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ORTHOGONAL DERIVATIONS ON $\Gamma-$SEMIRINGS

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Abstract. In this paper, we introduce the notion of orthogonal derivations on $\Gamma-$semirings. Some characterizations of semi prime $\Gamma-$semirings are obtained by means of orthogonal derivations. And also obtained necessary and sufficient conditions for two derivations to be orthogonal.

1. Introduction

Semiring, the best algebraic structure, which is a common generalization of rings and distributive lattices was first introduced by American mathematician Vandiver [**19**] in 1934 but non trivial examples of semirings have appeared in the earlier studies on the theory of commutative ideals of rings by German mathematician Richard Dedekind in 19th century. Semiring is an universal algebra with two binary operations called addition and multiplication, where one of them distributive over the other, bounded distributive lattices are commutative semirings which are both additively idempotent and multiplicatively idempotent. A natural example of semiring is the set of all natural numbers under usual addition and multiplication of numbers. In particular, if I is the unit interval on the real line then $(I, \max, \min)$ is a semiring in which 0 is the additive identity and 1 is the mutilative identity. The theory of rings and the theory of semigroups have considerable impact on the development of the theory of semirings. In structure, semirings lie between semigroups and rings. The study of rings shows that multiplicative structure of ring is independent of additive structure whereas in semiring multiplicative structure of semiring is not independent of additive structure of semiring. Additive and multiplicative structures of a semiring play an important role in determining the structure of a semiring. Semiring, as the basic algebraic structure, was used in the areas of

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theoretical computer science as well as in the solutions of graph theory and optimization theory and in particular for studying automata, coding theory and formal languages. Semiring theory has many applications in other branches. The notion of $\Gamma-$ring was introduced by Nobusawa [**15**] as a generalization of ring in 1964. Sen [**17**] introduced the notion of $\Gamma-$semigroup in 1981. The notion of Ternary algebraic system was introduced by Lehmer [**5**] in 1932, Lister [**6**] introduced ternary ring. Dutta & Kar [**3**] introduced the notion of ternary semiring which is a generalization of ternary ring and semiring. In 1995, Murali Krishna Rao [**7, 8**] introduced the notion of $\Gamma-$semiring which is a generalization of $\Gamma-$ring, ring, ternary semiring and semiring. After the paper [**7, 8**] was published, many mathematicians obtained interesting results on $\Gamma-$semirings. Murali Krishna Rao and Venkteswarlu [**9**] introduced the unity element in $\Gamma-$semiring and studied properties of $\Gamma-$incline and field $\Gamma-$semiring.

Over the last few decades several authors have investigated the relationship between the commutativity of ring R and the existence of certain specified derivations of R. The first result in this direction is due to Posner [**16**] in 1957. In the year 1990, Bresar and Vukman [**2**] established that a prime ring must be a commutative if it admits a non zero left derivation. The notion of derivation of ring is useful for characterization of rings.

The notion of derivation of prime $\Gamma-$semirings was introduced by Javed et al. [**4**]. Suganthameena et al. [**18**] introduced the concept of orthogonal derivations on semirings. The concepts of $(f, g)-$derivations, $d_a$derivations and derivations of fuzzy ideals of ordered $\Gamma-$semirings were introduced by Murali Krishna Rao et al. [**10, 11, 12**]. Venkateswarlu et al. [**13**] orthogonal reverse derivations on $\Gamma-$semirings was studied.

In this paper, we introduce the notion of orthogonal derivations on $\Gamma-$semirings. Some characterizations of semi prime $\Gamma-$semirings are obtained by means of orthogonal derivations. And also obtained necessary and sufficient conditions for two derivations to be orthogonal.

2. Preliminaries

In this section, we recall some important definitions which are necessary for this paper.

DEFINITION 2.1. A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called a semiring provided

(i) addition is a commutative operation.
(ii) multiplication distributes over addition both from the left and from the right.
(iii) there exists $0 \in S$ such that $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$ for all $x \in S$.

DEFINITION 2.2. Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M as a $\Gamma-$semiring,if there exists a mapping $M \times \Gamma \times M \rightarrow M$ is written (x, α, y) as $x\alpha y$ such that it satisfying the following axioms for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$

(i) $x\alpha(y+z) = x\alpha y + x\alpha z$
(ii) $(x+y)\alpha z = x\alpha z + y\alpha z$
(iii) $x(\alpha+\beta)y = x\alpha y + x\beta y$
(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

Every semiring R is a $\Gamma-$semiring with $\Gamma = R$ and ternary operation $x\gamma y$ as the usual semiring multiplication.

We illustrate the definition of $\Gamma-$semiring by the following example

Example 2.1. Let S be a semiring and $M_{p,q}(S)$ denote the additive abelian semigroup of all $p \times q$ matrices with identity element whose entries are from S. Then $M_{p,q}(S)$ is a $\Gamma-$semiring with $\Gamma = M_{p,q}(S)$ ternary operation is defined by $x\alpha z = x(\alpha^t)z$ as the usual matrix multiplication, where α^t denotes the transpose of the matrix α, for all x, y and $\alpha \in M_{p,q}(S)$.

A $\Gamma-$semiring M is said to have zero element if there exists an element $0 \in M$ such that $0+x = x = x+0$ and $0\alpha x = x\alpha 0 = 0$, for all $x \in M, \alpha \in \Gamma$. A $\Gamma-$semiring M is said to be commutative $\Gamma-$semiring if $x\alpha y = y\alpha x$, for all $x, y \in M$ and $\alpha \in \Gamma$. An element $a \in M$ is said to be an idempotent of M if there exists $\alpha \in \Gamma$ such that $a = a\alpha a$ and $a + a = a$. Every element of M is an idempotent of M then M is said to be idempotent $\Gamma-$semiring M. An element $1 \in M$ is said to be an unity if for each $x \in M$ there exists $\alpha \in \Gamma$ such that $x\alpha 1 = 1\alpha x = x$.

A non-empty subset A of $\Gamma-$semiring M is called a $\Gamma-$subsemiring M if $(A,+)$ is a subsemigroup of $(M,+)$ and $a\alpha b \in A$ for all $a, b \in A$ and $\alpha \in \Gamma$. A $\Gamma-$semiring M is said to be prime if $a\Gamma M\Gamma b = 0$ then $a = 0$ or $b = 0$, for all $a, b \in M$. A $\Gamma-$semiring M is said to be 2-torsion free if $2x = 0$ then $x = 0$, for all $x \in M$. A$\Gamma-$semirings M is said to be semiprime if $a\Gamma M\Gamma a = 0$ then $a = 0$, for all $a \in M$. Every prime $\Gamma-$semiring is obviously semiprime. We write $[x,y]_\alpha = x\alpha y - y\alpha x$. For commutative $\Gamma-$semirings,$[x,y]_\alpha = 0$, for every $x, y \in M$ and $\alpha \in \Gamma$. An additive mapping d from M into M is called a derivation if $d(x\alpha y) = d(x)\alpha y + x\alpha d(y)$, for all $x, y \in M, \alpha \in \Gamma$.

3. Orthogonal derivation of Γ−semirings

In this section, we introduce the notion of orthogonal derivations on Γ - semirings. Some characterizations of semi prime $\Gamma-$semirings are obtained by means of orthogonal derivations. And also obtained necessary and sufficient conditions for two derivations to be orthogonal.

Definition 3.1. Let M be a $\Gamma-$semiring. The derivations d and g of M into M is said to be orthogonal if $d(x)\Gamma M\Gamma g(y) = 0 = g(x)\Gamma M\Gamma d(y)$, for all $x, y \in M$.

Example 3.1. Let $M = \Gamma = \{0, a, b, c\}$ and define addition and multiplication as follows

+	0	a	b	c
0	0	a	b	c
a	a	b	c	c
b	b	c	c	c
c	c	c	c	c

α	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	b	c	c
c	0	c	c	c

.

Then M is a $\Gamma-$semiring. Define d and g of M into itself such that

$$d(x) \;=\; \begin{cases} 0, & \text{if } x=0 \\ c, & \text{if } x=a,b,c \end{cases} \quad \text{and } g(x) \;=\; \begin{cases} 0, & \text{if } x=0 \\ c, & \text{if } x=b,c \\ b, & \text{if } x=a \end{cases}.$$

Clearly d and g are derivations on M. Let $M_1 = M \times M$ and $\Gamma_1 = \Gamma \times \Gamma$. Define addition and multiplication on M_1 and Γ_1 on the following ray: for any $x_1, x_2, y_1, y_2 \in M$ and $\alpha, \beta \in \Gamma$ let be

$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1 \; x_2 + y_2)$$
$$(x_1, x_2)(\alpha, \beta)(y_1, y_2) = (x_1 \alpha y_1 \; x_2 \beta y_2).$$

Then M_1 under these conditions is a Γ_1-semiring.

Now define $d_1 : M_1 \to M_1$ and $g_1 : M_1 \to M_1$ by

$$d_1(x, y) = (d(x), 0) \text{ and } g_1(x, y) = (0, g(y))$$

respectively. Then d_1 and g_1 are orthogonal.

THEOREM 3.1. *Let a and b be two elements of $2-$torsion free semiprime $\Gamma-$semiring M. Then the following are equivalent*

(i) $a\Gamma x\Gamma b = 0$.
(ii) $b\Gamma x\Gamma a = 0$.
(iii) $a\Gamma x\Gamma b + b\Gamma x\Gamma a = 0$, *for all* $x \in M$.

If one of these conditions are fulfilled then $a\Gamma b = b\Gamma a = 0$.

PROOF. Let a and b be two elements of 2-torsion free semiprime $\Gamma-$semiring M.

(i)$\Rightarrow$(ii) : Assume $a\Gamma x\Gamma b = 0$, for all $x \in M$.
Pre and post multiplying by $b\Gamma x\Gamma$ and $\Gamma x\Gamma a$ then
$(b\ \Gamma x\Gamma a)\Gamma x\Gamma\ (b\Gamma x\Gamma a) = 0$.
Therefore $b\Gamma x\Gamma a = 0$, since M is a semiprime, for all $x \in M$.

(ii)$\Rightarrow$(i) : Suppose $b\Gamma x\Gamma a = 0$, for all $x \in M$.
Pre and post multiplying by $a\Gamma x\Gamma$ and $\Gamma x\Gamma b$ then
$(a\Gamma x\Gamma b)\Gamma x\Gamma(a\Gamma x\Gamma b) = 0$.
Therefore $a\Gamma x\Gamma b = 0$, for all $x \in M$.

(ii)$\Rightarrow$(iii) : Suppose $b\Gamma x\Gamma a = 0$, for all $x \in M$.
Then $a\Gamma x\Gamma b = 0$, for all $x \in M$.
There fore $a\Gamma x\Gamma b + b\Gamma x\Gamma a = 0$, for all $x \in M$.

(iii)$\Rightarrow$(i): Suppose $a\Gamma x\Gamma b + b\Gamma x\Gamma a = 0$, for all $x \in M \;\cdots\; (1)$
Pre multiplying by $b\Gamma x\Gamma$ then $b\Gamma x\Gamma a\Gamma x\Gamma b + b\Gamma x\Gamma b\Gamma x\Gamma a = 0$.
Again pre multiplying by $a\Gamma x\Gamma$ then
$(a\Gamma x\Gamma b)\Gamma x\Gamma(a\Gamma x\Gamma b) + (a\Gamma x\Gamma b)\Gamma x\Gamma(b\Gamma x\Gamma a) = 0 \;\cdots\; (2)$
Post multiplying (1) by $\Gamma x\Gamma a$ then $a\Gamma x\Gamma b\Gamma x\Gamma a + b\Gamma x\Gamma a\Gamma x\Gamma a = 0$.
Again post multiplying by $\Gamma x\Gamma b$ then
$(a\Gamma x\Gamma b)\Gamma x\Gamma(a\Gamma x\Gamma b) + (b\Gamma x\Gamma a)\Gamma x\Gamma(a\Gamma x\Gamma b) = 0 \;\cdots\; (3)$
Adding (2) and (3) then using (1), we get
$2(a\Gamma x\Gamma b)\Gamma x\Gamma(a\Gamma x\Gamma b) = 0$, for all $x \in M$.

Since M is a 2-torsion free and semi prime then $a\Gamma x\Gamma b=0$, for all $x\in M$.

Let $a\Gamma x\Gamma b=0$, for all $x\in M$.
Pre and post multiplying by $b\Gamma$ and Γa respectively then $b\Gamma a\Gamma x\Gamma b\Gamma a=0$.
Since M is a semiprime, $b\Gamma a=0$.
Similarly, from $b\Gamma x\Gamma a=0$, we can show that $b\Gamma a=0$.

□

THEOREM 3.2. *Let M be a 2-torsion free semi prime $\Gamma-$semiring. If additive mappings d and g of M into itself satisfy $d(x)\Gamma M\Gamma g(x)=0$, for all $x\in M$ then $d(x)\Gamma M\Gamma g(y)=0$, for all $x,y\in M$.*

PROOF. Suppose $d(x)\Gamma m\Gamma g(x)=0$, for all $x,m\in M\cdots(1)$.
Let $y\in M$.

$$\begin{aligned}0&=d(x+y)\Gamma m\Gamma g(x+y)\\&=d(x)\Gamma m\Gamma g(x)+d(x)\Gamma m\Gamma g(y)+d(y)\Gamma m\Gamma g(x)+d(y)\Gamma m\Gamma g(y)\\&=d(x)\Gamma m\Gamma g(y)+d(y)\Gamma m\Gamma g(x).\end{aligned}$$

Pre multiplying by $d(x)\Gamma m\Gamma g(y)\Gamma s\Gamma$, where $s\in M$,

$$\begin{aligned}0=&\,[d(x)\Gamma m\Gamma g(y)]\Gamma s\Gamma[d(x)\Gamma m\Gamma g(y)]\\&+d(x)\Gamma m\Gamma[g(y)\Gamma s\Gamma d(y)]\Gamma m\Gamma g(x),\ \text{for all} s\in M\cdots(2).\end{aligned}$$

$$\begin{aligned}\text{But from (1), }&0=d(x)\Gamma s\Gamma g(x),\ \text{for all } s\in M\\&\Rightarrow 0=g(x)\Gamma s\Gamma d(x),\text{By Theorem 3.1, for all } x,s\in M.\\\text{Now from (2), }&0=d(x)\Gamma m\Gamma g(y)\Gamma s\Gamma d(x)\Gamma m\Gamma g(y)\\&\Rightarrow\ 0=d(x)\Gamma m\Gamma g(y),\ \text{since } M \text{ is a semiprime, for all } x,y,m\in M.\end{aligned}$$

Hence the theorem. □

THEOREM 3.3. *Let M be a 2-torsion free semiprime $\Gamma-$semiring, d and g derivations of M into itself. Then derivations d and g are orthogonal if and only if $d(x)\alpha g(y)+g(x)\alpha d(y)=0$, for all $x,y\in M,\alpha\in\Gamma$.*

PROOF. Suppose $d(x)\alpha g(y)+g(x)\alpha d(y)=0$, for all $x,y\in M,\alpha\in\Gamma$.
Replace y by $y\beta x$, where $\beta\in\Gamma$, we get

$$\begin{aligned}&0=d(x)\alpha[g(y)\beta x+y\beta g(x)]+g(x)\alpha[d(y)\beta x+y\beta d(x)]\\\Rightarrow\ &0=d(x)\alpha g(y)\beta x+d(x)\alpha y\beta g(x)+g(x)\alpha d(y)\beta x+g(x)\alpha y\beta d(x).\\\Rightarrow\ &0=\{d(x)\alpha g(y)+g(x)\alpha d(y)\}\beta x+d(x)\alpha y\beta g(x)+g(x)\alpha y\beta d(x).\\\Rightarrow\ &0=0+d(x)\alpha y\beta g(x)+g(x)\alpha y\beta d(x).\end{aligned}$$

By Theorem 3.1, $d(x)\alpha y\beta g(x)=0=g(x)\alpha y\beta d(x)$, for all $x,y\in M,\alpha,\beta\in M$. By Theorem 3.2, $d(x)\alpha y\beta g(z)=g(x)\alpha y\beta d(z)$, for all $x,y,z\in M,\alpha,\beta\in M$. Thus d and g are orthogonal.

Conversely assume that d and g are orthogonal. Then

$$d(x)\Gamma m\Gamma g(y)=g(x)\Gamma m\Gamma d(y)=0,\ \text{for all } x,y,m\in M.$$

By Theorem 3.1, $d(x)\Gamma g(y) = 0 = g(x)\Gamma d(y)$ and $d(x)\Gamma g(y) + g(x)\Gamma d(y) = 0$, for all $x, y \in M$. Hence the theorem. □

THEOREM 3.4. *Let M be a 2-torsion free semi prime $\Gamma-$semiring. Suppose d and g are derivations of M into M. Then d and g are orthogonal if and only if $dg = 0$.*

PROOF. Let M be a 2-torsion free semi prime, $\Gamma-$semiring. Suppose $dg = 0$.

$$
\begin{aligned}
& 0 = dg(x\alpha y) \\
\Rightarrow\ & 0 = d[g(x)\alpha y + x\alpha g(y)] \\
\Rightarrow\ & 0 = d[g(x)]\alpha y + g(x)\alpha d(y) + d(x)\alpha g(y) + x\alpha d[g(y)] \\
\Rightarrow\ & 0 = g(x)\alpha d(y) + d(x)\alpha g(y).
\end{aligned}
$$

By Theorem 3.3, d and g are orthogonal.

Conversely, d and g are orthogonal. Then

$$
\begin{aligned}
& 0 = d(x)\alpha y\beta g(z), \text{for all } x, y, z \in M, \alpha, \beta \in \Gamma. \\
\Rightarrow\ & 0 = d[d(x)\alpha y\beta g(z)] \\
\Rightarrow\ & 0 = d[d(x)]\alpha y\beta g(z) + d(x)\alpha d(y)\beta g(z) + d(x)\alpha y\beta d[g(z)].
\end{aligned}
$$

Since d and g are orthogonal, then first and second summands are zero. Therefore, we obtain, $d(x)\alpha y\beta dg(z) = 0$, for all $x, y, z \in M, \alpha, \beta \in \Gamma$. Now x replace by $g(z)$ then $dg(z)\alpha y\beta dg(z) = 0$. Since M is a semi prime, we get $dg(z) = 0$, for all$z \in M$. Hence $dg = 0$. □

COROLLARY 3.1. *Let M be a 2-torsion free semi prime $\Gamma-$semiring. Suppose d and g are derivations of M into M. Then d and g are orthogonal if and only if $gd = 0$.*

THEOREM 3.5. *Let M be a 2-torsion free semiprime $\Gamma-$semiring and d and g be derivations of M into M. Then d and g are orthogonal if and only if $dg + gd = 0$.*

PROOF. Let M be a 2-torsion free semiprime $\Gamma-$semiring and d and g be derivations of M into M. Suppose $0 = dg + gd$. We have

$$
\begin{aligned}
0 &= dg + gd. \\
0 &= (dg + gd)(x\alpha y) \\
&= dg(x\alpha y) + gd(x\alpha y) \\
&= d[g(x)\alpha y + x\alpha g(y)] + g[d(x)\alpha y + x\alpha d(y)]. \\
&= d[g(x)\alpha y] + d[x\alpha g(y)] + g[d(x)\alpha y] + g[x\alpha d(y)] \\
&= dg(x)\alpha y + g(x)\alpha d(y) + d(x)\alpha g(y) + x\alpha dg(y) \\
&\quad + gd(x)\alpha y + d(x)\alpha g(y) + g(x)\alpha d(y) + x\alpha gd(y). \\
0 &= [dg + gd](x)\alpha y + 2d(x)\alpha g(y) + 2g(x)\alpha d(y) + x\alpha[dg + gd](y), \text{for all } x, y \in M. \\
0 &= 2d(x)\alpha g(y) + 2g(x)\alpha d(y) \\
0 &= d(x)\alpha g(y) + g(x)\alpha d(y), \text{ since } M \text{ is a 2-torsion free.}
\end{aligned}
$$

By Theorem 3.3, d and g are orthogonal. The proof of the converse follows from Theorem 3.4 and Corollary 3.1 □

THEOREM 3.6. *Let M be a 2-torsion free semiprime $\Gamma-$semiring. Suppose d and g are derivations of M into M. Then d and g are orthogonal if and only if dg is a derivation.*

Let M be a 2-torsion free semiprime $\Gamma-$semiring. Suppose d and g are derivations of M into M. Assume that dg is a derivation.

$$\begin{aligned} \text{Now } dg(x\alpha y) &= d[g(x\alpha y)] \\ \Rightarrow\ dg(x)\alpha y + x\alpha dg(y) &= d[g(x)\alpha y + x\alpha g(y)] \\ &= dg(x)\alpha y + g(x)\alpha d(y) + d(x)\alpha g(y) + x\alpha d(g(y)) \\ \Rightarrow\ 0 &= g(x)\alpha d(y) + d(x)\alpha g(y). \end{aligned}$$

Therefore, by Theorem 3.3, d and g are orthogonal.

Conversely suppose that d and g are orthogonal. Then By Theorem 3.4, $dg = 0$. Thus dg is a derivation.

COROLLARY 3.2. *Let M be a 2-torsion free semiprime $\Gamma-$semiring. Suppose that d and g are derivations of M into itself. Then d and g are orthogonal if and only if gd is a derivation.*

COROLLARY 3.3. *Let M be 2-torsion free semiprime $\Gamma-$semiring.Suppose d and g are derivations on M. Then the following are equivalent.*

(i). *d and g are orthogonal*
(ii). $dg = 0$
(iii). $gd = 0$
(iv). $dg + gd = 0$
(v). *dg is derivation*
(vi). *gd is derivation*

The proof of the following corollary from Theorem 3.1 and Corollary 3.3.

COROLLARY 3.4. *Let M be a 2-torsion free semiprime $\Gamma-$semiring. Suppose d and g are orthogonal on M. Then either $d = 0$ or $g = 0$.*

THEOREM 3.7. *Let M be a 2-torsion free semiprime $\Gamma-$semiring. If d is a derivation of M into M such that d^2 is a derivation then $d = 0$.*

PROOF. Suppose d^2 is a derivations 2-torsion free semiprime $\Gamma-$semiring M and $x, y \in M, \alpha \in \Gamma$. Now

$$d^2(x\alpha y) = d(d(x\alpha y))$$
$$\Rightarrow\ d^2(x)\alpha y + x\alpha d^2(y) = d[d(x)\alpha y + x\alpha d(y)]$$
$$= d^2(x)\alpha y + d(x)\alpha d(y) + d(x)\alpha d(y) + x\alpha d^2(y).$$
$$\Rightarrow\ 2d(x)\alpha d(y) = 0$$
$$\Rightarrow\ d(x)\alpha d(y) = 0, \text{ since } M \text{ is a 2-torsion free, for all } x, y \in M, \alpha \in \Gamma \cdots (1)$$

Replace x by $x\beta z$ in (1), $z \in M, \beta \in \Gamma$, we get

$$\begin{aligned} 0 &= d(x\beta z)\alpha d(y) \\ &= [d(x)\beta z + x\beta d(z)]\alpha d(y) \\ &= (d(x)\beta z)\alpha d(y) + x\beta(d(z)\alpha d(y)) \\ &= (d(x)\beta z)\alpha d(y) + 0, \text{ from } (1) \\ 0 &= (d(x)\beta z)\alpha d(y) \cdots (2) \end{aligned}$$

Replace y by $x + y$ in (2), we get

$$\begin{aligned} 0 &= (d(x)\beta z)\alpha d(x + y) \\ &= d(x)\beta z\alpha d(x) + d(x)\beta z\alpha d(y) \\ &= d(x)\beta z\alpha d(x), \text{from ,for all } x, z \in M, \alpha, \beta \in \Gamma. \\ \Rightarrow\ 0 &= d(x), \text{since } M \text{ is a semiprime, for all } x \in M. \end{aligned}$$

This completes the proof. □

THEOREM 3.8. *Let M be a 2-torsion free semiprime $\Gamma-$semiring. Suppose that d and g are derivations of M into M. Then d and g are orthogonal if and only if there exist $a, b \in M$ such that $dg(x) = a\alpha x + x\alpha b$, for all $x \in M, \alpha \in \Gamma$.*

PROOF. Let $a, b \in M$. Assume that $dg(x) = a\alpha x + x\alpha b$, for all $x, y \in M, \alpha, \beta \in \Gamma$. Replace x by $x\beta y$, we get

$$\begin{aligned} dg(x\beta y) &= a\alpha(x\beta y) + (x\beta y)\alpha b. \\ \Rightarrow\ d[g(x)\beta y + x\beta g(y)] &= a\alpha(x\beta y) + (x\beta y)\alpha b. \\ \Rightarrow\ dg(x)\beta y + g(x)\beta d(y) + d(x)\beta g(y) + x\beta dg(y) &= a\alpha(x\beta y) + (x\beta y)\alpha b. \\ \Rightarrow\ (a\alpha x + x\alpha b)\beta y + g(x)\beta d(y) + d(x)\beta g(y) + x\beta(a\alpha y + y\alpha b) &= a\alpha x\beta y + x\beta y\alpha b. \\ \Rightarrow\ (a\alpha x + x\alpha b)\beta y + g(x)\beta d(y) + d(x)\beta g(y) + x\beta a\alpha y + x\beta y\alpha b & \\ &= a\alpha x\beta y + x\beta y\alpha b. \end{aligned}$$

Comparing, then

$$x\alpha b\beta y + x\beta a\alpha y + g(x)\beta d(y) + d(x)\beta g(y) = 0 \ \cdots (1)$$

Replace y by $y\gamma x, \gamma \in \Gamma$ we have

$$\begin{aligned} 0 &= x\alpha b\beta y\gamma x + x\beta a\alpha y\gamma x + g(x)\beta d(y\gamma x) + d(x)\beta g(y\gamma x) \\ 0 &= x\alpha b\beta y\gamma x + x\beta a\alpha y\gamma x + g(x)\beta d(y)\gamma x + g(x)\beta y\gamma d(x) \\ &\quad + d(x)\beta g(y)\gamma x + d(x)\beta y\gamma g(x) \\ 0 &= \{x\alpha b\beta y + x\beta a\alpha y + g(x)\beta d(y)) + d(x)\beta g(y)\}\gamma x \\ &\quad + g(x)\beta y\gamma d(x) + d(x)\beta y\gamma g(x) \\ 0 &= 0\gamma x + g(x)\beta y\gamma d(x) + d(x)\beta y\gamma g(x), \text{ from } (1) \\ 0 &= g(x)\beta y\gamma d(x) + d(x)\beta y\gamma g(x). \end{aligned}$$

By Theorems 3.1 and 3.2, we have $g(x)\beta y\gamma d(x) = 0 = d(x)\beta y\gamma g(x)$. This proves that d and g are orthogonal.

Conversely,suppose d and g are orthogonal. By Theorem 3.4, $dg = 0$. Then we can choose $a = 0 = b$, so that $dg(x) = a\alpha x + x\alpha b$, for all $x \in M, \alpha \in \Gamma$. □

THEOREM 3.9. *Let M be a 2-torsion free semiprime $\Gamma-$semiring. Suppose d and g are derivations on M. If $d^2 = g^2$ then the following are holds*

(i). *$(d + g)$ and $(d - g)$ are orthogonal.*

(ii). *either $d = -g$ or$d = g$.*

PROOF. Let M be a 2-torsion free semiprime $\Gamma-$semiring. Suppose d and$g$ are derivations on M. Suppose $d^2 = g^2$.

(i).

$[(d + g)(d - g) + (d - g)(d + g)](x)$
$= (d + g)(d(x) - g(x)) + (d - g)(d(x) + g(x))$
$= d(d(x)) - dg(x) + gd(x) - g(g(x)) + d(d(x) + d(g(x) - gd(x) - g(g(x))$
$= 0.$

By Theorem 3.5, $d + g$ and $d - g$ are orthogonal.

(ii). From (i), $d + g$ and $d - g$ are orthogonal. By Theorem 3.4, $d + g = 0$ or $d - g = 0$. Then $d = -g$ or$d = g$.

The proof of the following corollary from Theorems 3.3 and 3.9. □

COROLLARY 3.5. *Let M be a 2-torsion free semiprime $\Gamma-$semiring and d and b derivations on M. If $d(x)\alpha d(x) = g(x)\alpha g(x)$, for all $x \in M$ then $d + g$ and $d - g$ are orthogonal.*

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