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PROPERTIES OF Q-INTUITIONISTIC L-FUZZY NORMAL SUBSEMIRING OF A SEMIRING

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Abstract. In this paper, we made an attempt to study the algebraic nature of Q-intuitionistic L-fuzzy normal subsemiring of a semiring.

1. Introduction

After the introduction of fuzzy sets by L. A. Zadeh [**26**], several researchers explored on the generalization of the concept of fuzzy sets. The concept of intuitionistic fuzzy subset was introduced by K. T. Atanassov [**4, 5**], as a generalization of the notion of fuzzy set. The notion of fuzzy subnearrings and ideals was introduced by S. Abou Zaid [**1**]. A. Solairaju and R. Nagarajan [**22, 23**] have introduced and defined a new algebraic structure called Q-fuzzy subgroups.

In this paper, we introduce the some theorems in Q-intuitionistic L-fuzzy normal subsemiring of a semiring and established some results.

2. Preliminaries

We start with the necessary definitions.

Definition 2.1 ([**22, 23**]). Let X be a non-empty set and $L = (L, \leqslant)$ be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A (Q, L)-fuzzy subset A of X is a function $A : X \times Q \to L$.

Definition 2.2 ([**18**]). Let $(R, +, \bullet)$ be a semiring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a (Q, L)-fuzzy subsemiring (QLFSSR) of R if the following conditions are satisfied:

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(i) $A(x+y,q) \geqslant A(x,q) \wedge A(y,q)$,

(ii) $A(xy,q) \geqslant A(x,q) \wedge A(y,q)$, for all x and y in R and q in Q.

EXAMPLE 2.1. Let $(N,+,\bullet)$ be a semiring and $Q=\{p\}$. Then the (Q,L)-Fuzzy Set A of N is defined by

$$A(x) = \begin{cases} 0.82 & \text{if x is even} \\ 0.18 & \text{if x is odd.} \end{cases}$$

Clearly A is an (Q,L)-Fuzzy subsemiring.

DEFINITION 2.3 ([**4, 5**]). An intuitionistic fuzzy subset(IFS) A in X is defined as an object of the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$, where $\mu_A : X\to[0,1]$ and $\nu_A : X\to[0,1]$ define the degree of membership and the degree of non-membership of the element x in X respectively and for every x in X,

$$0\leqslant \mu_A(x)+\nu_A(x)\leqslant 1.$$

DEFINITION 2.4 ([**20**]). Let $(L,\leqslant)$ be a complete lattice with an involutive order reversing operation $N : L\to L$ and Q a non-empty set. A Q-intuitionistic L-fuzzy subset (QLIFS) A of a set X is defined as an object of the form

$$A=\{\langle (x,q)\mu_A(x,q),\nu_A(x,q)\rangle : x\in X \text{and} q\in Q\},$$

where $\mu_A : X\times Q\to L$ and $\nu_A : X\times Q\to L$ define the degree of membership and the degree of non-membership of the element $x\in X$ respectively, and for every $x\in X$ satisfying $\mu_A(x)\leqslant N(\nu_A(x))$.

DEFINITION 2.5. Let A and B be any two Q-intuitionistic L-fuzzy subsets of a set X. We define the following operations.

(i) $A\cap B=\{\langle (x,q),\mu_A(x,q)\wedge\mu_B(x,q),\nu_A(x,q)\vee\nu_B(x,q)\rangle : x\in X$ and $q\in Q\}$.

(ii) $A\cup B=\{\langle (x,q),\mu_A(x,q)\vee\mu_B(x,q),\nu_A(x,q)\wedge\nu_B(x,q)\rangle : x\in X$ and $q\in Q\}$.

(iii) $\Box A=\{\langle (x,q),\mu_A(x,q),1-\nu_A(x,q)\rangle : x\in X$ and $q\in Q\}$.

(iv) $\Diamond A=\{\langle (x,q),1-\nu_A(x,q),\nu_A(x,q)\rangle : x\in X\}$, for all $x\in X$ and $q\in Q$.

DEFINITION 2.6. Let R be a semiring. A Q-intuitionistic L-fuzzy subset A of R is said to be a Q-intuitionistic L-fuzzy subsemiring (QILFSSR) of R if it satisfies the following conditions:

(i) $\mu_A(x+y,q)\geqslant\mu_A(x,q)\wedge\mu_A(y,q)$

(ii) $\mu_A(xy,q)\geqslant\mu_A(x,q)\wedge\mu_A(y,q)$

(iii) $\nu_A(x+y,q)\leqslant\nu_A(x,q)\vee\nu_A(y,q)$ *(iv)* $\nu_A(xy,q)\leqslant\nu_A(x,q)\vee\nu_A(y,q)$,

for all x and $y\in R$ and $q\in Q$.

EXAMPLE 2.2. Let L be the complete lattice and $A : Z\to L$ be an Q-intuitionistic L-fuzzy subset $A=\{<(x,q),\mu_A(x,q),\nu_A(x,q)>\,|\,x\in X, q\in Q\}$ defined as

$$\mu_A(x,q)=\begin{cases} 0.82 & \text{if } x=<4>, \\ 0.51 & \text{if } x\in<2>-<4>, \\ 0 & \text{if otherwise.} \end{cases} \text{ and } \nu_A(x,q)=\begin{cases} 0.33 & \text{if } x=<4>, \\ 0.72 & \text{if } x\in<2>-<4>, \\ 1 & \text{if otherwise.} \end{cases}$$

Clearly A is an Q-Intuitionistic L-fuzzy normal subsemiring.

Definition 2.7. Let A and B be any two Q-intuitionistic L-fuzzy normal subsemiring of a semiring G and H, respectively. The product of A and B, denoted by $A\times B$, is defined as $A\times B = \{< (x,y),q), \mu_{A\times B}((x,y),q), \nu_{A\times B}((x,y),q) > /$for all x in G and y in H and q in Q $\}$, where $\mu_{A\times B}((x,y),q) = \mu_A(x,q)\wedge\mu_B(y,q)$ and $\nu_{A\times B}((x,y),q) = \nu_A(x,q)\vee\nu_B(y,q)$.

Definition 2.8. Let A be an Q-intuitionistic L-fuzzy subset in a set S, the strongest Q-intuitionistic L-fuzzy relation on S, that is a Q-intuitionistic L-fuzzy relation on A is V given by $\mu_V((x,y),q) = \mu_A(x,q)\wedge\mu_A(y,q)$ and $\nu_V((x,y),q) = \nu_A(x,q)\vee\nu_A(y,q)$, for all x and y in S and q in Q.

Definition 2.9. Let $(R,+,\bullet)$ and $(R',+,\bullet)$ be any two semirings. Let $f : R\to R'$ be any function and A be an Q-intuitionistic L-fuzzy normal subsemiring in R, V be an Q-intuitionistic L-fuzzy normal subsemiring in $f(R) = R'$, defined by $\mu_V(y,q) = \sup_{x\in F^{-1}(y)}\mu_A(x,q)$ and $\nu_V(y,q) = \inf_{x\in f^{-1}(y)}\nu_A(x,q)$, for all x in R and y in R' and q in Q. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

Definition 2.10. Let A be an Q-intuitionistic L-fuzzy subsemiring of a semiring $(R,+,\bullet)$ and a in R. Then the pseudo Q-intuitionistic L-fuzzy normal coset $(aA)^p$ is defined by $((a\mu_A)^p)(x,q) = p(a)\mu_A(x,q)$ and $((a\nu_A)^p)(x,q) = p(a)\nu_A(x,q)$, for every x in R and for some p in P and q in Q.

3. Properties Q-Intuitionistic L-Fuzzy normal subsemiring of a semiring

Theorem 3.1. *Let $(R,+,\bullet)$ be a semiring. If A and B are two Q-intuitionistic L-fuzzy normal subsemirings of R, then their intersection $A\cap B$ is a Q-intuitionistic L-fuzzy normal subsemiring of R.*

Proof. Let x and $y\in R$. Let $A = \{\langle (x,q), \mu_A(x,q), \nu_A(x,q)\rangle /x\in R, q\in Q\}$ and $B = \{\langle (x,q), \mu_B(x,q), \nu_B(x,q)\rangle /x\in R, q\in Q\}$ be an Q-intuitionistic L-fuzzy normal subsemirings of a semiring R.

Let $C = A\cap B$ and $C = \{\langle (x,q), \mu_C(x,q), \nu_C(x,q)\rangle /x\in R, q\in Q\}$, where $\mu_C(x,q) = \mu_A(x,q)\wedge\mu_B(x,q)$ and $\nu_C(x,q) = \nu_A(x,q)\vee\nu_B(x,q)$. Then, C is a Q-intuitionistic L-fuzzy subsemiring of a semiring R, since A and B are two Q-intuitionistic L-fuzzy subsemirings of a semiring R. Now

(i) $\mu_C(x+y,q) = \mu_A(x+y,q)\wedge\mu_B(x+y,q), = \mu_A(y+x,q)\wedge\mu_B(y+x,q), = \mu_C(y+x,q)$. Therefore, $\mu_C(x+y,q) = \mu_C(y+x,q)$, for all x and y in R and q in Q.

(ii) $\mu_C(xy,q) = \mu_A(xy,q)\wedge\mu_B(xy,q), = \mu_A(yx,q)\wedge\mu_B(yx,q), = \mu_C(yx,q)$, Therefore, $\mu_C(xy,q) = \mu_C(yx,q)$, for all x and y in R and q in Q.

(iii) $\nu_C(x+y,q) = \nu_A(x+y,q)\vee\nu_B(x+y,q), = \nu_A(y+x,q)\vee\nu_B(y+x,q), = \nu_C(y+x,q)$, Therefore, $\nu_C(x+y,q) = \nu_C(y+x,q)$, for all x and y in R and q in Q.

(iv) $\nu_C(xy,q) = \nu_A(xy,q) \vee \nu_B(xy,q), = \nu_A(yx,q) \vee \nu_B(yx,q), = \nu_C(yx,q)$, Therefore, $\nu_C(xy,q) = \nu_C(yx,q)$, for all x and y in R and q in Q. Hence $A \cap B$ is a Q-intuitionistic L-fuzzy normal subsemiring of a semiring R. □

THEOREM 3.2. *Let $(R,+,\bullet)$ be a semiring. The intersection of a family of Q-intuitionistic L-fuzzy normal subsemirings of R is a Q-intuitionistic L-fuzzy normal subsemiring of R.*

PROOF. Let $\{A_i\}_{i\in I}$ be a family of Q-intuitionistic L-fuzzy normal subsemirings of a semiring R and let $A = \bigcap_{i\in I} A_i$. Then for x and $y \in R$ and $q \in Q$. Clearly the intersection of a family of Q-intuitionistic L-fuzzy subsemiring of a semiring R is a Q-intuitionistic L-fuzzy subsemiring of a semiring R.

(i) $\mu_A(x+y,q) = \inf_{i\in I} \mu_{A_i}(x+y,q) = \inf_{i\in I} \mu_{A_i}(y+x,q) = \mu_A(y+x,q)$.
Therefore, $\mu_A(x+y,q) = \mu_A(y+x,q)$, for all x and y in R and q in Q.

(ii)$\mu_A(xy,q) = \inf_{i\in I} \mu_{A_i}(xy,q) = \inf_{i\in I} \mu_{A_i}(yx,q) = \mu_A(yx,q)$.
Therefore, $\mu_A(xy,q) = \mu_A(yx,q)$, for all x and y in R and q in Q.

(iii) $\nu_A(x+y,q) = \sup_{i\in I} \nu_{A_i}(x+y,q) = \sup_{i\in I} \nu_{A_i}(y+x,q) = \nu_A(y+x,q)$.
Therefore, $\nu_A(x+y,q) = \nu_A(y+x,q)$, for all x and y in R and q in Q.

(iv)$\nu_A(xy,q) = \sup_{i\in I} \nu_{A_i}(xy,q) = \sup_{i\in I} \nu_{A_i}(yx,q) = \nu_A(yx,q)$.
Therefore, $\nu_A(xy,q) = \nu_A(yx,q)$, for all x and y in R and q in Q. Hence the intersection of a family of Q-intuitionistic L-fuzzy normal subsemiring of a semiring R is a Q-intuitionistic L-fuzzy normal subsemiring of R. □

THEOREM 3.3. *Let A and B be Q-intuitionistic L-fuzzy subsemirings of the semirings G and H, respectively. If A and B are Q-intuitionistic L-fuzzy normal subsemirings, then $A\times B$ is a Q- intuitionistic L-fuzzy normal subsemiring of $G\times H$.*

PROOF. Let A and B be Q-intuitionistic L-fuzzy normal subsemirings of the semirings G and H respectively. Clearly $A \times B$ is a Q-intuitionistic L-fuzzy subsemiring of $G \times H$. Let x_1 and x_2 be in G, y_1 and y_2 be in H and $q \in Q$. Then $(x_1, y_1)$and (x_2, y_2) are in $G \times H$. Now

(i) $\mu_{(A\times B)}((x_1,y_1)+(x_2,y_2),q) = \mu_{(A\times B)}((x_1+x_2,y_1+y_2),q) = \mu_A(x_1+x_2,q)\wedge\mu_B(y_1+y_2,q) = \mu_A(x_2+x_1,q)\wedge\mu_B(y_2+y_1,q) = \mu_{(A\times B)}((x_2+x_1,y_2+y_1),q) = \mu_{(A\times B)}((x_2,y_2)+(x_1,y_1),q)$. Therefore,

$$\mu_A((x_1,y_1)+(x_2,y_2),q) = \mu_{(A\times B)}((x_2,y_2)+(x_1,y_1),q),$$

for all x and y in R and q in Q.

(ii) $\mu_{(A\times B)}((x_1,y_1)(x_2,y_2),q) = \mu_{(A\times B)}((x_1x_2,y_1y_2),q)$
$= \mu_A(x_1x_2,q) \wedge \mu_B(y_1y_2,q) = \mu_A(x_2x_1,q) \wedge \mu_B(y_2y_1,q) = \mu_{(A\times B)}((x_2x_1,y_2y_1),q)$
$= \mu_{(A\times B)}((x_2,y_2)(x_1,y_1),q)$.
Therefore, $\mu_A((x_1,y_1)(x_2,y_2),q) = \mu_{(A\times B)}((x_2,y_2)(x_1,y_1),q)$, for all x and y in R and q in Q.

(iii) $\nu_{(A\times B)}((x_1,y_1)+(x_2,y_2),q) = \nu_{(A\times B)}((x_1+x_2,y_1+y_2),q)$
$= \nu_A(x_1+x_2,q) \vee \nu_B(y_1+y_2,q) = \nu_A(x_2+x_1,q) \vee \nu_B(y_2+y_1,q)$
$= \nu_{(A\times B)}((x_2+x_1,y_2+y_1),q) = \nu_{(A\times B)}((x_2,y_2)+(x_1,y_1),q)$.

Therefore, $\nu_A((x_1, y_1) + (x_2, y_2), q) = \nu_{(A\times B)}((x_2, y_2) + (x_1, y_1), q)$, for all x and y in R and q in Q.

(iv) $\nu_{(A\times B)}((x_1, y_1)(x_2, y_2), q) = \nu_{(A\times B)}((x_1x_2, y_1y_2), q)$
$= \nu_A(x_1x_2, q) \vee \nu_B(y_1y_2, q) = \nu_A(x_2x_1, q) \vee \nu_B(y_2y_1, q)$
$= \nu_{(A\times B)}((x_2x_1, y_2y_1), q) = \nu_{(A\times B)}((x_2, y_2)(x_1, y_1), q)$.
Therefore, $\nu_A((x_1, y_1)(x_2, y_2), q) = \nu_{(A\times B)}((x_2, y_2)(x_1, y_1), q)$, for all x and y in R and q in Q. Hence $A \times B$ is a (Q, L)-intuitionistic fuzzy normal subsemiring of $G \times H$. □

Theorem 3.4. *If A_i are (Q, L)-intuitionistic fuzzy normal subsemirings of the semirings R_i, then $\prod A_i$ is a (Q, L)-intuitionistic fuzzy normal subsemiring of $\prod R_i$.*

Proof. It is trivial. □

Theorem 3.5. *Let A be a Q-intuitionistic L-fuzzy subset in a semiring R and V be the strongest Q-intuitionistic L-fuzzy relation on R. Then A is a Q-intuitionistic L-fuzzy normal subsemiring of R if and only if V is a Q-intuitionistic L-fuzzy normal subsemiring of $R \times R$.*

Proof. Suppose that A is a Q-intuitionistic L-fuzzy normal subsemiring of$R$. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $R \times R$
Clearly V is a Q-intuitionistic L-fuzzy subsemiring of a semiring R. We have

(i) $\mu_V(x + y, q) = \mu_V[(x_1, x_2) + (y_1, y_2), q] = \mu_V((x_1 + y_1, x_2 + y_2), q)$
$= \mu_A((x_1 + y_1), q) \wedge \mu_A((x_2 + y_2), q) = \mu_A((y_1 + x_1), q) \wedge \mu_A((y_2 + x_2), q)$
$= \mu_V((y_1 + x_1, y_2 + x_2), q) = \mu_V[(y_1, y_2) + (x_1, x_2), q] = \mu_V(y + x, q)$.
Therefore $\mu_V(x + y, q) = \mu_V(y + x, q)$, for all x and y in $R \times R$ and q in Q.

(ii) $\mu_V(xy, q) = \mu_V[(x_1, x_2)(y_1, y_2), q] = \mu_V((x_1y_1, x_2y_2), q)$
$= \mu_A((x_1y_1), q)\wedge\mu_A((x_2y_2), q) = \mu_A((y_1x_1), q)\wedge\mu_A((y_2x_2), q) = \mu_V((y_1x_1, y_2x_2), q)$
$= \mu_V[(y_1, y_2)(x_1, x_2), q] = \mu_V(yx, q)$.
Therefore $\mu_V(xy, q) = \mu_V(yx, q)$, for all x and y in $R \times R$ and q in Q.

(iii) $\nu_V(x + y, q) = \nu_V[(x_1, x_2) + (y_1, y_2), q] = \nu_V((x_1 + y_1, x_2 + y_2), q)$
$= \nu_A((x_1 + y_1), q) \vee \nu_A((x_2 + y_2), q) = \nu_A((y_1 + x_1), q) \vee \nu_A((y_2 + x_2), q)$
$= \nu_V((y_1 + x_1, y_2 + x_2), q) = \nu_V[(y_1, y_2) + (x_1, x_2), q] = \nu_V(y + x, q)$.
Therefore $\nu_V(x + y, q) = \nu_V(y + x, q)$, for all x and y in $R \times R$ and q in Q.

(iv) $\nu_V(xy, q) = \nu_V[(x_1, x_2)(y_1, y_2), q] = \nu_V((x_1y_1, x_2y_2), q)$
$= \nu_A((x_1y_1), q)\vee\nu_A((x_2y_2), q) = \nu_A((y_1x_1), q)\vee\nu_A((y_2x_2), q) = \nu_V((y_1x_1, y_2x_2), q)$
$= \nu_V[(y_1, y_2)(x_1, x_2), q] = \nu_V(yx, q)$.
Therefore $\nu_V(xy, q) = \nu_V(yx, q)$, for all x and y in $R \times R$ and q in Q. This proves that V is a Q-intuitionistic L-fuzzy normal subsemiring of $R \times R$.

Conversely, assume that V is a Q-intuitionistic L-fuzzy normal subsemiring of $R \times R$. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $R \times R$, we know that A is a Q-intuitionistic L-fuzzy subsemiring of R. If $\mu_A(x_1 + y_1, q) \leqslant \mu_A(x_2 + y_2, q)$ and

(i) $\mu_A(x_1 + y_1, q) = \mu_A((x_1 + y_1, q) \wedge \mu_A(x_2 + y_2, q) = \mu_V[(x_1 + y_1, x_2 + y_2), q]$
$= \mu_V[(x_1, x_2) + (y_1, y_2), q] = \mu_V(x + y, q) = \mu_V(y + x, q) = \mu_V[(y_1, y_2) + (x_1, x_2), q]$
$= \mu_V((y_1 + x_1, y_2 + x_2), q) = \mu_A((y_1 + x_1, q) \wedge \mu_A(y_2 + x_2, q) = \mu_A(y_1 + x_1, q)$.

Therefore $\mu_V(x_1+y_1,q)=\mu_V(y_1+x_1,q)$, for all x and y in $R\times R$ and q in Q. We have, $\mu_A((x_1+y_1),q)=\mu_A((y_1+x_1),q)$, for all x_1 and $y_1\in R$ and $q\in Q$.

If $\mu_A(x_1y_1,q)\leqslant\mu_A(x_2y_2,q)$, and

(ii) $\mu_A(x_1y_1,q)=\mu_A((x_1y_1,q)\wedge\mu_A(x_2y_2,q))=\mu_V[(x_1y_1,x_2y_2),q]$
$=\mu_V[(x_1,x_2)(y_1,y_2),q]=\mu_V(xy,q)=\mu_V(yx,q)=\mu_V[(y_1,y_2)(x_1,x_2),q]$
$=\mu_V((y_1x_1,y_2x_2),q)=\mu_A((y_1x_1,q)\wedge\mu_A(y_2x_2,q))=\mu_A(y_1x_1,q)$
Therefore $\mu_V(x_1y_1,q)=\mu_V(y_1x_1,q)$, for all x and y in $R\times R$ and q in Q. We have, $\mu_A((x_1y_1),q)=\mu_A((y_1x_1),q)$, for all x_1 and $y_1\in R$ and $q\in Q$.

If $\nu_A(x_1+y_1,q)\geqslant\nu_A(x_2+y_2,q)$, and

(iii) $\nu_A(x_1+y_1,q)=\nu_A((x_1+y_1,q)\vee\nu_A(x_2+y_2,q)=\nu_V[(x_1+y_1,x_2+y_2),q]$
$=\nu_V[(x_1,x_2)+(y_1,y_2),q]=\nu_V(x+y,q)=\nu_V(y+x,q)=\nu_V[(y_1,y_2)+(x_1,x_2),q]$
$=\nu_V((y_1+x_1,y_2+x_2),q)=\nu_A((y_1+x_1,q)\vee\nu_A(y_2+x_2,q)=\nu_A(y_1+x_1,q)$
Therefore $\nu_V(x_1+y_1,q)=\nu_V(y_1+x_1,q)$, for all x and y in $R\times R$ and q in Q. We have, $\nu_A((x_1+y_1,q)=\nu_A((y_1+x_1,q)$, for all x_1 and $y_1\in R$ and $q\in Q$.

If $\nu_A(x_1y_1,q)\geqslant\nu_A(x_2y_2,q)$, and

(iv) $\nu_A(x_1y_1,q)=\nu_A((x_1y_1,q)\vee\nu_A(x_2y_2,q))=\nu_V[(x_1y_1,x_2y_2),q]$
$=\nu_V[(x_1,x_2)(y_1,y_2),q]=\nu_V(xy,q)=\nu_V(yx,q)=\nu_V[(y_1,y_2)(x_1,x_2),q]$
$=\nu_V((y_1x_1,y_2x_2),q)=\nu_A((y_1x_1,q)\vee\nu_A(y_2x_2,q))=\nu_A(y_1x_1,q)$
Therefore $\nu_V(x_1y_1,q)=\nu_V(y_1x_1,q)$, for all x and y in $R\times R$ and q in Q. We have, $\nu_A((x_1y_1),q)=\nu_A((y_1x_1),q)$, for all x_1 and $y_1\in R$ and $q\in Q$.

Therefore A is a Q-intuitionistic L-fuzzy normal subsemiring of R. □

THEOREM 3.6. *Let $(R,+,\bullet)$ and $(R^{'},+,\bullet)$ be two semirings. The homomorphic image of a Q-intuitionistic L-fuzzy normal subsemiring of R is a Q-intuitionistic L-fuzzy normal subsemiring of $R^{'}$.*

PROOF. Let $(R,+,\bullet)$ and $(R^{'},+,\bullet)$ be any two semirings and $f:R\to R^{'}$ be a homomorphism. Let $V=f(A)$, where A is a Q-intuitionistic L-fuzzy normal subsemiring of a semiring R. We have to prove that V is a Q-intuitionistic L-fuzzy normal subsemiring of a semiring $R^{'}$.

Now, for $f(x),f(y)\in R^{'}$, clearly V is a Q-intuitionistic L-fuzzy subsemiring of a semiring $R^{'}$, since A is a Q-intuitionistic L-fuzzy subsemiring of a semiring R.

(i) $\mu_v(f(x)+f(y),q)=\mu_v(f(x+y),q),\geqslant\mu_A(x+y,q)=\mu_A(y+x,q)\leqslant\mu_v(f(y+x),q)=\mu_v(f(y,q)+f(x,q))$.
Therefore $\mu_v(f(x,q)+f(y,q))=\mu_v(f(y,q)+f(x,q))$, for all $f(x)$ and $f(y)$ in R' and q in Q.

(ii) $\mu_v(f(x,q)f(y,q))=\mu_v(f(yx),q),\geqslant\mu_A(yx,q)=\mu_A(xy,q)\leqslant\mu_v(f(xy),q)$ $=\mu_v(f(y,q)f(x,q))$.
Therefore $\mu_v(f(x,q)f(y,q))=\mu_v(f(y,q)f(x,q))$, for all $f(x)$ and $f(y)$ in R' and q in Q.

(iii) $\nu_v(f(x,q)+f(y,q))=\nu_v(f(x+y),q),\leqslant\mu_A(x+y,q)=\nu_A(y+x,q)$ $\geqslant\mu_v(f(y+x),q)=\nu_v(f(y,q)+f(x,q))$.
Therefore $\nu_v(f(x,q)+f(y,q))=\nu_v(f(y,q)+f(x,q))$, for all $f(x)$ and $f(y)$ in R' and q in Q.

(iv)$\nu_v(f(x,q)f(y,q))=\nu_v(f(xy),q),\leqslant\nu_A(xy,q)=\nu_A(yx,q)\geqslant\mu_v(f(yx),q)$ $=\nu_v(f(y,q)f(x,q))$.

Therefore $\nu_v(f(x,q)f(y,q)) = \nu_v(f(y,q)f(x,q))$, for all $f(x)$ and $f(y)$ in R' and q in Q.

Hence V is a Q-intuitionistic L-fuzzy normal subsemiring of a semiring $R^{'}$. □

THEOREM 3.7. *Let $(R, +, \bullet)$ and $(R^{'}, +, \bullet)$ be any two semirings. The anti-homomorphic preimage of a Q-intuitionistic L-fuzzy normal subsemiring of $R^{'}$ is a Q-intuitionistic L-fuzzy normal subsemiring of R.*

PROOF. Let $(R, +, \bullet)$ and $(R^{'}, +, \bullet)$ be any two semirings and $f : R \to R^{'}$ be an anti-homomorphism. Let $V = f(A)$, where V is a Q-intuitionistic L-fuzzy normal subsemiring of a semiring $R^{'}$. We have to prove that A is a Q-intuitionistic L-fuzzy normal subsemiring of a semiring R. Let x and $y \in R$ and $q \in Q$, then A is a Q-intuitionistic L-fuzzy subsemiring of a semiring R, since V is a Q-intuitionistic L-fuzzy subsemiring of a semiring $R^{'}$.

(i) $\mu_A(x+y,q) = \mu_v(f(x+y,q)), = \mu_v(f(y,q)+f(x,q))$
$= \mu_v(f(x,q)+f(y,q)) = \mu_v(f(y+x,q)) = \mu_A(y+x,q)$.
Therefore $\mu_A(x+y,q) = \mu_A(y+x,q)$, for all x and y in R and q in Q.

(ii) $\mu_A(xy,q) = \mu_v(f(xy,q)) = \mu_v(f(y,q)f(x,q))$
$= \mu_v(f(x,q)f(y,q)) = \mu_A(yx,q)$.
Therefore $\mu_A(xy,q) = \mu_A(yx,q)$, for all x and y in R and q in Q.

(iii) $\nu_A(x+y,q) = \nu_v(f(x+y,q)) = \nu_v(f(y,q)+f(x,q)) = \nu_v(f(x,q)+f(y,q))$
$= \nu_v(f(y+x,q)) = \nu_A(y+x,q)$. Therefore $\nu_A(x+y,q) = \nu_A(y+x,q)$, for all x and y in R and q in Q.

(iv) $\nu_A(xy,q) = \nu_v(f(xy,q)) = \nu_v(f(y,q)f(x,q)) = \nu_v(f(x,q)f(y,q))$
$= \nu_v(f(yx,q)) = \nu_A(yx,q)$.
Therefore $\nu_A(xy,q) = \nu_A(yx,q)$, for all x and y in R and q in Q.

Hence A is a Q-intuitionistic L-fuzzy normal subsemiring of a semiring R. □

In the following Theorem *circ* is the composition operation of functions:

THEOREM 3.8. *Let A be a Q-intuitionistic L-fuzzy subsemiring of a semiring H and f is an isomomorphism from a semiring R onto H. If A is a Q-intuitionistic L-fuzzy normal subsemiring of the semiring H, then $A \circ f$ is a Q-intuitionistic L-fuzzy normal subsemiring of the semiring$R$.*

PROOF. Let x and $y \in R, q \in Q$ and A a Q-intuitionistic L-fuzzy normal subsemiring of a semiring H. Then, $A \circ f$ is a Q-intuitionistic L-fuzzy subsemiring of a semiring R. Now

(i) $(\mu_A \circ f)(x+y,q) = \mu_A(f(x+y,q)) = \mu_A(f(x,q)+f(y,q)) = \mu_A(f(y,q)+f(x,q)) = \mu_A(f(y+x,q)) = (\mu_A \circ f)(y+x,q)$.
Therefore $(\mu_A \circ f)(x+y,q) = (\mu_A \circ f)(y+x,q)$, for all x and y in R and q in Q.

(ii) $(\mu_A \circ f)(xy,q) = \mu_A(f(xy,q)) = \mu_A(f(x,q)f(y,q)) = \mu_A(f(y,q)f(x,q))$
$= \mu_A(f(yx,q)) = (\mu_A \circ f)(y+x,q)$.
Therefore $(\mu_A \circ f)(xy,q) = (\mu_A \circ f)(yx,q)$, for all x and y in R and q in Q.

(iii) $(\nu_A \circ f)(x+y,q) = \nu_A(f(x+y,q)) = \nu_A(f(x,q)+f(y,q)) = \nu_A(f(y,q)+f(x,q)) = \nu_A(f(y+x,q)) = (\nu_A \circ f)(y+x,q)$.
Therefore $(\nu_A \circ f)(x+y,q) = (\nu_A \circ f)(y+x,q)$, for all x and y in R and q in Q.

(iv) $(\nu_A \circ f)(xy, q) = \nu_A(f(xy, q)) = \nu_A(f(x, q)f(y, q)) = \nu_A(f(y, q)f(x, q)) = \nu_A(f(yx, q)) = (\nu_A \circ f)(y + x, q)$.
Therefore $(\nu_A \circ f)(xy, q) = (\nu_A \circ f)(yx, q)$, for all x and y in R and q in Q.

Hence $A \circ f$ is a (Q, L)-intuitionistic fuzzy normal subsemiring of a semiring R. □

THEOREM 3.9. *Let A be a Q-intuitionistic L-fuzzy subsemiring of a semiring H and f is an anti-isomomorphism from a semiring R onto H. If A is a Q-intuitionistic L-fuzzy normal subsemiring of the semiring H, then $A \circ f$ is a Q-intuitionistic L-fuzzy normal subsemiring of the semiring$R$.*

PROOF. Let x and $y \in R, q \in Q$ and A a Q-intuitionistic L-fuzzy normal subsemiring of a semiring H. Then, $A \circ f$ is a Q-intuitionistic L-fuzzy subsemiring of a semiring R. Now

(i) $(\mu_A \circ f)(x + y, q) = \mu_A(f(x + y, q)) = \mu_A(f(y, q) + f(x, q)) = \mu_A(f(x, q) + f(y, q)) = \mu_A(f(y + x, q)) = (\mu_A \circ f)(y + x, q)$.
Therefore $(\mu_A \circ f)(x + y, q) = (\mu_A \circ f)(y + x, q)$, for all x and y in R and q in Q.

(ii) $(\mu_A \circ f)(xy, q) = \mu_A(f(xy, q)) = \mu_A(f(y, q)f(x, q)) = \mu_A(f(x, q)f(y, q)) = \mu_A(f(yx, q)) = (\mu_A \circ f)(y + x, q)$.
Therefore $(\mu_A \circ f)(xy, q) = (\mu_A \circ f)(yx, q)$, for all x and y in R and q in Q.

(iii) $(\nu_A \circ f)(x + y, q) = \nu_A(f(x + y, q)) = \nu_A(f(y, q) + f(x, q)) = \nu_A(f(x, q) + f(y, q)) = \nu_A(f(y + x, q)) = (\nu_A \circ f)(y + x, q)$.
Therefore $(\nu_A \circ f)(x + y, q) = (\nu_A \circ f)(y + x, q)$, for all x and y in R and q in Q.

(iv) $(\nu_A \circ f)(xy, q) = \nu_A(f(xy, q)) = \nu_A(f(y, q)f(x, q)) = \nu_A(f(x, q)f(y, q)) = \nu_A(f(yx, q)) = (\nu_A \circ f)(y + x, q)$.
Therefore $(\nu_A \circ f)(xy, q) = (\nu_A \circ f)(yx, q)$, for all x and y in R and q in Q.

Hence $A \circ f$ is a (Q, L)-intuitionistic fuzzy normal subsemiring of a semiring R. □

THEOREM 3.10. *Let A be a Q-intuitionistic L-fuzzy normal subsemiring of a semiring $(R, +, \bullet)$*

(i) *If $\mu_A(x + y, q) = 1$, then $\mu_A(x, q) = \mu_A(y, q)$, for a and $y \in R$ and $q \in Q$.*

(ii) *If $\nu_A(x + y, q) = 0$, then $\nu_A(x, q) = \nu_A(y, q)$, for a and $y \in R$ and $q \in Q$.*

PROOF. It is trivial. □

THEOREM 3.11. *Let A be a Q-intuitionistic L-fuzzy normal subsemiring of a semiring $(R, +, .)$. Then*

(i) *If $\mu_A(x + y, q) = 0$, then either $\mu_A(x, q) = 0$ or $\mu_A(y, q) = 0$, for all x and $y \in R$ and $q \in Q$.*

(ii) *If $\nu_A(x + y, q) = 1$, then either $\nu_A(x, q) = 1$ or $\nu_A(y, q) = 1$, for all x and $y \in R$ and $q \in Q$.*

PROOF. It is trivial. □

THEOREM 3.12. *If A is a Q-intuitionistic L-fuzzy normal subsemiring of a semiring $(R, +, \bullet)$,*

(i) *If* $\mu_A(x,q) < \mu_A(y,q)$ *for some* x *and* $y \in R$ *and* $q \in Q$ *then*

$$\mu_A(x+y,q) = \mu_A(x,q) = \mu_A(y+x,q),$$

for some x *and* $y \in R$ *and* $q \in Q$.

(ii) *If* $\nu_A(y,q) < \nu_A(x,q)$ *for some* x *and* $y \in R$ *and* $q \in Q$ *then*

$$\nu_A(x+y,q) = \nu_A(x,q) = \nu_A(y+x,q),$$

for some x *and* $y \in R$ *and* $q \in Q$.

PROOF. It is trivial. □

THEOREM 3.13. *Let* A *be a* Q*-intuitionistic* L*-fuzzy normal subsemiring of a semiring* $(R,+,\bullet)$.

(i) *If* $\mu_A(x,q) > \mu_A(y,q)$ *for some* x *and* $y \in R$ *and* $q \in Q$ *then*

$$\mu_A(x+y,q) = \mu_A(y,q) = \mu_A(y+x,q),$$

for some x *and* $y \in R$ *and* $q \in Q$.

(ii) *If* $\nu_A(y,q) > \nu_A(x,q)$ *for some* x *and* $y \in R$ *and* $q \in Q$ *then*

$$\nu_A(x+y,q) = \nu_A(y,q) = \nu_A(y+x,q),$$

for some x *and* $y \in R$ *and* $q \in Q$.

PROOF. It is trivial. □

THEOREM 3.14. *Let* A *be a* Q*-intuitionistic* L*-fuzzy normal subsemiring of a semiring* $(R,+,\bullet)$ *such that* $Im\mu_A = \{\alpha\}$ *and* $Im\nu_A = \{\beta\}$, *where* α *and* $\beta \in L$. *If* $A = B \cup C$, *where* B *and* C *are* Q*-intuitionistic* L*-fuzzy normal subsemiring of a semiring of* R, *then either* $B \subseteq C$ *or* $C \subseteq B$.

PROOF. It is trivial. □

THEOREM 3.15. *If* A *is a* Q*-intuitionistic* L*-fuzzy normal subsemiring of a semiring* $(R,+,\bullet)$, *then* $\Box A$ *is a* Q*-intuitionistic* L*-fuzzy normal subsemiring of* R.

PROOF. Let A be a Q-intuitionistic L-fuzzy normal subsemiring of a semiring R. Consider $A = \{\langle (x,q), \mu_A(x,q), \nu_A(x,q)\rangle\}$, for all x in R and q in Q, we take $\Box A = B = \{\langle (x,q), \mu_B(x,q), \nu_B(x,q)\rangle\}$, where $\mu_B(x,q) = \mu_A(x,q), \nu_B(x,q) = 1-\mu_A(x,q)$. Clearly A is a (Q,L)-intuitionistic fuzzy subsemiring of R, We have

(i) $\mu_B(x+y,q) = \mu_A(x+y,q) = \mu_A(y+x,q) = \mu_B(y+x,q)$.
Therefore $\mu_B(x+y,q) = \mu_B(y+x,q)$ for all x and $y \in R$ and $q \in Q$.

(ii) $\mu_B(xy,q) = \mu_A(xy,q) = \mu_A(yx,q) = \mu_B(yx,q)$.
Therefore $\mu_B(xy,q) = \mu_B(yx,q)$ for all x and $y \in R$ and $q \in Q$.

Since A is a Q-intuitionistic L-fuzzy subsemiring of R, we have $\mu_A(x+y,q) = \mu_A(y+x,q)$, for all x and $y \in R$ and $q \in Q$, which implies that $1-\mu_A(x+y,q) = 1-\mu_A(y+x,q)$ which implies that $\nu_B(x+y,q) = \nu_B(y+x,q)$. Therefore, $\nu_B(x+y,q) = \nu_B(y+x,q)$, for all x and $y \in R$ and $q \in Q$. And $\mu_A(xy,q) = \mu_A(yx,q)$, for all x and $y \in R$ and $q \in Q$, which implies that $1-\mu_A(xy,q) = 1-\mu_A(yx,q)$ which implies that $\nu_B(xy,q) = \nu_B(yx,q)$. Therefore, $\nu_B(xy,q) = \nu_B(yx,q)$, for all x and $y \in R$ and $q \in Q$.

Hence $B = \Box A$ is a Q-intuitionistic L-fuzzy normal subsemiring of a semiring R. $\Box$

REMARK 3.1. The converse of the above theorem is not true.

THEOREM 3.16. *Let A be a Q-intuitionistic L-fuzzy normal subsemiring of a semiring $(R, +, \bullet)$, then the pseudo Q-intuitionistic L-fuzzy coset $(aA)^p$ is a Q-intuitionistic L-fuzzy normal subsemiring of a semiring R, for every $a \in R$.*

PROOF. Let A be a Q-intuitionistic L-fuzzy normal subsemiring of a semiring R. For every x and $y \in R$ and $q \in Q$, clearly $(aA)^p$ is a Q-intuitionistic L-fuzzy subsemiring of a semiring R and

(i) $((a\mu_A)^p)(x+y, q) = p(a)\mu_A(x+y, q) = p(a)\mu_A(y+x, q) = ((a\mu_A)^p)(y+x, q)$. Therefore $((a\mu_A)^p)(x+y, q) = ((a\mu_A)^p)(y+x, q)$ for all x and $y \in R$ and $q \in Q$. Therefore, $((a\mu_A)^p)(x+y, q) = ((a\mu_A)^p)(y+x, q)$, for all x and $y \in R$ and $q \in Q$.

(ii) $((a\mu_A)^p)(xy, q) = p(a)\mu_A(xy, q) = p(a)\mu_A(yx, q) = ((a\mu_A)^p)(yx, q)$, Therefore $((a\mu_A)^p)(xy, q) = ((a\mu_A)^p)(yx, q)$ for all x and $y \in R$ and $q \in Q$.

(iii) $((a\nu_A)^p)(x+y, q) = p(a)\nu_A(x+y, q) = p(a)\nu_A(y+x, q) = ((a\nu_A)^p)(y+x, q)$. Therefore, $((a\nu_A)^p)(x+y, q) = ((a\nu_A)^p)(y+x, q)$, for all x and $y \in R$ and $q \in Q$.

(iv) $((a\nu_A)^p)(xy, q) = p(a)\nu_A(xy, q) = p(a)\nu_A(yx, q) = ((a\nu_A)^p)(yx, q)$, Therefore, $((a\nu_A)^p)(xy, q) = ((a\nu_A)^p)(yx, q)$, for all x and $y \in R$ and $q \in Q$.

Hence $(aA)^p$ is a Q-intuitionistic L-fuzzy normal subsemiring of a semiring R. $\Box$

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