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POSITIVE IMPLICATIVE FILTERS AND ASSOCIATIVE FILTERS OF IMPLICATIVE ALMOST DISTRIBUTIVE LATTICES

Berhanu Assaye, Mihret Alamneh and Tilahun Mekonnen

Abstract. In this paper, we introduce the concept of positive implicative filters and associative filters of implicative almost distributive lattices. We prove that every positive implicative filter is an implicative filter and every associative filter is a filter. We give equivalent conditions for both a positive implicative filter and associative filter in implicative almost distributive lattices (IADLs).

1. Introduction

In order to research the logical system whose propositional value is given in a lattice Xu, Y. [**10**] proposed the concept of lattice implication algebras, and discussed their some properties of lattice implication algebra. Xu, Y. and Qin, K. Y. [**12**] introduced the notions of a filter and an implicative filter in a lattice implication algebra, and investigate their properties. Jun, Y. B. [**4, 5**] gave some equivalent conditions that a filter is an implicative filter in lattice implication algebra and an extension property for implicative filter. They introduced the concept of a positive implicative filter and an associative filter in lattice implication algebra. They also proved that every positive implicative filter is a filter. They provided equivalent condition for a positive implicative filter and an associative filter. Venkateswarlu Kolluru, V. and Berhanu, B. [**7**] introduced the concept of implicative algebras and obtained certain properties. They also proved that every implicative algebra is a lattice implication algebra. The concept of an almost distributive lattice (ADL) was introduced in 1981 by Swamy, U. M. and Rao, G.C. [**9**] as a common abstraction to

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most of the existing ring theoretic and lattice theoretic generalization of Boolean algebra. For our discussion we used implicative algebra in order to convert to an ADL concept. Formerly, Berhanu, A., Mihret, A. and Tilahun. M. [**1**] introduced the concept of an implicative almost distributive lattices (IADLs) as a generalization of implicative algebra in the class of ADLs. We proved some properties and equivalence condition in an implicative almost distributive lattice. In this paper, we introduce filter, implicative filter, positive implicative filter and associative filter in an implicative almost distributive lattice. We prove that every positive filter is an implicative filter and hence a filter. We give example to show that a filter may not be an associative filter. We provide equivalent conditions for both a positive implicative filter and an associative filter.

In the following, we give some important definitions and results that will be useful in this study.

2. Preliminaries

DEFINITION 2.1. ([**9**]) An algebra $(L, \vee, \wedge, 0)$ of type $(2, 2, 0)$ is called an almost distributive lattice (ADL) with 0 if it satisfies the following axioms:

(1) $(x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z)$
(2) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
(3) $(x \vee y) \wedge y = y$
(4) $(x \vee y) \wedge x = x$
(5) $x \vee (x \wedge y) = x$
(6) $0 \wedge x = 0$, for all $x, y, z \in L$.

If $(L, \vee, \wedge, 0)$ is an ADL, for any $x, y \in L$, define $x \leqslant y$ if and only if $x = x \wedge y$ or equivalently $x \vee y = y$, then $\leqslant$ is a partial ordering on L.

DEFINITION 2.2. ([**9**]) Let L be an ADL. An element $m \in L$ is called maximal if for any $x \in L, m \leqslant x$ implies m = x.

DEFINITION 2.3. ([**9**]) A non empty subset F of an ADL L is called a filter of L if it satisfies

(1) $x, y \in F$ implies $x \wedge y \in F$
(2) $x \in F$ and $y \in L$ implies $y \vee x \in F$, for all $x, y \in L$.

THEOREM 2.1 ([**9**]). *Let F is filter of an ADL L and $x, y \in L$. Then $x \vee y \in F$ if and only if $y \vee x \in F$.*

. The following definitions, lemma and theorems are in an implicative algebra that is useful in our study.

DEFINITION 2.4. ([**7**]) An algebra $(L, \rightarrow, ', 0, 1)$ of type $(2, 1, 0, 0)$ is called implicative algebra if it satisfies the following conditions:

(1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$
(2) $1 \rightarrow x = x$
(3) $x \rightarrow 1 = 1$
(4) $x \rightarrow y = y' \rightarrow x'$

(5) $(x \to y) \to y = (y \to x) \to x$
(6) $0' = 1$, for $x, y, z \in L$

DEFINITION 2.5. ([**7**]) A relation $\leqslant$ on an implicative algebra L is defined as follows: $x \leqslant y \Leftrightarrow x \to y = 1$, for all $x, y \in L$.

THEOREM 2.2 ([**7**]). *Let* $(L, \to, ', 0, 1)$ *be an implicative algebra. Then* $(L, \vee, \wedge, \to, ', 0, 1)$ *is a lattice implication algebra.*

DEFINITION 2.6. ([**5**]) Let L be an implicative algebra.

a . A subset F of L is called a filter of L if it satisfies
  (F_1) $1 \in F$
  (F_2) $x \in F$ and $x \to y \in F$ implies $y \in F$, for all $x, y \in L$.

b . A subset F of L is called an implicative filter of L, if it satisfies
  (F_1) $1 \in F$
  (I) $x \to y \in F$ and $x \to (y \to z) \in F$ implies $x \to z \in F$, for all $x, y, z \in L$.

DEFINITION 2.7. ([**1**]) Let $(L, \vee, \wedge, 0, m)$ be an ADL with 0 and maximal element m. Then an algebra $(L, \vee, \wedge, \to, ', 0, m)$ of type $(2, 2, 2, 1, 0, 0)$ is called implicative almost distributive Lattice (IADL)if it satisfies the following conditions:

(1) $x \vee y = (x \to y) \to y$
(2) $x \wedge y = [(x \to y) \to x']'$
(3) $x \to (y \to z) = y \to (x \to z)$
(4) $m \to x = x$
(5) $x \to m = m$
(6) $x \to y = y' \to x'$
(7) $0' = m$, for all $x, y, z \in L$.

Now we define the relation $\leqslant$ on an IADL L as follows:

$$x \leqslant y \Leftrightarrow x \to y = m, \text{ for all } x, y \in L.$$

The relation $\leqslant$ on L is a partial ordering.Thus $(L, \leqslant)$ is a poset.

THEOREM 2.3 ([**1**]). *In an IADL L the following conditions hold:*

(1) $[(x \to y) \to y] \wedge m = [(y \to x) \to x] \wedge m$
(2) $[((x \to y) \to x')'] \wedge m = [((y \to x) \to y')'] \wedge m$
(3) $x \to x = m$
(4) $m' = 0$
(5) $(x')' = x$
(6) $x' = x \to 0$
(7) $0 \to x = m$
(8) $x \to y = m = y \to x$ *implies* $x = y$.
(9) *If* $x \to y = m$ *and* $y \to z = m$*, then* $x \to z = m$
(10) $x \leqslant y$ *if and only if* $z \to x \leqslant z \to y$ *and* $y \to z \leqslant x \to z$
(11) $((x \to y) \to y) \to y = x \to y$
(12) $(x \to y) \to ((y \to z) \to (x \to z)) = m$
(13) $(x \to z) \to (x \to y) = (z \to x) \to (z \to y)$.

(14) $x \to y \leqslant (y \to z) \to (x \to z)$
(15) $(x \wedge y)' = x' \vee y'$, *for all* $x, y, z \in L$
(16) $x \leqslant y$ *implies* $y' \leqslant x'$, *for all* $x, y \in L$.

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Theorem 2.4 ([**1**]). *In an IADL L, the following conditions hold:*

(1) $x \leqslant y$, $x \leqslant z$ *implies* $x \leqslant (y \wedge z)$
(2) $y \leqslant x$, $z \leqslant x$ *implies* $(y \vee z) \leqslant x$
(3) $(x \vee y) \to z \leqslant x \to z$ *and* $(x \vee y) \to z \leqslant y \to z$
(4) $x \to z \leqslant (x \wedge y) \to z$ *and* $y \to z \leqslant (x \wedge y) \to z$
(5) $(x \vee y) \to z = (x \to z) \wedge (y \to z)$
(6) $(x \wedge y) \to z = (x \to z) \vee (y \to z)$
(7) $x \to (y \wedge z) = (x \to y) \wedge (x \to z)$
(8) $x \to (y \vee z) = (x \to y) \vee (x \to z)$

3. The main results

In this section we introduce filter, implicative filter, positive implicative filters and associative filters in an IADL. We discuss some properties and theorems based on these ideas.

Definition 3.1. . Let L be an IADL.

(1) A subset F of L is called a filter of L if it satisfies
 (F_1) $m \in F$
 (F_2) $x \in F$ and $x \to y \in F$ implies $y \in F$, for all $x, y \in L$.
(2) A subset F of L is called implicative filter of L if it satisfies
 (F_1) $m \in F$
 (I) $x \to y \in F$ and $x \to (y \to z) \in F$ implies $x \to z \in F$, for all $x, y, z \in L$, .

We can observe that every implicative filter of an IADL L is a filter of L. Since $m \to x = x \in F$ and $m \to (x \to y) = x \to y \in F$ for $x, y \in L$ implies $m \to y = y \in F$ by the definition of implicative filter.

Lemma 3.1. *Let F be a non empty subset of an IADL L. Then F is a filter of L if and only if it satisfies for all* $x, y \in F$ *and* $z \in L$*:*
$x \leqslant y \to z$ *implies* $z \in F$

Proof. Let L be an IADL. Assume F is a filter of L and $x \leqslant y \to z$, for $x, y \in F$ and $z \in L$. Then $x \to (y \to z) = m \in F$. Applying definition of filter in L twice we get $z \in F$. Conversely, assume $x \leqslant y \to z$ implies $z \in F$,for all $x, y \in F$. Clearly $m = x \to (y \to z) \in F$ and $x, y \in F$ implies $z \in F$. So by definition of filter, F is a filter of L. b □

Lemma 3.2. *Every filter F of an IADL L has the following property:* $x \leqslant y$ *and* $x \in F$ *implies* $y \in F$.

Proof. Let F be filter of IADL L. Assume $x \leqslant y$ and $x \in F$. Since $x \to y = m \in F$ and $x \in F$, by definition of filter we get $y \in F$. □

In the following we discuss positive implicative filters and some of their properties in an IADL.

Definition 3.2. A subset F of an IADL L is called a positive implicative filter of L if it satifies
(F_1) $m \in F$
(P_1) $x \in F$ and $x \to ((y \to z) \to y) \in F$ implies $y \in F$, for $x, y, z \in L$.

Example 3.1. Let $L = \{0, x, y, z, w, m\}$ be the underlining set with partial ordering

$$\leqslant = \{(0,0),(x,x),(y,y),(z,z),(w,w),(m,m),(0,w),(0,x),(0,m),(0,z),\ (0,y),\ (w,x),\ (w,m),\ (w,y),\ (x,m),(z,y),(z,m),(y,m)\}.$$

Define a unary operation $'$ and a binary operation $\to$ on L as shown in Table 3.1 and Table 3. 2 respectively.

x	x'
0	m
x	z
y	w
z	x
w	y
m	0

Table 3.1.

$\to$	o	x	y	z	w	m
o	m	m	m	m	m	m
x	z	m	m	m	y	m
y	w	x	m	y	x	m
z	x	x	m	m	x	m
w	y	m	m	y	m	m
m	o	x	y	z	w	m

Table 3.2.

Define $\vee$ and $\wedge$ operation on L as follows

$$x \vee y = (x \to y) \to y$$

$$x \wedge y = ((x \to y) \to x')',\ \text{for all} x, y \in L.$$

Then L is an IADL. Clearly $F = \{y, z, m\}$ is a positive implicative filter of L.

Theorem 3.1. *Every positive implicative filter of an IADL L is a filter.*

Proof. Let F be a positive implicative filter of IADL L. Let $x \to y \in F$ and $x \in F$ for all $y, z \in L$. Putting z = y in definition 3.4, we have that

$$x \to ((y \to y) \to y) = x \to (m \to y) = x \to y \in F$$

implies $y \in F$. Hence F is a filter. □

Remark 3.1. The converse of Theorem 3.6 may not be true. Consider Example 3.5, $\{m\}$ is a filter of L but it is not positive implicative filter of L, since

$$m \to ((z \to x) \to z) = m \in \{m\} \text{ and } z \notin \{m\}.$$

Now we give equivalent condition that every filter is a positive implicative filter.

Theorem 3.2. . *Let F be a filter of an IADL L. Then F is a positive implicaive filter of L if and only if for all $x, y \in L$ the following holds*
(P_2) $(x \to y) \to x \in F$ *implies* $x \in F$.

Proof. Let F be a filter of IADL L. Assume that F is a positive implicative filter of L and $(x \to y) \to x \in F$ for all $x, y \in L$. Now $m \in F$ and $m \to ((x \to y) \to x) = (x \to y) \to x \in F$ implies $x \in F$ (...by (P_1) of definition 3.4). Hence (P_2) holds.

Conversely, assume that F satisfies the condition (P_2). Let $x \in F$ and $x \to ((y \to z) \to y) \in F$ for all $y, z \in L$. Then $(y \to z) \to y \in F$ (by (F_2)) which implies $y \in F$ (by $((P_2))$. Hence F is a positive implicative filter of L. □

Theorem 3.3. *Let F be a non - empty subset of an IADL L. Then F is a positive implicative filter of L if and only if it is an implicative filter of L.*

Proof. Suppose F be a posive implicative filter of an IADL L. Let $x, y, z \in L$, $x \to (y \to z) \in F$ and $x \to y \in F$. We need to prove $x \to z \in F$. Now $x \to (y \to z) = y \to (x \to z) \leqslant (x \to y) \to (x \to (x \to z))$ (by 14 of theorem 2.10 and definition 2.9) . Since F is a positive implicative filter of L, it is a filter of L (by theorem 3.6) and applying Lemma 3.2 and definition of filter, we get $x \to (x \to z) \in F$. On the other hand using definition 2.9 and Theorem 2.10, we have

$$((x \to z) \to z) \to (x \to z) = x \to (((x \to z) \to z) \to z) = x \to (x \to z) \in F$$

(by theorem 2.10). This implies $x \to z \in F$ (by Theorem 3.8). Therefore, F is an implicative filter of L.

Conversely, suppose F is implicative filter of L such that $x, y, z \in L$. Then we can easily verify that F is filter of L. Let $x \in F$ and $x \to [(y \to z) \to y] \in F$. Since $x \in F$ and F is a filter, we get that $[(y \to z) \to y] \in F$. Since $y' \leqslant y \to z$, we have $(y \to z) \to y \leqslant y' \to y = y' \to (y' \to 0)$ (by Theorem 2.10). Since F is an implicative filter of L and hence filter of L, we get $y' \to (y' \to 0) \in F$ and $y' \to y' = m \in F$. This yields that $y = (y')' = y' \to 0 \in F$ and clearly $m \in F$. Hence, F is Positive implicative filter of L. □

In the following we discuss associative filters and some properties of associative filters in an IADL L.

Definition 3.3. Let L be an IADL and $x \in L$ be fixed. A subset F of L is called an associative filter of L with respect to x if it satisfies

(F_1) $m \in F$

(A_1) $x \to y \in F$ and $x \to (y \to z) \in F$ implies $z \in F$, for all $x, y \in L$.

An associative filter of L with respect to all $x \neq 0$ is called an associative filter of L.

Remark 3.2. (1) An associative filter with respect to 0 is equal to L.

(2) An associative filter with respect to m coincide with a filter.

Proof. Let F be an associative filter of L and $x, y, z \in L$.

(1) Clearly $F \subseteq L$. Let $z \in L$. Since $0 \to (y \to z) = m \in F$ and $0 \to y = m \in F$ implies $z \in F$ (by definition of associative filter), then $L \subseteq F$. Therefore, F = L.

(2) Clearly $m \in F$. Assume $x \in F$ and $x \to y \in F$. We need to prove $y \in F$. By definition of associative filter, $m \to (x \to y) = (x \to y) \in F$ and $m \to x = x \in F$ implies $y \in F$. Therefore, F is a filter of L. □

EXAMPLE 3.2. Let $L = \{0, x, y, z, w, m\}$ be a set with partial ordering

$$\leqslant= \{(0,0),(x,x),(y,y),(z,z),(w,w),(m,m),(0,y),(0,x),(0,m),(0,w),(0,z),(y,x),(y,m),(w,x),(w,m),(w,z),(z,m),\}.$$

Define a unary operation $''$ and a binary operation $\rightarrow$ on L by Table 3.3 and Table 3.4 respectively.

x	x'
0	m
x	w
y	z
z	y
w	x
m	0

Table 3.3.

$\rightarrow$	0	x	y	z	w	m
0	m	m	m	m	m	m
x	w	m	x	z	z	m
y	z	m	m	z	z	m
z	y	x	y	m	a	m
w	x	m	x	m	m	m
m	o	x	y	z	w	m

Table 3.4.

and define $\vee$ and $\wedge$ operation on L as follows

$x \vee y = (x \rightarrow y) \rightarrow y$

$x \wedge y = ((x \rightarrow y) \rightarrow x')'$, for all $x, y \in L$.

Then L is an IADl.

It is easy to verify that $F = \{m, x, y\}$ is an associative filter with respect to x and y but not with respect to z and w, since $z \rightarrow (y \rightarrow w) = z \rightarrow z = m \in F$, $z \rightarrow y = y \in F$ but $w \notin F$ and $w \rightarrow (y \rightarrow z) = w \rightarrow z = m \in F$ and $w \rightarrow y = x \in F$ but $z \notin F$.

PROPOSITION 3.1. *Every associative filter of an IADL L with respect to x contains x itself.*

PROOF. Let F be an associative filters of an IADL L and $x \in L$. If x = 0, m then $0 \rightarrow (y \rightarrow 0) = m \in F$ and $0 \rightarrow y \in F$ implies $0 \in F$ and $m \rightarrow (y \rightarrow m) \in F$ and $m \rightarrow y \in F$ implies $m \in F$. Assume $x \neq 0$. Since F is an associative filter of L with respect to x, we have $x \rightarrow (m \rightarrow x) = x \rightarrow x = m \in F$ and $x \rightarrow m = m \in F$. This implies from (A_1) that $x \in F$. □

THEOREM 3.4. *Every associative filter of an IADL L is a filter of L.*

PROOF. Let F be an associative filter of an IADL L, $x \in F$ and $x \rightarrow y \in F$. we need to prove $y \in F$. Since $m \rightarrow x = x \in F$ and $m \rightarrow (x \rightarrow y) = x \rightarrow y \in F$, it follows from (A_1) that $y \in F$. Hence, F is a filter of L. □

REMARK 3.3. The converse of theorem 3.14 may not be true. In example 3.5 $x \in \{m\}$ and $x \rightarrow y \in \{m\}$ implies $y \in \{m\}$. Therefore $\{m\}$ is a filter of L. But it is not an associative filter of L, since $x \rightarrow (y \rightarrow z) = x \rightarrow y = m \in \{m\}$, but $z \notin \{m\}$.

Now we give equivalent conditions that every filter is an assocaitve filter.

THEOREM 3.5. *Let F be a filter of an IADL L. Then F is an assocaitive filter of L if and only if it satisfies*
(A_2) $x \to (y \to z) \in F$ *implies* $(x \to y) \to z \in F$ *for all* $x, y, z \in L$.

PROOF. Let F be a filter of IADL L that satisfies the condition (A_2). Assume $x \to (y \to z) \in F$ and $x \to y \in F$. We need to prove $z \in F$. Now $x \to (y \to z) \in F$ implies $(x \to y) \to z \in F$ and $x \to y \in F$ these implies $z \in F$ (...by definition of filter F of L and (A_2)). Thus, F is an associative filter of L. Conversely, let F be an associative filter of an IADL L and let $x \to (y \to z) \in F$ for all $x, y, z \in L$. Now $x \to [(y \to z) \to ((x \to y) \to z)] = (y \to z) \to [(x \to y) \to (x \to z)] = (x \to y) \to [(y \to z) \to (x \to z)] = (x \to y) \to [(z \to y) \to (x \to y)] = (z \to y) \to m = m \in F$ which implies from (A_1) that $(x \to y) \to z \in F$. □

THEOREM 3.6. *Let F be a filter on an IADL L.Then F is an associative filter of L if and only if it satisfies*
(A_3) $x \to (x \to y) \in F$ *implies* $y \in F$ *for all* $x, y \in L$.

PROOF. Let F be a filter of an IADL L. It is enough to show (A_2) and (A_3) are equivalent. Assume (A_2), putting y = x in (A_2) and using condition of definition 2.14 we obtain (A_3). Assume (A_3) holds and let $x \to (y \to z) \in F$ for all $x, y, z \in L$. Using definition 2.9, we have

$(x \to (y \to z)) \to (x \to (x \to ((x \to y) \to z))) = (x \to (y \to z)) \to ((x \to y) \to (x \to (x \to z))) = (y \to (x \to z)) \to ((x \to y) \to (x \to (x \to z))) = (x \to y) \to ((y \to (x \to z)) \to (x \to (x \to z)) = (x \to y) \to (x \to ((x \to z) \to y) \to y) = (x \to y) \to (((x \to z) \to y) \to (x \to y)) = ((x \to z) \to y) \to ((x \to y) \to (x \to y) = ((x \to z) \to y) \to m = m \in F$. Therefore, $(x \to (y \to z)) \to (x \to (x \to ((x \to y) \to z))) = m \in F$. Since $(x \to (y \to z)) \in F$ and F is a filter, it follows that $(x \to (x \to ((x \to y) \to z))) \in F$ By using (A_3) ,we have $(x \to y) \to z \in F$. Hence, F is an associative filter. □

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B. Assaye: Department of Mathematics, College of science, Bahir Dar University, Bahir Dar, Ethiopia,
M. Alamneh: Department of Mathematics, Andhra University, Visakhapatnam, Andhra Pradesh 530003, India,
T. Mekonnen: Department of Mathematics, College of science, Bahir Dar University, Bahir Dar, Ethiopia
E-mail address: tilametekek@yahoo.com (Tilahun Mekonnen)