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A STUDY OF Γ-SEMIRING AS A GENERALIZATION OF SOFT SEMIRING (F, Γ) OVER M

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ABSTRACT. In 1999, Moladstov introduced soft set theory for modeling uncertainty. In 1995, M. Murali Krishna Rao introduced the notion of a Γ-semiring as a generalization of Γ-ring, ring, ternary semiring and semiring. In this paper, we study Γ-semiring M as a generalization of soft semiring over M.

1. Introduction

The notion of a semiring is an algebraic structure with two associative binary operations where one distributes over the other, was first introduced by H. S. Vandiver [**16**] in 1934, but semirings had appeared in studies on the theory of ideals of rings. In structure, semirings lie between semigroups and rings. The results which hold in rings but not in semigroups hold in semirings since semiring is a generalization of ring. The study of rings shows that multiplicative structure of ring is an independent of additive structure whereas in semiring multiplicative structure of semiring is not an independent of additive structure of semiring.

Semirings play an important role in studying matrices and determinants. Semirings are useful in the areas of theoretical computer science as well as in the solution of graph theory, optimization theory, in particular for studying automata, coding theory and formal languages. Semiring theory has many applications in other branches.

As a generalization of ring, the notion of a $\Gamma-$ring was introduced by N. Nobusawa [**14**] in 1964. In 1981, M. K. Sen [**15**] introduced the notion of a $\Gamma-$semigroup as a generalization of semigroup. The notion of a ternary algebraic system was introduced by Lehmer [**5**] in 1932, Lister introduced the notion of a ternary ring . M. Murali Krishna Rao and B. Venkateswarlu [**12**] studied regular $\Gamma-$incline and field $\Gamma-$semiring.

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Molodtsov introduced the concept of Soft set theory as a new mathematical tool for dealing with uncertainties. Aktas and Cagman [**2**] defined the notion of soft groups. Feng et. al [**3**] initiated the study of soft semirings. Soft rings are defined by Acar et.al [**1**] and Jayanth Ghosh et.al initiated the study of Fuzzy soft rings and Fuzzy soft ideals. In this paper, we study $\Gamma-$semiring M as soft semiring over M.

2. Preliminaries

In this section, we recall some definitions introduced by the pioneers in this field earlier.

Definition 2.1. A set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) will be called semiring provided

(i) Addition is a commutative operation.
(ii) Multiplication distributes over addition both from the left and from the right.
(iii) There exists $0 \in S$ such that $x+0=x$ and $x\cdot 0 = 0\cdot x = 0$ for each $x \in S$.

Definition 2.2. Let M and Γ be additive abelian groups. If there exists a mapping $M \times \Gamma \times M \to M$ (images of (x, α, y) will be denoted by $x\alpha y$, for any $x, y \in M$, and $\alpha \in \Gamma$) satisfying the following conditions for all $x, y, z \in M$, $\alpha, \beta \in \Gamma$

(i) $x\alpha(y\beta z) = (x\alpha y)\beta z$
(ii) $x\alpha(y+z) = x\alpha y + x\alpha z$
(iii) $x(\alpha+\beta)y = x\alpha y + x\beta y$
(iv) $(x+y)\alpha z = x\alpha z + y\alpha z$.

Then M is called a $\Gamma-$ ring.

Definition 2.3. Let $(M, +)$ and $(\Gamma, +)$ be commutative semigroups. Then we call M a $\Gamma-$semiring, if there exists a mapping $M\times\Gamma\times M \to M$ (images of (x, α, y) will be denoted by $x\alpha y, x, y \in M, \alpha \in \Gamma$) such that it satisfies the following axioms for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$

(i) $x\alpha(y+z) = x\alpha y + x\alpha z$
(ii) $(x+y)\alpha z = x\alpha z + y\alpha z$
(iii) $x(\alpha+\beta)y = x\alpha y + x\beta y$
(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$.

Every semiring R is a $\Gamma-$semiring with $\Gamma = R$ and ternary operation $x\gamma y$ defined as the usual semiring multiplication.

We illustrate the definition of $\Gamma-$semiring by the following example.

Example 2.1. Let S be a semiring and $M_{p,q}(S)$ denote the additive abelian semigroup of all $p \times q$ matrices whose entries are from S. Then $M_{p,q}(S)$ is a $\Gamma-$semiring with $\Gamma = M_{p,q}(S)$ and the ternary operation defined by the usual matrix multiplication as $x\alpha y = x(\alpha^t)y$, where α^t denotes the transpose of the matrix α; for all x, y and $\alpha \in M_{p,q}(S)$.

DEFINITION 2.4. A $\Gamma-$semiring M is said to have zero element if there exist an element $0 \in M$ such that $0+x = x = x+0$ and $0\alpha x = x\alpha 0 = 0,$ for all $x \in M, \alpha \in \Gamma$.

DEFINITION 2.5. A $\Gamma-$semiring M is said to be commutative $\Gamma-$semiring if $x\alpha y = y\alpha x$, for all $x, y \in M$ and $\alpha \in \Gamma$.

DEFINITION 2.6. Let M be a $\Gamma-$semiring. An element $a \in M$ is said to be α idempotent if $a = a\alpha a$; an element of M is said to be an idempotent of M if it is α idempotent for some $\alpha \in \Gamma$.

DEFINITION 2.7. Let M be a $\Gamma-$semiring. If every element of M is an idempotent of M, M is said to be an idempotent $\Gamma-$semiring.

DEFINITION 2.8. Let M be a $\Gamma-$semiring. An element $a \in M$ is said to be a regular element of M if there exist $x \in M, \alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$.

DEFINITION 2.9. Let M be a $\Gamma-$semiring. If every element of M is a regular, then M is said to be a regular $\Gamma-$semiring.

DEFINITION 2.10. A non-empty subset A of $\Gamma-$semiring M is called a Γ-subsemiring M if $(A, +)$ is a subsemigroup of $(M, +)$ and $a\alpha b \in A$ for all $a, b \in A$ and $\alpha \in \Gamma$.

DEFINITION 2.11. An additive subsemigroup I of a $\Gamma-$semiring M is said to be a left (right) ideal of M if $M\Gamma I \subseteq I$ $(I\Gamma M \subseteq I)$. If I is both left and right ideal then I is called an ideal of $\Gamma-$semiring M.

DEFINITION 2.12. Let U be an initial Universe set and E be the set of parameters. Let $P(U)$ denotes the power set of U. A pair (f, E) is called soft set over U where f is a mapping given by $f : E \to P(U)$.

DEFINITION 2.13. Let U be an initial Universe set and E be the set of parameters. Let $A \subseteq E$. A pair (f, A) is called fuzzy soft set over U where f is a mapping given by $f : A \to I^U$ where I^U denotes the collection of all fuzzy subsets of U.

DEFINITION 2.14. Let X be a group and (f, A) be a soft set over X. Then (f, A) is said to be soft group over X if and only if $f(a)$ is a subgroup of X for each $a \in A$.

DEFINITION 2.15. For a soft set (f, A), the set $\{x \in A \mid f(x) \neq \emptyset\}$ is called Support of (f, A) denoted by $Supp(f, A)$. If $\text{Supp}(f, A) \neq \emptyset$ then (f, A) is called non null soft set.

DEFINITION 2.16. Let $(f, A), (g, B)$ be two soft sets over U then (f, A) is said to be a soft subset of (g, B) denoted by $(f, A) \subseteq (g, B)$ if $A \subseteq B$ and $f(a) \subseteq g(a)$ for all $a \in A$.

DEFINITION 2.17. Let $(f, A), (g, B)$ be non null soft sets. The intersection of soft sets (f, A) and (g, B) is denoted by $(f, A) \cap (g, B) = (h, C)$ where $C = A \cup B$ is defined as

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B; \\ g_c, & \text{if } c \in B \setminus A; \\ f_c \cap g_c, & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.18. Let $(f, A), (g, B)$ be non null soft sets. The Union of soft sets (f, A) and (g, B) is denoted by $(f, A) \cup (g, B) = (h, C)$ where $C = A \cup B$ is defined as

$$h_c = \begin{cases} f_c, & \text{if } c \in A \setminus B; \\ g_c, & \text{if } c \in B \setminus A; \\ f_c \cup g_c, & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.19. A function $f : R \to S$ where R and S are $\Gamma-$semirings is said to be a $\Gamma-$semiring homomorphism if $f(a+b) = f(a)+f(b)$ and $f(a\alpha b) = f(a)\alpha f(b)$ for all $a, b \in R, \alpha \in \Gamma$.

Definition 2.20. Let S be a $\Gamma-$semiring and E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to P(S)$ where $P(S)$ is the power set of S. Then (f, A) is called a soft $\Gamma-$semiring over S if and only if for each $a \in A$, $f(a)$ is $\Gamma-$subsemiring of S. i.e. (i) $x, y \in S \Rightarrow x + y \in f(a)$ (ii) $x, y \in S, \alpha \in \Gamma \Rightarrow x\alpha y \in f(a)$.

Definition 2.21. Let S be a $\Gamma-$semiring and E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to P(S)$. Then (f, A) is called a soft left(right) ideal over S if and only if for each $a \in A, f(a)$ is a left(right) ideal of S. i.e., (i) $x, y \in f(a) \Rightarrow x + y \in f(a)$ (ii) $x, y \in f(a), \alpha \in \Gamma, r \in S \Rightarrow r\alpha x(x\alpha r) \in f(a)$.

Definition 2.22. Let S be a $\Gamma-$semiring and E be a parameter set, $A \subseteq E$ and $f : A \to P(R)$. Then (f, A) is called a soft ideal over S if and only if for each $a \in A, f(a)$ is an ideal of S. i.e., (i) $x, y \in f(a) \Rightarrow x + y \in f(a)$ (ii) $x \in f(a), \alpha \in \Gamma, r \in S \Rightarrow r\alpha x \in f(a)$ and $x\alpha r \in f(a)$.

3. Main Results

In this section, we study $\Gamma-$semiring M as soft semiring over M..

Definition 3.1. Let S be a semiring and E be a parameter set and $A \subseteq E$. Let f be a mapping given by $f : A \to P(S)$ where $P(S)$ is the power set of S. Then (f, A) is called a soft semiring over S if and only if for each $a \in A$, $f(a)$ is subsemiring of S. i.e. (i) $x, y \in S \Rightarrow x + y \in f(a)$, (ii) $x, y \in S, \Rightarrow xy \in f(a)$.

Theorem 3.1. *Every* $\Gamma-$*semiring* M *is a soft semiring over* M.

Proof. Let M be a $\Gamma-$semiring and $\alpha \in \Gamma$. Define a mapping $* : M \times M \to M$ such that $a * b = a\alpha b$, for all $a, b \in M$. Then $(M, +, *)$ is a semiring of M. It is denoted by M_α. Define $F : \Gamma \to \mathbb{P}(M)$, where $\mathbb{P}(M)$ is the power set of M by $F(\alpha) = M_\alpha$. Hence $\Gamma-$semiring M can be consider as a soft semiring over M. It is denoted by (F, Γ). □

Example 3.1. Let $M = \{0, 1\}$ and $\Gamma = \{\alpha, \beta\}$. We define operations with the following tables:

$+$	0	1
0	0	0
1	0	1

$+$	α	β
α	α	α
β	α	β

α	0	1
0	0	0
1	1	1

β	0	1
0	0	0
1	1	1

.

Then $(M,+),(\Gamma,+)$ are semigroups and M is a $\Gamma-$semiring and $\Gamma-$semiring M is a soft semiring over M. It is denoted by (F,Γ).

Converse of this Theorem 3.1 need not be true.

EXAMPLE 3.2. Let $M=\{0,1\}$ and $\Gamma=\{\alpha,\beta\}$. We define operations with the following tables:

$+$	0	1
0	0	0
1	0	1

$+$	α	β
α	α	α
β	α	β

α	0	1
0	0	0
1	1	1

β	0	1
0	0	1
1	1	1

.

Then $(M,+),(\Gamma,+)$ are semigroups and M is not a $\Gamma-$semiring since M is not a $\Gamma-$semigroup. Here (F,Γ) is a soft semiring but not a $\Gamma-$semiring.Hence Γ -semiring M is a generalization of soft semiring over M.

THEOREM 3.2. *Let M be a $\Gamma-$semiring in which $(M,+)$ is right singular semigroup. Then M holds $a+b\alpha c=(a+b)\alpha(a+c)$, for all $a,b\in M,\alpha\in\Gamma$.*

PROOF. Let $a,bc\in M$ and $\alpha\in\Gamma$. Then $a+b\alpha c=b\alpha c$ and $(a+b)\alpha(a+c)=b\alpha c$. There $a+b\alpha c=(a+b)\alpha(a+c)$. □

THEOREM 3.3. *Let M be a $\Gamma-$semiring with identities $a+b\alpha c=(a+b)\alpha(a+c)$ and $a=a+a\alpha b=a+b\alpha a$, for all $a,b\in M,\alpha\in\Gamma$. Then $(M,+)$ and $\Gamma-$semiring M are bands.*

PROOF. Let $a,b\in M$ and $\alpha\in\Gamma$. We have

$$\begin{aligned} a &= a+a\alpha a \\ \Rightarrow a &= (a+a)\alpha(a+a) \\ \Rightarrow a &= 4a\alpha a. \\ 5a = 4a+a &= 4a+4a\alpha a \\ &= 4(a+a\alpha a) \\ &= 4a \\ \Rightarrow 8a &= 4a. \end{aligned}$$

Thus for every $a\in M$, $4a$ is an idempotent.

$$\begin{aligned} a+a &= 4a\alpha a+4a\alpha a \\ &= 8a\alpha a \\ &= (4a+4a)\alpha a. \\ &= 4a\alpha a \\ &= a. \end{aligned}$$

Therefore a is an additive idempotent. Now $a=a+a\alpha a=(a+a)\alpha(a+a)=a\alpha a$. Hence the theorem. □

THEOREM 3.4. *Let M be a $\Gamma-$semiring with identity $a+a\alpha b=a+b\alpha a=a$. If $\Gamma-$semiring M is a band then $a+b\alpha c=(a+b)\alpha(a+c)$, for all $a,b,c\in M$ and $\alpha\in\Gamma$.*

Proof. Let $a, b, c \in M$ and $\alpha \in \Gamma$.

$$\begin{aligned} a + b\alpha c &= a + b\alpha a + b\alpha c \\ &= a + a\alpha c + b\alpha a + b\alpha c \\ &= a\alpha a + a\alpha c + b\alpha a + b\alpha c \\ &= (a + b)\alpha(a + c). \end{aligned}$$

Hence the theorem. □

Theorem 3.5. *Let M be a $\Gamma-$semiring with identity $a + a\alpha b = a + b\alpha a = a$. If $\Gamma-$semiring M is a band then $(M, +)$ and $\Gamma-$semiring M are semilattices.*

Proof. Let $a \in M$ and $\alpha \in \Gamma$. Then $a + a\alpha a = a \Rightarrow a + a = a$.
Thus $(M, +)$ is band.
For every $a, b \in M, \alpha \in \Gamma$, we have $a\alpha(a + b) = a\alpha a + a\alpha b = a + a\alpha b = a$.
and $a\alpha(b + a) = a\alpha b + a\alpha a = a\alpha b + a = a$.
Similarly $b\alpha(b + a) = b$.

$$\begin{aligned} a + b &= a\alpha(b + a) + b\alpha(b + a) \\ &= (a + b)\alpha(b + a) \\ &= a\alpha b + b\alpha b + a\alpha a + b\alpha a \\ &= a\alpha b + b + a + b\alpha a \\ &= b + a. \end{aligned}$$

Hence $(M, +)$ is commutative.

$$\begin{aligned} a\alpha b &= (a + b\alpha a)a(b + b\alpha a) \\ &= a\alpha(b + b\alpha a) + (b\alpha a)\alpha(b + b\alpha a \\ &= a\alpha b + a\alpha(b\alpha a) + b\alpha a\alpha b + (b\alpha a)\alpha(b\alpha a) \\ &= a\alpha b + b\alpha a. \end{aligned}$$

Similarly we can prove $b\alpha a = b\alpha a + a\alpha b = a\alpha b + b\alpha a$. Therefore $a\alpha b = b\alpha a$. Thus $\Gamma-$semiring M is commutative. □

Theorem 3.6. *Let M be a $\Gamma-$semiring with*
(i) $a + b\alpha c = (a + b)\alpha(a + c)$
(ii) $a + a\alpha b = a + b\alpha a = a$
(iii) $a\alpha b + a = b\alpha a + a = a$, *for all* $a, b \in M$.

Then soft semiring (F, Γ) is $\Gamma-$semiring M is a soft distributive lattice structure.

Proof. By Theorem 3.5, semigroup $(M, +)$ and $\Gamma-$semiring M are semilattices. Let $\alpha \in \Gamma$.

Define $a \leqslant b$ by $a + b = b$ and $a\alpha b = a$. Obviously $\leqslant$ is a partial order. If $a \leqslant c$ and $b \leqslant c$ then $a + c = c, b + c = c, a = a\alpha c, b = b\alpha c$. Then

$$\begin{aligned} c &= c + c \\ &= a + c + b + c \\ &= (a + b) + c + c \\ &= a + b + c. \\ (a + b)\alpha c &= a\alpha c + b\alpha c = a + b. \end{aligned}$$

Hence $a + b \leqslant c$. Thus $a \vee b = a + b$.

If $c \leqslant a$ and $c \leqslant b$ then $c + a = a, c + b = b$ and $c\alpha a = c, c\alpha b = c$. Now

$$\begin{aligned} c + a\alpha b &= (c + a)\alpha(c + b) \\ &= a\alpha b \\ c\alpha a\alpha b &= (c\alpha a)\alpha b \\ &= c\alpha b \\ &= c. \end{aligned}$$

Therefore $c \leqslant a\alpha b$. Thus $a \wedge b = a\alpha b$.

$$\begin{aligned} a \vee (b \wedge c) &= a + (b\alpha c) \\ &= (a + b)\alpha(a + c) \\ &= (a \vee b) \wedge (a \vee c). \end{aligned}$$

Therefore

$$\begin{aligned} a \vee (b \wedge c) &= (a \vee b) \wedge (a \vee c). \\ a \wedge (b \vee c) &= a\alpha(b + c) \\ &= a\alpha b + a\alpha c \\ &= (a \wedge b) \vee (a \wedge c). \end{aligned}$$

Hence, for each $\alpha \in \Gamma$, semiring M_α admits a distributive lattice structure (M_α, $\vee$, $\wedge$). Then soft semiring (F, Γ) over M in $\Gamma-$semiring with identity is a soft distributive lattice structure over M. □

THEOREM 3.7. *Let M be a $\Gamma-$semiring with zero. Then the following are equivalent.*

(i) *M is a $0-$simple $\Gamma-$semiring*
(ii) *M_α is a $0-$simple semiring, for any $\alpha \in \Gamma$*
(iii) *M_α is a $0-$simple semiring, for some $\alpha \in \Gamma$*
(iv) *$M\alpha a\beta M = M$, for any $a \in M \setminus \{0\}$, $\alpha, \beta, \in \Gamma$*

PROOF. Let M be a $\Gamma-$semiring with zero.

(i) $\Rightarrow$ (ii): Since M_α is an ideal of M_α, for every $\alpha \in \Gamma, a \in M \setminus \{0\}$.
Then $M\alpha a\alpha M = M$. Therefore M_α is a $0-$simple semiring.

(ii)$\Rightarrow$ (iii): Obvious

(iii)⇒(iv): Suppose M_α is $0-$simple, for some $\alpha \in \Gamma$.
$(M\gamma a\beta M)\alpha M \leqslant M\gamma a\beta M$ and $M\alpha(M\gamma a\beta M) \leqslant M\gamma a\beta M, \gamma, \beta \in M$.
$M\gamma a\beta M$ is an ideal of M_α.
Therefore $M\gamma a\beta M = M_\alpha$, for any $a \in M \setminus \{0\}$.

(iv) ⇒(i): Let A be an ideal of M containing an element a. Then

$$\begin{aligned} &M\alpha a\beta M \subseteq A \\ \Rightarrow &M \subseteq A \\ \Rightarrow &M = A. \end{aligned}$$

Hence M is a $0-$simple $\Gamma-$semiring.

□

Theorem 3.8. *Let M be a $\Gamma-$semiring. Then M_α is a $0-$simple semiring for some $\alpha \in \Gamma$ if and only if (F, Γ) is a soft $0-$simple semiring.*

Definition 3.2. A division semiring M is a semiring M which is a multiplicative group.

Definition 3.3. A $\Gamma-$semiring M is called a division $\Gamma-$semiring if M_α is a division semiring, for some $\alpha \in \Gamma$.

Theorem 3.9. *Let M be a $\Gamma-$semiring. Then the following are equivalent*
(i) *M is a division $\Gamma-$semiring*
(ii) *M_α is a division semiring, for some $\alpha \in \Gamma$*
(iii) *M_α is a division semiring, for all $\alpha \in \Gamma$*

Proof. Let M be a $\Gamma-$semiring.

(i) ⇒ (ii): From the definition of division $\Gamma-$semiring, proof is obvious.

(ii)⇒ (iii): Let M_α be a division semiring, for some $\alpha \in \Gamma$ and $\beta \in \Gamma, a \in M$.
Then $(a\beta M)\alpha M \subseteq a\beta M$ or $M\alpha(M\beta a) \subseteq M\beta a$.
Therefore $a\beta M$ and $M\beta a$ are right ideal and left ideal respectively of M_α.
Since M_α is a division semiring, we have $a\beta M = M\beta a = M$.
Therefore M_β is a division semiring

(iii)⇒(i): Obvious.

□

Proof of the following theorem follows from Theorem 3.9.

Theorem 3.10. *Let M be a $\Gamma-$semiring. Then M_α is a division semiring for some $\alpha \in \Gamma$ if and only if (F, Γ) is a soft division semiring.*

Definition 3.4. An element x of a Γ-semiring M is called a left zeroid (right zeroid) if for each $y \in M$, $\alpha \in \Gamma$, there exists $a \in M$ such that $a\alpha y = x$ $(y\alpha a = x)$.

Theorem 3.11. *Let M be a Γ-semiring. If e is a left zeroid of a semiring M_α, for some $\alpha \in \Gamma$ then e is a left zeroid of a soft semiring (F, Γ).*

Proof. Suppose e is a left zeroid of a semiring M_α, $\alpha, \beta \in \Gamma$, $x \in M$. Therefore $x\beta x \in M$. Then there exists $z \in M$ such that $z\alpha(x\beta x) = e$, since e is a left zeroid of a semiring M_α, $\Rightarrow (z\alpha x)\beta x = e$. Hence e is a left zeroid of a semiring M_β. Therefore e is a left zeroid of a semiring M_β for all $\beta \in \Gamma$. Hence the Theorem. □

THEOREM 3.12. *Let M be a Γ-semiring , x is a left zeroid element of a semiring M_α, for some $\alpha \in \Gamma$ and $e\alpha e = e$ for all $\alpha \in \Gamma$. Then e is a right identity of a soft semiring (F, Γ) over$M$. i.e.$x\alpha e = x$ for all $\alpha \in \Gamma$.*

PROOF. Let x be a left zeroid element of a semiring M_α, for some $\alpha \in \Gamma$. Then there exists $a \in M$ such that $a\alpha e = x$. Therefore

$$\begin{aligned} x\alpha e &= a\alpha e\alpha e \\ &= a\alpha e \\ &= x. \end{aligned}$$

By Theorem 3.16, x is a left zeroid of a soft semiring (F, Γ). Therefore $x\alpha e = x$ for all $\alpha \in \Gamma$. □

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