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ON PAIRWISE FUZZY SEMI WEAKLY VOLTERRA SPACES

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Abstract. In this paper, by using pairwise fuzzy semi G_δ-sets and pairwise fuzzy semi dense sets, the concept of pairwise fuzzy semi weakly Volterra space is introduced and studied. Example is given for pairwise fuzzy semi weakly Volterra spaces. Several characterizations of pairwise fuzzy semi weakly Volterra spaces are also given in this paper. The inter-relations between pairwise fuzzy semi Volterra and pairwise fuzzy semi weakly Volterra spaces, are also established.

1. Introduction

L. A. Zadeh [**11**] published his first famous research paper on fuzzy sets in 1965. The theory of fuzzy topological spaces was introduced and developed by C. L. Chang [**2**] in 1968. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed. Today fuzzy topology has been firmly established as one of the basic disciplines of fuzzy mathematics. The concept of fuzzy semi-open sets and fuzzy semi-continuous mappings in fuzzy topological spaces was studied by K. K. Azad [**1**]. In 1989, A. Kandil [**7**] introduced the notion of fuzzy bitopological spaces. The concepts of Volterra spaces have been studied extensively in classical topology in [**3, 4, 5, 6**]. The concept of pairwise semi Volterra spaces in fuzzy setting was introduced and studied by the authors in [**9**]. The objective of this paper, we introduce and study the notion of pairwise semi weakly Volterra spaces

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in fuzzy setting. Several characterizations of pairwise fuzzy semi weakly Volterra spaces are also established in this paper.

2. Preliminaries

Now we give some basic notions and results used in the sequel. In this work by (X,T) or simply by X, we will denote a fuzzy topological space due to Chang (1968). By a fuzzy bitopological space (Kandil, 1989) we mean an ordered triple (X,T_1,T_2), where T_1, T_2 are two fuzzy topologies on a non-empty set X.

Definition 2.1. ([**11**]) A fuzzy set λ in a set X is a function from X to $[0,1]$, that is., $\lambda : X \to [0,1]$.

Definition 2.2. ([**8**]) Let (X,τ_1,τ_2) be a fuzzy bitopological space. The (i,j)-semi closure (denoted by (i,j)-scl) and (i,j)-semi interior (denoted by (i,j)-sint) of a fuzzy set A are defined as follows:
(i,j)-$scl(A) = \inf\{B : B \geqslant A : \text{B is (i,j)-fuzzy semi-closed}\}$
(i,j)-$sint(A) = \sup\{B : B \leqslant A, \text{B is (i,j)-fuzzy semi-open}\}$.

Definition 2.3. ([**8**]) A fuzzy set A of a fuzzy bitopological space (X,τ_1,τ_2) is called

(a) (i,j)-fuzzy semi-open if there exists a τ_i-fuzzy open set U such that $U \leqslant A \leqslant \tau_j$-$Cl(U)$.
(b) (i,j)-fuzzy semi-closed if there exists a τ_i-fuzzy closed set F such that τ_j-$Int(F) \leqslant A \leqslant F$.

Definition 2.4. ([**8**]) A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy semi open set if $\lambda \leqslant scl_{T_1} sint_{T_2}(\lambda)$ and $\lambda \leqslant scl_{T_2} sint_{T_1}(\lambda)$ in (X,T_1,T_2).

Definition 2.5. ([**8**]) A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy semi closed set if $sint_{T_1} scl_{T_2}(\lambda) \leqslant \lambda$ and $sint_{T_2} scl_{T_1}(\lambda) \leqslant \lambda$ in (X,T_1,T_2).

Definition 2.6. ([**9**]) A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy semi G_δ-set if $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy semi open sets in (X,T_1,T_2).

Definition 2.7. ([**9**]) A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy semi F_σ-set if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy semi closed sets in (X,T_1,T_2).

Definition 2.8. ([**9**]) A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a pairwise fuzzy semi dense set if $scl_{T_1} scl_{T_2}(\lambda) = 1 = scl_{T_2} scl_{T_1}(\lambda)$ in (X,T_1,T_2).

Definition 2.9. ([**9**]) A fuzzy set λ in a fuzzy bitopological space (X,T_1,T_2) is called a T_i $(i = 1,2)$-fuzzy semi dense set if $scl_{T_1}(\lambda) = 1$ and $scl_{T_2}(\lambda) = 1$ in (X,T_1,T_2).

Definition 2.10. ([**10**]) A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy semi nowhere dense set if $sint_{T_1} scl_{T_2}(\lambda) = 0$ and $sint_{T_2} scl_{T_1}(\lambda) = 0$ in (X, T_1, T_2).

Definition 2.11. ([**10**]) Let (X, T_1, T_2) be a fuzzy bitopological space. A fuzzy set λ in (X, T_1, T_2) is called a pairwise fuzzy semi first category set if $\lambda = \vee_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy semi nowhere dense sets in (X, T_1, T_2). Any other fuzzy set in (X, T_1, T_2) is said to be a pairwise fuzzy semi second category set in (X, T_1, T_2).

Definition 2.12. ([**9**]) If λ is a pairwise fuzzy semi first category set in a fuzzy bitopological space (X, T_1, T_2), then the fuzzy set $1 - \lambda$ is called a pairwise fuzzy semi residual set in (X, T_1, T_2).

Definition 2.13. ([**9**]) A fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy semi σ-nowhere dense set if λ is a pairwise fuzzy semi F_σ-set in (X, T_1, T_2) such that $sint_{T_1} sint_{T_2}(\lambda) = 0$ and $sint_{T_2} sint_{T_1}(\lambda) = 0$.

Definition 2.14. A fuzzy bitopological space (X, T_1, T_2) is said to be a pairwise fuzzy semi Volterra space if $scl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 1$, $(i = 1, 2)$ where (λ_k)'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2).

Theorem 2.1 ([**8**]). *Let A and B be fuzzy sets in a fuzzy bitopological space (X, τ_1, τ_2). Then*

(a) (i, j)-$scl(A) \leqslant i$-$cl(A)$.
(b) (i, j)-$scl(A)$ *is* (i, j)-*fuzzy semi closed.*
(c) A *is* (i, j)-*fuzzy semi closed if and only if* $A = (i, j)$-$scl(A)$.
(d) $A \leqslant B \Rightarrow (i, j)$-$scl(A) \leqslant (i, j)$-$scl(B)$.
(e) τ_i-$int(A) \leqslant (i, j)$-$sint(A)$.
(f) (i, j)-$sint(A)$ *is* (i, j)-*fuzzy semi open.*
(g) A *is* (i, j)-*fuzzy semi open if and only if* $A = (i, j)$-$sint(A)$.
(h) $A \leqslant B \Rightarrow (i, j)$-$sint(A) \leqslant (i, j)$-$sint(B)$.

Theorem 2.2 ([**8**]). *Let (X, τ_1, τ_2) be a fuzzy bitopological space. Then*

(a) A *is* (i, j)-*fuzzy semi open if and only if* $A \leqslant \tau_j$-$cl(\tau_i$-$int(A))$,
(b) F *is* (i, j)-*fuzzy semi closed if and only if* τ_j-$int(\tau_i$-$cl(F)) \leqslant F$.

Theorem 2.3 ([**8**]). *Let (X, τ_1, τ_2) be a fuzzy bitopological space. Then*

(a) *If A is (i, j)-fuzzy semi open and $A \leqslant B \leqslant \tau_j$-$cI(A)$, then B is (i, j)-fuzzy semi open;*
(b) *If A is (i, j)-fuzzy semi closed and τ_j-$int(A) \leqslant B \leqslant A$, then B is (i, j)-fuzzy semi closed.*

Theorem 2.4 ([**10**]). *If λ is a pairwise fuzzy semi nowhere dense set in a fuzzy bitopological space (X, T_1, T_2), then $1 - \lambda$ is a pairwise fuzzy semi dense set in (X, T_1, T_2).*

Theorem 2.5 ([**9**]). *If a pairwise fuzzy semi G_δ-set λ in a fuzzy bitopological space (X, T_1, T_2) such that $scl_{T_i}(\lambda) = 1$, $(i = 1, 2)$, then λ is a pairwise fuzzy semi residual set in (X, T_1, T_2).*

THEOREM 2.6 ([**9**]). *In a fuzzy bitopological space* (X, T_1, T_2)*, a fuzzy set* λ *is a pairwise fuzzy semi* σ*-nowhere dense set in* (X, T_1, T_2) *if and only if* $1 - \lambda$ *is a pairwise fuzzy semi dense and pairwise fuzzy semi* G_δ*-set in* (X, T_1, T_2).

3. Pairwise Fuzzy Semi Weakly Volterra Spaces

DEFINITION 3.1. A fuzzy bitopological space (X, T_1, T_2) is said to be a pairwise fuzzy semi weakly Volterra space if $\wedge_{k=1}^{N}(\lambda_k) \neq 0$, where (λ_k)'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2).

EXAMPLE 3.1. Let $X = \{a, b, c\}$. The fuzzy sets λ, μ, ν and γ are defined on X as follows:

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 1,\ \lambda(b) = 0.2,\ \lambda(c) = 0.7$;
$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.3,\ \mu(b) = 1,\ \mu(c) = 0.2$;
$\nu : X \to [0, 1]$ is defined as $\nu(a) = 0.7,\ \nu(b) = 0.4, \nu(c) = 1$;
$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.2,\ \gamma(b) = 1,\ \gamma(c) = 0.4$.

Then, clearly $T_1 = \{0, \lambda, \mu, \nu, \lambda \vee \mu, \lambda \vee \nu, \mu \vee \nu, \lambda \wedge \mu, \lambda \wedge \nu, \mu \wedge \nu, \lambda \vee (\mu \wedge \nu), \mu \vee (\lambda \wedge \nu), \nu \wedge (\lambda \vee \mu), \lambda \vee \mu \vee \nu, 1\}$ and $T_2 = \{0, \lambda, \gamma, \nu, \lambda \vee \gamma, \lambda \vee \nu, \gamma \vee \nu, \lambda \wedge \gamma, \lambda \wedge \nu, \gamma \wedge \nu, \lambda \vee (\gamma \wedge \nu), \gamma \vee (\lambda \wedge \nu), \nu \wedge (\lambda \vee \gamma), \lambda \vee \gamma \vee \nu, 1\}$ are fuzzy topologies on X. The fuzzy sets $\lambda, \nu, \lambda \vee \mu, \lambda \vee \gamma, \lambda \vee \nu, \mu \vee \nu, \nu \vee \gamma, \lambda \wedge \nu, \lambda \vee (\mu \wedge \nu), \lambda \vee (\gamma \wedge \nu), \mu \vee (\lambda \wedge \nu), \gamma \vee (\lambda \wedge \nu), \nu \wedge (\lambda \vee \mu), \nu \wedge (\lambda \vee \gamma), \lambda \vee \mu \vee \nu, \lambda \vee \gamma \vee \nu, 1$ are pairwise fuzzy semi open sets in (X, T_1, T_2).

Now $\alpha = \lambda \wedge (\lambda \vee \mu) \wedge (\lambda \vee \nu) \wedge [\lambda \vee (\mu \wedge \nu)] \wedge [\mu \vee (\lambda \wedge \nu)] \wedge [\nu \wedge (\lambda \wedge \nu)]$, $\beta = \nu \wedge (\lambda \vee \nu) \wedge [\lambda \vee (\mu \wedge \nu)] \wedge [\nu \wedge (\lambda \wedge \nu)]$ and $\delta = \lambda \wedge (\lambda \vee \mu) \wedge (\lambda \vee \nu) \wedge [\lambda \vee (\mu \wedge \nu)] \wedge (\lambda \vee \mu \vee \nu)$ are pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Also, $scl_{T_1} scl_{T_2}(\alpha) = scl_{T_1}(\alpha) = 1$, $scl_{T_1} scl_{T_2}(\beta) = scl_{T_1}(\beta) = 1$ and $scl_{T_1} scl_{T_2}(\delta) = scl_{T_1}(\delta) = 1$. Then α, β and δ are pairwise fuzzy semi dense sets in (X, T_1, T_2). Hence α, β and δ are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Now $\alpha \wedge \beta \wedge \delta = \lambda \wedge \nu \neq 0$. This implies that the fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy semi weakly Volterra space.

4. Characterizations of pairwise fuzzy semi weakly Volterra spaces

PROPOSITION 4.1. *If* λ *is a pairwise fuzzy semi* G_δ*-set such that* $scl_{T_i}(\lambda) = 1$, $(i = 1, 2)$ *in a fuzzy bitopological space* (X, T_1, T_2)*, then* λ *is a pairwise fuzzy semi residual set in* (X, T_1, T_2).

PROOF. Let λ be a pairwise fuzzy semi G_δ-set such that $scl_{T_i}(\lambda) = 1,\ (i = 1, 2)$ in (X, T_1, T_2). Then by theorem 2.5, $1 - \lambda$ is a pairwise fuzzy semi first category set in (X, T_1, T_2) and hence $1 - (1 - \lambda)$ is a pairwise fuzzy semi residual set in (X, T_1, T_2). That is., λ is a pairwise fuzzy semi residual set in (X, T_1, T_2).

PROPOSITION 4.2. *If* $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$*, where the fuzzy sets* (λ_k)*'s are pairwise fuzzy semi residual sets in a fuzzy bitopological space* (X, T_1, T_2)*, then* (X, T_1, T_2) *is a pairwise fuzzy semi weakly Volterra space.*

PROOF. Let (λ_k)'s $(k = 1$ to $\infty)$ be pairwise fuzzy semi residual sets in (X, T_1, T_2) such that $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$. Let (μ_j)'s $(j = 1$ to $N)$ be pairwise fuzzy semi

G_δ-sets such that $scl_{T_i}(\mu_j) = 1,\ (i = 1, 2)$ in (X, T_1, T_2). Then by proposition 4.1, (μ_j)'s are pairwise fuzzy semi residual sets in (X, T_1, T_2). Let us take the first N pairwise fuzzy semi residual sets in (λ_k)'s as (μ_j)'s. Then $\wedge_{k=1}^{\infty}(\lambda_k) \leqslant \wedge_{j=1}^{N}(\mu_j)$. But $\wedge_{k=1}^{\infty}(\lambda_k) \neq 0$ implies that $\wedge_{j=1}^{N}(\mu_j) \neq 0$. Now $scl_{T_1}(\mu_j) = 1$ and $scl_{T_2}(\mu_j) = 1$, implies that $scl_{T_1} scl_{T_2}(\mu_j) = 1 = scl_{T_2} scl_{T_1}(\mu_j)$. Thus, (μ_j)'s are pairwise fuzzy semi dense sets in (X, T_1, T_2). Hence $\wedge_{j=1}^{N}(\mu_j) \neq 0$, where (μ_j)'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Therefore (X, T_1, T_2) is a pairwise fuzzy semi weakly Volterra space.

Proposition 4.3. *A fuzzy bitopological space (X, T_1, T_2) is a pairwise fuzzy semi weakly Volterra space if and only if $\vee_{k=1}^{N}(\mu_k) \neq 1$, where (μ_k)'s are pairwise fuzzy semi σ-nowhere dense sets in (X, T_1, T_2).*

Proof. Let (X, T_1, T_2) be a pairwise fuzzy semi weakly Volterra space. Then $\wedge_{k=1}^{N}(\lambda_k) \neq 0$, where (λ_k)'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Now $1 - \wedge_{k=1}^{N}(\lambda_k) \neq 1$. This implies that $\vee_{k=1}^{N}(1 - \lambda_k) \neq 1$. Since (λ_k)'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2) and by theorem 2.6, $(1 - \lambda_k)$'s are pairwise fuzzy semi σ-nowhere dense sets in (X, T_1, T_2). Let $\mu_k = 1 - \lambda_k$. Then $\vee_{k=1}^{N}(\mu_k) \neq 1$, where (μ_k)'s are pairwise fuzzy semi σ-nowhere dense sets in (X, T_1, T_2).

Conversely, let $\vee_{k=1}^{N}(\mu_k) \neq 1$, where (μ_k)'s are pairwise fuzzy semi σ-nowhere dense sets in (X, T_1, T_2). Now $1 - \vee_{k=1}^{N}(\mu_k) \neq 0$. Then $\wedge_{k=1}^{N}(1 - \mu_k) \neq 0$. Since (μ_k)'s are pairwise fuzzy semi σ-nowhere dense sets in (X, T_1, T_2) and by theorem 2.6, $(1 - \mu_k)$'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Hence $\wedge_{k=1}^{N}(1 - \mu_k) \neq 0$, where $(1 - \mu_k)$'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Therefore (X, T_1, T_2) is a pairwise fuzzy semi weakly Volterra space.

Proposition 4.4. *If $\vee_{k=1}^{N}(\lambda_k) = 1$, where (λ_k)'s are pairwise fuzzy semi F_σ-sets in a pairwise fuzzy semi weakly Volterra space (X, T_1, T_2), then $sint_{T_1} sint_{T_2}(\lambda_k) \neq 0$ and $sint_{T_1} sint_{T_2}(\lambda) = 0$ and $sint_{T_2} sint_{T_1}(\lambda) = 0$ for at least one λ_k.*

Proof. Suppose that $sint_{T_1} sint_{T_2}(\lambda_k) = 0$ and $sint_{T_2} sint_{T_1}(\lambda_k) = 0$ for all pairwise fuzzy semi F_σ-sets (λ_k), $(k = 1$ to $N)$ in a pairwise fuzzy semi weakly Volterra space (X, T_1, T_2) such that $\vee_{k=1}^{N}(\lambda_k) = 1$. Then $1 - sint_{T_1} sint_{T_2}(\lambda_k) = 1$ and $1 - sint_{T_2} sint_{T_1}(\lambda_k) = 1$. Then $scl_{T_1} scl_{T_2}(1 - \lambda_k) = scl_{T_2} scl_{T_1}(1 - \lambda_k) = 1$. Hence $(1 - \lambda_k)$'s are pairwise fuzzy semi dense sets in (X, T_1, T_2). Since (λ_k)'s $(k = 1$ to $N)$ are pairwise fuzzy semi F_σ-sets in (X, T_1, T_2), $(1 - \lambda_k)$'s are pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Now consider $\wedge_{k=1}^{N}(1 - \lambda_k) = 1 - \vee_{k=1}^{N}(\lambda_k) = 1 - 1 = 0$. Hence $\wedge_{k=1}^{N}(1 - \lambda_k) = 0$, where $(1 - \lambda_k)$'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). But this is a contradiction to (X, T_1, T_2) being a pairwise fuzzy semi weakly Volterra space. Hence it must be $sint_{T_1} sint_{T_2}(\lambda_k) \neq 0$ and $sint_{T_2} sint_{T_1}(\lambda_k) \neq 0$ for at least one λ_k.

Proposition 4.5. *If $\vee_{k=1}^{N}(\lambda_k) = 1$, where (λ_k)'s are pairwise fuzzy semi F_σ-sets in a fuzzy bitopological space (X, T_1, T_2) such that $sint_{T_1} sint_{T_2}(\lambda_k) \neq 0$ and*

$sint_{T_2} sint_{T_1}(\lambda_k) \neq 0$ for at least one λ_k, then (X, T_1, T_2) is a pairwise fuzzy semi weakly Volterra space.

PROOF. Suppose that $\wedge_{k=1}^{N}(\mu_k) = 0$, where (μ_k)'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Then $1 - \wedge_{k=1}^{N}(\mu_k) = 1$, implies that $\vee_{k=1}^{N}(1 - \mu_k) = 1$. Since (μ_k)'s are pairwise fuzzy semi G_δ-sets in (X, T_1, T_2), $(1 - \mu_k)$'s are pairwise fuzzy semi F_σ-sets in (X, T_1, T_2). Since (μ_k)'s are pairwise fuzzy semi dense sets, $scl_{T_1} scl_{T_2}(\mu_k) = 1 = scl_{T_2} scl_{T_1}(\mu_k)$. Then $sint_{T_1} sint_{T_2}(1 - \mu_k) = 0$ and $sint_{T_2} sint_{T_1}(1 - \mu_k) = 0$ for all $k = 1$ to N. Let $1 - \mu_k = \lambda_k$. Thus, for the pairwise fuzzy semi F_σ-sets λ_k in (X, T_1, T_2), $\vee_{k=1}^{N}(\lambda_k) = 1$ and $sint_{T_1} sint_{T_2}(\lambda_k) = 0$ and $sint_{T_2} sint_{T_1}(\lambda_k) = 0$ for all $k = 1$ to N. But this is a contradiction to the hypothesis. Hence we must have $\wedge_{k=1}^{N}(\mu_k) \neq 0$, where (μ_k)'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Therefore, (X, T_1, T_2) is a pairwise fuzzy semi weakly Volterra space.

DEFINITION 4.1. A non-zero fuzzy set λ in a fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy semi somewhere dense set if $sint_{T_1} scl_{T_2}(\lambda) \neq 0$ and $sint_{T_2} scl_{T_1}(\lambda) \neq 0$.

PROPOSITION 4.6. *If $\wedge_{k=1}^{N}(\lambda_k)$ is a pairwise fuzzy semi somewhere dense set in (X, T_1, T_2) where (λ_k)'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2), then (X, T_1, T_2) is a pairwise fuzzy semi weakly Volterra space.*

PROOF. Let $\wedge_{k=1}^{N}(\lambda_k)$ be a pairwise fuzzy semi somewhere dense set in (X, T_1, T_2). Then $sint_{T_1} scl_{T_2}\left(\wedge_{k=1}^{N}(\lambda_k)\right) \neq 0$ and $sint_{T_2} scl_{T_1}\left(\wedge_{k=1}^{N}(\lambda_k)\right) \neq 0$. Suppose that $\wedge_{k=1}^{N}(\lambda_k) = 0$, where (λ_k)'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Then $sint_{T_1} scl_{T_2}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 0$ and $sint_{T_2} scl_{T_1}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 0$, shows that $\wedge_{k=1}^{N}(\lambda_k)$ is a pairwise fuzzy semi nowhere dense set in (X, T_1, T_2). But this is a contradiction to $\wedge_{k=1}^{N}(\lambda_k)$ being a pairwise fuzzy semi somewhere dense set in (X, T_1, T_2). Hence it must be $\wedge_{k=1}^{N}(\lambda_k) \neq 0$, where (λ_k)'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Therefore (X, T_1, T_2) is a pairwise fuzzy semi weakly Volterra space.

PROPOSITION 4.7. *If (λ_k)'s ($k = 1$ to N) are pairwise fuzzy semi G_δ-sets such that $scl_{T_i}(\lambda_k) = 1$, $(i = 1, 2)$ in a pairwise fuzzy semi weakly Volterra space (X, T_1, T_2), then $\vee_{k=1}^{N}(1 - \lambda_k) \neq 1$.*

PROOF. Let (λ_k)'s ($k = 1$ to N) be pairwise fuzzy semi G_δ-sets such that $scl_{T_i}(\lambda_k) = 1$, $(i = 1, 2)$. Now $scl_{T_1}(\lambda_k) = 1$ and $scl_{T_2}(\lambda_k) = 1$ implies that $scl_{T_1} scl_{T_2}(\lambda_k) = 1 = scl_{T_1} scl_{T_2}(\lambda_k)$. Hence (λ_k)'s are pairwise fuzzy semi dense sets in (X, T_1, T_2). Suppose that $\vee_{k=1}^{N}(1 - \lambda_k) = 1$. Then $1 - \wedge_{k=1}^{N}(\lambda_k) = 1$. This will imply that $\wedge_{k=1}^{N}(\lambda_k) = 0$, a contradiction to (X, T_1, T_2) being a pairwise fuzzy semi weakly Volterra space in which $\wedge_{k=1}^{N}(\lambda_k) \neq 0$ for the pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets (λ_k)'s in (X, T_1, T_2). Therefore we must have $\vee_{k=1}^{N}(1 - \lambda_k) \neq 1$ for the pairwise fuzzy semi G_δ-sets (λ_k)'s with $scl_{T_i}(\lambda_k) = 1$, $(i = 1, 2)$.

Proposition 4.8. *If* $\vee_{k=1}^{N}(\lambda_k) \neq 1$, *where* (λ_k)*'s are pairwise fuzzy semi nowhere dense and pairwise fuzzy semi* F_σ*-sets in a fuzzy bitopological space* (X, T_1, T_2), *then* (X, T_1, T_2) *is a pairwise fuzzy semi weakly Volterra space.*

Proof. Let (λ_k)'s ($k = 1$ to N) be pairwise fuzzy semi nowhere dense and pairwise fuzzy semi F_σ-sets in (X, T_1, T_2) such that $\vee_{k=1}^{N}(\lambda_k) \neq 1$. Then $1 - \vee_{k=1}^{N}(\lambda_k) \neq 0$. This implies that $\wedge_{k=1}^{N}(1 - \lambda_k) \neq 0$. Since (λ_k)'s are pairwise fuzzy semi F_σ-sets in (X, T_1, T_2), $(1 - \lambda_k)$'s are pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Since (λ_k)'s are pairwise fuzzy semi nowhere dense sets in (X, T_1, T_2) and by theorem 2.4, $(1 - \lambda_k)$'s are pairwise fuzzy semi dense sets in (X, T_1, T_2). Hence $\wedge_{k=1}^{N}(1 - \lambda_k) \neq 0$, where $(1 - \lambda_k)$'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Therefore (X, T_1, T_2) is a pairwise fuzzy semi weakly Volterra space.

5. Inter-relations between pairwise fuzzy Volterra and pairwise fuzzy weakly Volterra spaces

Proposition 5.1. *If a fuzzy bitopological space* (X, T_1, T_2) *is a pairwise fuzzy semi Volterra space, then* (X, T_1, T_2) *is a pairwise fuzzy semi weakly Volterra space.*

Proof. Let (X, T_1, T_2) be a pairwise fuzzy semi Volterra space. Let (λ_k)'s ($k = 1$ to N) be pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Suppose that $\wedge_{k=1}^{N}(\lambda_k) = 0$. Then $scl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 0$, $(i = 1, 2)$, a contradiction to the hypothesis that (X, T_1, T_2) being a pairwise fuzzy semi Volterra space in which $scl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 1$, $(i = 1, 2)$ for the pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets (λ_k)'s in (X, T_1, T_2). Thus we must have $\wedge_{k=1}^{N}(\lambda_k) \neq 0$ in (X, T_1, T_2). Hence (X, T_1, T_2) is a pairwise fuzzy semi weakly Volterra space.

Remark 5.1. The converse of the above proposition does not hold. That is., a pairwise fuzzy semi weakly Volterra space need not be a pairwise fuzzy semi Volterra space. For, in example 3.1, $scl_{T_1}(\alpha \wedge \beta \wedge \delta) = 1 - (\mu \wedge \nu) \neq 1$ and $scl_{T_2}(\alpha \wedge \beta \wedge \delta) = 1$. That is., $scl_{T_i}(\alpha \wedge \beta \wedge \delta) \neq 1$, $(i = 1, 2)$ where α, β and δ are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in (X, T_1, T_2). Hence (X, T_1, T_2) is not a pairwise fuzzy semi Volterra space whereas (X, T_1, T_2) is a pairwise fuzzy semi weakly Volterra space.

Definition 5.1. A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy semi P-space if every non-zero pairwise fuzzy semi G_δ-set in (X, T_1, T_2) is a pairwise fuzzy semi open set in (X, T_1, T_2). That is., if (X, T_1, T_2) is a pairwise fuzzy semi P-space if $\lambda \in T_i$, $(i = 1, 2)$ for $\lambda = \wedge_{k=1}^{\infty}(\lambda_k)$, where (λ_k)'s are pairwise fuzzy semi open sets in (X, T_1, T_2).

Definition 5.2. A fuzzy bitopological space (X, T_1, T_2) is called a pairwise fuzzy semi hyperconnected space if λ is a pairwise fuzzy semi open set in (X, T_1, T_2), then $scl_{T_i}(\lambda) = 1$, $(i = 1, 2)$.

Proposition 5.2. *If a pairwise fuzzy semi weakly Volterra space* (X, T_1, T_2) *is a pairwise fuzzy semi* P*-space and pairwise fuzzy semi hyperconnected space, then* (X, T_1, T_2) *is a pairwise fuzzy semi Volterra space.*

PROOF. Let (λ_k)'s ($k = 1$ to N) be pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-sets in a pairwise fuzzy semi weakly Volterra space (X, T_1, T_2). Then $\wedge_{k=1}^{N}(\lambda_k) \neq 0$. Since (X, T_1, T_2) is a pairwise fuzzy semi P-space, the pairwise fuzzy semi G_δ-sets (λ_k)'s are pairwise fuzzy semi open sets in (X, T_1, T_2) and hence $\wedge_{k=1}^{N}(\lambda_k)$ is a pairwise fuzzy semi open set in (X, T_1, T_2). Also, since (X, T_1, T_2) is a pairwise fuzzy semi hyperconnected space and $\wedge_{k=1}^{N}(\lambda_k)$ is a non-zero pairwise fuzzy semi open set in (X, T_1, T_2), $scl_{T_i}\left(\wedge_{k=1}^{N}(\lambda_k)\right) = 1$, $(i = 1, 2)$ where (λ_k)'s are pairwise fuzzy semi dense and pairwise fuzzy semi G_δ-set in (X, T_1, T_2). Hence (X, T_1, T_2) is a pairwise fuzzy semi Volterra space.

REMARK 5.2. Inter-relations between pairwise fuzzy semi Volterra and pairwise fuzzy semi weakly Volterra spaces, can be summarized as follows:

Pairwise fuzzy semi Volterra space	$\rightarrow$ $\not\leftarrow$	**Pairwise fuzzy semi weakly Volterra space**

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