BULLETIN OF THE INTERNATIONAL MATHEMATICAL VIRTUAL INSTITUTE
ISSN (p) 2303-4874, ISSN (o) 2303-4955
www.imvibl.org /JOURNALS / BULLETIN
Vol. 8(2018), 553-559
DOI: 10.7251/BIMVI1803553J

Former
BULLETIN OF THE SOCIETY OF MATHEMATICIANS BANJA LUKA
ISSN 0354-5792 (o), ISSN 1986-521X (p)

$^*\lambda_\mu$-CONNECTEDNESS IN GENERALIZED TOPOLOGICAL SPACES

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ABSTRACT. In this paper, we introduce the concept of $^*\lambda_\mu$-connectedness in generalized topological spaces by means of $^*\lambda_\mu$-open sets and investigate their properties.

1. Introduction

In 1997, Császár [**2**] introduced the concept of a generalization of topological spaces, which is a generalized topological space. A generalized topology (briefly GT) μ on a non-empty set X is a collection of subsets of X such that $\phi \in \mu$ and μ is closed under arbitrary union. Elements of μ are called μ-open sets. A set X with a GT μ is called a generalized topological space (briefly GTS), denoted by (X, μ). If A is a subset of (X, μ), then $c_\mu(A)$ is the smallest μ-closed set containing A and $i_\mu(A)$ is the largest μ-open set contained in A. Clearly, A is μ-open if and only if $A = i_\mu(A)$ and A is μ-closed if and only if $A = c_\mu(A)$ [**4, 3**]. A GTS (X, μ) is called a strong generalized topological space if $X \in \mu$. The concept of γ-connectedness was also introduced by Császár, further studied by several authors including Shen [**10**] and Baskaran et al. [**1**]. In this paper, we introduce the concept of $^*\lambda_\mu$-connectedness in generalized topological spaces and give some characterizations of these spaces.

DEFINITION 1.1. ([**6**]) Let (X, μ) be a GTS and $A \subseteq X$. Then the subsets $\wedge_\mu(A)$ and $\vee_\mu(A)$ are defined as follows:

$$\wedge_\mu(A) = \begin{cases} \cap\{G : A \subseteq G, G \in \mu\} & \text{if there exists } G \in \mu \text{ such that } A \subseteq G; \\ X & \text{otherwise.} \end{cases}$$

2010 *Mathematics Subject Classification.* Primary 54 A 05.
Key words and phrases. $^*\lambda_\mu$-separateness, $^*\lambda_\mu$-connectedness, $^*\lambda_\mu$-closed, $^*\lambda_\mu$-open.

and

$$\vee_\mu(A) = \begin{cases} \cup\{H : H \subseteq A, H^c \in \mu\} & \text{if there exists } H^c \in \mu \textit{ suchthat } H \subseteq A; \\ \emptyset & \text{otherwise.} \end{cases}$$

DEFINITION 1.2. ([**6**]) In a GTS (X, μ), a subset B is called a $\wedge_\mu$-set (resp. $\vee_\mu$-set) if $B = \wedge_\mu(B)$ (resp. $B = \vee_\mu(B)$).

DEFINITION 1.3. ([**9**]) A subset A of a GTS (X, μ) is said to be λ_μ-closed set if $A = T \cap C$, where T is a $\wedge_\mu$-set and C is a μ-closed set. The complement of a λ_μ-closed set is called a λ_μ-open set.

For $A \subseteq X$, we denote by $c_{^*\lambda_\mu}(A)$ the intersection of all $^*\lambda_\mu-$closed subsets of X containing A.

DEFINITION 1.4. ([**8**]) Let (X, μ) be a GTS. A subset A of X is called a $^*\wedge_\mu$-set if $A =^* \wedge_\mu (A)$, where $^* \wedge_\mu (A) = \cap\{G : A \subset G, G \in \lambda_\mu O(X, \mu)\}$.

DEFINITION 1.5. A subset A of a GTS (X, μ) is called a $^*\lambda_\mu$-closed set if $A = T \cap C$, where T is a $^*\wedge_\mu$-set and C is λ_μ-closed. The complement of a $^*\lambda_\mu$-closed set is called a $^*\lambda_\mu$-open set.

We denote the collection of all λ_μ-open (resp.λ_μ-closed, $^*\lambda_\mu$-open. $^*\lambda_\mu$-closed) sets of X by $\lambda_\mu O(X, \mu)$ (resp.$\lambda_\mu C(X, \mu)$, $^*\lambda_\mu O(X, \mu)$, $^*\lambda_\mu C(X, \mu)$).

DEFINITION 1.6. A GTS (X, μ) is μ-connected [**10**] (γ-connected [**5**]) if there are no non-empty disjoint sets $U, V \in \mu$ such that $U \cup V = X$.

DEFINITION 1.7. ([**1**]) Two subsets A and B in a GTS (X, μ) are said to be μ-separated if and only if $A \cap c_\mu(B) = \emptyset$ and $B \cap c_\mu(A) = \emptyset$.

DEFINITION 1.8. ([**7**]) If (X, μ) is a GTS and Y is a subset of X, then the collection $\mu|_Y = \{U \cap Y : U \in \mu\}$ is a GT on Y called the subspace generalized topology and $(Y, \mu|_Y)$ is the subspace of X.

2. $^*\lambda_\mu$-Separateness

In this section, we introduce the notion of $^*\lambda_\mu$-separated sets and discuss its properties.

DEFINITION 2.1. Two subsets A and B of a GTS (X, μ) are said to be $^*\lambda_\mu$-separated if and only if $A \cap c_{^*\lambda_\mu}(B) = \emptyset$ and $c_{^*\lambda_\mu}(A) \cap B = \emptyset$.

From the fact that $c_{^*\lambda_\mu}(A) \subseteq c_\mu(A)$, for every subset A of (X, μ), every μ-separated set is $^*\lambda_\mu$-separated. But the converse may not be true as shown in the following example.

EXAMPLE 2.1. Let $X = R$ and $\mu = \{\emptyset, Q\}$, where R and Q denote the set of all real numbers and rational numbers, respectively. The family of all $^*\lambda_\mu$-closed sets is $\{\emptyset, Q, R\backslash Q, R\}$. Then $Q \cap c_{^*\lambda_\mu}(R\backslash Q) = c_{^*\lambda_\mu}(Q) \cap (R\backslash Q) = \emptyset$ but $c_\mu(Q) \cap (R\backslash Q) \neq \emptyset$. Hence Q and $R\backslash Q$ are $^*\lambda_\mu$-separated but not μ-separated. $\square$

REMARK 2.1. Since $A\cap B \subseteq A\cap c_{^*\lambda_\mu}(B)$, $^*\lambda_\mu$-separated sets are always disjoint. The converse may not be true in general. □

EXAMPLE 2.2. Let $X = R$ and $\mu = \{\emptyset, Q\}$. The subsets $\{\sqrt{2}, \sqrt{3}\}$, $\{\sqrt{5}, \sqrt{7}\}$ are disjoint but not $^*\lambda_\mu$-separated.

□

THEOREM 2.1. *Let A and B be non-empty subsets in a GTS (X, μ). The following statements are hold:*

(i) *If A and B are $^*\lambda_\mu$-separated, $A_1 \subseteq A$ and $B_1 \subseteq B$, then A_1 and B_1 are also $^*\lambda_\mu$-separated.*

(ii) *If A and B are $^*\lambda_\mu$-closed sets such that $A \cap B = \emptyset$, then A and B are $^*\lambda_\mu$-separated.*

(iii) *If A and B are $^*\lambda_\mu$-open, $H = A \cap (X\backslash B)$ and $G = B \cap (X\backslash A)$, then H and G are $^*\lambda_\mu$-separated.*

PROOF. (i) Since $A_1 \subseteq A$, $c_{^*\lambda_\mu}(A_1) \subseteq c_{^*\lambda_\mu}(A)$. Therefore $B \cap c_{^*\lambda_\mu}(A) = \emptyset$ implies $B_1 \cap c_{^*\lambda_\mu}(A) = \emptyset$ and $B_1 \cap c_{^*\lambda_\mu}(A_1) = \emptyset$. Similarly $A_1 \cap c_{^*\lambda_\mu}(B_1) = \emptyset$. Hence A_1 and B_1 are $^*\lambda_\mu$-separated.

(ii) Since A and B are $^*\lambda_\mu$-closed, $A = c_{^*\lambda_\mu}(A)$ and $B = c_{^*\lambda_\mu}(B)$. Now $A \cap B = \emptyset$ implies $c_{^*\lambda_\mu}(A) \cap B = \emptyset$ and $c_{^*\lambda_\mu}(B) \cap A = \emptyset$. Hence A and B are $^*\lambda_\mu$-separated.

(iii) Since $H \subseteq (X\backslash B)$, $c_{^*\lambda_\mu}(H) \subseteq c_{^*\lambda_\mu}(X\backslash B) = X\backslash B$ and hence $c_{^*\lambda_\mu}(H) \cap B = \emptyset$. Also $G \subseteq B$ implies $c_{^*\lambda_\mu}(H) \cap G = \emptyset$. Similarly, $H \cap c_{^*\lambda_\mu}(G) = \emptyset$. Hence H and G are $^*\lambda_\mu$-separated. □

COROLLARY 2.1. *Let A and B be non-empty sets in a GTS (X, μ). The following statements are hold:*

(i) *If A and B are $^*\lambda_\mu$-open sets such that $A \cap B = \emptyset$, then A and B are $^*\lambda_\mu$-separated.*

(ii) *If A and B are $^*\lambda_\mu$-closed, $H = A \cap (X\backslash B)$ and $G = B \cap (X\backslash A)$, then H and G are $^*\lambda_\mu$-separated.*

THEOREM 2.2. *The subsets A and B of a GTS (X, μ) are $^*\lambda_\mu$-separated if and only if there exist $U, V \in {^*\lambda_\mu}O(X, \mu)$ such that $A \subseteq U$, $B \subseteq V$ and $A \cap V = \emptyset$, $B \cap U = \emptyset$.*

PROOF. Let A and B be $^*\lambda_\mu$-separated sets. Let $V = X\backslash c_{^*\lambda_\mu}(A)$ and $U = X\backslash c_{^*\lambda_\mu}(B)$. Then $U, V \in {^*\lambda_\mu}O(X, \mu)$ such that $A \subset U$, $B \subseteq V$ and $A \cap V = \emptyset$, $B \cap U = \emptyset$. On the otherhand, let $U, V \in {^*\lambda_\mu}O(X, \mu)$ such that $A \subseteq U$, $B \subseteq V$ and $A \cap V = \emptyset$, $B \cap U = \emptyset$. Since $X\backslash V$ and $X\backslash U$ are $^*\lambda_\mu$-closed, $c_{^*\lambda_\mu}(A) \subseteq c_{^*\lambda_\mu}(X\backslash V) = X\backslash V \subseteq X\backslash B$. Thus, $c_{^*\lambda_\mu}(A) \cap B = \emptyset$. Similarly, $A \cap c_{^*\lambda_\mu}(B) = \emptyset$. Hence A and B are $^*\lambda_\mu$-separated sets. □

3. $^*\lambda_\mu$-Connectedness

In this section, we introduce the notion of $^*\lambda_\mu$-connectedness and discuss their properties.

DEFINITION 3.1. A subset S of a GTS (X,μ) is said to be ${}^*\lambda_\mu$-connected if there exist no ${}^*\lambda_\mu$-separated subsets A and B and $S = A \cup B$. Otherwise S is said to be ${}^*\lambda_\mu$-disconnected.

It is clear that each ${}^*\lambda_\mu$-connected set is μ-connected. The converse may not be true in general as shown in the following example. In other world, each μ-disconnected is ${}^*\lambda_\mu$-disconnected.

EXAMPLE 3.1. Let $X = [1,2]$ and $\mu = \{\emptyset, \{1\}, \{1,2\}\}$. The family of all ${}^*\lambda_\mu$-closed sets is $\{\emptyset, \{1\}, \{2\}, \{1,2\}, (1,2), [1,2), (1,2], [1,2]\}$. Thus, $\{1,2\}$ is μ-connected but not ${}^*\lambda_\mu$-separated.

□

THEOREM 3.1. *A GTS (X,μ) is ${}^*\lambda_\mu$-disconnected if and only if there exists a non-empty proper ${}^*\lambda_\mu$-clopen subset.*

PROOF. Assume that (X,μ) is ${}^*\lambda_\mu$-disconnected. There exist ${}^*\lambda_\mu$-separated sets A and B such that $A \cup B = X$, $A \cap B = \emptyset$. Hence $A = X \backslash B$ and $B = X \backslash A$. Since $A \cup B = X$ and $B \subseteq c_{^*\lambda_\mu}(B)$, $X \subseteq A \cup c_{^*\lambda_\mu}(B)$. But $A \cup c_{^*\lambda_\mu}(B) \subseteq X$. Thus, $A \cup c_{^*\lambda_\mu}(B) = X$. We have $A \cap c_{^*\lambda_\mu}(B) = \emptyset$ and $B \cap c_{^*\lambda_\mu}(A) = \emptyset$ which implies $A = X \backslash c_{^*\lambda_\mu}(B)$ and $B = X \backslash c_{^*\lambda_\mu}(A)$. Since $c_{^*\lambda_\mu}(A)$ and $c_{^*\lambda_\mu}(B)$ are ${}^*\lambda_\mu$-closed, $X \backslash c_{^*\lambda_\mu}(A)$ and $X \backslash c_{^*\lambda_\mu}(B)$ are ${}^*\lambda_\mu$-open. Thus, A and B are ${}^*\lambda_\mu$-open. Since $A = X \backslash B$ and $B = X \backslash A$, A and B are ${}^*\lambda_\mu$-closed. Conversely, assume that there exists non-empty proper ${}^*\lambda_\mu$-clopen subset A of X. Let $B = X \backslash A$. Then $A \cap B = \emptyset$ and $A \cup B = X$. Since $A \cap B = \emptyset$, $c_{^*\lambda_\mu}(A) \cap B = \emptyset$ and $A \cap c_{^*\lambda_\mu}(B) = \emptyset$. Thus, A and B are ${}^*\lambda_\mu$-separated. Hence (X,μ) is ${}^*\lambda_\mu$-disconnected. □

THEOREM 3.2. *A GTS (X,μ) is ${}^*\lambda_\mu$-disconnected if and only if any one of the following statements holds:*
(i) *X is the union of two non-empty disjoint ${}^*\lambda_\mu$-open sets.*
(ii) *X is the union of two non-empty disjoint ${}^*\lambda_\mu$-closed sets.*

PROOF. Assume that (X,μ) is ${}^*\lambda_\mu$-disconnected. By Theorem 3.1, there exists a non-empty proper ${}^*\lambda_\mu$-clopen subset A of X. Also, $A \cup (X \backslash A) = X$. Hence A and $X \backslash A$ satisfy the conditions (i) and (ii). Conversely, assume that $A \cup B = X$ and $A \cap B = \emptyset$, where A and B are non-empty ${}^*\lambda_\mu$-open sets. Then $A = X \backslash B$ is ${}^*\lambda_\mu$-closed. Since B is non-empty, A is a proper subset of X. Thus, A is a non-empty proper ${}^*\lambda_\mu$-clopen subset of X. By Theorem 3.1, X is ${}^*\lambda_\mu$-disconnected. Let $X = C \cup D$ and $C \cap D = \emptyset$, where C and D are non-empty ${}^*\lambda_\mu$-closed sets. Then $C = X \backslash D$ so that C is ${}^*\lambda_\mu$-open. Since D is non-empty, C is a proper ${}^*\lambda_\mu$-clopen subset of X. By Theorem 3.1, X is ${}^*\lambda_\mu$-disconnected. □

THEOREM 3.3. *If E is a ${}^*\lambda_\mu$-connected subset of a GTS (X,μ) such that $E \subseteq A \cup B$, where A and B are ${}^*\lambda_\mu$-separated sets, then either $E \subseteq A$ or $E \subseteq B$.*

PROOF. Since A and B are ${}^*\lambda_\mu$-separated sets, $A \cap c_{^*\lambda_\mu}(B) = \emptyset$ and $B \cap c_{^*\lambda_\mu}(A) = \emptyset$. $E \subseteq A \cup B$ implies $E = E \cap (A \cup B) = (E \cap A) \cup (E \cap B)$. Suppose $E \cap A \neq \emptyset$ and $E \cap B \neq \emptyset$. Then $(E \cap A) \cap c_{^*\lambda_\mu}(E \cap B) \subseteq (E \cap A) \cap (c_{^*\lambda_\mu}(E) \cap c_{^*\lambda_\mu}(B)) = (E \cap c_{^*\lambda_\mu}(E)) \cap (A \cap c_{^*\lambda_\mu}(B)) = \emptyset$. Similarly, $(E \cap B) \cap c_{^*\lambda_\mu}(E \cap A) = \emptyset$.

Hence $E \cap A$ and $E \cap B$ are ${}^*\lambda_\mu$-separated. Thus, E is ${}^*\lambda_\mu$-disconnected, which is a contradiction. Hence at least one of the sets $E \cap A$ and $E \cap B$ is empty. If $E \cap A = \emptyset$, then $E = E \cap B$ which implies that $E \subseteq B$. Similarly if $E \cap B = \emptyset$, then $E \subseteq A$. Therefore, either $E \subseteq A$ or $E \subseteq B$. □

Corollary 3.1. *If E is a ${}^*\lambda_\mu$-connected subset of a GTS (X, μ) such that $E \subseteq A \cup B$, where A and B are disjoint ${}^*\lambda_\mu$-open (resp, ${}^*\lambda_\mu$-closed) subsets of X, then A and B are ${}^*\lambda_\mu$-separated.*

Proof. Since $A \subseteq X \backslash B$, $c_{{}^*\lambda_\mu}(A) \subseteq c_{{}^*\lambda_\mu}(X \backslash B) = X \backslash B$. Thus, $B \cap c_{{}^*\lambda_\mu}(A) = \emptyset$. Similarly, $A \cap c_{{}^*\lambda_\mu}(B) = \emptyset$. Hence A and B are ${}^*\lambda_\mu$-separated. □

Theorem 3.4. *If E is a ${}^*\lambda_\mu$-connected subset of a GTS (X, μ) and C is a subset such that $E \subseteq C \subseteq c_{{}^*\lambda_\mu}(E)$, then C is also ${}^*\lambda_\mu$-connected.*

Proof. Suppose that C is not ${}^*\lambda_\mu$-connected. There exist ${}^*\lambda_\mu$-separated sets A and B such that $C = A \cup B$. Since $E \subseteq C$, $E \subseteq A \cup B$. By Theorem 3.3, $E \subseteq A$ or $E \subseteq B$. Let $E \subseteq A$, then $c_{{}^*\lambda_\mu}(E) \subseteq c_{{}^*\lambda_\mu}(A)$ which implies $c_{{}^*\lambda_\mu}(E) \cap B \subseteq c_{{}^*\lambda_\mu}(A) \cap B = \emptyset$. Since $C \subseteq c_{{}^*\lambda_\mu}(E)$, $B \subseteq C \subseteq c_{{}^*\lambda_\mu}(E)$ and hence $c_{{}^*\lambda_\mu}(E) \cap B = B$. Thus, $c_{{}^*\lambda_\mu}(E) \cap B = \emptyset$ and $c_{{}^*\lambda_\mu}(E) \cap B = B$ imply $B = \emptyset$. Similarly, if we consider $E \subseteq B$, we obtain $A = \emptyset$, which contradicts A and B are non-empty. Therefore C is ${}^*\lambda_\mu$-connected. □

Corollary 3.2. *If E is a ${}^*\lambda_\mu$-connected subset of a GTS (X, μ), $c_{{}^*\lambda_\mu}(E)$ is also ${}^*\lambda_\mu$-connected.*

Proof. This is obvious by Theorem 3.4. □

Theorem 3.5. *Let E be a subset of a GTS (X, μ). If any two points of E are contained in some ${}^*\lambda_\mu$-connected subset of E, E is a ${}^*\lambda_\mu$-connected subset of X.*

Proof. Suppose E is not ${}^*\lambda_\mu$-connected. Then there exist non-empty subsets A and B of X such that $A \cap c_{{}^*\lambda_\mu}(B) = \emptyset$, $B \cap c_{{}^*\lambda_\mu}(A) = \emptyset$ and $E = A \cup B$. Since A, B are non-empty, there exists a point $a \in A$ and a point $b \in B$. By hypothesis, a and b must be contained in some ${}^*\lambda_\mu$-connected subset F of E. Since $F \subseteq A \cup B$ and F is ${}^*\lambda_\mu$-connected, either $F \subseteq A$ or $F \subseteq B$. It follows that either a, $b \in A$ or $a, b \in B$. Let a, $b \in A$. Then $A \cap B \neq \emptyset$, which is a contradiction. Hence E is a ${}^*\lambda_\mu$-connected subset of X. □

Theorem 3.6. *The union of any family of ${}^*\lambda_\mu$-connected sets having a non-empty intersection is a ${}^*\lambda_\mu$-connected set.*

Proof. Let $\{E_\alpha\}$ be any family of ${}^*\lambda_\mu$-connected sets such that $\cap\{E_\alpha\} \neq \emptyset$. Let $E = \cup\{E_\alpha\}$. Suppose E is not ${}^*\lambda_\mu$-connected. Therefore, there exist ${}^*\lambda_\mu$-separated sets A and B such that $E = A \cup B$. Since $\cap\{E_\alpha\} \neq \emptyset$, $x \in \cap\{E_\alpha\}$. Then x belongs to each E_α and so $x \in E$. Consequently, $x \in A$ or $x \in B$. Without loss of generality, assume that $x \in A$. Then $E_\alpha \subseteq A$ for each α. Hence $\cup E_\alpha \subseteq A$ and so $E \subseteq A$. Thus, $A \cup B \subseteq A$. Therefore $A = E$ which implies $B = \emptyset$ which is a contradiction. Thus, E is ${}^*\lambda_\mu$-connected. □

THEOREM 3.7. *The union of any family of* $^*\lambda_\mu$*-connected subsets of a GTS* (X,μ) *with the property that one of the members of the family, intersects every other members is a* $^*\lambda_\mu$*-connected set.*

PROOF. Let $\{E_\alpha\}$ be any family of $^*\lambda_\mu$-connected sets of a GTS (X,μ) with the property that one of the member say, E_{α_0} intersects every other members. By Theorem 3.6, $E_{\alpha_0} \cup E_\alpha$ is $^*\lambda_\mu$-connected. Now, let E_{α_p} and E_{α_q} be any two members of the family. Then $E_{\alpha_0} \cap E_{\alpha_p} \neq \emptyset$, $E_{\alpha_0} \cap E_{\alpha_q} \neq \emptyset$ and hence $(E_{\alpha_0} \cap E_{\alpha_p}) \cup (E_{\alpha_0} \cap E_{\alpha_q}) = E_{\alpha_0} \cup (E_{\alpha_p} \cap E_{\alpha_q}) \neq \emptyset$. By Theorem 3.6, $\cup(E_{\alpha_0} \cap E_\alpha)$ for each α is $^*\lambda_\mu$-connected. Hence $\cup E_\alpha$ is $^*\lambda_\mu$-connected. □

THEOREM 3.8. *If* $A \subseteq B \cup C$ *such that* A *is a non-empty* $^*\lambda_\mu$*-connected set in a GTS* (X,μ) *and* B, C *are* $^*\lambda_\mu$*-separated, then one of the following conditions holds:*
(i) $A \subseteq B$ *and* $A \cap C = \emptyset$.
(ii) $A \subseteq C$ *and* $A \cap B = \emptyset$.

PROOF. This is obvious by Theorem 3.3. □

DEFINITION 3.2. Let (X,μ) and $(X,\mu^{'})$ be two GTS. A mapping $f : (X,\mu) \to (Y,\mu^{'})$ is said to be $(^*\lambda_\mu, \mu^{'})$-continuous if for each $\mu^{'}$-open set V, $f^{-1}(V)$ is $^*\lambda_\mu$-open.

THEOREM 3.9. *Let* $f : (X,\mu) \to (Y,\mu^{'})$ *be a* $(^*\lambda_\mu, \mu^{'})$*-continuous function. If* K *is* $^*\lambda_\mu$*-connected in* X*, then* $f(K)$ *is* $\mu^{'}$*-connected in* Y*.*

PROOF. Suppose that $f(K)$ is $\mu^{'}$-disconnected in Y. There exist $\mu^{'}$-separated sets G and H of Y such that $f(K) = G \cup H$. Set $A = K \cap f^{-1}(G)$ and $B = K \cap f^{-1}(H)$. Since $f(K) = G \cup H$, $K \cap f^{-1}(G) \neq \emptyset$ and hence $A \neq \emptyset$. Similarly, $B \neq \emptyset$. Now, $A \cap B = (K \cap f^{-1}(G)) \cap (K \cap f^{-1}(H)) = K \cap (f^{-1}(G) \cap f^{-1}(H)) = K \cap (f^{-1}(G \cap H)) = \emptyset$. Thus, $A \cap B = \emptyset$ and $A \cup B = K$. Now, $A \cap c_{^*\lambda_\mu}(B) \subseteq f^{-1}(G) \cap c_{^*\lambda_\mu}(f^{-1}(H))$. Since f is $(^*\lambda_\mu, \mu^{'})$-continuous, $A \cap c_{^*\lambda_\mu}(B) \subseteq f^{-1}(G) \cap f^{-1}(c_{\mu'}(H)) \subseteq f^{-1}(G \cap c_{\mu'}(H)) = \emptyset$. Therefore, $A \cap c_{^*\lambda_\mu}(B) = \emptyset$. Similarly, $B \cap c_{^*\lambda_\mu}(A) = \emptyset$. Thus, A and B are $^*\lambda_\mu$-separated in X which is a contradiction. Therefore $f(K)$ is $\mu^{'}$-connected in Y. □

COROLLARY 3.3. *Let* $f : (X,\mu) \to (Y,\mu^{'})$ *be a* $(^*\lambda_\mu, \mu^{'})$*-continuous surjection. If* K *is* μ'*-disconnected in* Y*, then* $f^{-1}(K)$ *is* $^*\lambda_\mu$*-disconnected in* X*.*

PROOF. Let $f^{-1}(K)$ be not $^*\lambda_\mu$-disconnected in X. Then $f^{-1}(K)$ is $^*\lambda_\mu$-connected in X and by Theorem 3.10, $f(f^{-1}(K)) = K$ is μ'-connected. Hence K is not μ'-disconnected in Y. Therefore, the proof is completed. □

Acknowledgement

We would like to thank the referees for his suggestions to improve the presentation of the paper.

References

[1] R. Baskaran, M. Murugalingam and D. Sivaraj. Separated sets in generalized topological spaces. *J. Adv. Res. Pure Math.*, **2**(1)(2010), 74–83.

[2] Á. Császár. Generalized open sets. *Acta Math.Hungar.*, **75**(1-2) (1997), 65–87.

[3] Á. Császár. On the γ-interior and γ-closure of a set. *Acta Math.Hungar.*, **80**(1-2) (1998), 89–93.

[4] Á. Császár. Generalized open sets in generalized topologies. *Acta Math.Hungar.*,**106**(1-2)(2005), 53–66.

[5] Á. Császár. γ-connected spaces. *Acta Math.Hungar.*, **101**(4)(2003), 273–279.

[6] E. Ekici and B. Roy. New generalized topologies on generalized topological spaces due to Császár. *Acta Math.Hungar.*, **132**(1-2) (2011), 117–124.

[7] J. Li. Generalized topologies generated by subbases. *Acta Math.Hungar.*, **114** (1-2)(2007), 1–12.

[8] P. Jeyanthi, P. Nalayini and T. Noiri. $^*\wedge_\mu$- sets and $^*\vee_\mu$- sets in generalized topological spaces. Bol. Soc. Paran. Mat. (3s), **35**(1)(2017), 33–41.

[9] B. Roy and E. Ekici. On $(\wedge, \mu)$-closed sets in generalized topological spaces. *Methods Func. Anal. Top.*, **17**(2)(2011), 174–179.

[10] R. X. Shen. A note on generalized connectedness. *Acta Math.Hungar.*, **122**(3)(2009), 231–235.

Received by editors 07.10.2017; Revised version 01.10.2018; Available online 15.10.2018.

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