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INTERVAL VALUED FUZZY IDEALS OF GAMMA NEAR-RINGS

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Abstract. In this paper, we introduce the concept of interval valued fuzzy ideals of Γ-near-rings, We also characterize some of its properties and illustrate with examples of interval valued fuzzy ideals of Γ-near-rings .

1. Introduction

The notion of fuzzy sets was introduced by Zadeh [**12**] in 1965, and he [**13**] also generalized it to interval valued fuzzy subsets (shortly i.v fuzzy subsets), whose of membership values are closed subinterval of $[0,1]$. Near-ring was introduced by Pilz [**8**] and Γ -near-ring was introduced by Satyanarayana [**9**] in 1984. The idea of fuzzy ideals of near-rings was presented by Kim *et al.* [**6**]. Fuzzy ideals in Gamma-near-rings was proposed by Jun *et al.* [**5**] in 1998. Moreover, Thillaigovindan *at al.* [**10**] studied the interval valued fuzzy quasi-ideals of semigroups. Chinnadurai *et al.* [**3**] characterized of fuzzy weak bi-ideals of Γ-near-rings. Thillaigovindan *et al.* [**11**] worked on interval valued fuzzy ideals of near-rings. Rao [**7**] carried out a study on anti-fuzzy k-ideals and anti-homomorphism of Γ-near-rings. In this paper, we define a new notion of interval valued fuzzy ideals of Γ- near-rings, which is a generalized concept of an interval valued fuzzy ideals of near-rings. We also investigate some of its properties and illustrate with examples.

2. Preliminaries

In this section, we list some basic definitions.

Definition 2.1. ([**13**]) Let X be any set. A mapping $\eta : X \to D[0,1]$ is called an interval valued fuzzy subset (briefly, an i.v fuzzy subset) of X, where $D[0,1]$ denotes the family of closed subintervals of $[0,1]$ and $\tilde{\eta}(x) = [\eta^-(x), \eta^+(x)]$ for all

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$x \in X$, where $\eta^-(x)$ and $\eta^+(x)$ are fuzzy subsets of X such that $\eta^-(x) \leqslant \eta^+(x)$ for all $x \in X$.

Definition 2.2. ([**12**]) An interval number $\tilde{a}$, we mean an interval $[a^-, a^+]$ such that $0 \leqslant a^- \leqslant a^+ \leqslant 1$ and where a^- and a^+ are the lower and upper limits of $\tilde{a}$ respectively. The set of all closed subintervals of $[0, 1]$ is denoted by $D[0, 1]$. We also identify the interval $[a, a]$ by the number $a \in [0, 1]$. For any interval numbers $\tilde{a}_j = [a_j^-, a_j^+], \tilde{b}_j = [b_j^-, b_j^+] \in D[0, 1], j \in \Omega$ (where Ω is index set), we define

$\max^i\{\tilde{a}_j, \tilde{b}_j\} = [\max\{a_j^-, b_j^-\}, \max\{a_j^+, b_j^+\}]$,
$\min^i\{\tilde{a}_j, \tilde{b}_j\} = [\min\{a_j^-, b_j^-\}, \max\{a_j^+, b_j^+\}]$,
$\inf^i \tilde{a}_j = [\cap_{j\in\Omega} a_j^-, \cap_{j\in\Omega} a_j^+]$,
$\sup^i \tilde{a}_j = [\cup_{j\in\Omega} a_j^-, \cup_{j\in\Omega} a_j^+]$.

and let

(i) $\tilde{a} \leqslant \tilde{b} \Leftrightarrow a^- \leqslant b^-$ and $a^+ \leqslant b^+$,
(ii) $\tilde{a} = \tilde{b} \Leftrightarrow a^- = b^-$ and $a^+ = b^+$,
(iii) $\tilde{a} < \tilde{b} \Leftrightarrow \tilde{a} \leqslant \tilde{b}$ and $\tilde{a} \neq \tilde{b}$,
(iv) $k\tilde{a} = [ka^-, ka^+]$, whenever $0 \leqslant k \leqslant 1$.

Definition 2.3. ([**10**]) Let $\tilde{\eta}$ be an i.v fuzzy subset of X and $[t_1, t_2] \in D[0, 1]$. Then the set $\tilde{U}(\tilde{\eta} : [t_1, t_2]) = \{x \in X | \tilde{\eta}(x) \geqslant [t_1, t_2]\}$ is called the upper level subset of $\tilde{\eta}$.

Definition 2.4. ([**8**]) A near-ring is an algebraic system $(R, +, \cdot)$ consisting of a non empty set R together with two binary operations called $+$ and $\cdot$ such that $(R, +)$ is a group not necessarily abelian and $(R, \cdot)$ is a semigroup connected by the following distributive law: $(x + z) \cdot y = x \cdot y + z \cdot y$ valid for all $x, y, z \in R$. We use the word ‘near-ring ’to mean ‘right near-ring ’. We denote xy instead of $x \cdot y$.

Definition 2.5. ([**9**]) A Γ- near-ring is a triple $(M, +, \Gamma)$ where
(i) $(M, +)$ is a group,
(ii) Γ is a nonempty set of binary operations on M such that for each $\alpha \in \Gamma$, $(M, +, \alpha)$ is a near-ring,
(iii) $x\alpha(y\beta z) = (x\alpha y)\beta z$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

In what follows , let M denote a Γ- near-ring unless otherwise specified.

Definition 2.6. ([**9**]) A subset A of a Γ-near-ring M is called a left(resp. right) ideal of M if
(i) $(A, +)$ is a normal divisor of $(M, +)$, (i.e) $x - y \in A$ for all $x, y \in A$ and $y + x - y \in A$ for $x \in A, y \in M$
(ii) $u\alpha(x + v) - u\alpha v \in A$ (resp. $x\alpha u \in A$) for all $x \in A, \alpha \in \Gamma$ and $u, v \in M$.

Definition 2.7. ([**9**]) Let M be a Γ-near-ring. Given two subsets A and B of M, we define $A\Gamma B = \{a\alpha b | a \in A, b \in B$ and $\alpha \in \Gamma\}$ and also define another operation $*$ on the class of subset of M as

$$A\Gamma * B = \{a\gamma(a^{'} + b) - a\gamma a^{'} | a, a^{'} \in A, \gamma \in \Gamma, b \in B\}.$$

DEFINITION 2.8. ([**10**]) Let I be a subset of a near-ring M. Define a function $\tilde{f}_I : M \to D[0,1]$ by

$$f_I(x) = \begin{cases} \tilde{1} \ if\, x \in I \\ 0 \ otherwise. \end{cases}$$

DEFINITION 2.9. ([**4**]) If $\tilde{\eta}$ and $\tilde{\lambda}, \tilde{\eta}_i (i \in \Omega)$ are i.v fuzzy subsets of X. The following are defined by

(i)$\tilde{\eta} \leqslant \tilde{\lambda} \Leftrightarrow \tilde{\eta}(x) \leqslant \tilde{\lambda}(x)$.
(ii)$\tilde{\eta} = \tilde{\lambda} \Leftrightarrow \tilde{\eta}(x) = \tilde{\lambda}(x)$.
(iii) $(\tilde{\eta} \cap \tilde{\lambda})(x) = \min^i\{\tilde{\eta}(x),\ \tilde{\lambda}(x)\}$.
(iv) $(\tilde{\eta} \cup \tilde{\lambda})(x) = \max^i\{\tilde{\eta}(x),\ \tilde{\lambda}(x)\}$.
(v) $\bigcup_{i\in\Omega} \tilde{\eta}(x) = \sup^i\{\tilde{\eta}(x)|i \in \Omega\}$.
(vi) $\bigcap_{i\in\Omega} \tilde{\eta}(x) = \inf^i\{\tilde{\eta}(x)|i \in \Omega\}$ forall $x \in X$.

where $\inf^i\{\tilde{\eta}_i|i \in \Omega\} = [\inf_{i\in\Omega}\{\eta_i^-(x)\}, \inf_{i\in\Omega}\{\eta_i^+(x)\}]$ is the i.v infimum norm and $\sup^i\{\tilde{\eta}_i|i \in \Omega\} = [\sup_{i\in\Omega}\{\eta_i^-(x)\}, \sup_{i\in\Omega}\{\eta_i^+(x)\}]$ is the i.v supremum norm.

3. Interval valued fuzzy ideals of Γ-near-rings

In this section, we introduce the notion of i.v fuzzy left(right)ideal of M and discuss some of its properties.

DEFINITION 3.1. An i.v fuzzy subset $\tilde{\eta}$ in a Γ-near-ring M is called an i.v fuzzy left (resp. right) ideal of M if

(i) $\tilde{\eta}$ is an i.v fuzzy normal divisor with respect to addition,
(ii) $\tilde{\eta}(c\alpha(p+d) - c\alpha d) \geqslant \tilde{\eta}(p)$, (resp. $\tilde{\eta}(p\alpha c) \geqslant \tilde{\eta}(p)$ for all $p, c, d \in M$ and $\alpha \in \Gamma$.

The condition (i) of definition 3.1 means that $\tilde{\eta}$ satisfies:
(i) $\tilde{\eta}(p - q) \geqslant \min^i\{\tilde{\eta}(p), \tilde{\eta}(q)\}$,
(ii) $\tilde{\eta}(q + p - q) \geqslant \tilde{\eta}(p)$, for all $p, q \in M$

Note that $\tilde{\eta}$ is an i.v fuzzy left (resp. right) ideal of Γ-near-ring M, then $\tilde{\eta}(0) \geqslant \tilde{\eta}(p)$ for all $p \in M$, where 0 is the zero element of M.

EXAMPLE 3.1. Let $M = \{0, a, b, c\}$ be a non-empty set with binary operation + and $\Gamma = \{\alpha, \beta\}$ be the non-empty set of binary operations as shown in the following tables:

$+$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

α	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	b	b
c	a	a	c	c

β	0	a	b	c
0	0	0	0	0
a	0	0	0	0
b	0	a	c	b
c	0	a	b	c

Table 1.

Let $\tilde{\eta} : M \to D[0,1]$ be an i.v fuzzy subset defined by $\tilde{\eta}(0) = [0.8, 0.9]$, and $\tilde{\eta}(a) = [0.6, 0.7], \tilde{\eta}(b) = \tilde{\eta}(c) = [0.2, 0.3]$. Then $\tilde{\eta}$ is an i.v fuzzy ideal of M.

Theorem 3.1. *Let $\tilde{\eta} = [\eta^-, \eta^+]$ be an i.v fuzzy subset of a Γ-near-ring M, then $\tilde{\eta}$ is an i.v fuzzy left(right) ideal of M if and only if η^-, η^+ are fuzzy left (right) ideal of M.*

Proof. Let $\tilde{\eta}$ be an i.v fuzzy left ideal of M. For any $p, q, r \in M$. Now

$$\begin{aligned}
[\eta^-(p-q), \eta^+(p-q)] &= \tilde{\eta}(p-q) \\
&\geqslant min^i\{\tilde{\eta}(p), \tilde{\eta}(q)\} \\
&= min^i\{[\eta^-(p), \eta^+(p)], [\eta^-(q), \eta^+(q)]\} \\
&= min^i\{[\eta^-(p), \eta^-(q)]\}, \overset{i}{\min}\{[\eta^+(p), \eta^+(q)]\}
\end{aligned}$$

It follows that $\eta^-(p) \geqslant min^i\{\eta^-(p), \eta^-(q)$ and

$$\begin{aligned}
[\eta^-(p\alpha q), \eta^+(p\alpha q)] &= \tilde{\eta}(p\alpha q) \\
&\geqslant min^i\{\tilde{\eta}(p), \tilde{\eta}(q)\} \\
&= min^i\{[\eta^-(p), \eta^+(p)], [\eta^-(q), \eta^+(q)]\} \\
&= min^i\{[\eta^-(p), \eta^-(q)]\}, \overset{i}{\min}\{[\eta^+(p), \eta^+(q)]\}
\end{aligned}$$

Then

$$\eta^-(p\alpha q) \geqslant min^i\{\tilde{\eta}(p), \tilde{\eta}(q)$$

and

$$\eta^+(p\alpha q) \geqslant min^i\{\tilde{\eta}(p), \tilde{\eta}(q).$$

Now

$$[\eta^-(q+p-q), \eta^+(q+p-q)] = \tilde{\eta}(q+p-q) = \tilde{\eta}(p) = [\eta^-(p), \eta^+(q)].$$

It follows that $\eta^-(q+p-q)] = \eta^-(p)$ and $\eta^+(q+p-q)] = \eta^+(p)$. Also

$$\begin{aligned}
[\eta^-(p\alpha q), \eta^+(p\alpha q)] &\geqslant \tilde{\eta}(p\alpha q) \\
&\geqslant \tilde{\eta}(q) \\
&= [\eta^-(q), \eta^+(q)]
\end{aligned}$$

So

$$\eta^-(p\alpha q) \geqslant \eta^-(q) \text{ and } \eta^+(p\alpha q) \geqslant \eta^+(q).$$

Now

$$\begin{aligned}
[\eta^-((p+r)\alpha q - p\beta q), \eta^+((p+r)\alpha q - p\beta q)] &= \tilde{\eta}((p+r)\alpha q - p\beta q \\
&\geqslant \tilde{\eta}(r) \\
&= [\eta^-(r), \eta^+(r)].
\end{aligned}$$

It follows that $\eta^-((p+r)\alpha q - p\beta q) \geqslant \eta^-(r)$ and $\eta^+((p+r)\alpha q - p\beta q) \geqslant \eta^+(r)$.

Conversely, assume that η^-, η^+ are fuzzy left(right) ideals of M.Let $p, q, r \in M$ Now

$$\begin{aligned}
\tilde{\eta}(p-q) &= [\eta^-(p-q), \eta^+(p-q)] \\
&\geqslant min^i\{[\eta^-(p)\eta^-(q)]\}, min^i\{[\eta^+(p)\eta^+(q)]\} \\
&=\geqslant min^i\{[\eta^-(p)\eta^+(p)]\}, min^i\{[\eta^-(q)\eta^+(q)]\} \\
&= min^i\{[\tilde{\eta}(p), \tilde{\eta}(q)]\} \\
\tilde{\eta}(p\alpha q) &= [\eta^-(p\alpha q), \eta^+(p\alpha q)] \\
&\geqslant min^i\{[\eta^-(p)\eta^-(q)]\}, min^i\{[\eta^+(p)\eta^+(q)]\} \\
&=\geqslant min^i\{[\eta^-(p)\eta^+(p)]\}, min^i\{[\eta^-(q)\eta^+(q)]\} \\
&= min^i\{[\tilde{\eta}(p), \tilde{\eta}(q)]\}
\end{aligned}$$

$$\begin{aligned}
\tilde{\eta}(q+p-q) &= [\eta^-(q+p-q), \eta^+(q+p-q)] \\
&= [\eta^-(p), \eta^+(p)] \\
&= \tilde{\eta}(p) \\
\tilde{\eta}(p\alpha q) &= [\eta^-(p\alpha q), \eta^+(p\alpha q)] \\
&\geqslant [\eta^-(q), \eta^+(q)] \\
&= \tilde{\eta}(q) \\
\tilde{\eta}((p+r)\alpha q - p\beta q) &= [\eta^-((p+r)\alpha q - p\alpha q), \eta^+((p+r)\alpha q - p\alpha q)] \\
&\geqslant [\eta^-(r), \eta^+(r)] \\
&= \tilde{\eta}(r)
\end{aligned}$$

Hence $\tilde{\eta}$ is an i.v fuzzy left(right) ideal of M. □

THEOREM 3.2. *Let I be a left (right) ideal of Γ-near-ring M. Then for any $\tilde{c} \in D[0,1]$, there exists an i.v fuzzy left (right) ideal $\tilde{\eta}$ of M such that $\tilde{U}(\tilde{\eta} : \tilde{c}) = I$.*

PROOF. Let I be a left (right)ideal of M. Let $\tilde{\eta}$ be an i.v fuzzy subset of M defined by

$$\tilde{\eta}(p) = \begin{cases} \tilde{c} & \text{if } p \in I \\ \tilde{0} & \text{otherwise.} \end{cases}$$

Then $\tilde{U}(\tilde{\eta}; \tilde{c}) = I$. If $p, q \in I$, then $p, q \in I$ and

$$\tilde{\eta}(p-q) = \tilde{c} = \overset{i}{\min}\{\tilde{c}, \tilde{c}\} = \min{}^i\{\tilde{\eta}(p), \tilde{\eta}(q)\}.$$

If $p, q \notin I$, then $\tilde{\eta}(p) = \tilde{0} = \tilde{(}q)$ and thus

$$\tilde{\eta}(p-q) \geqslant \tilde{0} = \min{}^i\{\tilde{0}, \tilde{0}\} = \min{}^i\{\tilde{\eta}(p), \tilde{\eta}(q)\}.$$

Suppose that $p, q \in I$. Then

$$\tilde{\eta}(p-q) \geqslant \tilde{0} = \overset{i}{\min}\{\tilde{c}, \tilde{0}\} = \min{}^i\{\tilde{\eta}(p), \tilde{\eta}(q)\}.$$

If $p \in I$ and $q \in M$, then $q+p-q \in I$ and so $\tilde{\eta}(q+p-q) = \tilde{c} = \tilde{\eta}(p)$. If $p \notin I$ and $q \in M$,then $\tilde{\eta}(p) = \tilde{0}$ and thus $\tilde{\eta}(q+p-q) \geqslant \tilde{0} = \tilde{\eta}(p)$. If $q \in I$ and $p \in M$, then $p\alpha q \in I$ and so $\tilde{\eta}(p\alpha q) = \tilde{c} = \tilde{\eta}(q)$. If $q \notin I$ and $p \in M$, then $\tilde{\eta}(q) = \tilde{0}$ and thus $\tilde{\eta}(p\alpha q) \geqslant \tilde{0} = \tilde{\eta}(q)$.

If $r \in I$ and $p, q \in M$, then $((p+r)\alpha q - p\beta q) \in I$ and so $\tilde{\eta}((p+r))\alpha q - p\beta q) = \tilde{c} = \tilde{\eta}(r)$. If $z \notin I$ and $p, q \in M$, then $\tilde{\eta}(r) = \tilde{0}$ and $\tilde{\eta}((p+r))\alpha q - p\beta q) \geqslant \tilde{0} = \tilde{\eta}(r)$. Hence $\tilde{\eta}$ is an i.v fuzzy left(right) ideal of tha Γ-near-ring M. □

THEOREM 3.3. *Let M be a Γ-near-ring and $\tilde{\eta}$ is an i.v fuzzy left (right) ideal of M, then the set $M_{\tilde{\eta}} = \{p \in M \mid \tilde{\eta}(p) = \tilde{\eta}(0)\}$ is left (right) ideal of M.*

PROOF. Let $\tilde{\eta}$ be an i.v fuzzy left ideal of M. Let $p, q \in M_{\tilde{\eta}}$. Then

$$\tilde{\eta}(p) = \tilde{\eta}(0), \tilde{\eta}(q) = \tilde{\eta}(0)$$

and

$$\tilde{\eta}(p-q) \geqslant \overset{i}{\min}\{\tilde{\eta}(p), \tilde{\eta}(q)\} = \overset{i}{\min}\{\tilde{\eta}(0), \tilde{\eta}(0)\} = \tilde{\eta}(0).$$

So $\tilde{\eta}(p-q) = \tilde{\eta}(0)$. Thus $p-q \in M_{\tilde{\eta}}$. For every $q \in M$ and $p \in M_{\tilde{\eta}}$ and $\alpha, \beta \in \Gamma$ we have $\tilde{\eta}(q+p-q) \geqslant \tilde{\eta}(p) = \tilde{\eta}(0)$. Hence $q+p-q \in M_{\tilde{\eta}}$ which shows that $M_{\tilde{\eta}}$ is a normal divisor of M with respect to the addition. Let $p \in M_{\tilde{\eta}}, \alpha \in \Gamma$ and $c, d \in M$. Then $\tilde{\eta}(c\alpha(p+d) - c\alpha d) \geqslant \tilde{\eta}(p) = \tilde{\eta}(0)$ and hence $\tilde{\eta}(c\alpha(p+d) - c\alpha d) = \tilde{\eta}(0)$.(i.e) $c\alpha(p+d) - c\alpha d \in M_{\tilde{\eta}}$. Therefore $M_{\tilde{\eta}}$ is a left ideal of M. □

THEOREM 3.4. *Let H be a non empty subset of a Γ-near-ring M and $\tilde{\eta}_H$ be an i.v fuzzy set M defined by*

$$\tilde{\eta}_H(p) = \begin{cases} \tilde{s} \ if\, p \in H \\ \tilde{t} \ \ otherwise \end{cases}$$

for $p \in M$ and $\tilde{s}, \tilde{t} \in D[0,1]$ and $\tilde{s} > \tilde{t}$. Then $\tilde{\eta}_H$ is an i.v fuzzy left(right) ideal of M if and only if H is a left ideal of M. Also $M_{\tilde{\eta}_H} = H$.

PROOF. $\tilde{\eta}_H$ be an i.v fuzzy left(right) ideal of M and let $p, q \in H$. Then $\tilde{\eta}_H(p) = \tilde{s} = \tilde{\eta}_H(q)$. Consider

$$\begin{aligned} \tilde{\eta}_H(p-q) &\geqslant min^i\{\tilde{\eta}_H(p), \tilde{\eta}_H(q)\} \\ &= min^i\{\tilde{s}, \tilde{s}\} \\ &= \tilde{s} \end{aligned}$$

and so $\tilde{\eta}_H(p-q) = \tilde{s}$ which implies that $p-q \in H$. For any $p \in H, \alpha \in \Gamma$ and $c, d \in M$. Then $\tilde{\eta}_H(c\alpha(p+d) - c\beta d) \geqslant \tilde{\eta}_H(p) = \tilde{s}$ (resp. $\tilde{\eta}_H(p\alpha c) \geqslant \tilde{\eta}_H(p) = \tilde{s}$) and hence $\tilde{\eta}_H(c\alpha(p+d) - c\beta d) = \tilde{s}$ (resp. $\tilde{\eta}_H(p\alpha c) = \tilde{s}$). This shows that M is a left(right) ideal of M.

Conversely assume that H is a left (right) ideal of M. Let $p, q \in M$, if at least one of p and q does not belong to H then $\tilde{\eta}_H(p-q) \geqslant \tilde{t} = min^i\{\tilde{\eta}_p, \tilde{\eta}_q\}$.

If $p, q \in H$,then $p-q \in H$ and so $\tilde{\eta}_H(p-q) = \tilde{s} = min^i\{\tilde{\eta}_H(p), \tilde{\eta}_H(q)\}$.

If $p \in H$,then $q+p-q \in M$ and hence $\tilde{\eta}_H(q+p-q) = \tilde{s} = \tilde{\eta}_H(p)\}$.

Clearly $\tilde{\eta}_H(q+p-q) \geqslant \tilde{t} = \tilde{\eta}_H(p)\}$ for all $p \notin M$ and $q \in M$. This shows that $\tilde{\eta}_H$ is an i.v fuzzy normal divisor w.r.to addition. Now let $p, c, d \in M$ and $\alpha \in \Gamma$.

If $p \in H$,then $c\alpha(p+d)-c\alpha d \in A$ (resp.$p\alpha c \in A$) and thus $\tilde{\eta}_H(c\alpha(p+d)-c\beta d) = \tilde{s} = \tilde{\eta}_H(p)$ (resp. $\tilde{\eta}_H(p\alpha c) = \tilde{s} = \tilde{\eta}_H(p)$.

If $p \notin H$, then clearly

$$\tilde{\eta}_H(c\alpha(p+d) - c\beta d) = \tilde{t} \geqslant \tilde{\eta}_H(p)$$

respective

$$\tilde{\eta}_H(p\alpha c) \geqslant \tilde{t} = \tilde{\eta}_H(p).$$

Hence $\tilde{\eta}_H$ is an i.v fuzzy left(right) ideal of M. Also

$$\begin{aligned} M_{\tilde{\eta}_H} &= \{p \in M | \tilde{\eta}_H(p) = \tilde{\eta}_H(0)\} \\ &= \{p \in M | \tilde{\eta}_H(p) = \tilde{s}\} \\ &= \{p \in M | p \in H\} \\ &= H \end{aligned}$$

□

The following theorem given the relation between fuzzy subsets and crisp subsets of a Γ-near-ring.

THEOREM 3.5. *Let H be a subset of a Γ-near-ring M. The characteristic function $\tilde{\eta}_H : M \to D[0,1]$ is an i.v fuzzy left(right) ideal of M if and only if H is a left(right) ideal of M.*

PROOF. Let H be a left ideal of M, using the theorem 3.4. Conversely , assume hat $\tilde{\eta}_H$ is an i.v fuzzy left(right) ideal of M.

Let $p, q \in H$, then $\tilde{\eta}_H(p) = \tilde{1} = \tilde{\eta}_H(q)$ and so

$$\begin{aligned} \tilde{\eta}_H(p-q) &\geqslant min^i\{\tilde{\eta}_H(p), \tilde{\eta}_H(q)\} \\ &= min^i\{\tilde{1}, \tilde{1}\} \\ &= \tilde{1} \end{aligned}$$

so $\tilde{\eta}_H(p-q) = \tilde{1}$. Therefore $p - q \in H$. Hence H is an additive subgroup of M.

Let $p \in H$ and $q \in M$, then $\tilde{\eta}_H(q+p-q) = \tilde{\eta}_H(p) = \tilde{1}$. Hence $\tilde{\eta}_H(q+p-q) = \tilde{1}$ this implies that $q+p-q \in H$. Thus H is a normal subgroup of M. Let $p \in M$ and $q \in H$, then $\tilde{\eta}_H(p\alpha q) \geqslant \tilde{\eta}_H(q) = \tilde{1}$. Hence $\tilde{\eta}_H(p\alpha q) = \tilde{1}$ this implies $p\alpha q \in H$ and H is a left ideal of M. Let $p, q \in M$ and $r \in H$. Then $\tilde{\eta}_H((p+r)\alpha q) - p\beta q \geqslant \tilde{\eta}_H(r) = \tilde{1}$. Hence $\tilde{\eta}_H((p+r)\alpha q - p\beta) = \tilde{1}$ this implies that $(p+r)\alpha q - p\beta q) \in H$.Hence H is a left(right) ideal of M. □

THEOREM 3.6. *Let $\{\tilde{\eta}_i | i \in \Omega\}$ be family of i.v fuzzy ideals of a Γ- near-ring M, then $\bigcap_{i\in\Omega} \tilde{\eta}_i$ is also an i.v fuzzy ideal of M, where Ω is any index set.*

PROOF. Let $\{\tilde{\eta}_i | i \in \Omega\}$ be a family of i.v fuzzy ideals of M. Let $p, q, r \in M, \alpha, \beta \in \Gamma$ and $\tilde{\eta} = \bigcap_{i\in\Omega} \tilde{\eta}_i$. Then,

$$\tilde{\eta}(p) = \bigcap_{i\in\Omega} \tilde{\eta}_i(p) = \left(\inf^i_{i\in\Omega} \tilde{\eta}_i\right)(p) = \inf^i_{i\in\Omega} \tilde{\eta}_i(p).$$

Now

$$
\begin{aligned}
\tilde{\eta}(p-q) &= \inf^i_{i\in\Omega}\ \tilde{\eta}_i(p-q)\\
&\geqslant \inf^i_{i\in\Omega}\ \min^i\{\tilde{\eta}_i(p),\tilde{\eta}_i(q)\}\\
&= \min^i\left\{\inf^i_{i\in\Omega}\tilde{\eta}_i(p),\inf^i_{i\in\Omega}\tilde{\eta}_i(q)\right\}\\
&= \min^i\left\{\bigcap_{i\in\Omega}\tilde{\eta}_i(p),\bigcap_{i\in\Omega}\tilde{\eta}_i(q)\right\}\\
&= \min^i\{\tilde{\eta}(p),\tilde{\eta}(q)\}.\\
\tilde{\eta}(p\alpha q) &= \inf^i\{\ \tilde{\eta}_i(p\alpha q): i\in\Omega\}\\
&\geqslant \inf^i\ \{\min^i\{\tilde{\eta}_i(p),\tilde{\eta}_i(q)\}: i\in\Omega\}\\
&= \min^i\left\{\inf^i\{\tilde{\eta}_i(p): i\in\Omega\},\inf^i_{i\in\Omega}\{\tilde{\eta}_i(q): i\in\Omega\}\right\}\\
&= \min^i\left\{\bigcap_{i\in\Omega}\tilde{\eta}_i(p),\bigcap_{i\in\Omega}\tilde{\eta}_i(q)\right\}\\
\bigcap_{i\in\Omega}\tilde{\eta}(q+p-q) &= \inf^i\{\ \tilde{\eta}_i(q+p-q): i\in\Omega\}\\
&= \inf^i\{\tilde{\eta}_i(p): i\in\Omega\}\\
&= \left\{\bigcap_{i\in\Omega}\tilde{\eta}_i(p)\right\}.
\end{aligned}
$$

$$
\begin{aligned}
\bigcap_{i\in\Omega}\tilde{\eta}(p\alpha q) &= \inf^i\{\ \tilde{\eta}_i(p\alpha q): i\in\Omega\}\\
&\geqslant \inf^i\{\tilde{\eta}_i(q): i\in\Omega\}\\
&= \left\{\bigcap_{i\in\Omega}\tilde{\eta}_i(q)\right\}\\
\bigcap_{i\in\Omega}\tilde{\eta}((p+r)\alpha q-p\beta q) &= \inf^i\{\ \tilde{\eta}_i(p+r\alpha q-p\beta q): i\in\Omega\}\\
&\geqslant \inf^i\{\tilde{\eta}_i(r): i\in\Omega\}\\
&= \left\{\bigcup_{i\in\Omega}\tilde{\eta}_i(r)\right\}
\end{aligned}
$$

Therefore $\bigcap_{i\in\Omega}\tilde{\eta}_i$ is an i.v fuzzy ideal of M. □

THEOREM 3.7. *Let $\{\tilde{\eta}_i | i\in\Omega\}$ be family of i.v fuzzy ideals of a Γ- near-ring M, then $\bigcup_{i\in\Omega}\tilde{\eta}_i$ is also an i.v fuzzy ideal of M, where Ω is any index set.*

PROOF. Let $\{\tilde{\eta}_i | i \in \Omega\}$ be a family of i.v fuzzy ideals of M. Let $p, q, r \in M, \alpha, \beta \in \Gamma$ and $\tilde{\eta} = \bigcup_{i\in\Omega} \tilde{\eta}_i$. Then,

$$\tilde{\eta}(p) = \bigcup_{i\in\Omega} \tilde{\eta}_i(p) = \left(\sup^i_{i\in\Omega} \tilde{\eta}_i\right)(p) = \sup^i_{i\in\Omega} \tilde{\eta}_i(p).$$

Now

$$\begin{aligned}
\tilde{\eta}(p-q) &= \sup^i_{i\in\Omega} \ \tilde{\eta}_i(p-q) \\
&\geqslant \sup^i_{i\in\Omega} \ \max{}^i\{\tilde{\eta}_i(p), \tilde{\eta}_i(q)\} \\
&= \max{}^i \left\{\sup^i_{i\in\Omega} \tilde{\eta}_i(p), \sup^i_{i\in\Omega} \tilde{\eta}_i(q)\right\} \\
&= \max{}^i \left\{\bigcup_{i\in\Omega} \tilde{\eta}_i(p), \bigcup_{i\in\Omega} \tilde{\eta}_i(q)\right\} \\
&= \max{}^i\{\tilde{\eta}(p), \tilde{\eta}(q)\}. \\
\tilde{\eta}(p\alpha q) &= \sup{}^i\{\ \tilde{\eta}_i(p\alpha q) : i \in \Omega\} \\
&\geqslant \sup{}^i\ \{\max{}^i\{\tilde{\eta}_i(p), \tilde{\eta}_i(q)\} : i \in \Omega\} \\
&= \max{}^i \left\{\sup{}^i\{\tilde{\eta}_i(p) : i \in \Omega\}, \sup^i_{i\in\Omega}\{\tilde{\eta}_i(q) : i \in \Omega\}\right\} \\
&= \max{}^i \left\{\bigcup_{i\in\Omega} \tilde{\eta}_i(p), \bigcup_{i\in\Omega} \tilde{\eta}_i(q)\right\} \\
\bigcup_{i\in\Omega} \tilde{\eta}(q+p-q) &= \sup{}^i\{\ \tilde{\eta}_i(q+p-q) : i \in \Omega\} \\
&= \sup{}^i\{\tilde{\eta}_i(p) : i \in \Omega\} \\
&= \left\{\bigcup_{i\in\Omega} \tilde{\eta}_i(p)\right\}.
\end{aligned}$$

$$\begin{aligned}
\bigcup_{i\in\Omega} \tilde{\eta}(p\alpha q) &= \sup{}^i\{\ \tilde{\eta}_i(p\alpha q) : i \in \Omega\} \\
&\geqslant \sup{}^i\{\tilde{\eta}_i(q) : i \in \Omega\} \\
&= \left\{\bigcup_{i\in\Omega} \tilde{\eta}_i(q)\right\} \\
\bigcup_{i\in\Omega} \tilde{\eta}((p+r)\alpha q - p\beta q) &= \sup{}^i\{\ \tilde{\eta}_i(p + r\alpha q - p\beta q) : i \in \Omega\} \\
&\geqslant \sup{}^i\{\tilde{\eta}_i(r) : i \in \Omega\} \\
&= \left\{\bigcup_{i\in\Omega} \tilde{\eta}_i(r)\right\}
\end{aligned}$$

Therefore $\bigcup_{i\in\Omega} \tilde{\eta}_i$ is an i.v fuzzy ideal of M. □

Theorem 3.8. *Let $\tilde{\eta}$ be an i.v fuzzy subset of M.$\tilde{\eta}$ is an i.v fuzzy left (right) ideal of M if and only if $\tilde{U}(\tilde{\eta}:[t_1,t_2])$ is left (right) ideal of M, for all $[t_1,t_2]\in D[0,1]$.*

Proof. Assume that $\tilde{\eta}$ is an i.v fuzzy left(right) ideal of M.

Let $[t_1,t_2]\in D[0,1]$ such that $p,q\in\tilde{U}(\tilde{\eta}:[t_1,t_2])$.

Then $\tilde{\eta}(p-q)\geqslant \min^i\{\tilde{\eta}(p),\tilde{\eta}(q)\}\geqslant \min^i\{[t_1,t_2],[t_1,t_2]\}=[t_1,t_2]$. Thus $p-q\in\tilde{U}(\tilde{\eta}:[t_1,t_2])$. Let $p\in\tilde{U}(\tilde{\eta}:[t_1,t_2])$ and $q\in M$ and $\alpha,\beta\in\Gamma$. We have $\tilde{\eta}(q+p-q)=\tilde{\eta}(p)\geqslant[t_1,t_2]$. Therefore $q+p-q\in\tilde{U}(\tilde{\eta}:[t_1,t_2])$. Hence $\tilde{U}(\tilde{\eta}:[t_1,t_2])$ is a normal subgroup of M. Let $q\in\tilde{U}(\tilde{\eta}:[t_1,t_2])$ and $p\in M$, thus $\tilde{(}p\alpha q)\geqslant\tilde{(}q)[t_1,t_2]$. Hence $p\alpha q\in\tilde{U}(\tilde{\eta}:[t_1,t_2])$. Let $z\in\tilde{U}(\tilde{\eta}:[t_1,t_2])$ and $p,q\in M$, thus $\tilde{\eta}((p+r)\alpha q-p\beta q)\geqslant\tilde{\eta}(r)\geqslant[t_1,t_2]$, and $((p+r)\alpha q-p\beta q)\in\tilde{U}(\tilde{\eta}:[t_1,t_2])$. Hence $\tilde{U}(\tilde{\eta}:[t_1,t_2])$ is a left (right) ideal of a Γ-near-ring of M.

Conversely, assume $\tilde{U}(\tilde{\eta}:[t_1,t_2])$ is a left(right) ideal of M, for all $[t_1,t_2]\in D[0,1]$. Let $p,q\in M$. Suppose $\tilde{\eta}(p-q)<\min^i\{\tilde{\eta}(p),\tilde{\eta}(q)\}$.

Choose $\tilde{c}=[c_1,c_2]\in D[0,1]$ such that $\tilde{\eta}(p-q)<[c_1,c_2]<\min^i\{\tilde{\eta}(p),\tilde{\eta}(q)\}$. This implies that $\tilde{\eta}(p)>[c_1,c_2]$ and $\tilde{\eta}(q)>[c_1,c_2]$, then we have $p,q\in\tilde{U}(\tilde{\eta}:[t_1,t_2])$, and since $\tilde{U}(\tilde{\eta}:[c_1,c_2])$ is a left(right) ideal of M then $(p-q)\in\tilde{U}(\tilde{\eta}:[c_1,c_2])$. Hence $\tilde{(}p-q)\geqslant[c_1,c_2]$ is a contradiction, this implies that $\tilde{\eta}(p-q)\geqslant\min^i\{\tilde{\eta}(p),\tilde{\eta}(q)\}$.

Suppose $\tilde{\eta}(q+p-q)<\tilde{p}$, choose an interval $\tilde{c}=[c_1,c_2]\in D[0,1]$ such that $\tilde{\eta}(q+p-q)<[c_1,c_2]\tilde{\eta}(p)$. This implies that $\tilde{\eta}(p)>[c_1,c_2]$ then we have $p\in\tilde{U}(\tilde{\eta}:[c_1,c_2])$, and since $\tilde{U}(\tilde{\eta}:[c_1,c_2])$ is a normal subgroup of $(M,+)$ then $(q+p-q)\in\tilde{U}(\tilde{\eta}:[c_1,c_2])$. Hence $\tilde{(}q+p-q)\geqslant[c_1,c_2]$ is a contradiction. Hence $\tilde{\eta}(q+p-q)\geqslant\tilde{\eta}(p)$. Similarly $\tilde{\eta}(q+p-q)\leqslant\tilde{\eta}(p)$. Hence $\tilde{\eta}(q+p-q)=\tilde{\eta}(p)$.

Suppose $\tilde{\eta}(p\alpha q)<\tilde{q}$, choose an interval $\tilde{c}=[c_1,c_2]\in D[0,1]$ such that $\tilde{\eta}(p\alpha q)<[c_1,c_2]\tilde{\eta}(q)$. This implies that $\tilde{\eta}(q)>[c_1,c_2]$ then we have $q\in\tilde{U}(\tilde{\eta}:[c_1,c_2])$, and since $\tilde{U}(\tilde{\eta}:[c_1,c_2])$ is a left(right) ideal of M then $(p\alpha q)\in\tilde{U}(\tilde{\eta}:[c_1,c_2])$. Hence $\tilde{(}p\alpha q)\geqslant[c_1,c_2]$ is a contradiction. Hence $\tilde{\eta}(p\alpha q)=\tilde{\eta}(q)$.

Suppose $\tilde{\eta}((p+r)\alpha q-p\beta q)<\tilde{r}$, choose an interval $\tilde{c}=[c_1,c_2]\in D[0,1]$ such that $\tilde{\eta}((p+r)\alpha q-p\beta q))<[c_1,c_2]<\tilde{\eta}(r)$. This implies that $\tilde{\eta}(r)>[c_1,c_2]$ we have $q\in\tilde{U}(\tilde{\eta}:[c_1,c_2])$, and since $\tilde{U}(\tilde{\eta}:[c_1,c_2])$ is a left(right) ideal of M it follows that $((p+r)\alpha q-p\beta q))\in\tilde{U}(\tilde{\eta}:[c_1,c_2])$. Hence $\tilde{(}(p+r)\alpha q-p\beta q))\geqslant[c_1,c_2]$ which is a contradiction. Hence $\tilde{\eta}((p+r)\alpha q-p\beta q))\geqslant\tilde{\eta}(r)$. Thus $\tilde{\eta}$ is an i.v fuzzy left(right) ideal of M. □

4. Homomorphism of interval valued fuzzy ideals of Γ-near-rings

In this section, we characterize i.v fuzzy ideals of Γ-near-rings using homomorphism.

Definition 4.1. ([**9**]) Let f be a mapping from a set M to a set S. Let $\tilde{\eta}$ and $\tilde{\delta}$ be i.v fuzzy subsets of M and S respectively. Then $f(\tilde{\eta})$, the image of $\tilde{\eta}$ under f is an i.v fuzzy subset of S defined by

$$f(\tilde{\eta})(y)=\begin{cases}\sup^i_{x\in f^{-1}(y)}\tilde{\eta}(x) & \text{if } f^{-1}(y)\neq\emptyset,\\ 0 & \text{otherwise}\end{cases}$$

and the pre-image of $\tilde{\eta}$ under f is an i.v fuzzy subset of M defined by $f^{-1}(\tilde{\delta}(x)) = \tilde{\delta}(f(x))$, for all $x \in M$ and $f^{-1}(y) = \{x \in M | f(x) = y\}$.

Definition 4.2. ([**5**]) Let M and S be Γ-near-rings. A map $\theta : M \to S$ is called a (Γ-near-ring)homomorphism if $\theta(x + y) = \theta(x) + \theta(y)$ and $\theta(x\alpha y) = \theta(x)\alpha\theta(y)$ for all $x, y \in M$ and $\alpha \in \Gamma$.

Theorem 4.1. *Let $f : M_1 \to M_2$ be a homomorphism between Γ-near-rings M_1 and M_2. If $\tilde{\delta}$ is an i.v fuzzy ideal of M_2, then $f^{-1}(\tilde{\delta})$ is an i.v fuzzy left (right) ideal of M_1.*

Proof. Let $\tilde{\delta}$ is an i.v fuzzy ideal of M_2. Let $p, q, r \in M_1$ and $\alpha, \beta \in \Gamma$. Then

$$\begin{aligned}
f^{-1}(\tilde{\delta})(p - q) &= \tilde{\delta}(f(p - q)) \\
&= \tilde{\delta}(f(p) - f(q)) \\
&\geqslant \min{}^i\{\tilde{\delta}(f(p)), \tilde{\delta}(f(q))\} \\
&= \min{}^i\{f^{-1}(\tilde{\delta}(p)), f^{-1}(\tilde{\delta}(q))\}. \\
f^{-1}(\tilde{\delta})(p\alpha q) &= \tilde{\delta}(f(p\alpha q)) \\
&= \tilde{\delta}(f(p), f(q)) \\
&\geqslant \min{}^i\{\tilde{\delta}(f(p)), \tilde{\delta}(f(q))\} \\
&= \min{}^i\{f^{-1}(\tilde{\delta}(p)), f^{-1}(\tilde{\delta}(q))\}. \\
f^{-1}(\tilde{\delta})(q + p - q) &= \tilde{\delta}(f(q + p - q)) \\
&= \tilde{\delta}(f(q) + f(p) - f(q)) \\
&= \tilde{\delta}(f(p)) \\
&= f^{-1}(\tilde{\delta}(p)). \\
f^{-1}(\tilde{\delta})(p\alpha q) &= \tilde{\delta}(f(p\alpha q)) \\
&= \tilde{\delta}(f(p)\alpha f(q)) \\
&\geqslant \tilde{\delta}(f(q)) \\
&= f^{-1}(\tilde{\delta}(q)) \\
f^{-1}(\tilde{\delta})((p + r)\alpha q - p\beta q) &= \tilde{\delta}(f((p + r)\alpha q - p\beta q)) \\
&= \tilde{\delta}(f(p) + f(r))\alpha f(q) - f(p)\beta f(q) \\
&\geqslant \tilde{\delta}(f(r)) \\
&= f^{-1}(\tilde{\delta}(r))
\end{aligned}$$

Therefore $f^{-1}(\tilde{\delta})$ is an i.v fuzzy left(right) ideal of M_1. □

Theorem 4.2. *Let $f : M_1 \to M_2$ be an onto homomorphism of Γ- near-rings M_1 and M_2. Let $\tilde{\delta}$ be an i.v fuzzy subset of M_2. If $f^{-1}(\tilde{\delta})$ is an i.v fuzzy left (right)ideal of M_1, then $\tilde{\delta}$ is an i.v fuzzy left(right) ideal of M_2.*

Proof. Let $p, q, r \in M_2$. Then $f(a) = p, f(b) = q$ and $f(c) = r$ for some $a, b, c \in M_1$ and $\alpha, \beta \in \Gamma$. It follows that

$$\begin{aligned}
\tilde{\delta}(p-q) &= \tilde{\delta}(f(a)-f(b)) \\
&= \tilde{\delta}(f(a-b)) \\
&= f^{-1}(\tilde{\delta})(a-b) \\
&\geqslant \min^i\{f^{-1}(\tilde{\delta})(a), f^{-1}(\tilde{\delta})(b)\} \\
&= \min^i\{\tilde{\delta}(f(a)), \tilde{\delta}(f(b))\} \\
&= \min^i\{\tilde{\delta}(p), \tilde{\delta}(q)\}.
\end{aligned}$$

$$\begin{aligned}
\tilde{\delta}(p\alpha q) &= \tilde{\delta}(f(a)\alpha f(b)) \\
&= \tilde{\delta}(f(a\alpha b)) \\
&= f^{-1}(\tilde{\delta})(a\alpha b) \\
&\geqslant \min^i\{f^{-1}(\tilde{\delta})(a), f^{-1}(\tilde{\delta})(b)\} \\
&= \min^i\{\tilde{\delta}(f(a)), \tilde{\delta}(f(b))\} \\
&= \min^i\{\tilde{\delta}(p), \tilde{\delta}(q)\}.
\end{aligned}$$

$$\begin{aligned}
\tilde{\delta}(q+p-q) &= \tilde{\delta}(f(b)+f(a)-f(b)) \\
&= \tilde{\delta}(f(b+a-b)) \\
&= f^{-1}(\tilde{\delta})(b+a-b) \\
&= f^{-1}\tilde{\delta}(a) \\
&= \tilde{\delta}(f(a)) \\
&= \tilde{\delta}(p).
\end{aligned}$$

$$\begin{aligned}
\tilde{\delta}(p\alpha q) &= \tilde{\delta}(f(a)\alpha f(b)) \\
&= \tilde{\delta}(f(a\alpha b)) \\
&= f^{-1}(\tilde{\delta})(a\alpha b) \\
&\geqslant f^{-1}(\tilde{\delta})(b) \\
&= \tilde{\delta}(f(b)) \\
&= \tilde{\delta}(q)\}
\end{aligned}$$

$$\begin{aligned}
\tilde{\delta}(p+r)\alpha q - p\beta q) &= \tilde{\delta}(f(a)+f(c))\alpha f(b) - f(a)\beta f(b)) \\
&= \tilde{\delta}(f(a+c)\alpha b - a\beta b)) \\
&= f^{-1}(\tilde{\delta})(a+c)\alpha b - a\beta b) \\
&\geqslant f^{-1}(\tilde{\delta})(c) \\
&= \tilde{\delta}(f(c)) = \tilde{\delta}(f(r)).
\end{aligned}$$

Hence $\tilde{\delta}$ is an i.v fuzzy left(right)ideal of M_1 □

THEOREM 4.3. *Let $f : M_1 \to M_2$ be an onto Γ-near-ring homomorphism. If $\tilde{\eta}$ is an i.v fuzzy left (right)ideal of M_1, then $f(\tilde{\eta})$ is an i.v fuzzy left (right)ideal of M_2.*

PROOF. Let $\tilde{\eta}$ be an i.v fuzzy ideal of M_1. Since $f(\tilde{\eta})(p') = \sup^i_{f(p)=p'}(\tilde{\eta}(p))$, for $p' \in M_2$ and hence $f(\tilde{\eta})$ is nonempty. Let $p', q' \in M_2$ and $\alpha, \beta \in \Gamma$.
Then we have $\{p|p \in f^{-1}(p'-q')\} \supseteq \{p-q|x \in f^{-1}(p')$ and $q \in f^{-1}(q')\}$ and $\{p|p \in f^{-1}(p'q')\} \supseteq \{p\alpha q|p \in f^{-1}(p')$ and $q \in f^{-1}(q')\}$.

$$\begin{aligned}
f(\tilde{\eta})(p'-q') &= \sup^i_{f(r)=p'-q'}\{\tilde{\eta}(r)\} \\
&\geqslant \sup^i_{f(p)=p',f(q)=q'}\{\tilde{\eta}(p-q)\} \\
&\geqslant \sup^i_{f(p)=p',f(q)=q'}\{\min^i\{\tilde{\eta}(p),\tilde{\eta}(q)\}\} \\
&= \min^i\{\sup^i_{f(p)=p'}\{\tilde{\eta}(p)\}, \sup^i_{f(q)=q'}\{\tilde{\eta}(q)\}\} \\
&= \min^i\{f(\tilde{\eta})(p'), f(\tilde{\eta})(q')\}. \\
f(\tilde{\eta})(p'\alpha q') &= \sup^i_{f(r)=p'\alpha q'}\{\tilde{\eta}(r)\} \\
&\geqslant \sup^i_{f(p)=p',f(q)=q'}\{\tilde{\eta}(p\alpha q)\} \\
&\geqslant \sup^i_{f(p)=p',f(q)=q'}\{\min^i\{\tilde{\eta}(p),\tilde{\eta}(q)\}\} \\
&= \min^i\{\sup^i_{f(p)=p'}\{\tilde{\eta}(p)\}, \{\sup^i_{f(q)=q'}\{\tilde{\eta}(q)\}\} \\
&= \min^i\{f(\tilde{\eta})(p'), f(\tilde{\eta})(q')\}.
\end{aligned}$$

$$\begin{aligned}
f(\tilde{\eta})(q'+p'-q') &= \sup^i_{f(r)=q'+p'-q'}\{\tilde{\eta}(r)\} \\
&\geqslant \sup^i_{f(p)=p',f(q)=q'}\{\tilde{\eta}(q+p-q)\} \\
&= \sup^i_{f(p)=p'}\{\tilde{\eta}(p)\}. \\
f(\tilde{\eta})(p'\alpha q') &= \sup^i_{f(r)=p'\alpha q'}\{\tilde{\eta}(r)\} \\
&\geqslant \sup^i_{f(p)=p',f(q)=q'}\{\tilde{\eta}(p\alpha q)\} \\
&\geqslant \sup^i_{f(q)=q'}\{\tilde{\eta}(q)\} \\
&= f(\tilde{\eta})(q'). \\
f(\tilde{\eta})((p'+r')\alpha q' - p'\beta q') &= \sup^i_{f(r)=(p'+r')\alpha q'-p'\beta q'}\{\tilde{\eta}(r)\} \\
&\geqslant \sup^i_{f(p)=p',f(q)=q',f(r)=r'}\{\tilde{\eta}((p+r)\alpha q - p\beta q))\} \\
&\geqslant \sup^i_{f(r)=r'}\{\tilde{\eta}(r)\} \\
&= f(\tilde{\eta})(r').
\end{aligned}$$

Therefore $f(\tilde{\eta})$ is an i.v fuzzy left (right)ideal of M_2. □

5. Anti-homomorphism of interval valued fuzzy ideals of Γ-near-rings

In this section, we characterize i.v fuzzy ideals of Γ-near-rings using anti-homomorphism.

DEFINITION 5.1. ([**7**]) Let M and S be Γ-near-rings. A map $\theta : M \to S$ is called a (Γ-near-ring) anti-homomorphism if $\theta(x+y) = \theta(y) + \theta(x)$ and $\theta(x\alpha y) = \theta(y)\alpha\theta(x)$ for all $x, y \in M$ and $\alpha \in \Gamma$.

THEOREM 5.1. *Let $f : M_1 \to M_2$ be an anti-homomorphism between Γ-near-rings M_1 and M_2. If $\tilde{\delta}$ is an i.v fuzzy ideal of M_2, then $f^{-1}(\tilde{\delta})$ is an i.v fuzzy left (right)ideal of M_1.*

THEOREM 5.2. *Let $f : M_1 \to M_2$ be an onto anti-homomorphism of Γ- near-rings M_1 and M_2. Let $\tilde{\delta}$ be an i.v fuzzy subset of M_2. If $f^{-1}(\tilde{\delta})$ is an i.v fuzzy left (right)ideal of M_1, then $\tilde{\delta}$ is an i.v fuzzy left(right) ideal of M_2.*

THEOREM 5.3. *Let $f : M_1 \to M_2$ be an onto Γ-near-ring anti-homomorphism. If $\tilde{\eta}$ is an i.v fuzzy left (right)ideal of M_1, then $f(\tilde{\eta})$ is an i.v fuzzy left (right)ideal of M_2.*

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