl(1-\lambda) \neq 1$ in (X,T). Hence $1-\lambda$ is not a fuzzy dense set in (X,T). □

4. Fuzzy B^* sets and Fuzzy weakly Baire spaces, Fuzzy Baire spaces

Proposition 4.1. *If $int\{\bigvee_{i=1}^{\infty}([1 - intcl(\lambda_i)] \wedge cl(\lambda_i))\} = 0$, where (λ_i)'s are fuzzy B^* sets in a fuzzy perfectly disconnected space (X,T),then (X,T) is a fuzzy weakly Baire space.*

Proof. Let (λ_i)'s $(i = 1 to \infty)$ be fuzzy B* sets in a fuzzy perfectly disconnected space (X,T). Then by Proposition. 3.6, $[cl(\lambda_i)]$'s are fuzzy pre closed sets in (X,T) and hence $[1 - cl(\lambda_i)]$'s are fuzzy regular open sets in (X,T). Let

$$\mu_i = \{[1 - intcl(\lambda_i)] \wedge cl(\lambda_i)\} = cl[1 - cl(\lambda_i)] \wedge cl(\lambda_i) = cl[1 - cl(\lambda_i)] \wedge [1 - (1 - cl(\lambda_i)].$$

Let $1 - cl(\lambda_i) = \delta_i$ and then (δ_i)'s are fuzzy regular open sets in (X,T). Then $\mu_i = cl(\delta_i) \wedge (1-\delta_i)$, in (X,T). Now $int\{\bigvee_{i=1}^{\infty}([1 - intcl(\lambda_i)] \wedge cl(\lambda_i))\} = 0$, implies that $int\{\bigvee_{i=1}^{\infty}(\mu_i)\} = 0$, where $\mu_i = cl(\delta_i) \wedge (1-\delta_i)$ and (δ_i)'s are fuzzy regular open sets and hence (X,T) is a fuzzy weakly Baire space. □

Proposition 4.2. *If $int\{\bigvee_{i=1}^{\infty}([1 - intcl(\lambda_i)] \wedge cl(\lambda_i))\} = 0$, where (λ_i)'s are fuzzy B^* sets in a fuzzy perfectly disconnected and fuzzy open hereditarily irresolvable space (X,T), then (X,T) is a fuzzy Baire space.*

Proof. Let (X,T) is a fuzzy weakly Baire space.By hypothesis (X,T) is a fuzzy open hereditarily irresolvable space. Then, by Theorem.2.1, (X,T) is a fuzzy Baire space. □

Proposition 4.3. *If $int\{\bigvee_{i=1}^{\infty}([1 - intcl(\lambda_i)] \wedge cl(\lambda_i))\} = 0$, where (λ_i)'s are fuzzy B^* sets in a fuzzy perfectly disconnected and fuzzy open hereditarily irresolvable space (X,T), then (X,T) is a fuzzy second category space.*

Proof. Since each fuzzy Baire is a fuzzy second category space, the proof follows from the Proposition 4.2. and Theorem 2.2. □

Proposition 4.4. *If each fuzzy dense set is a fuzzy B^* set in a fuzzy topological space (X, T),then (X, T) is a fuzzy irresolvable space.*

Proof. Let λ be a fuzzy dense set in (X, T).By hypothesis λ is a fuzzy B* set in (X, T). Since fuzzy B* sets are fuzzy faintly open sets in fuzzy topological spaces [by Remark 3.1]λ is a fuzzy faintly open set in (X, T). Hence the fuzzy dense set λ is a fuzzy faintly open set in (X, T). Then, by Theorem 2.3, (X, T) is a fuzzy irresolvable space. □

Proposition 4.5. *If $cl\{\wedge_{i=1}^{\infty}(\lambda_i)\} = 1$, where (λ_i)'s are fuzzy B^* sets in a fuzzy hyper connected space (X, T), then (X, T) is a fuzzy Baire space.*

Proof. Let $(\lambda_i)(i = 1to\infty)$ be fuzzy B* sets in (X, T). Since (X, T) is a fuzzy hyperconnected space, by proposition 3.7,$(1 - \lambda_i)$'s are fuzzy nowhere dense sets in (X, T). Now $int\{\vee_{i=1}^{\infty}(1 - \lambda_i)\} = int\{1 - \wedge_{i=1}^{\infty}(\lambda_i)\} = 1 - cl\{\wedge_{i=1}^{\infty}(\lambda_i)\} = 1 - 1 = 0$. Hence (X, T) is a fuzzy Baire space. □

Proposition 4.6. *If (X, T) is a fuzzy second category (but not a fuzzy Baire) and fuzzy hyperconnected space (X, T), then (X, T) is a fuzzy irresolvable space.*

Proof. Let λ be a fuzzy B* sets in (X, T). Since (X, T) is a fuzzy hyper connected space, by proposition 3.7, $(1 - \lambda)$ is a fuzzy nowhere dense set in (X, T) and then $incl(1 - \lambda) = 0$ in (X, T). But $int(1 - \lambda) \leqslant intcl(1 - \lambda)$ implies that $int(1 - \lambda) = 0$ and thus $1 - cl(\lambda) = 0$ in (X, T). This implies that $cl(\lambda) = 1$ in (X, T). By proposition 3.11, $1 - \lambda$ is not a fuzzy dense set in (X, T). That is, $cl(1 - \lambda) \neq 1$ in (X, T). Thus $cl(\lambda) = 1$ and $cl(1 - \lambda) \neq 1$ in (X, T) shows that (X, T) is a fuzzy irresolvable space. □

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