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FUZZY GRAPH OF SEMIGROUP

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Abstract. The main objective of this paper is to connect fuzzy theory and graph theory with an algebraic structure semigroup.In this paper,we introduce the notion of fuzzy graph of semigroup, the notion of isomorphism of fuzzy graphs of semigroups,the notion of regular fuzzy graph of semigroup and the notion of anti fuzzy ideal graph of semigroup as a generalization of anti fuzzy ideal of semigroup, fuzzy graph and graph. We study some of their properties and prove that $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ be fuzzy graphs of semigroups are isomorphic if and only if their complements are isomorphic..

1. Introduction

The formal study of semigroups begin in the early 20th century. Semigroups are basic algebraic structures in many braches of engineering like automata, formal languages, coding theory, finite state machines. The major role of graph theory in computer applications is the development of graph algorithms. A number of algorithms are used to solve problems that are modeled in the form of graphs. In 1965, Zadeh [24] introduced the fuzzy theory. The aim of this theory is to develop theory which deals with problem of uncertainity Zadeh [25] introduced the notion of interval-valued fuzzy sets and Atanassov [3] introduced the concept of intuitionistic fuzzy sets as an extension of Zadeh's fuzzy set for representing vagueness and uncertainty.. The concept of fuzzy set was applied to theory of subgroups by Rosenfeld [21]. After that Kuroki [9] and Mordeson et al. [11] studied theory of fuzzy semigroups. Jun et al. [5, 7] studied theory of fuzzy semigroups and fuzzy $\Gamma-$ rings. Murali Krishna Rao [8-16] studied Anti fuzzy kideals and anti homomorphisms of Γ-semiring and fuzzy soft $\Gamma-$ semirings. The first definition

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of fuzzy graph was introduced by Kauffman [9] in 1973 based on Zadeh's fuzzy relations. In 1975, Rosenfeld [22] considered fuzzy relations on fuzzy subsets and developed the theory of fuzzy graphs as a generalization of Euler's graph theory obtaining analogs of several graph theoretical concepts. Rosenfeld introduced fuzzy graph to model real life situations.

If there is a vagueness in the description of objects or in its relationships or in both then we need to a assign a fuzzy graph model. Fuzzy graphs are useful to represent relationships which deal with uncertainty. Fuzzy graph theory is useful in solving the combinatorial problems in data structure theory, data mining, neural networks, cluster analysis and etc. Mordeson and Peng [10] defined the concept of complement of fuzzy graph and described some operations on fuzzy graphs. Akram [1, 2] introduced many new concepts including bipolar fuzzy graphs, interval-valued line fuzzy graphs and strong intuitionistic fuzzy graphs. Bhattacharya [4], Sunitha et al. [23] studied fuzzy graphs.In this paper,we introduce the notion of fuzzy graph of semigroup, the notion of isomorphism of fuzzy graphs of semigroups,the notion of regular fuzzy graph of semigroup and the notion of anti fuzzy ideal graph of semigroup as a generalization of anti fuzzy ideal of semigroup, fuzzy graph and graph. The main objective of this paper is to connect fuzzy theory and graph theory with algebraic structure.

2. Preliminaries

In this section we will recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1. A graph is a pair (V, E), where V is a non-empty set and E is a set of unordered pairs of elements of V.

Definition 2.2. A simple graph is an undirected graph without loops and multiple edges.

Definition 2.3. A complete graph is a simple graph in which every pair of distinct vertices is connected by an edge.

Definition 2.4. A graph $G(V, E)$ is connected if there exists a path between every two vertices a and b of V.

Definition 2.5. The number of vertices in a graph $G(V, E)$ is called an order of $G(V, E)$ and it is denoted by $|V|$.

Definition 2.6. The number of edges in a graph $G(V, E)$ is called a size of graph $G(V, E)$ and it is denoted by $|E|$.

Definition 2.7. The neighbor set of a vertex x of graph $G(V, E)$ is the set of all elements in V which are adjacent to x and it is denoted by $N(x)$.

Definition 2.8. The *degree* of vertex x of graph $G(V, E)$ is defined as the number of edges incident on x and it is denoted by $d(x)$ or equivalently $deg(x) = |N(x)|$

The first theorem of graph theory
Let $G(V,E)$ be a graph. Then twice the number of edges of graph $G(V,E)$ is sum of the degrees of all vertices belong to V.

Definition 2.9. A graph $G(V,E)$ is said to be $k-regular$ graph if $deg(v)=k$ for all $v \in V$.

Definition 2.10. Let S be a non-empty set.A mapping $f : S \to [0,1]$ is called a fuzzy subset of S.

Definition 2.11. Let S be a semigroup. A fuzzy subset μ of S is said to be fuzzy subsemigroup of S if it satisfies $\mu(xy) \geqslant \min\{\mu(x),\mu(y)\}$ for all $x,y \in S$.

Definition 2.12. A fuzzy subset μ of a semigroup S is called a fuzzy left(right) ideal of S if $\mu(xy) \geqslant \mu(y)(\mu(x))$ for all $x,y \in S$.

Definition 2.13. A fuzzy subset μ of a semigroup S is called a fuzzy ideal of S if $\mu(xy) \geqslant \max\{\mu(x),\mu(y)\}$ for all $x,y \in S$

Definition 2.14. A fuzzy subset μ of a semigroup S is called an anti fuzzy ideal of S if $\mu(xy) \leqslant \min\{\mu(x),\mu(y)\}$ for all $x,y \in S$

Definition 2.15. A map $\sigma : X \times X \to [0,1]$ is called a fuzzy relation on a fuzzy subset μ of X if $\sigma(x,y) \leqslant \min\{\mu(x),\mu(y)\}$, for all $x,y \in X$. A fuzzy relation σ is symmetric if $\sigma(x,y)=\sigma(y,x)$, for all $x,y \in X$. A fuzzy relation σ is reflexive if $\sigma(x,x)=\mu(x)$, for all $x \in X$.

Definition 2.16. Let V be a non-empty finite set, μ and σ be fuzzy subsets on V and $V \times V$ respectively. If $\sigma(x,y) \leqslant \min\{\mu(x),\mu(y)\}$ for all $\{u,v\} \in E$ then the pair $G=(\mu,\sigma)$ is called a fuzzy graph over the set V. Here μ and σ are called fuzzy vertex and fuzzy edge of the fuzzy graph G respectively.

Definition 2.17. The underlying crisp graph of a fuzzy graph $G=(\mu,\sigma)$ is denoted by $G=(\mu^*,\sigma^*)$, where $\mu^*=\{x \in V \mid \mu(x)>0\}$ and $\sigma^*=\{(x,y) \in V \times V \mid \sigma(x,y)>0\}$.

Definition 2.18. A fuzzy graph $G=(\mu,\sigma)$ is called a strong fuzzy graph if $\sigma(x,y)=\min\{\mu(x),\mu(y)\}$, for all $\{u,v\} \in E$.

Definition 2.19. A fuzzy graph $G=(\mu,\sigma)$ is called a complete fuzzy graph if $\sigma(x,y)=\min\{\mu(x),\mu(y)\}$, for all $x,y \in V$.

Definition 2.20. The order and the size of a fuzzy graph $G=(\mu,\sigma)$ are defined as $O(G)=\sum_{x\in V}\mu(x)$ and $S(G)=\sum_{(x,y)\in E}\sigma(x,y)$ respectively.

Definition 2.21. Let $H=(\delta,\gamma)$ and $G=(\mu,\sigma)$ be fuzzy graphs over the set V. Then H is called a fuzzy subgraph of fuzzy graph G if $\delta(x) \leqslant \mu(x)$ for all $x \in V$ and $\gamma(x,y) \leqslant \sigma(x,y)$, for all $\{u,v\} \in E$.

3. Fuzzy graph of semigroup

In this section, we introduce the notion of fuzzy graph of semigroup as a generalization of fuzzy graph and graph. We study some of their properties.Through out this paper we will consider only simple graphs with finite number of vertices and edges.

Definition 3.1. Let $G(V, E)$ be a graph, $(V, \cdot)$ be a finite commutative semigroup and μ be a fuzzy subset of V such that $\mu(uv) \leqslant \min\{\mu(u), \mu(v)\}$, for all $\{u, v\} \in E$. Then $G(V, E)$ is called a fuzzy graph of semigroup. It is denoted by $G(V, E, \mu)$.

Definition 3.2. If $G(V, E)$ be a complete graph.Then fuzzy graph of semigroup $G(V, E, \mu)$ is called an anti fuzzy ideal graph of semigroup V.

Remark 3.1. Let $G(V, E, \mu)$ be an anti fuzzy ideal graph of semigroup.Define σ as a fuzzy subset of $V \times V$ such that $\sigma(x, y) = \mu(xy)$, for all $x, y \in V$. Then $G = (\sigma, \mu)$ is a fuzzy graph in the sense of Rosenfeld. Then fuzzy ideal graph $G(V, E, \mu)$ is a generalization of anti fuzzy ideal of semigroup, fuzzy graph $G = (\sigma, \mu)$ and the graph $G(V, E)$.

Definition 3.3. Let $G(V, E, \mu)$ be a fuzzy graph of semigroup .

(1). The order of $G(V, E, \mu)$ is defined as $\sum\limits_{x \in V} \mu(x)$. It is denoted by p.

(2). The size of $G(V, E, \mu)$ is defined as $\sum\limits_{\{x,y\} \in E} \mu(xy)$. It is denoted by q.

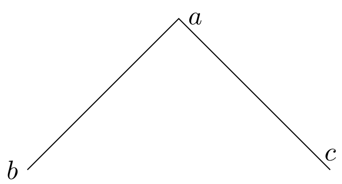
(3). The degree of vertex v of $G(V, E, \mu)$ is defined as $\sum\limits_{u \neq v, \{u,v\} \in E} \mu(uv)$. It is denoted by $D(v)$.

Example 3.1. Let $V = \{a, b, c\}$. The binary operation $' \cdot '$ on V is defined by

.	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

.

Let $G(V, E)$ be a graph where $E = \{(a, b), (a, c)\}$



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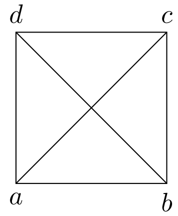
Let $\mu : V \to [0, 1]$ be a fuzzy subset defined by $\mu(a) = 0.5, \mu(b) = 0.4, \mu(c) = 0.3$. By Definition 3.1, $G(V, E, \mu)$ is a fuzzy graph of semigroup V.

EXAMPLE 3.2. Let $V = \{a, b, c, d\}$. The binary operation $' \cdot '$ on V is defined by

.	a	b	c	d
a	a	b	c	d
b	b	b	c	d
c	c	c	c	d
d	d	d	d	c

.

Then $(V, \cdot)$ is a commutative semigroup. Let $G(V, E)$ be a complete graph where $E = \{(a, b), (b, c), (c, d), (d, a), (a, c), (d, b)\}$.



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Let $\mu : V \to [0, 1]$ be a fuzzy subset defined by

$$\mu(x) = \begin{cases} 0.4, & \text{if } x = a; \\ 0.2, & \text{if } x \neq 0. \end{cases}$$

Obviously μ is an anti fuzzy ideal of semigroup V. By Definition 3.1, $G(V, E, \mu)$ is an anti fuzzy ideal graph of semigroup

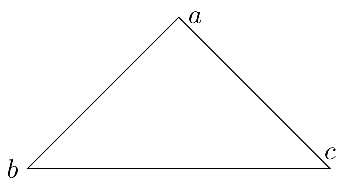
EXAMPLE 3.3. Let $G(V, E)$ be a graph with

$$V = \{a, b, c\} \text{ and } E = \{(a, b), (b, c), (c, a)\}$$

and binary operation $' \cdot '$ on V be defined by

.	a	b	c
a	a	a	c
b	a	b	c
c	c	c	c

.



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Obviously $(V, \cdot)$ is a finite commutative semigroup. Define $\mu : V \to [0, 1]$ by $\mu(a) = \frac{1}{2}, \mu(b) = \frac{3}{4}, \mu(c) = \frac{1}{3}$. Then μ is an anti fuzzy ideal of semigroup V. Therefore, $G(V, E, \mu)$ is an anti fuzzy ideal graph of semigroup. Order of an anti fuzzy ideal graph is equal $\sum_{v \in V} \mu(v) = \mu(a) + \mu(b) + \mu(c) = \frac{19}{12}$. Size of an anti fuzzy ideal graph is equal $\sum_{\{u,v\} \in E} \mu(uv) = \mu(ab) + \mu(bc) + \mu(ca) = \frac{1}{2} + \frac{1}{3} + \frac{1}{3} = \frac{7}{6}$.

DEFINITION 3.4. Let $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ be fuzzy graphs of semigroups V_1 and V_2 respectively. Then a map $h : V_1 \to V_2$ such that

(i) h is an isomorphism of semigroups

(ii) $\mu_1(x) = \mu_2(h(x))$, for all $x \in V_1$

(iii) $\mu_1(xy) = \mu_2(h(x)h(y))$ for all $\{x, y\} \in E_1$ and $\{h(x), h(y)\} \in E_2$

if and only if h is said to be isomorphism of fuzzy graphs of semigroups. It is denoted by $G(V_1, E_1, \mu_1) \cong G(V_2, E_2, \mu_2)$.

THEOREM 3.1. *Let $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ be isomorphic fuzzy graphs of semigroup. Then the degree of their vertices are preserved.*

PROOF. Let h be the isomorphism of fuzzy graphs of semigroups $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$. Then there exists an isomorphism $h : V_1 \to V_2$ of semigroups such that $\mu_1(xy) = \mu_2(h(x)h(y))$, for all $\{x, y\} \in E_1$ and $\{h(x), h(y)\} \in E_2$. Therefore

$$D(u) = \sum_{u\neq v, \{u,v\}\in E_1} \mu_1(uv) = \sum_{u\neq v, \{h(u),h(v)\}\in E_2} \mu_2(h(u)h(v)) = D(h(u)).$$

Hence the theorem. □

THEOREM 3.2. *Let $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ be isomorphic fuzzy graphs of semigroups V_1 and V_2 respectively. Then their orders and sizes are same.*

PROOF. Suppose $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ are isomorphic fuzzy graphs of semigroups. Then there exists an isomorphism $h : V_1 \to V_2$ such that

(i) $\mu_1(x) = \mu_2(h(x))$ for all $x \in V_1$

(ii) $\mu_1(xy) = \mu_2(h(x)h(y))$, for all $\{x, y\} \in E_1$. and $\{h(x), h(y)\} \in E_2$

$$\begin{aligned}\text{Order of } G(V_1, E_1, \mu_1) &= \sum_{v\in V} \mu_1(v)\\ &= \sum_{v\in V} \mu_2 h((v))\\ &= \text{ order of } G(V_2, E_2, \mu_2).\\ \text{Size of } G(V_1, E_1, \mu_1) &= \sum_{\{x,y\}\in E_1} \mu_1(xy)\\ &= \sum_{\{x,y\}\in E_1} \mu_2(h(xy))\\ &= \sum_{\{h(x),h(y)\}\in E_2} \mu_2(h(x)h(y))\\ &= \text{ size of } G(V_2, E_2, \mu_2).\end{aligned}$$

Hence the theorem. □

THEOREM 3.3. *Let $G(V, E, \mu)$ be a fuzzy graph of semigroup. Then $\sum_{v\in V} D(v) \leqslant \sum_{v\in V} d(v)\mu(v)$.*

PROOF. Let $v_1, v_2, \cdots v_n$ be vertices of fuzzy graph of semigroup $G(V, E, \mu)$. Then

$$\begin{aligned} D(v_i) &= \sum_{v_i \neq v_j, \{v_i, v_j\} \in E} \mu_1(v_i v_j) \\ &\leqslant d(v_i)\mu(v_j) \\ \Rightarrow \sum_{v_i \in V} D(v_i) &\leqslant \sum d(v_i)\mu(v_j). \end{aligned}$$

Hence the theorem. □

COROLLARY 3.1. *Let $G(V, E, \mu)$ be a fuzzy regular graph of semigroup. Then* $\sum_{v_i \in V} D(v_i) \leqslant d(v_i) \sum \mu(v_j)$.

DEFINITION 3.5. Let $G(V, E, \mu)$ be a fuzzy graph of semigroup. Then the complement of $G(V, E, \mu)$ is defined as $G(V, E, \bar{\mu})$, where $\bar{\mu}(xy) = \min\{\mu(x), \mu(y)\} - \mu(xy)$, for all $\{x, y\} \in E$.

THEOREM 3.4. *$G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ be isomorphic fuzzy graphs of semigroups if and only if their complements are isomorphic.*

PROOF. Suppose $G(V_1, E_1, \mu_1)$ and $G(V_2, E_2, \mu_2)$ are isomorphic fuzzy graphs of semigroups Then there exists an isomorphism of semigroups, $h : V_1 \to V_2$ such that

$$\begin{aligned} \mu_1(x) &= \mu_2(h(x)), \text{ for all } \ x \in V \\ \text{and } \ \mu_1(xy) &= \mu_2(h(x)h(y)), \{x, y\} \in E_1, \{h(x), h(y)\} \in E_2 \\ \bar{\mu}_1(xy) &= \min\{\mu_1(x), \mu_1(y)\} - \mu_1(xy) \\ &= \min\{\mu_2(h(x)), \mu_2 h((y))\} - \mu_2(h(x)h(y)) \\ &= \bar{\mu_2}(h(x)h(y)), \text{ for all } \{x, y\} \in E_1, , \{h(x), h(y)\} \in E_2 \end{aligned}$$

Therefore $G(V_1, E_1, \bar{\mu}_1) \cong G(V_2, E_2, \bar{\mu}_2)$. Similarly we can prove the converse. Hence the theorem. □

THEOREM 3.5. *The complement of complement of fuzzy graph of semigroup $G(V, E, \mu)$ is itself.*

PROOF. Suppose the complement of $G(V, E, \mu)$ is $G(V, E, \bar{\mu})$, where $\bar{\mu}(xy) = \min\{\mu(x), \mu(y)\} - \mu(xy)$, for all $\{x, y\} \in E$. Then

$$\begin{aligned} \overline{(\bar{\mu})}(xy) &= \min\{\mu(x), \mu(y)\} - \bar{\mu}(xy) \\ &= \min\{\mu(x), \mu(y)\} - \{\min\{\mu(x), \mu(y)\} - \mu(xy)\} \\ &= \mu(xy), \text{ for all } \{x, y\} \in E. \end{aligned}$$

Hence the theorem. □

THEOREM 3.6. *Let $G(V, E, \mu)$ be an anti fuzzy ideal graph of semigroup. Then $\mu(xy) = \frac{1}{2}\min\{\mu(x), \mu(y)\}$ for all $x, y \in V$ if and only if $G(V, E, \mu)$ is self complementary anti fuzzy ideal graph of semigroup.*

PROOF. Let the complement of $G(V,E,\mu)$ be $G(V,E,\bar{\mu})$, where

$$\bar{\mu}(xy) = \min\{\mu(x), \mu(y)\} - \mu(xy).$$

Suppose

$$\mu(xy) = \frac{1}{2}\min\{\mu(x), \mu(y)\}, \quad \text{for all } x, y \in V.$$

Thus $2\mu(xy) = \min\{\mu(x), \mu(y)\}$ and $\bar{\mu(xy)} = \mu(xy)$, for all $x, y \in V$. Therefore $\bar{\mu}(xy) = \mu(xy)$, for all $x, y \in V$. Hence $G(V,E,\mu)$ is self complementary anti fuzzy ideal graph of semigroup.

Conversely suppose that $G(V,E,\mu)$ is self complementary anti fuzzy ideal graph of semigroup where $\bar{\mu}(xy) = \min\{\mu(x), \mu(y)\} - \mu(xy)$. Then $\bar{\mu}(xy) = \mu(xy)$ for all $x, y \in V$.. Thus $2\mu(xy) = \min\{\mu(x), \mu(y)\}$ and $\mu(xy) = \frac{1}{2}\min\{\mu(x), \mu(y)\}$. □

COROLLARY 3.2. *Let $G(V,E,\mu)$ be a self complementary anti fuzzy ideal graph of semigroup . Then size of $G(V,E,\mu) = \frac{1}{2}\sum\limits_{x \neq y} \min\{\mu(x), \mu(y)\}$, $x, y \in V$.*

THEOREM 3.7. *Let $G(V,E,\mu)$ be an anti fuzzy ideal graph of semigroup with 0 element and 1 element. Then*

(i) $\mu(0) \leqslant \mu(u)$, *for all* $u \in V$

(ii) $\mu(0) \leqslant \mu(1)$.

PROOF. Let $G(V,E,\mu)$ be an anti fuzzy ideal graph of semigroup V with 0 element and 1 element. Then

$$\mu(0) = \mu(ov) \leqslant \mu(v)$$
$$\text{and } \mu(0) = \mu(o1) \leqslant \mu(1)$$

Hence the theorem. □

DEFINITION 3.6. Let $G(V,E,\mu)$ be a fuzzy graph of semigroup . If $D(v) = k$, for all $v \in V$ then $G(V,E,\mu)$ is said to be regular fuzzy graph of semigroup .

DEFINITION 3.7. Let $G(V,E,\mu)$ be a fuzzy graph of semigroup . Total degree of a vertex $u \in V$ is defined as $D(u) + \mu(u)$. It is denoted by $TD(u)$.

DEFINITION 3.8. If each vertex of fuzzy graph of semigroup $G(V,E,\mu)$ has the same total degree k then $G(V,E,\mu)$ is said to be totally regular fuzzy graph of semigroup of total degree k.

THEOREM 3.8. *The size of a $k-$regular anti fuzzy ideal graph of semigroup $G(V,E,\mu)$ is $\frac{|V|k}{2}$.*

PROOF. Size of $G(V,E,\mu)$ is equal $\sum\limits_{\{u,v\} \in E} \mu(uv)$. We have

$$\sum_{v \in V} D(v) = 2\sum_{\{u,v\} \in E} \mu(uv) = 2\text{size of}\, G(V,E,\mu).$$

and $2S(G) = \sum k = \mid V \mid k$. Therefore $S(G) = \frac{|V|k}{2}$. □

THEOREM 3.9. *Let $G(V, E, \mu)$ be fuzzy graph of semigroup. Then fuzzy subset μ is a constant function if and only if the following are equivalent*

(i) *fuzzy graph of semigroup $G(V, E, \mu)$ is regular*

(ii) *fuzzy graph of semigroup $G(V, E, \mu)$ is totally regular .*

PROOF. Let $G(V, E, \mu)$ be a fuzzy graph of semigroup and fuzzy subset μ be a constant function.

$(i) \Rightarrow (ii)$: Suppose fuzzy graph of semigroup $G(V, E, \mu)$ is regular, $D(u) = k$, and $\mu(u) = c$. for all $u \in V$.
$TD(u) = D(u) + \mu(u) = k + c$, for all $u \in V$. Hence $(i) \Rightarrow (ii)$.

$(ii) \Rightarrow (i)$: Suppose a fuzzy graph of semigroup $G(V, E, \mu)$ is totally regular and $TD(u) = k$, for all $u \in V$

$$\Rightarrow D(u) + \mu(u) = k, \text{ for all } u \in V$$
$$\Rightarrow D(u) + c = k$$
$$\Rightarrow D(u) = k - c.$$

Therefore fuzzy graph of semigroup $G(V, E, \mu)$ is regular.

Converse is obvious. Hence the theorem. □

4. Conclusion

We introduced the notion of fuzzy graph of semigroup, the notion of isomorphism of fuzzy graphs of semigroup, the notion of regular fuzzy graph of semigroups and the notion of anti fuzzy ideal graph of semigroup as a generalization of anti fuzzy ideal of semigroup, fuzzy graph and graph. We studied some of their properties.

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