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3-DIFFERENCE CORDIALITY OF CORONA OF DOUBLE ALTERNATE SNAKE GRAPHS

R.Ponraj, M.Maria Adaickalam, and R.Kala

Abstract. Let G be a (p, q) graph. Let $f : V(G) \rightarrow \{1, 2, \ldots, k\}$ be a map where k is an integer $2 \leqslant k \leqslant p$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called k-difference cordial labeling of G if $\left|v_f(i) - v_f(j)\right| \leqslant 1$ and $\left|e_f(0) - e_f(1)\right| \leqslant 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph. In this paper we investigate 3-difference cordial labeling behavior of $DA(T_n) \odot K_1$, $DA(T_n) \odot 2K_1$, $DA(T_n) \odot K_2$, $DA(Q_n) \odot K_1$, $DA(Q_n) \odot 2K_1$.

1. Introduction

Graphs considered here are finite and simple. Graph labeling is used in several areas of science and technology like coding theory, astronomy, circuit design etc. For more details refer Gallian [**1**]. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. Recently Ponraj et al. [**3**], introduced the concept of k-difference cordial labeling of graphs and studied the 3-difference cordial labeling behavior of of star, m copies of star etc. In [**4, 5, 6**] they discussed the 3-difference cordial labeling behavior of path, cycle, complete graph, complete bipartite graph, star, bistar, comb, double comb, quadrilateral snake, $C_4^{(t)}$, $S(K_{1,n})$, $S(B_{n,n})$ and some more graphs. In this paper we investigate 3-difference cordial labeling behavior of $DA(T_n) \odot K_1$, $DA(T_n) \odot 2K_1$, $DA(T_n) \odot K_2$, $DA(Q_n) \odot K_1$, $DA(Q_n) \odot 2K_1$. Terms are not defined here follows from Harary [**2**].

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2. k-Difference cordial labeling

DEFINITION 1. Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \ldots, k\}$ be a map. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leqslant 1$ and $|e_f(0) - e_f(1)| \leqslant 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph.

A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path. That is a double alternate triangular snake is obtained from a path $u_1u_2...u_n$ by joining u_i and u_{i+1} (alternatively) to two new vertices v_i and w_i.

Theorem 2.1-2.3 the 3-difference cordial behavior of

$$DA(T_n) \odot K_1,\ DA(T_n) \odot 2K_1 \text{ and } DA(T_n) \odot K_2.$$

THEOREM 2.1. $DA(T_n) \odot K_1$ is 3-difference cordial.

PROOF. Let

$V(DA(T_n) \odot K_1) = V(DA(T_n)) \cup \{u'_i : 1 \leqslant i \leqslant n\} \cup \{v'_i, w'_i : 1 \leqslant i \leqslant \frac{n}{2}\}$

and

$E(DA(T_n) \odot K_1) = E(DA(T_n)) \cup \{u_iu'_i : 1 \leqslant i \leqslant n\} \cup \{v_iv'_i, w_iw'_i : 1 \leqslant i \leqslant \frac{n}{2}\}$.

Case 1. The two triangles stars from u_1 and ends with u_n.

First we consider the path vertices u_i. Assign the label 1 to the path vertices $u_1, u_3, u_5, ...$ Now we assign the label 3 to the path vertices u_2 and u_4. For all the values of $i = 0, 1, 2, 3, ...$ assign the label 2 to the path vertices u_{12i+6}. Now we assign the label 3 to the path vertices $u_8, u_{20}, u_{32}, ...$ and the sequence of vertices $u_{10}, u_{22}, u_{34}, ...$ Then we assign the label 3 to the path vertices vertices u_{12i} for all the values of $i = 1, 2, 3, ...$ an we assign the label 3 to the path vertices u_{12i+2} for $i = 1, 2, 3, ...$ For all the values of $i = 1, 2, 3, ...$ assign the label 3 to the path vertices u_{12i+4}. Next we move to the vertices v_i and w_i. Assign the label 2 to the vertices $v_1, v_2, v_3, ...$ and we assign the label 1 to the vertices $w_1, w_2, w_3, ...$ Consider the vertices v'_i. Assign the label 1,3 to the vertices v'_1 and v'_2 respectively. Now we assign the labels $1, 1, 3$ to the vertices v'_3, v'_4, v'_5 respectively. Then we assign the labels $1, 1, 3$ to the next three vertices v'_6, v'_7, v'_8 respectively. Continuing like this assign the label to the next three vertices and so on. If all the vertices are labeled, then we stop the process. Otherwise there are some nonlabeled vertices are exist. If the number of nonlabeled vertices are less than or equal to 2 then assign the label 1,1 to the nonlabeled vertices. If only one nonlabeled vertices exist assign the label 1 only. Next we move to the vertices w'_i. Assign the $2, 2, 3$ to the vertices w'_1, w'_2, w'_3 respectively. Then we assign the label 3 to the vertices $w'_9, w'_{15}, w'_{21}...$ Now we assign the label 2 to the vertices w'_{6i+4} and w'_{6i+5} for all the values of $i = 0, 1, 2, 3, ...$ for all the values of $i = 1, 2, 3, ...$ assign the label 2 to the vertices w'_{6i}, w'_{6i+1} and w'_{6i+2}. Finally we consider the vertices u'_i. Assign the label 2 to the vertices u'_1 and u'_3. Then we assign the label 3 to the vertices $u'_2, u'_4, u'_6...$ Next we assign the label 3 to the vertices $u'_5, u'_{17}, u'_{29}, ...$ and the vertices $u'_7, u'_{19}, u'_3, ...$ Now we assign the label 2 to the vertices u'_{12i+1} and u'_{12i+3}. The edge condition of this case $e_f(0) = \frac{5n-2}{2}$ and $e_f(1) = \frac{5n}{2}$. Also the vertex condition is given in Table 1.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0, 6 \pmod{12}$	$\frac{4n}{3}$	$\frac{4n}{3}$	$\frac{4n}{3}$
$n \equiv 2 \pmod{12}$	$\frac{4n+1}{3}$	$\frac{4n+1}{3}$	$\frac{4n-2}{3}$
$n \equiv 4 \pmod{12}$	$\frac{4n-1}{3}$	$\frac{4n+2}{3}$	$\frac{4n-1}{3}$
$n \equiv 8 \pmod{12}$	$\frac{4n+1}{3}$	$\frac{4n-2}{3}$	$\frac{4n+1}{3}$
$n \equiv 10 \pmod{12}$	$\frac{4n-1}{3}$	$\frac{4n-1}{2}$	$\frac{4n+2}{3}$

TABLE 1

Case 2. The two triangles starts from u_2 and ends with u_{n-1}.

Consider the path vertices u_i. Assign the label 3 to the path vertices u_{12i+1}, u_{12i+3}, u_{12i+5} and u_{12i+7} for all the values of i=0,1,2,3,... Then assign the label 2 to th path vertices u_{12i+9}, u_{12i+10} and u_{12i+11} for all the values of i=0,1,2,3,... For all the values of i=0,1,2,3,... assign the label 1 to the path vertices $u_{12i+2}, u_{12i+4}, u_{12i+6}$ and u_{12i+8}. Then assign the label to the path vertices u_{12i} for i=1,2,3,... Next we move to the vertices v_i and w_i. Assign the label 2 to the vertices v_1, v_2, v_3. Then we assign the label 1 to the vertices $v_4, v_{10}, v_{16}, ...$ Now we assign the label 2 to the vertices v_{6+5} for $i = 0, 1, 2, 3...$ Then for all values of $i = 1, 2, 3, ...$ assign the label 2 to the vertices $v_{6i}, u_{6i+1}, u_{6i+2}$ and u_{6i+3}. Now we assign the label 1 to the vertices w_1, w_2, w_3 and assign the label 3 to the vertices $w_4, w_{10}, w_{16}, ...$ for all the values of $i = 0, 1, 2, 3...$ assign the label 1 to the vertices w_{6i+5}. Then assign the label 1 to the vertices $w_{6i}, w_{6i+1}, w_{6i+2}, w_{6i+3}$ for $i = 1, 2, 3...$ Now we consider the vertices v'_i. Assign the label 1 to th vertices v'_1. Then we assign the label 1 to the vertices $u_2, u_4, u_6, ...$ and assign the label 3 to the vertices $u_3, u_5, u_7, ...$ Next we move to the vertices u'_i. Assign the label 2 to the vertices u'_1, u'_2, u'_6 and u'_8 and assign the label 3 to the vertices u'_3, u'_4, u'_5 and u'_7. Then assign the label 3 to the vertices u'_{12i+9} and u'_{12i+11} for all the values of $i = 0, 1, 2, 3, ...$ For all the values of $i = 1, 2, 3, ...$ assign the label 3 to the vertices $u'_{12i}, u'_{12i+1}, u'_{12i+3}, u'_{12i+5}$ and u'_{12i+7}. Now we assign the label 1 to the vertices u'_{12i+10} for $i = 0, 1, 2, 3...$ Then we assign the label 2 to the vertices $u'_{12i+2}, u'_{12i+4}, u'_{12i+6}$ and u'_{12i+8} for all the values of $i = 1, 2, 3, ...$ Now we move to the vertices w'_i. Assign the label 2 to the vertices w'_1, w'_2, w'_3. Then we assign the label 3 to the vertices $w'_4, w'_{10}, w'_{16}, ...$ and assign the label 1 to the vertices $w'_5, w'_{11}, w'_{17}, ...$ For all the values of $i = 1, 2, 3, ...$ assign the label 2 to the vertices $w'_{6i}, w'_{6i+1}, w'_{6i+2}$ and w'_{6i+3}. Note that in this case the edge condition is $e_f(0) = \frac{5n-6}{2}$ and $e_f(1) = \frac{5n-8}{2}$. Also the vertex condition is given in Table 2.

Case 3. The two triangles starts from u_2 and ends with u_n.

First we consider the path vertices u_i. Assign the label 1 to the path vertices $u_2, u_4, u_6, u_8, ...$ and we assign the label 2 to the vertices $u_{11}, u_{23}, u_{35}, u_{47}, ...$ Then for all the values of $i = 0, 1, 2, 3...$ assign the label 3 to the vertices u_{12i+1}, u_{12i+3}, u_{12i+5}, u_{12i+7} and u_{12i+9}. Now we consider the vertices v_i and w_i. Assign the label

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 0 \pmod{12}$	$\frac{4n-3}{3}$	$\frac{4n-6}{3}$	$\frac{4n-3}{3}$
$n \equiv 2 \pmod{12}$	$\frac{4n-2}{3}$	$\frac{4n-5}{3}$	$\frac{4n-5}{3}$
$n \equiv 4, 10 \pmod{12}$	$\frac{4n-4}{3}$	$\frac{4n-4}{3}$	$\frac{4n-4}{3}$
$n \equiv 6 \pmod{12}$	$\frac{4n-3}{3}$	$\frac{4n-3}{3}$	$\frac{4n-6}{3}$
$n \equiv 8 \pmod{12}$	$\frac{4n-5}{3}$	$\frac{4n-2}{3}$	$\frac{4n-5}{3}$

Table 2

to the vertices v_i $(1 \leqslant i \leqslant n-1)$ and w_i $(1 \leqslant i \leqslant n-1)$ as in case 1. Next we move to the vertices v'_i. Assign the label 1 to the vertices v'_1 and v'_3 and we assign the label 3 to the vertices v'_2 and v'_4. Now we assign the labels 1,1,3 to the vertices v'_5, v'_6, v'_7 respectively. Then we assign the label 1,1,3 to the vertices v'_8, v'_9, v'_{10} respectively. Continuing like this assign the label to the next three vertices and so on. If all the vertices are labeled then we stop the process. Otherwise there are some non labeled vertices are exist. If the number of non labeled vertices are less than or equal to 2, then assin the labels 1,1 to the non labeled vertices. If only one non labeled vertex is exist then assign the label 1 to that vertex. Now we consider the vertices u'_i. Assign the label 2 to the vertices u'_1, u'_4, u'_6, u'_8 and we assign the label 1 to the vertex u'_5. Then we assign the label 3 to the vertices u'_2, u'_3, u'_7, u'_9. Assign the label u'_{12i+10} and u'_{12i+11} for all the values of i=0,1,2,3,... For all the values of i=1,2,3,... assign the label 3 to the vertices $u'_{12i+1}, u'_{12i}, u'_{12i+3}, u'_{12i+5}$ and u'_{12i+7}. Now we assign the label 2 to the vertices u'_{12i+2}, u'_{12i+4} and u'_{12i+6} for i=1,2,3,... Next we move to the vertices w'_i. Assign the label 2 to the vertices w'_1, w'_2, w'_3, w'_4. Then we assign the label 3 to the vertices $w'_5, w'_{11}, w'_{17}, \ldots$ For all the values of i=1,2,3,... assign the label 2 to the vertices $w'_{6i}, w'_{6i+1}, w'_{6i+2}, u'_{6i+3}$ and u'_{6i+4}. Note that in this case the edge condition is $e_f(0) = \frac{5n-3}{2}$ and $e_f(1) = \frac{5n-5}{2}$. Also the vertex codition of this case is given in Table 3.

Nature of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
$n \equiv 1 \pmod{12}$	$\frac{4n-1}{3}$	$\frac{4n-4}{3}$	$\frac{4n-1}{3}$
$n \equiv 3 \pmod{12}$	$\frac{4n-3}{3}$	$\frac{4n-3}{3}$	$\frac{4n}{3}$
$n \equiv 5, 11 \pmod{12}$	$\frac{4n-2}{3}$	$\frac{4n-2}{3}$	$\frac{4n-2}{3}$
$n \equiv 7 \pmod{12}$	$\frac{4n-1}{3}$	$\frac{4n-1}{3}$	$\frac{4n-4}{3}$
$n \equiv 9 \pmod{12}$	$\frac{4n-3}{3}$	$\frac{4n}{3}$	$\frac{4n-3}{3}$

Table 3

□

A 3-difference cordial labeling of $DA(T_8) \odot K_1$ where the two triangle starts from u_1 and ends with u_8 is shown in Figure 1.

A 3-difference cordial labeling of $DA(T_{10}) \odot K_1$ where the two triangle starts from u_2 and ends with u_9 is shown in Figure 2.

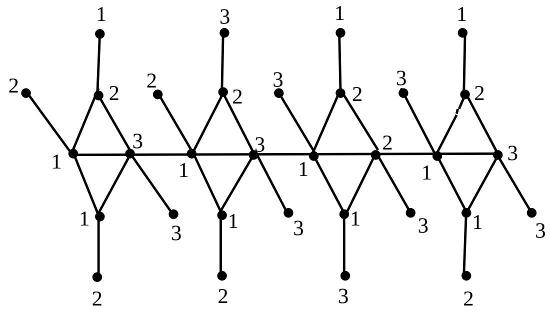

FIGURE 1

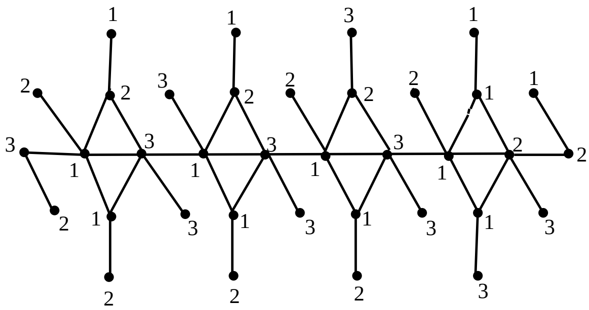

FIGURE 2

A 3-difference cordial labeling of $DA(T_9) \odot K_1$ where the two triangles starts from u_2 and ends with u_9 is shown in Figure 3.

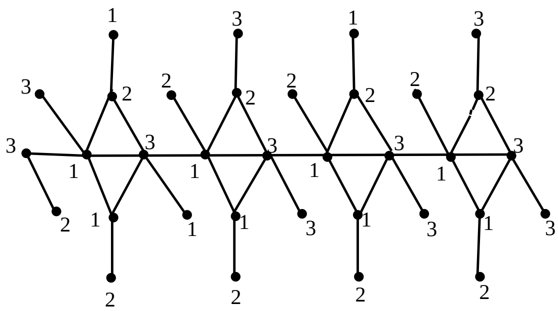

FIGURE 3

THEOREM 2.2. $DA(T_n) \odot 2K_1$ is 3-difference cordial.

PROOF. Let
$V(DA(T_n) \odot 2K_1)$
$= V(DA(T_n)) \cup \{u_i', u_i'' : 1 \leqslant i \leqslant n\} \cup \{v_i', w_i', v_i'', w_i'' : 1 \leqslant i \leqslant \frac{n}{2}\}$
and
$E(DA(T_n) \odot 2K_1)$
$= E(DA(T_n)) \cup \{u_iu_i', u_iu_i'' : 1 \leqslant i \leqslant n\} \cup \{v_iv_i', v_iv_i'', w_iw_i', w_iw_i'' : 1 \leqslant i \leqslant \frac{n}{2}\}$.

Case 1. The two triangles starts from u_1 and ends with u_n.

First we consider the path vertices u_i. Assign the labels 1,2,2,1 to the first four path vertices u_1, u_2, u_3, u_4 respectively. Then we assign the labels 1,2,2,1 to the next four path vertices u_5, u_6, u_7, u_8 respectively. Continuing like this we assign the label to the next four vertices and so on. Note that in this case the last vertex u_n received the label 1 or 2 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Next we move to the vertices v_i and w_i. Assign the label 2 to the vertices v_1, v_2, v_3, v_4... and we assign the label 1 to the vertices w_1, w_2, w_3, w_4... Now we consider the vertices v_i' and v_i''. Assign the label 1 to all the vertices of v_i' $(1 \leqslant i \leqslant \frac{n}{2})$ and assign the label 3 to all the vertices of v_i'' $(1 \leqslant i \leqslant \frac{n}{2})$. Next we assign the label 2 to the vertices w_1', w_2', w_3', w_4'... and assign the label 3 to the vertices $w_1'', w_2'', w_3'', w_4''$... Next we move to the vertices u_i' and u_i''. Assign the label 1 to the vertices u_{4i+1}' for all the values of i=0,1,2,3,... and we assign the label 1 to u_{4i}' for i=1,2,3,... For all the values of i=0,1,2,3... assign the label 2 to the vertices u_{4i+2}' and u_{4i+3}'. Finally assign the label 3 to the vertices $u_1'', u_2'', u_3'', u_4''$... The verte and edge condition is given by $v_f(1) = v_f(2) = v_f(3) = 2n$ and $e_f(0) = \frac{7n-2}{2}$ and $e_f(1) = \frac{7n}{2}$.

Case 2. The two triangles starts from u_2 and ends with u_{n-1}.

Firt we consider the path vertices u_i. Assign the label 1 to the vertices u_1, u_2 and u_6. Then assign the label 2 to the vertices u_1, u_4 and u_5. Now we assign the labels 1,2,2,1 to the vertices u_7, u_8, u_9, u_{10} respectively. Then we assign the labels 1,2,2,1 to the vertices $u_{11}, u_{12}, u_{13}, u_{14}$ respectively. Proceeding like this we assign the label to the next four vertices and so on. Note that in this case the last vertex u_n received the label 2 or 1 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Now we assign the label to the vertices $v_i, w_i, v_i', v_i'', w_i', w_i''$ $(1 \leqslant i \leqslant \frac{n}{2})$ as in case 1. Next we move to the vertices u_i' and u_i''. Assign the label 1 to the vertices u_2', u_3' and assign the label 2 to the vertex u_2'. For all the values of i=1,2,3,... assign the label 2 to the vertices u_{4i}' and u_{4i+1}'. Then assign the label 1 to the vertices u_{4i+2}' and u_{4i+3}' for all the values of i=1,2,3... Finally assign the label $u_i''(1 \leqslant i \leqslant n)$ as in case 1. Clearly the vertex and edge condition of this case is $v_f(1) = v_f(2) = v_f(3) = 2n-2$ and $e_f(0) = \frac{7n-10}{2}$ and $e_f(1) = \frac{7n-8}{2}$.

Case 3. The two triangles starts from u_2 and ends with u_n.

Label the vertices $v_i, v_i', v_i'', w_i, w_i', w_i''(1 \leqslant i \leqslant \frac{n}{2})$ as in case 1. Then we assign the label 2 to the vertex u_1 and assign the label 3 to vertex u_3. Assign the label 1 to the vertices u_2 and u_4. Now we assign the labels 2,2,1,1 to the path vertices u_5, u_6, u_7, u_8 respectively. Then we assign the labels 2,2,1,1 to the next four path vertices $u_9, u_{10}, u_{11}, u_{12}$ respectively. Continuing like this we assign the label to the next four path vertices and so on. Clearly the last four vertices $u_{n-3}, u_{n-2}, u_{n-1}, u_n$ received the label by the integers 2,2,1,1 respectively. Next we move to the vertices u'_i and u''_i. Assign the labels 1,2 to the vertices u'_1 and u'_2 respectively. Then assign the label 1 to the vertices u'_{4i+1} and u'_{4i+2} for all the values of i=1,2,3,... For all the values of i=1,2,3,... assign the label 1 to th vertices u'_{4i}. Finally assign the label 2 to the vertex u''_1 and assign the label 3 to the vertices $u''_2, u'_3, u''_4, ...$ Clearly the vertex condition is $v_f(1) = v_f(2) = v_f(3) = 2n - 1$ and the edge condition is $e_f(0) = e_f(1) = \frac{7n-5}{2}$. □

THEOREM 2.3. $DA(T_n) \odot K_2$ is 3-difference cordial.

PROOF. Let
$V(DA(T_n) \odot K_2)$
$= V(DA(T_n)) \cup \{u'_i, u''_i : 1 \leqslant i \leqslant n\} \cup \{v'_i,\ w'_i,\ v''_i,\ w''_i\ :\ 1 \leqslant i \leqslant \frac{n}{2}\}$
and
$E(DA(T_n) \odot K_2)$
$= E(DA(T_n)) \cup \{u_iu'_i, u_iu''_i, u'_iu''_i : 1 \leqslant i \leqslant n\} \cup \{v_iv'_i, v_iv''_i, v'_iv''_i, w_iw'_i, w_iw''_i, w'_iw''_i :$
$1 \leqslant i \leqslant \frac{n}{2}\}$.

Case 1. The two triangles starts from u_1 and ends with u_n.

Consider the path vertices u_i. Assign the label 1,1,2,2 to the path vertices $u_1,\ u_2,\ u_3,\ u_4$ respectively. Then we assign the label 1,1,2,2 to the next four path vertices u_5, u_6, u_7, u_8 respectively. Continuing like this we assign the label to the next four path vertices and so on. Clearly the last vertex u_n received the label 2 or 1 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Next we move to the vertices v_i and w_i. Assign the label 2 to the vertices $v_1, v_2, v_3, v_4, ...$ and assign the label 3 to the vertices $w_1, w_2, w_3, ...$ Now we consider the vertices u'_i and u''_i. Assign the label 3 to all the vertices of $u''_i (1 \leqslant i \leqslant n)$. Then assign the label 1 to the vertices u'_1, u'_2, u'_3, u'_4. Assign the label 2 to the vertices $u'_5, u'_7, u'_9, ...$ and assign the label 1 to the vertices $u'_6, u'_8, u'_{10}, ...$ Next we move to the vertices v'_i and v''_i. Assign the label 1 to all the vertices of $v'_i (1 \leqslant i \leqslant \frac{n}{2})$ and assign the label 3 to all the vertices of $v''_i (1 \leqslant i \leqslant \frac{n}{2})$. Now we consider the vertices w'_i and w''_i. Assign the label 2 to the vertices w'_1, w'_2, w''_1 and w''_2. Assign the label 2 to the vertices w'_{2i+1} and w''_{2i+1} for all the values of i=0,1,2,3... For all the values of i=1,2,3,... assign the label 1 to the vertices w'_{2i} and w''_{2i}. Note that in this case the vertex condition is $v_f(1) = v_f(2) = v_f(3) = 2n$. Also the edge condition is $e_f(0) = \frac{9n}{2}$ and $e_f(1) = \frac{9n-2}{2}$.

Case 2. The two triangles starts from u_2 and ends with u_{n-1}.

Assign the label 1 to the vertex u_1. Now we assign the label 1,1,2,2 to the vvertices u_2, u_3, u_4, u_5 respectively. Then we assign the label 1,1,2,2 to the next path vertices $u_{6,7}, u_8, u_9$ respectively. Proceeding like this we assign the label to the next four vertices and so on. If all the vertices are labeled then we stop the process. Otherwise there are some non labeled vertices exist. If the number of non labeled vertices are less than or equal to 3 then assign the labels 1,1,2 to the non labeled vertices. If it is two, then assign the labels 1,1 to the non labeled vertices. If only non labeled vertices exist then assign the label 1 to that vertex. Assign the label to the vertices $v_i, w_i (1 \leqslant i \leqslant n-1)$ as in case 1. Next we move to the vertices v'_i and v''_i. Assign the label 1 to the vertices $v'_1, v'_3, v'_5, ...$ and assign the label 2 to the vertices $v'_2, v'_4, v'_6, ...$ Now we assign the label 3 to the vertices $v''_1, v''_3, v''_5, ...$ and assign the label 1 to the vertices $v''_2, v''_4, v''_6, ...$ Consider the vertices u'_i and u''_i. Assign the label 2 to the vertex u'_1 and assign the label 3 to the vertex u''_1. Now we assign the label 2 to the vertices $u'_2, u'_4, u'_6, ...$ and assign the label 1 to the vertices $u'_3, u'_5, u'_7, ...$ Assign the label 3 to the vertices $u''_i (1 \leqslant i \leqslant n)$ Next we move to the vertices w'_i and w''_i. Assign the label 1 to the vertices $w'_1, w'_2, w'_3, ...$ Then assign the label 2 to the verticces $w''_1, w''_3, w''_5, ...$ and we assign the label 3 to the vertices $w''_2, w''_4, w''_6, ...$ Clearly the vertex and edge condition of this case is $v_f(1) = v_f(2) = v_f(3) = 2n-2$ and $e_f(0) = \frac{9n-12}{2}$ and $e_f(1) = \frac{9n-10}{2}$.

Case 3. The two triangles starts from u_2 and ends with u_n.

Assign the label to the vertices $v_i, w_i (1 \leqslant i \leqslant \lfloor \frac{n}{2} \rfloor)$ and $v'_i, v''_i (1 \leqslant i \leqslant \lfloor \frac{n}{2} \rfloor)$ as in case 1. Now we consider the path vertices u_i. Assign the label 2 to the path vertex u_1. Assign the labels 1,1,2,2 to the path vertices u_3, u_4, u_5, u_6 respectively. Then we assign the labels 1,1,2,2 to the next four path vertices u_7, u_8, u_9, u_{10} respectively. Continuing like this, we assign the label to the next four vertices and so on. Clearly the last vertex u_n received the label 1 and 2 according $n \equiv 3 \pmod 4$ or $n \equiv 1 \pmod 4$. Next we move to the vertices u'_i and u''_i. Assign the label 1 to the vertices $u'_1, u'_3, u'_5, ...$ and we assign the label 3 to the vertices $u'_2, u'_4, u'_6, ...$ Then we assign the label 3 to all the vertices $u''_i (1 \leqslant i \leqslant n)$. Now we consider the vertices w'_i and w''_i. Assign the label 2 to the vertices w'_{2i+1} and w''_{2i+1} for i=0,1,2,3,... and we assign the label 1 to the vertices w'_{2i} and w''_{2i} for all the values of i=1,2,3... In this case $v_f(1) = v_f(2) = v_f(3) = 2n-1$ and $e_f(1) = \frac{9n-5}{2}$ and $e_f(0) = \frac{9n-7}{2}$. □

A double alternate quadrilateral snake $DA(Q_n)$ consists of two alternate triangular snakes that have a common path. That is a double alternate quadrilateral snake is obtained from a path $u_1 u_2 ... u_n$ by joining u_i and u_{i+1} (alternatively) to new vertices v_i, x_i and w_i, y_i respectively and then joining v_i, w_i and x_i, y_i.

Theorem 2.4-2.6 the 3-difference cordial behavior of $DA(Q_n) \odot K_1$, $DA(Q_n) \odot 2K_1$ and $DA(Q_n) \odot K_2$.

THEOREM 2.4. $DA(Q_n) \odot K_1$ is 3-difference cordial.

PROOF. Let
$V(DA(Q_n) \odot K_1)$
$= V(DA(Q_n)) \cup \{u'_i : 1 \leqslant i \leqslant n\} \cup \{v'_i, w'_i, x'_i, y'_i : 1 \leqslant i \leqslant \frac{n}{2}\}$
and
$E(DA(Q_n) \odot K_1)$
$= E(DA(Q_n)) \cup \{u_i u'_i : 1 \leqslant i \leqslant n\} \cup \{v_i v'_i, w_i w'_i, x_i x'_i, y_i y'_i : 1 \leqslant i \leqslant \frac{n}{2}\}$.

Case 1. The two squares starts from u_1 and ends with u_n.

Consider the path vertices u_i. Assign the labels 1,1,1,2 to the first four path vertices u_1, u_2, u_3, u_4 respectively. Then we assign the labels 1,1,1,2 to the next four path vertices u_5, u_6, u_7, u_8 respectively. Proceeding like this we assign the label to the next four path vertices and so on. Note that in this case the last vertex u_n received the label 1 or 2 according as $n \equiv 2 \pmod 4$ or $n \equiv 0 \pmod 4$. Next we move to the vertices v_i and w_i. Assign the label 2 to the vertices $v_1, v_3, v_5, ...$ and assign the label 1 to the vertices $v_2, v_4, v_6, ...$ then assign the label 3 to the vertices $w_1, w_2, w_3, ...$ Now we consider the vertices x_i and y_i. Assign the label to the vertices x_i and y_i as same as assign the label to the vertices v_i and w_i. Next we move to the vertices v'_i and w'_i. Assign the label 1 to the vertices $v'_1, v'_3, v'_5, ...$ and assign the label 2 to the vertices $v'_2, v'_4, v'_6, ...$ Then assign the label 3 to the vertices w'_{2i+1} for all the values of i=0,1,2,3,... For all the values of i=1,2,3... assign the label 2 to the vertices w'_{2i}. Next we move to the vertices u'_i. Assign the labels 2,3,1,3 to the vertices u'_1, u'_2, u'_3, u'_4 respectively. Then we assign the labels 2,3,1,3 to the next four vertices u'_5, u'_6, u'_7, u'_8 respectively. Continuing like this we assign the label to the next four vertices and so on. Clearly the last vertex u'_n received the label 3. Now we consider the vertices x'_i and y'_i. Assign the label 2 to the vertices $x'_i (1 \leqslant i \leqslant \frac{n}{2})$. Then assign the label 1 to the vertices $y'_1, y'_3, y'_5, ...$ and assign the label 3 to the vertices $y'_2, y'_4, y'_6, ...$ Clearly $v_f(1) = v_f(2) = v_f(3) = 2n$ and $e_f(0) = \frac{7n}{2}$ and $e_f(1) = \frac{7n-2}{2}$.

Case 2. The two squares starts from u_2 and ends with u_{n-1}.

Assign the label 1 to the vertices u_1 and u_2. Then we assign the label 2 to the vertices u_3 and u_4. Assign the labels 1,1,1,2 to the vertices u_5, u_6, u_7, u_8 respectively. Then we assign the label 1,1,1,2 to the vertices $u_9, u_{10}, u_{11}, u_{12}$ respectively. Proceeding like this we assign the label to the next four vertices and so on. Clearly the last vertex u_n received the label 2 or 1 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Next we move to the vertices v_i, w_i, x_i and y_i. Assign the label 1 to the vertices v_1 and x_1 and we assign the label 3 to the vertices w_1 and y_1. Assign the label 1 to the vertices v_{2i+1} and x_{2i+1} for all the values of i=1,2,3,... For all the values of i=2,3,4... assign the label 2 to the vertices v_{2i} and x_{2i}. Then assign the label 3 to the vertices $w_3, w_4, w_5, ...$ and $y_2, y_3, y_4, ...$ Now we consider the vertices v'_i and w'_i. Assign the label 2 to the vertices v'_1 and w'_1. Assign the label 2 to the vertices v'_{2i} and w'_{2i} for all the values of i=1,2,3,... For all the values of i=1,2,3,... assign the label 1 to the vertices v'_{2i+1}. Then assign the label 3 to the vertices w'_{2i+1} for all the values of i=1,2,3,... Next we move to the vertices u'_i. Assign the label 1 to the vertices u'_1 and u'_2. Then assign the label 3 to the vertices u'_3 and u'_4. For all the values of i=1,2,3,... assign the label 2 to the vertices u'_{4i+1}. Now we assign the label 3 to the vertices u'_{4i} for all the values of i=2,3,4,... and assign the label 1 to the vertices u'_{4i+3} for all the values of i=2,3,4,... For all the values of i=1,2,3,... assign the label 3 to the vertices u'_{4i+2}. Consider the vertices x'_i and y'_i. Assign the label 2 to the vertex x'_1 and assign the label 3 to the vertex y'_1. Assign the label 2 to the vertices $x'_2, x'_3, x'_4, ...$ Then assign the label 3 to the vertices $y'_2, y'_4, y'_6, ...$ and assign the label 1 to the vertices $y'_1, y'_3, y'_5, ...$ Clearly $v_f(1) = \frac{6n-6}{3}$ and $v_f(2) = v_f(3) = \frac{6n-9}{3}$ and $e_f(0) = \frac{7n-10}{2}$ and $e_f(1) = \frac{7n-12}{2}$.

Case 3. The two squares starts from u_2 and ends with u_n.

Assign the label to the vertices $v_i, w_i, x_i, y_i, v'_i, w'_i, x'_i, y'_i (1 \leqslant i \leqslant \lfloor \frac{n}{2} \rfloor)$ as in case 1. Consider the path u_i. Assign the label 2 to the vertex u_1. Then assign the label 1,1,1,2 to the path vertices u_2, u_3, u_4, u_5 respectively. Assign the label 1,1,1,2 to the next four path vertices u_6, u_7, u_8, u_9 respectively. Proceeding like this assign the label to the next four vertices and so on. Clearly the last verte u_n received the label 1 or 2 according as $n \equiv 3 \pmod 4$ or $n \equiv 1 \pmod 4$. Next we move to the vertices u'_i. Assign the label 1 to the vertex u'_1 and assign the label 3 to the vertices u'_2 and u'_3. Assign the label 1 to the vertices u'_{4i} for all the values of i=1,2,3,... For all the values of i=1,2,3... assign the label 3 to the vertices u'_{4i+1} and u'_{4i+3}. Then assign the label 2 to the vertices u'_{4i+2} for i=1,2,3,... Since $v_f(1) = v_f(3) = 2n - 1$ and $v_f(2) = 2n - 2$, $e_f(0) = \frac{7n-5}{2}$ and $e_f(1) = \frac{7n-7}{2}$, this labeling is a 3-difference cordial labeling of $DA(Q_n) \odot K_1$. □

A 3-difference cordial labeling of $DA(Q_8) \odot K_1$ where the two triangle starts from u_1 and ends with u_8 is shown in Figure 4.

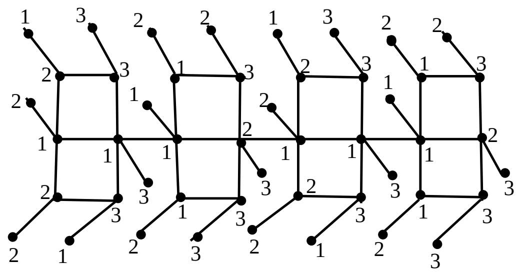

FIGURE 4

A 3-difference cordial labeling of $DA(Q_{10}) \odot K_1$ where the two triangle starts from u_2 and ends with u_9 is shown in Figure 5.

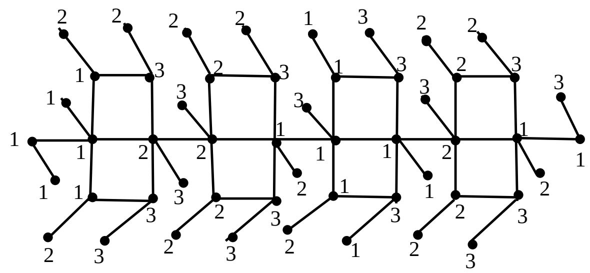

FIGURE 5

A 3-difference cordial labeling of $DA(Q_9) \odot K_1$ where the two triangle starts from u_2 and ends with u_9 is shown in Figure 6.

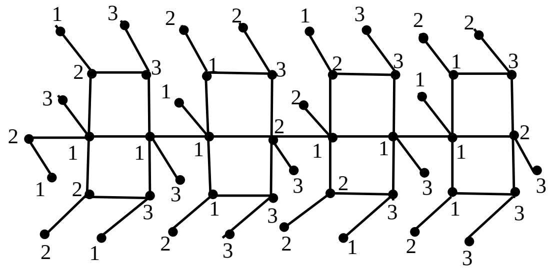

FIGURE 6

THEOREM 2.5. $DA(Q_n) \odot 2K_1$ is 3-difference cordial.

PROOF. Let
$V(DA(Q_n) \odot 2K_1)$
$= V(DA(Q_n)) \cup \{u_i', u_i'' : 1 \leqslant i \leqslant n\} \cup \{v_i', v_i'', w_i', w_i'', x_i', x_i'', y_i', y_i'' : 1 \leqslant i \leqslant \frac{n}{2}\}$
and
$E(DA(Q_n) \odot 2K_1) = E(DA(Q_n)) \cup \{u_iu_i', u_iu_i'' : 1 \leqslant i \leqslant n\}$
$\cup\{v_iv_i', v_iv_i'', w_iw_i', w_iw_i'', x_ix_i', x_ix_i'', y_iy_i', y_iy_i'' : 1 \leqslant i \leqslant \frac{n}{2}\}$.

Case 1. The two squares starts from u_1 and ends with u_n.

Consider the path vertices u_i. Assign the label 1 to vertices $u_1, u_3, u_5, ...$ and assign the label 2 to the path vertices $u_2, u_4, u_6, ...$ Clearly in this case the last vertex u_n received the label 2. Next we move to the vertices v_i and w_i. Assign the label 2 to all the vertices of $v_i (1 \leqslant i \leqslant \frac{n}{2})$ and assign the label 3 to all the vertices of $w_i (1 \leqslant i \leqslant \frac{n}{2})$. Now we consider the vertices x_i and y_i. Assign the label 1 to the vertices $x_1, x_2, x_3, ...$ and assign the label 3 to the vertices $y_1, y_2, y_3, ...$ Next we move to the vertices v_i', v_i'', w_i' and w_i''. Assign the label 2 to the vertices $v_i' (1 \leqslant i \leqslant \frac{n}{2})$ and assign the label 3 to the vertices $v_i'' (1 \leqslant i \leqslant \frac{n}{2})$. Then assign the label 1 to the vertices $w_i' (1 \leqslant i \leqslant \frac{n}{2})$ and assign the label 3 to the vertices $w_i'' (1 \leqslant i \leqslant \frac{n}{2})$. Consider the vertices u_i' and u_i''. Assign the label 1 to the vertices $u_i' (1 \leqslant i \leqslant n)$. Then assign the label 3 to the vvertices $u_1'', u_3'', u_5'', ...$ and assign the label 2 to the vertices $u_2'', u_4'', u_6'', ...$ Next we move to the vertices x_i', x_i'', y_i' and y_i''. Assign the label 2 to the vertices $x_i' (1 \leqslant i \leqslant \frac{n}{2})$ and assign the label 3 to the vertices $x_i'' (1 \leqslant i \leqslant \frac{n}{2})$. Then assign the label 1 to the vertices $y_1', y_2', y_3', ...$ and assign the label 2 to the vertices $y_1'', y_2'', y_3'', ...$ Clearly the vertex condition is $v_f(1) = v_f(2) = v_f(3) = 3n$. Also the edge condition is $e_f(0) = 5n$ and $e_f(1) = 5n - 1$.

Case 2. The two squares starts from u_2 and ends with u_{n-1}.

Assign the label to the vertices $v_i, w_i, x_i, y_i, v_i', w_i', x_i', y_i', v_i'', w_i'', x_i'', y_i''$ $(1 \leqslant i \leqslant \lfloor \frac{n}{2} \rfloor - 1)$ as in case 1. Consider the path vertices u_i. Assign the label 1 to the path vertex u_1. Then assign the label 1 to the path vertices $u_2, u_4, u_6, ...$ and assign the label 2 to the path vertices $u_3, u_5, u_7, ...$ Clearly the last vertex u_n received the label 1. Next we move to the vertices u_i' and u_i''. Assign the label 2 to the vertices u_1', u_4' and u_3''. Then assign the label 3 to the vertices u_1'', u_2'' and u_4''. Assign the label 1 to the vertices u'_2 and u'_3. Then assign the label 2 to the vertices u''_{2i+1} for all the values of i=2,3,4,5... and assign the label 3 to the vertices u''_{2i}. Now we assign the label 1 to the vertices $u'_5, u'_6, u'_7, ...$ Hence $v_f(1) = v_f(2) = v_f(3) = 3n-4$. Also the edge condition is $e_f(0) = 5n-7$ and $e_f(1) = 5n-8$.

Case 3. The two squares starts from u_2 and ends with u_n.

Consider the path vertices u_i. Assign the label 2 to the path vertex u_1 and assign the label 1 to path vertices u_2 and u_3. Then assign the label to the path vertices $u_4, u_6, u_8, ...$ and assign the label 2 to the path vertices $u_5, u_7, u_9, ...$ Next we move to the vertices v_i andd w_i. Assign the labels 2,1 to the vertices v_1, v_2 and assign the label 3 to the vertices w_1 and w_3. Then assign the label 2 to the path vertices $v_3, v_4, v_5, ...$ and assign the label 3 to the path vertices $w_3, w_4, w_5, ...$ Now we consider the vertices x_i and y_i. Assign the label 2,3 to the vertices x_1 and y_1 respectively. Thn we assign the label 1 to the vertices $x_2, x_3, x_4, ...$ and assign the label 3 to the vertices $y_2, y_3, y_4, ...$ Next we move to the vertices v'_i, v''_i, w'_i and w''_i. Assign the label 1 to the vertices v'_1 and w'_2. Then assign the label 2 to the vertices v'_2 and v'_3. Assign the label 3 to the vertices v'_1, w''_1 and v''_2 and we assign the label 2 to the vertex w''_2. Now we assign the label 2 to the vertices $v'_3, v'_4, v'_5, ...$ and assign the label 1 to the vertices $w'_3, w'_4, w'_5, ...$ Assign the label 3 to the vertices $v''_3, v''_4, v''_5, ...$ and $w''_3, w''_4, w''_5, ...$ Now we consider the vertices x'_i, x''_i, y'_i and y''_i. Assign the label 1 to the vertices x'_1 and assign the label 2 to he vertices x''_1. Then we assign the labels 1,3 to the vertices y'_1 and y''_1. Now we assign the label 2 to the vertices $x'_2, x'_3, x'_4, ...$ and assign the label 3 to the vertices $x''_2, x''_3, x''_4, ...$ Then assign the label 1 to the vertices $y'_2, y'_3, y'_4, ...$ and we assign the label 3 to the vertices $y''_2, y''_3, y''_4, ...$ Next we move to the vertices u'_i and u''_i. Assign the label 1 to the vertices u'_1, u'_2 and u'_5 and we assign the labe 2 to the vertices u''_1, u'_3 and u'_4. Then we assign the label 3 to the vertices u''_2, u''_3, u''_4 and u''_5. Now we assign the label 1 to the vertices $u'_6, u'_7, u'_8, ...$ Then we assign the label 3 to the vertices $u''_6, u''_8, u''_{10}, ...$ and assign the label 2 to the vertices $u''_7, u''_9, u''_{11}, ...$ Clearly $v_f(1) = v_f(2) = v_f(3) = 3n-2$ and $e_f(0) = e_f(1) = 5n-4$. □

THEOREM 2.6. $DA(Q_n) \odot K_2$ is 3-difference cordial.

PROOF. Let

$$V(DA(Q_n) \odot K_2)$$
$$= V(DA(Q_n)) \cup \{u'_i, u''_i : 1 \leqslant i \leqslant n\} \cup \{v'_i, v''_i, w'_i, w''_i, x'_i, x''_i, y'_i, y''_i : 1 \leqslant i \leqslant \tfrac{n}{2}\}$$

and

$$E(DA(Q_n) \odot K_2) = E(DA(Q_n)) \cup \{u_iu'_i, u_iu''_i, u'_iu''_i : 1 \leqslant i \leqslant n\}$$
$$\cup\{v_iv'_i,\, v_iv''_i,\, v'_iv''_i,\, w_iw'_i,\, w_iw''_i,\, w'_iw''_i,\, x_ix'_i,\, x_ix''_i,\, x'_ix''_i,\, y_iy'_i,\, y_iy''_i,\, y'_iy''_i : 1 \leqslant i \leqslant \tfrac{n}{2}\}.$$

Case 1. The two squares starts from u_1 and ends with u_n.

First we consider the path vertices u_i. Assign the labels 1,1,1,2 to vertices u_1, u_2, u_3, u_4 respectively. Then we assign the labels 1,1,1,2 to the next four path vertices u_5, u_6, u_7, u_8 respectively. Continuing like this we assign the label to the next four vertices and so on. Clearly the last vertex u_n received the label 2 or 1 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Next we move to the vertices v_i

and w_i. Assign the label 2 to the vertices $v_1, v_2, v_3, ...$ and assign the label 3 to the vertices $w_1, w_2, w_3, ...$ Now we consider the vertices x_i and y_i. Assign the label 2 to the vertices $x_1, x_3, x_5, ...$ and we assign the label 1 to the vertices $x_2, x_4, x_6, ...$ Then we assign the label 3 to the vertices $y_i(1 \leqslant i \leqslant \frac{n}{2})$. Next we move to the vertices w_i', w_i'', v_i' and v_i''. Assign the label 1 to the vertices $v_i', w_i'(1 \leqslant i \leqslant \frac{n}{2})$ and we assign the label 2 to the vertices $v_i''(1 \leqslant i \leqslant \frac{n}{2})$. Then we assign the label 3 to the vertices $w_i''(1 \leqslant i \leqslant \frac{n}{2})$. Now we consider the vertices x_i', x_i'', y_i' and y_i''. Assign the label 1 to the vertices $x_1', x_3', x_5', ...$ and we assign the label 2 to the vertices $x_2', x_4', x_6', ...$ Then we assign the label 2 to the vertices $x_1'', x_3'', x_5'', ...$ and we assign the label 3 to the vertices $x_2'', x_4'', x_6'', ...$ Now we assign the label 2 to the vertices $y_1', y_3', y_5', ...$ and we assign the label 1 to the vertices $y_2', y_4', y_6', ...$ Assign the label 3 to the vertices $y_1'', y_3'', y_5'', ...$ and we assign the label 2 to the vertices $y_2'', y_4'', y_6'', ...$ Next we move to the vertices u_i' and u_i''. Assign the label 2 to the vertices u_{4i+1}' for all the values of i=0,1,2,3... and assign the label 2 to the vertices u_{4i}' for i=1,2,3,... For all the values of i=0,1,2,3... assign the label 3 to the vertices u_{4i+2}' and u_{4i+3}'. Clearly $v_f(1) = v_f(2) = v_f(3) = 3n$ and $e_f(0) = \frac{13n}{2}$ and $e_f(1) = \frac{13n-2}{2}$.

Case 2. The two squares starts from u_2 and ends with u_{n-1}.

First we consider the path vertices u_i. Assign the label 2 to the vertex u_1 and assign the label 1 to the path vertices u_2 and u_3. Assign the labels 1,1,2,2 to the path vertices u_4, u_5, u_6, u_7 respectively. Then we assign the labels 1,1,2,2 to the next four path vertices u_8, u_9, u_{10}, u_{11} respectively. Continuing like this we assign the label to the next four vertices and so on. The last vertex u_n received the label 1 or 2 according as $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$. Next we move to the vertices v_i and w_i. Assign the label 2,3 to the vertices v_1 and w_1 respectively. Then we assign the label 2 to the vertices $v_2, v_4, v_6, ...$ and we assign the label 1 to the vertices $v_3, v_5, v_7, ...$ Now we assign the label 3 to the vertices $w_2, w_3, w_4, ...$ Consider the vertices x_i and y_i. Assign the label 2 to the vertex x_1. Then assign the label 1 to the vertices $x_i(1 \leqslant i \leqslant \frac{n}{2})$ and we assign the label 3 to the vertices $y_i(1 \leqslant i \leqslant \frac{n}{2})$. Next we move to the verices v_i', v_i'', w_i' and w_i''. Assign the label to the vertices v_i', v_i'', w_i' and w_i'' as in case 1. Now we consider the vertices x_i', x_i'', y_i' and y_i''. Assign the labels 1,2,2,3 to the vertices x_1', x_1'', y_1' and y_1'' respectively. Then assign the label 2 to the vertices $x_i'(2 \leqslant i \leqslant \frac{n}{2})$ and assign the label 3 to the vertices $x_i''(2 \leqslant i \leqslant \frac{n}{2})$. Now we assign the label 1 to the vertices y_{2i}' for all the values of i=1,2,3,.. and assign the label 2 to the vertices y_{2i+1}' for i=1,2,3,... For all the values of i=1,2,3,.. assign the label 2 to the vertices y_{2i}''. Finally assign the label 3 to the vertices y_{2i+1}'' for i=1,2,3,... Note that in this case the vertex condition is $v_f(1) = v_f(2) = v_f(3) = 3n - 4$ and the edge condition is $e_f(0) = \frac{13n-20}{2}$ and $e_f(1) = \frac{13n-18}{2}$.

Case 3. The two squares starts from u_2 and ends with u_n.

Assign the label to the vertices $v_i, w_i, x_i, y_i(1 \leqslant i \leqslant \lceil \frac{n}{2} \rceil)$ and v_i', w_i', x_i', y_i', v_i'', w_i'', x_i'', $y_i''(1 \leqslant i \leqslant \lfloor \frac{n}{2} \rfloor)$ as in case 1. Now we consider the path vertices u_i. Assign the label 1 to the path vertices vertex u_1 and assign the labels 1,1,1,2 to the path vertices u_2, u_3, u_4, u_5 respectively. Then we assign the labels 1,1,1,2 to the next four path vertices u_6, u_7, u_8, u_9 respectively. Proceeding like this we assign the label to the next four vertices and so on. Clearly the last vvertex u_n received the label 1 or 2 according as $n \equiv 3 \pmod 4$ or $n \equiv 1 \pmod 4$. Next we move to the vertices u'_i and u''_i. Assign the labels 1,2 to the vertices u'_1, u'_2 respectively and we assign the label 3 to the vertices $u''_i (1 \leqslant i \leqslant n)$. Then we assign the label 1 to the vertices u'_{4i+3} for all the values i=0,1,2,3,... and we assign the label 1 to the vertices u'_{4i} for i=1,2,3,... for all values of i=1,2,3... assign the label 2 to the vertices u'_{4i+1} and u'_{4i+2}. Since $v_f(1) = v_f(2) = v_f(3) = 3n - 2$, $e_f(0) = \frac{13n-11}{2}$ and $e_f(1) = \frac{13n-9}{2}$ this labeling is a 3-difference cordial labeling. □

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Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, India.

E-mail address: ponrajmaths@gmail.com

Department of Economics and Statistics, District Statistical office, Ramanathapuram - 623501, India.

E-mail address: mariaadaickalam@gmail.com

Department of Mathematics, Manonmaniam Sundaranar University, Tirunelveli-627012, India.

E-mail address: karthipyi91@yahoo.co.in.