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A STUDY ON ISOMORPHISM OF ALGEBRAIC GRAPHS

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Abstract. In this paper, we introduce the notion of algebraic graph, isomorphism of algebraic graphs and we study the properties of algebraic graphs.Our main objective is to connect graph theory with algebra.We prove that the number of edges of algebraic graph $G(V, E, F)$ is sum of the degrees of all functions belong to F

1. Introduction

In 1735, Euler introduced graph theory to solve Konigsberg bridge problem. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. Many problems that occur in the field of Computer Science, Information Technology, Electrical Engineering and many other areas can be analyzed by using techniques described in graph theory. Graph theory serves as a mathematical model for any system involving a binary relation.

The problem of determining whether or not two graphs are isomorphic is known as the isomorphism problem.The only known algorithms which guarantee a correct answer to the isomorphic problem require approximately 2^n operations where n is the number of vertices.Finding a simple and efficient method for showing the isomorphism of graphs is an important unsolved problem in graph theory.In this paper, we introduce the notion of algebraic graph,isomorphism of algebraic graphs and study the properties of algebraic graphs. We prove that the number of edges of algebraic graph $G(V, E, F)$ is sum of the degrees of all functions belong to F

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and verify the isomorphism of the graphs, using the definition of isomorphism of algebraic graphs

2. Preliminaries

A graph is a pair (V, E) where V is a nonempty set and E is a set of unordered pairs of elements of V. The graph (V, E) is denoted by $G(V, E)$. The number of vertices in $G(V, E)$ is called the order of G and it is denoted by $|V|$. The number of edges in $G(V, E)$ is called the size of $G(V, E)$ and it is denoted by $|E|$. Two vertices x and y in $G(V, E)$ are said to be *adjacent* or *neighbors* if $\{x, y\}$ is an edge of G.The neighbor set of a vertex x of $G(V, E)$ is the set of all elements in V which are adjacent to x and it is denoted by $N(x)$. The *degree* of vertex x is defined as the number of edges incident on x and it is denoted by $d(x)$ or equivalently $deg(x) = |N(x)|$. The degree of vertices of a graph arranged in non decreasing order is called the degree sequence of the graph.If $v_1, v_2, \ldots, v_n$ are vertices of V then the sequence $(d_1, d_2, \ldots, d_n)$ where d_i is deg (v_i), is the degree sequence of G and $\delta(G) = d_1 \leqslant d_2 \leqslant \ldots \leqslant d_n = \Delta(G)$ where $\delta(G)$ and $\Delta(G)$ are minimum and maximum of degrees of vertices respectively. A graph $G(V, E,)$ is said to be $k-regular$ graph if $deg(v) = k$ for all $v \in V$. A graph G is connected if there exists a path between every two vertices a and b of G.A bipartite graph is one whose vertices are partitioned into two disjoint parts such that the vertices of each edge belongs to different partitions. A complete graph on the n vertices, denoted by Kn, is a graph such that each pair of distinct vertices are adjacent.A graph $G(V, E)$ where $V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$, is called a *complete bi-partite graph* if there is an edge between every vertex in V_1 and every vertex in V_2.A *circuit* may have repeated vertices whereas a *cycle* is a *circuit* with no repeated vertices. A complete graph with n vertices has $\dfrac{n(n-1)}{2}$ edges and each of its vertices has degree $n-1$. A *cyclic graph* of order n is a *connected graph* whose edges form a cyclic of length n. Two graphs $G(V_1, E_1)$ and $G(V_2, E_2)$ are said to be *isomorphic* if there exists a bijection $f : V_1 \to V_2$ such that vertices u, v are adjacent in $G(V_1, E_1)$ if and only if vertices $f(u)$ and $f(v)$ are adjacent in $G(V_2, E_2)$.

A bijection mapping of a finite set V onto itself is called a *permutation.* If $V = \{v_1, v_2, \ldots, v_n\}$ is a finite set and f is a bijection on V then

$$f = \begin{pmatrix} v_1 & v_2 & \ldots & v_n \\ f(v_1) & f(v_2) & \ldots & f(v_n) \end{pmatrix}.$$

If $f : V \to M$ is a function then the number of elements of V is called the *degree* of f and it is denoted by d(f). Let $V = \{v_1, v_2, \ldots, v_n\}$ and a permutation $f = \begin{pmatrix} v_1 & v_2 & \ldots & v_k & v_{k+1} & \ldots & v_n \\ v_2 & v_3 & \ldots & v_{k+1} & v_{k+2} & \ldots & v_1 \end{pmatrix}$ is called a *cyclic permutation* of degree n. It is represented as $(v_1, v_2, \ldots, v_n)$ which is a cycle of length n. If f, g be two cycles on V such that they have no common element then f, g are *disjoint cycles.*

3. Algebraic graphs

In this section, we introduce the notion of algebraic graph,isomorphism of algebraic graphs and study the properties of algebraic graphs. We prove that the number of edges of algebraic graph $G(V, E, F)$ is sum of the degrees of all functions belong to F and verify the isomorphism of the graphs, using the definition of isomorphism of algebraic graphs

Definition 3.1. A graph $G(V, E)$ is said to be *algebraic graph* if there are functions $f_i : V_i \to M_i$, $(i = 1, 2, \ldots, n)$ where V_i and M_i are subsets of V and functions satisfying the following conditions.

(i) $d(f_1) \geqslant d(f_2) \geqslant \ldots \geqslant d(f_n)$ and $\bigcup\limits_{i=1}^{n} V_i \cup M_i = V$ and there is no function defined on M which is a subset of V if $o(M) > o(V_1)$,

(ii) If $\{a, b\} \in E$ then there exists a unique function f_i such that $f_i(a) = b$.

(iii) If there is a path $\{v_1, v_2\}, \{v_2, v_3\}, \{v_3, v_4\}, \ldots \{v_{n-1}, v_n\} \in E$ then there exists a function f_i such that $f_i(v_1) = v_2, f_i(v_2) = v_3, f_i(v_3) = v_4, \ldots, f_i(v_{n-1}) = v_n$.

The number of elements in a set M is denoted by $o(M)$. An algebraic graph of graph $G(V, E)$ is denoted by $G(V, E, F)$.

Example 3.1. Let $G(V, E)$ be a graph shown in the figure 1, where $V = \{a, b, c, d, e\}$ and $E = \{\{a, e\}, \{a, d\}, \{e, d\}, \{a, c\}, \{e, b\}, \{d, c\}, \{d, b\}\}$.

Define functions $f_1 : \{a, e, b, d, c\} \to V$ by $f_1 = \begin{pmatrix} a & e & b & d & c \\ e & b & d & c & a \end{pmatrix}$ and $f_2 : \{a, d\} \to V$ by $f_2 = \begin{pmatrix} a & d \\ d & e \end{pmatrix}$.

Define functions $g_1 : \{a, e, d, c\} \to V$ by $g_1 = \begin{pmatrix} a & e & d & c \\ e & d & c & a \end{pmatrix}$ and $g_2 : \{e, b, d\} \to V$ by $g_2 = \begin{pmatrix} e & b & d \\ b & d & a \end{pmatrix}$.

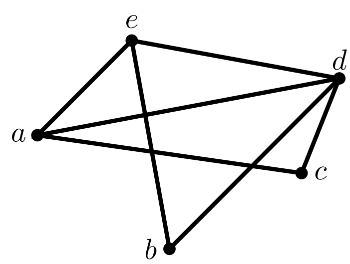

Figure 1. Diagram of graph $G(V, E)$

We observe that $G(V, E, F_1)$ where $F_1 = \{f_1, f_2\}$ is an algebraic graph and $G(V, E, F_2)$ where $F_2 = \{g_1, g_2\}$ is not an algebraic graph,since domain of g_1 is a proper subset of f_1.Hence F_2 is not satisfying the condition (i) in Definition 3.1

EXAMPLE 3.2. Let $G(V,E)$ be a bipartite graph, shown in the figure 2, with vertices $V=\{a,b,c,d,e\}$ and $E=\{\{a,d\},\{a,e\},\{b,d\},\{b,e\},\{c,d\},\{c,e\}\}$. We define functions by $f_1=\begin{pmatrix} a & d & b & e & c \\ d & b & e & c & d \end{pmatrix}$ and $f_2=\begin{pmatrix} a \\ e \end{pmatrix}$.

We observe that $G(V,E,F)$ where $F=\{f_1,f_2\}$ is an algebraic graph

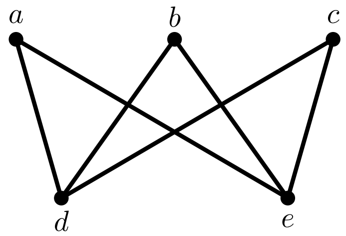

FIGURE 2. Bipartite graph $G(V,E)$.

EXAMPLE 3.3. Let $G(V,E)$ be a graph with $V=\{a,b,c,d\}$ and $E=\{\{a,b\},\{b,c\}\}$. We cannot define a function on any subset of V which contains d. Hence $G(V,E)$ is not an algebraic graph.

DEFINITION 3.2. Let $G(V,E,F)$ where $F=\{f_i/i=1,2,\ldots,n\}$ be an algebraic graph. *Size* of an algebraic graph is defined as n if $|E|=n$ and *order* is defined as $|V|$.

DEFINITION 3.3. Let $G(V,E,F)$ where $F=\{f_i/i=1,2,\ldots,n\}$ be an algebraic graph. The *degree* of function $f\in F$ is defined as the number of elements in the domain of $f$and it is denoted by d(f).

DEFINITION 3.4. *Diameter* of an algebraic graph $G(V,E,F)$ where $F=\{f_i/i=1,2,\ldots,n\}$ and $d(f_1)\geqslant d(f_2)\geqslant\ldots\geqslant d(f_n)$, is defined as degree of f_1 .

REMARK 3.1. An algebraic graph is a graph but a graph need not be an algebraic graph.

REMARK 3.2. Let $G(V,E,F)$ be an algebraic graph. If $f\in F$ and

$$f=\begin{pmatrix} a & b & c & d & e \\ b & c & d & e & a \end{pmatrix}$$

then by definition of algebraic graph there is a closed path from $a-b-c-d-e-a$.

REMARK 3.3. Let $G(V,E,F)$ be an algebraic graph. Suppose the functions $f,g\in F$ such that

$$f=\begin{pmatrix} a & b & c & d & e \\ b & c & d & e & b \end{pmatrix} \text{ and } g=\begin{pmatrix} b & d \\ d & a \end{pmatrix}.$$

By definition of algebraic graph there is a closed path from $a-b-c-d-e-b-d-a$.

REMARK 3.4. Let $G(V, E, F)$ be an algebraic graph. If all the degrees of vertices are two theemn there is a only one cyclic permutation on V.

REMARK 3.5. Let $G(V, E, F)$ be an algebraic graph. If all the degrees of vertices are two except one vertex of degree is 3 then there are only two functions in F and one function is a cyclic permutation on V.

REMARK 3.6. Let $G(V, E, F)$ be an algebraic graph.If the *degree* of function $f \in F$ is n then there are n edges $\in E$

EXAMPLE 3.4. Let $G(V, E)$ be a graph with $V = \{a, b, c, d, e, f\}$ and $E = \{\{a, b\}, \{b, c\}, \{c, d\}, \{d, e\}, \{e, f\}, \{f, a\}\}$. We define a cyclic permutation $f_1 : V \to V$ such that $f_1(a) = b, f_1(b) = c, f_1(c) = d, f_1(d) = e, f_1(e) = f, f_1(f) = a$. Hence $G(V, E, F)$ where $F = \{f_1\}$ is an algebraic graph and *diameter* of the algebraic graph $G(V, E, F)$ is 6.

EXAMPLE 3.5. Let $G(V, E)$ be a graph with $V = \{a, b, c, d, e\}$ and $E = \{\{a, c\}, \{a, d\}, \{d, b\}, \{b, e\}, \{c, b\}\}$.

Define a function f_1 on $V_1 = \{c, a, d, b\}$ such that $f_1(c) = a, f_1(a) = d, f_1(d) = b, f_1(b) = e$.

and a function f_2 on $V_2 = \{c\}$ by $f_2(c) = b$. Obviously degree of $f_1 = 4\ and\ degree\ of f_2 = 1$.

Therefore $G(V, E, F)$ where $F = \{f_1, f_2\}$ is an algebraic graph and diameter of an algebraic graph $G(V, E, F)$ is 4 and number of edges = 4+1=5.

DEFINITION 3.5. An algebraic graph $G(V, E, F)$ where $V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$, is called a *bi-partite algebraic graph* if there are edges between vertices in V_1 and vertices in V_2

DEFINITION 3.6. An algebraic graph $G(V, E, F)$ where $V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$, is called a *complete bi-partite algebraic graph* if there is an edge between every vertex in V_1 and every vertex in V_2

DEFINITION 3.7. An algebraic graph $G(V, E, F)$ is said to be $k-regular$ algebraic graph if $deg(v) = k$ for all $v \in V$.

THEOREM 3.1 (**The first theorem of algebraic graph theory**). *Let $G(V, E, F)$ be an algebraic graph. Then the number of edges of algebraic graph $G(V, E, F)$ is the sum of the degrees of all functions belong to F.*

PROOF. Let $G(V, E, F)$ be an algebraic graph and $F = \{f_i / i = 1, 2, \ldots, n\}$. Suppose the degree of each function f_i is d_i. By the Definition [3.3], if the *degree* of function $f \in F$ is n then there are n edges $\in E$. Therefore the number of edges in $E = d_1 + d_2 + \ldots + d_n$. Hence number of edges of algebraic graph is the sum of degrees of all functions belong to F. Hence the theorem. □

THEOREM 3.2. *If an algebraic graph $G(V, E, F)$ is a $2-regular$ algebraic graph then $F = \{f\}$ and $d(f) = |V|$.*

PROOF. Suppose $G(V, E, F)$ is a $2-regular$ algebraic graph. Then there exists only one cyclic permutation f on V. Hence by the first theorem of algebraic graph theory, $d(f) = |E| = |V|$. □

Corollary 3.1. *If an algebraic graph $G(V,E,F)$ has only cyclic permutations then all vertices of G are of even degree.*

Theorem 3.3. *If $G(V,E,F)$ is an algebraic bipartite graph then G has no cycle of odd length.*

Proof. Suppose $G(V,E,F)$ is an algebraic bipartite graph. Let $V=V_1\cup V_2$ and $V_1\cap V_2=\emptyset$. Let $v_1-v_2-v_3-\cdots-v_n-v_1$ be a cycle of length n and $v_1\in V_1$. By the definition of algebraic graph, there exists a function $f\in F$ such that $f(v_1)=v_2, f(v_2)=v_3,\cdots,f(v_n)=v_1$. Therefore $v_2,v_4,\cdots,v_n\in V_2$ and $v_1,v_3,\cdots,v_{n-1}\in V_1$ and the length of the cycle cannot be an odd number. Thus G has no cycle of odd length. □

Theorem 3.4. *If an algebraic graph $G(V,E,F)$ is a $k-regular$ algebraic graph then $k|V|=2\sum\limits_{f\in F}d(f)$.*

Proof. Suppose $G(V,E,F)$ is a $k-regular$ algebraic graph. By Definition3.7, we have $deg(v)=k$ for all $v\in V$.
$\Rightarrow \sum deg(v)=2|E|$
Therefore $k|V|=2\sum\limits_{f\in F}d(f)$.

□

Theorem 3.5. *Let $G(V,E,F)$ be an algebraic graph.Then*

$$\delta(G)\leqslant\frac{2\sum\limits_{f\in F}d(f)}{|V|}\leqslant\Delta(G)$$

where $\delta(G)$ and $\Delta(G)$ are minimum and maximum of degrees of vertices respectively.

Proof. Let $G(V,E,F)$ be an algebraic graph. We have $\delta(G)\leqslant deg(V)\leqslant\Delta(G)$ for all $v\in V$.
$\Rightarrow \sum\delta(G)\leqslant\sum deg(V)\leqslant\sum\Delta(G)$.
$\Rightarrow |V|\delta(G)\leqslant 2|E|\leqslant|V|\Delta(G)$.
Therefore $\delta(G)\leqslant\dfrac{2\sum\limits_{f\in F}d(f)}{|V|}\leqslant\Delta(G)$. □

Corollary 3.2. *If an algebraic graph $G(V,E,F)$ is a $k-regular$ then*
$\sum\limits_{f\in F}d(f)=\dfrac{k|V|}{2}$.

Definition 3.8. Let $G(V_1,E_1,F_1)$, $G_(V_2,E_2,F_2)$ be algebraic graphs. $f\in F_1$ and $g\in F_2$. Then f is said to be equivalent to g

(i) if $d(f)=d(g)$,
(ii) if $f=\begin{pmatrix} a & d & b & e & c\\ e & b & c & a & d\end{pmatrix}$ and

$g = \begin{pmatrix} p & q & s & r & t \\ r & s & p & t & q \end{pmatrix} \Rightarrow$ d(a) = d(p), d(d) = d(q), d(b) = d(s), d(e) = d(r), d(c) = d(t)

and

(iii) if f is a cyclic permutation then g is also a cyclic permutation

THEOREM 3.6. *Let $G(V_1, E_1, F_1)$and $G(V_2, E_2, F_2)$ be algebraic graphs and $|F_1| = |F_2|$ and f_i is an equivalent to g_i for all i, $f_i \in F_1$ and $g_i \in F_2$. Then $|V_1| = |V_2|$ and there exists a bijection $f : V_1 \to V_2$ such that vertices u, v are adjacent in $G(V_1, E_1, F_1)$ if and only if vertices $f(u)$ and $f(v)$ are adjacent in $|F_1| = |F_2|$.*

PROOF. Let the algebraic graphs be $G(V_1, E_1, F_1)$ and $G_(V_2, E_2, F_2)$ where $F_1 = \{f_1, f_2, \ldots, f_m\}$, $d(f_1) \geqslant d(f_2) \geqslant d(f_3) \geqslant \ldots \geqslant d(f_m)$, $F_2 = \{g_1, g_2, \ldots, g_m\}$, $d(g_1) \geqslant d(g_2) \geqslant d(g_3) \leqslant \ldots \geqslant d(g_m)$ $d(f_i) = d(g_i)$ for all i

Then $\sum d(f_i) = \sum d(g_i)$ for all i. Therefore by the first theorem of algebraic graph theory,we have $|E_1| = |E_2|$ which implies $\sum\limits_{v \in V_1} d(v) = \sum\limits_{u \in V_2} d(u)$. Therefore $|V_1| = |V_2|$ since f_i is an equivalent to g_i for all i.

Therefore there exists a bijection $f : V_1 \to V_2$ such that u, v are adjacent in $G(V_1, E_1, F_1)$ if and only if $f(u)$ and $f(v)$ are adjacent in $G(V_2, E_2, F_2)$ Hence the theorem. □

DEFINITION 3.9. The algebraic graph $G_1(V_1, E_1, F_1)$ where

$$F_1 = \{f_1, f_2, \ldots, f_m\}, \ d(f_1) \geqslant d(f_2) \geqslant d(f_3) \geqslant \ldots \geqslant d(f_m)$$

and the algebraic graph $G(V_2, E_2, F_2)$ where $F_2 = \{g_1, g_2, \ldots, g_m\}$, such that

$$d(g_1) \geqslant d(g_2) \geqslant d(g_3) \leqslant \ldots \geqslant d(g_m)$$

are said to be *isomorphic algebraic graphs*, if f_i is equivalent to g_i for all i.

Proof of the following theorem follows from Theorem 3.6 and the algebraic graph isomorphism Definition 3.9.

THEOREM 3.7. *If $G_1(V_1, E_1, F_1)$ and $G_2(V_2, E_2, F_2)$ are isomorphic algebraic graphs then the graphs $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ are isomorphic graphs.*

EXAMPLE 3.6. Let $G_1(V_1, E_1)$ and $G_2(V_2, E_2)$ be two graphs shown in the figure 3.

Define a function $\lambda = \begin{pmatrix} a & b & c & d & e & f \\ b & c & d & e & f & a \end{pmatrix}$; degree of $\lambda = 6$. Here λ is a cyclic permutation.

Define a function $\mu = \begin{pmatrix} f & b & e \\ b & e & c \end{pmatrix}$. Degree of $\mu = 3$. Here μ is not a cyclic permutation.

Then $G(V_1, E_1, F_1)$ where $F_1 = \{\lambda, \mu\}$.

Define a function $\alpha = \begin{pmatrix} p & t & q & u & r & s \\ t & q & u & r & s & p \end{pmatrix}$; degree of $\alpha = 6$. Here α is a cyclic permutation.

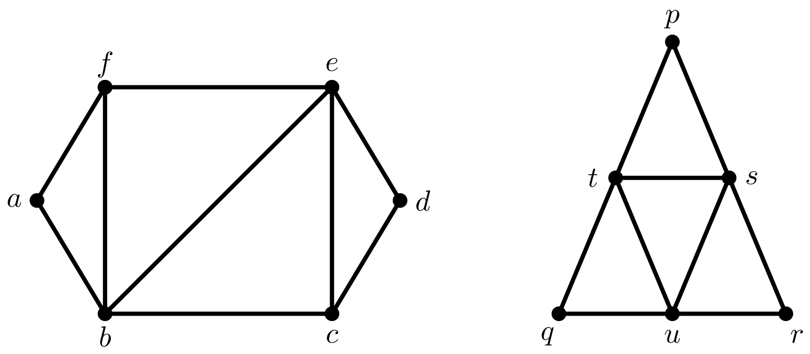

FIGURE 3. Diagram of non isomorphic graphs $G_1(V_1, E_1)$, $G_2(V_2, E_2)$

Define a function $\beta = \begin{pmatrix} t & u & s \\ u & s & t \end{pmatrix}$; degree of $\beta = 3$. Here β is a cyclic permutation. Thus $G(V_2, E_2, F_2)$ where $F_2 = \{\alpha.\beta\}.\mu$ *is not equivalent to* β.

Therefore graph $G_1(V_1, E_1)$ is not isomorphic to graph $G_2(V_2, E_2)$.

EXAMPLE 3.7. Consider the following graphs shown in the figure 4.

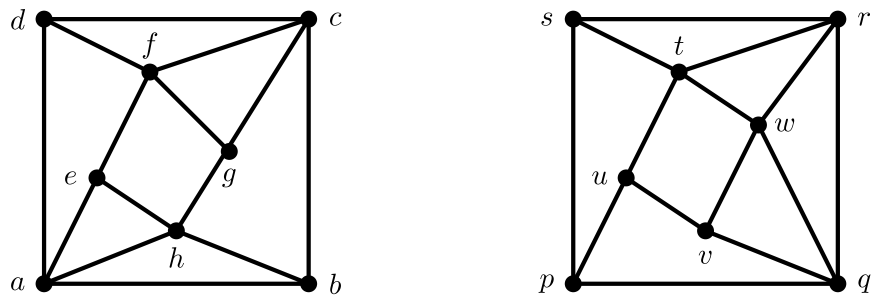

FIGURE 4. Non isomorphic graphs $G(V, E)$,$G'(V', E')$

Define $\lambda = \begin{pmatrix} d & c & g & h & b & a & e & f \\ c & g & h & b & a & e & f & d \end{pmatrix}$, $\delta = \begin{pmatrix} h & d & a & c \\ c & a & b & f \end{pmatrix}$, $G(V, E, F)$ where $F = \{\lambda, \delta\}$ and $d(\lambda) = 8$ and $d(\delta) = 4$.

Define $\lambda' = \begin{pmatrix} s & t & u & p & q & v & w & r \\ t & u & p & q & v & w & r & s \end{pmatrix}$, $\delta' = \begin{pmatrix} u & p & t & q \\ v & s & r & w \end{pmatrix}$.

$G'(V', E', F')$

where $F' = \{\lambda', \delta'\}$ and $d(\lambda') = 8$, $d(\delta'\} = 4$. λ, δ are not equivalent to λ',δ' respectively Hence by Definition of isomorphism of algebraic graphs, $G(V, E, F)$ and $G'(V', E', F')$ are not isomorphic i.e, $G \ncong G'$.Therefore graphs $G(V, E)$ and $G'(V', E')$ are not isomorphic.

EXAMPLE 3.8. Consider the following graphs shown in the figure 5.

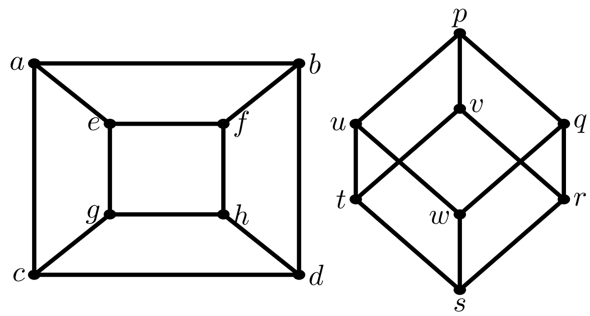

FIGURE 5. Isomorphic Graphs $G(V, E)$, $G(V', E')$

Define functions

$$\lambda = \begin{pmatrix} f & h & g & e & a & c & d & b \\ h & g & e & a & c & d & b & f \end{pmatrix} \mu = \begin{pmatrix} e & f & h & a \\ f & h & d & b \end{pmatrix}.$$

Therefore $d(\lambda) = 8$ and $d(\mu) = 4$. Then $G(V, E, F)$ where $F = \{\lambda, \mu\}$ is an algebraic graph.

Define functions

$$\lambda' = \begin{pmatrix} p & v & t & u & w & q & r & s \\ v & t & u & w & q & r & s & t \end{pmatrix}, \mu' = \begin{pmatrix} u & p & w & v \\ p & v & s & r \end{pmatrix}.$$

Therefore $d(\lambda') = 8$ and $d(\mu') = 4$. Then $G(V', E', F')$ where $F' = \{\lambda', \mu'\}$ is an algebraic graph.

λ, μ are equivalent to λ',μ' respectively Therefore $G(V, E, F)$ and $G(V', E', F')$ are isomorphic. Hence by Theorem 3.7, the graphs $G(V, E)$ and $G(V', E')$ are isomorphic.

EXAMPLE 3.9. Consider the graphs $G(V_1, E_1)$, $G(V_2, E_2)$ shown in the figure 6.

Define functions

$$f_1 = \begin{pmatrix} a & b & c & d & e & f & g \\ b & c & d & e & f & g & a \end{pmatrix} \text{ and } f_2 = \begin{pmatrix} a & c & e & g & b & d & f \\ c & e & g & b & d & f & a \end{pmatrix}.$$

Here f_1 and f_2 are cyclic permutations. Define functions

$$g_1 = \begin{pmatrix} p & q & r & s & t & u & v \\ q & r & s & t & u & v & p \end{pmatrix} \text{ and } g_2 = \begin{pmatrix} p & s & v & r & u & q & t \\ s & v & r & u & q & t & p \end{pmatrix}.$$

Here f_1 and f_2 are equivalent to g_1 and g_2 respectively. Therefore by Definition of isomorphism of algebraic graphs, $G(V, E, F)$ and $G(V', E', F')$ are isomorphic. Hence by Theorem 3.7, the graphs $G(V_1, E_1)$, $G(V_2, E_2)$ are isomorphic.

4. Conclusion

In this paper we introduced the notion of algebraic graph isomorphism of algebraic graphs. We studied the properties of algebraic graphs and we proved that

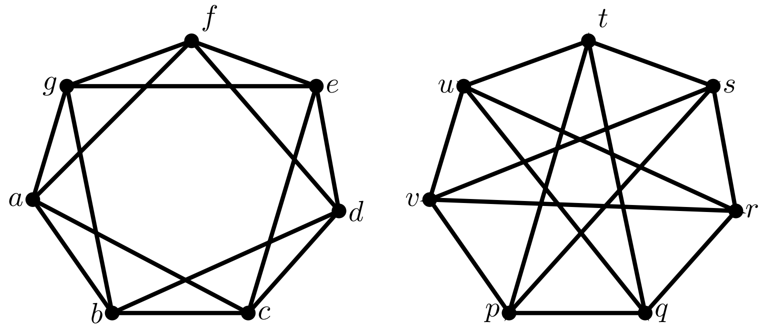

FIGURE 6. Isomorphic Graphs $G(V_1, E_1)$, $G(V_2, E_2)$

the number of edges of an algebraic graph $G(V, E, F)$ is sum of the degrees of all functions belong to F and verified the isomorphism of graphs, using the definition of isomorphism of algebraic graphs In continuation of this paper we propose to introduce and study subgraphs, trees and planar graphs of algebraic graphs.

References

[1] F.Harry. *Graph Theory*, Narora Publishing house, New Delhi, 1995.
[2] G.Chartrand and P.Zhang. *Introduction to Graph Theory*,McGraw Hill International, 2005.
[3] I.N.Herstain. *Topics in Algebra*, Blaindell publishing company, New York,1964.
[4] M.Murali Krishna Rao, *A Study On Eulerian and Hamiltonian algebraic graphs*, Communicated.

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