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SOLUTION OF SECOND ORDER COMPLEX EQUATIONS WITH ADOMIAN DECOMPOSITION METHOD

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Abstract. In this study, second order linear complex differential equations are solved by the Adomian decomposition method. A theorem is given for this method. Furthermore some examples are given, and the results obtained indicate this approach is indeed practical.

1. Introduction

The difficulties of some problems in real space have been overcome by the solution methods of complex equations.For example, in real, general solutions of some equations, especially type of elliptic, are not found. Real partial differential equation systems when number of independent variables are even can be transformed to a complex partial differential equations. The solving a complex equation can more easier with complex methods. For example,

$$u_{xx} + u_{yy} = 0.$$

Laplace equation hasn't got general solution in R^2, but it can be written

$$u_{z\overline{z}} = 0$$

with the relation

$$\Delta = \frac{\partial^2}{\partial z \partial \overline{z}}$$

and the solution of this equation is

$$u = f(z) + g(\overline{z}).$$

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where f is analytic, g is anti analytic arbitrary functions ([**14, 15**]).

The most basic works in the theory of complex differential equations are "Theory of Pseudo Analytic Functions" which is written by L. Bers ([**5**]) and "Generalized Analytic Functions" which is written by I, N. Vekua [**19**]. First order complex differential equations were solved by using laplace transform, elzaki transform and adomian decomposion method in [**14, 15, 16**]. Moreover in [**13**] some examples associated with complex equations from second order were solved by using differential transform method. We extended previous our paper [**13**] and we found a solution of this type equations satisfying certain conditions. In this study, we solved the second order linear complex differential equations.We give a theorem for this kind equations

Firstly, second order linear complex partial differential equation is transformed to real partial differential equation system by seperating to real and imaginer parts. Then, we put forward a solution which satisfy certain conditions of the complex equation with real equation system is solved by Adomian method.

1.1. Derivatives of Complex Functions. Let $w = w(z, z)$ be a complex function. Here $z = x + iy$, $w(z, z) = u(x, y) + i.v(x, y)$. First and second order derivatives according to z and $\overline{z}$ of $w(z, \overline{z})$ are defined as follows:

$$\frac{\partial w}{\partial z} = \frac{1}{2}(\frac{\partial w}{\partial x} - i\frac{\partial w}{\partial y}) \tag{1.1}$$

$$\frac{\partial w}{\partial \overline{z}} = \frac{1}{2}(\frac{\partial w}{\partial x} + i\frac{\partial w}{\partial y}) \tag{1.2}$$

$$\frac{\partial^2 w}{\partial z^2} = \frac{1}{4}\left[\frac{\partial^2 w}{\partial x^2} - 2i\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial^2 w}{\partial y^2}\right] \tag{1.3}$$

$$\frac{\partial^2 w}{\partial \overline{z}^2} = \frac{1}{4}\left[\frac{\partial^2 w}{\partial x^2} + 2i\frac{\partial^2 w}{\partial x \partial y} - \frac{\partial^2 w}{\partial y^2}\right] \tag{1.4}$$

$$\frac{\partial^2 w}{\partial z \partial \overline{z}} = \frac{1}{4}\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right] \tag{1.5}$$

If we write $u + iv$ in place of w in $(1.3, 1.4, 1.5)$ we get that

$$\frac{\partial^2 w}{\partial z \partial \overline{z}} = \frac{1}{4}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + i(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2})\right] \tag{1.6}$$

$$\frac{\partial^2 w}{\partial z^2} = \frac{1}{4}\left[\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} + i\left(\frac{\partial^2 v}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 v}{\partial y^2}\right)\right] \tag{1.7}$$

$$\frac{\partial^2 w}{\partial \overline{z}^2} = \frac{1}{4}\left[\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 v}{\partial x \partial y} - \frac{\partial^2 u}{\partial y^2} + i\left(\frac{\partial^2 v}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} - \frac{\partial^2 v}{\partial y^2}\right)\right] \tag{1.8}$$

$$\frac{\partial^2 w}{\partial z \partial \overline{z}} = \frac{1}{4}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + i\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)\right]. \tag{1.9}$$

1.2. Adomian Decomposion Method. In this section we mention from the Adomian Decomposition Method(ADM). The ADM is a iterative method which is used in several areas of mathematics. Recently a great deal of interest has been focused on the application of the ADM to solve a wide variety of linear and nonlinear problems. This method has been introduced by Adomian [**2**] and it can been used in the linear and nonlinear differential equations, in the differential equations systems, in the integral equations, in the difference equations, in the differential-difference equations, in the algebraic equations, in the fractional differential equations,couple system of two equation [**6, 7, 8, 9, 1, 17, 18, 12, 3, 4, 10, 11**]. This method generates a solution in the form of a series whose terms are determined by a recursive relationship using the Adomian polynomials [**16**].

We consider $F(y(x)) = g(x)$, where F represents a general differential operator involving both the linear and nonlinear terms. The linear term is decomposed into $L + R$, where L is the highest order differential operator and R is the remainder of the linear operator. Thus the equation can be written

$$Ly + Ry + Ny = g(x),$$

where Nu represents the nonlinear terms. Solving for Ly, we use

$$Ly = g(x) - Ry - Ny.$$

Because L is invertible, an equivalent expression is as follows

$$L^{-1}Ly = L^{-1}g - L^{-1}Ry - L^{-1}Ny.$$

If L is first order, L^{-1} is a integral operator. If L is second order, L^{-1} is two fold integration operator. The nonlinear term Ny will be equated to $\sum\limits_{n=0}^{\infty} A_n$, where A_n are the adomian polynomials. Thus it can be written

$$\sum_{n=0}^{\infty} y_n = y_0 - L^{-1}R\left(\sum_{n=0}^{\infty} y_n\right) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right),$$

where y_0 is solution of $Ly = g(x)$. Consequently we can write

$$\begin{aligned}
y_1 &= -L^{-1}y_0 - L^{-1}A_0 \\
y_2 &= -L^{-1}y_1 - L^{-1}A_1 \\
y_3 &= -L^{-1}y_2 - L^{-1}A_2 \\
&\vdots \\
y_{n+1} &= -L^{-1}y_n - L^{-1}A_n,
\end{aligned}$$

where A_n polynomials are determined as follows:

$$Ny = f(y)$$

$$A_0 = f(y_0)$$
$$A_1 = y_1 \frac{df(y_0)}{dy_0}$$
$$A_2 = y_2 \frac{df(y_0)}{dy_0} + \frac{y_1^2}{2} \frac{d^2 f(y_0)}{d^2 y_0}$$
$$A_3 = y_3 \frac{df(y_0)}{dy_0} + y_1 . y_2 \frac{d^2 f(y_0)}{d^2 y_0} + \frac{y_1^3}{3!} \frac{d^3 f(y_0)}{d^3 y_0}$$
$$\vdots$$

2. Solution of Complex Equations with ADM

THEOREM 2.1. *Let A, B, C, D, E, F, G be functions of $z, \overline{z}$ and let $w = u + iv$ be a complex function. We consider following problem*

$$A(z,\overline{z}) \frac{\partial^2 w}{\partial z^2} + B(z,\overline{z}) \frac{\partial^2 w}{\partial z \partial \overline{z}} + C(z,\overline{z}) \frac{\partial^2 w}{\partial \overline{z}^2} + D(z,\overline{z}) \frac{\partial w}{\partial z} + \tag{2.1}$$
$$E(z,\overline{z}) \frac{\partial w}{\partial \overline{z}} + F(z,\overline{z}) w = G(z,\overline{z})$$

$$w(x,0) = f(x) \tag{2.2}$$
$$\frac{\partial w}{\partial y}(x,0) = g(x). \tag{2.3}$$

The solution of above mentioned problem is $w = u + iv$, where $u = u_0 + \sum_{n=0}^{\infty} u_{n+1}$ and $v = v_0 + \sum_{n=0}^{\infty} v_{n+1}$, $u_0, v_0, u_{n+1}, v_{n+1}$ are as follows:
If $B_1 - A_1 - C_1 \neq 0$, then

$$u_0 = L_y^{-2}\left(\frac{4G_1}{-A_1 + B_1 - C_1}\right) + L_y^{-1}(Reg(x)) + Ref(x),$$
$$v_0 = L_y^{-2}\left(\frac{4G_2}{-A_1 + B_1 - C_1}\right) + L_y^{-1}(Img(x)) + Imf(x)$$

$$\begin{aligned} u_{n+1} = \; & L_y^{-2}\left(\frac{A_1 + B_1 + C_1}{A_1 - B_1 + C_1} L_x^2 u_n\right) + L_y^{-2}\left(\frac{2A_2 - 2C_2}{A_1 - B_1 + C_1} L_x L_y u_n\right) \\ & + L_y^{-2}\left(\frac{A_2 - B_2 + C_2}{A_1 - B_1 + C_1} L_y^2 v_n\right) + L_y^{-2}\left(\frac{-A_2 - B_2 - C_2}{A_1 - B_1 + C_1} L_x^2 v_n\right) \\ & + L_y^{-2}\left(\frac{2A_1 - 2C_1}{A_1 - B_1 + C_1} L_x L_y v_n\right) + L_y^{-2}\left(\frac{2D_1 + 2E_1}{A_1 - B_1 + C_1} L_x u_n\right) \\ & + L_y^{-2}\left(\frac{2D_2 - 2E_2}{A_1 - B_1 + C_1} L_y u_n\right) + L_y^{-2}\left(\frac{-2D_2 - 2E_2}{A_1 - B_1 + C_1} L_x v_n\right) \\ & + L_y^{-2}\left(\frac{2D_1 - 2E_1}{A_1 - B_1 + C_1} L_y v_n\right) + L_y^{-2}\left(\frac{4F_1 u_n - 4F_2 v_n}{A_1 - B_1 + C_1}\right). \end{aligned}$$

$$\begin{aligned}
v_{n+1} &= L_y^{-2}\left(\frac{A_1+B_1+C_1}{A_1-B_1+C_1}L_x^2 v_n\right)+L_y^{-2}\left(\frac{2A_2-2C_2}{A_1-B_1+C_1}L_xL_y v_n\right)\\
&+L_y^{-2}\left(\frac{-A_2+B_2-C_2}{A_1-B_1+C_1}L_y^2 u_n\right)+L_y^{-2}\left(\frac{A_2+B_2+C_2}{A_1-B_1+C_1}L_x^2 u_n\right)\\
&+L_y^{-2}\left(\frac{-2A_1+2C_1}{A_1-B_1+C_1}L_xL_y u_n\right)+L_y^{-2}\left(\frac{2D_1+2E_1}{A_1-B_1+C_1}L_x v_n\right)\\
&+L_y^{-2}\left(\frac{2D_2-2E_2}{A_1-B_1+C_1}L_y v_n\right)+L_y^{-2}\left(\frac{2D_2+2E_2}{A_1-B_1+C_1}L_x u_n\right)\\
&+L_y^{-2}\left(\frac{-2D_1+2E_1}{A_1-B_1+C_1}L_y u_n\right)+L_y^{-2}\left(\frac{4F_1v_n+4F_2u_n}{A_1-B_1+C_1}\right).
\end{aligned}$$

If $A_2-B_2+C_2\neq 0,$ *then*

$$\begin{aligned}
u_0(x,y) &= L_y^{-2}\left(\frac{-4G_2}{A_2-B_2+C_2}\right)+L_y^{-1}\left(Reg\left(x\right)\right)+Ref\left(x\right),\\
v_0(x,y) &= L_y^{-2}\left(\frac{4G_1}{A_2-B_2+C_2}\right)+L_y^{-1}\left(Img\left(x\right)\right)+Imf\left(x\right).
\end{aligned}$$

$$\begin{aligned}
v_{n+1} &= L_y^{-2}\left(\frac{-B_1+A_1+C_1}{A_2-B_2+C_2}L_y^2 u_n\right)+L_y^{-2}\left(-\frac{A_1+B_1+C_1}{A_2-B_2+C_2}L_x^2 u_n\right)\\
&+L_y^{-2}\left(\frac{-2A_2+2C_2}{A_2-B_2+C_2}L_xL_y u_n\right)+L_y^{-2}\left(\frac{A_2+B_2+C_2}{A_2-B_2+C_2}L_x^2 v_n\right)\\
&+L_y^{-2}\left(\frac{2C_1-2A_1}{A_2-B_2+C_2}L_xL_y v_n\right)+L_y^{-2}\left(\frac{-2D_1-2E_1}{A_2-B_2+C_2}L_x u_n\right)\\
&+L_y^{-2}\left(\frac{-2D_2+2E_2}{A_2-B_2+C_2}L_y u_n\right)+L_y^{-2}\left(\frac{2D_2+2E_2}{A_2-B_2+C_2}L_x v_n\right)\\
&+L_y^{-2}\left(\frac{-2D_1+2E_1}{A_2-B_2+C_2}L_y v_n\right)+L_y^{-2}\left(\frac{-4F_1u_n+4F_2v_n}{A_2-B_2+C_2}\right).
\end{aligned}$$

$$\begin{aligned}
u_{n+1} &= L_y^{-2}\left(\frac{B_1-A_1-C_1}{A_2-B_2+C_2}L_y^2 v_n\right)+L_y^{-2}\left(\frac{A_1+B_1+C_1}{A_2-B_2+C_2}L_x^2 v_n\right)\\
&+L_y^{-2}\left(\frac{2A_2-2C_2}{A_2-B_2+C_2}L_xL_y v_n\right)+L_y^{-2}\left(\frac{A_2+B_2+C_2}{A_2-B_2+C_2}L_x^2 u_n\right)\\
&+L_y^{-2}\left(\frac{-2A_1+2C_1}{A_2-B_2+C_2}L_xL_y u_n\right)+L_y^{-2}\left(\frac{2D_2-2E_2}{A_2-B_2+C_2}L_y v_n\right)\\
&+L_y^{-2}\left(\frac{2D_1+2E_1}{A_2-B_2+C_2}L_x v_n\right)+L_y^{-2}\left(\frac{-2D_1+2E_1}{A_2-B_2+C_2}L_y u_n\right)\\
&+L_y^{-2}\left(\frac{2D_2+2E_2}{A_2-B_2+C_2}L_x u_n\right)+L_y^{-2}\left(\frac{4F_1v_n+4F_2u_n}{A_2-B_2+C_2}\right).
\end{aligned}$$

$$\begin{aligned}
A_1 &= ReA\left(z,\overline{z}\right), A_2 = ImA\left(z,\overline{z}\right), B_1 = ReB\left(z,\overline{z}\right), B_2 = ImB\left(z,\overline{z}\right),\\
C_1 &= ReC(z,\overline{z}), C_2 = ImC(z,\overline{z}), D_1 = ReD\left(z,\overline{z}\right), D_2 = ImD\left(z,\overline{z}\right),\\
E_1 &= ReE\left(z,\overline{z}\right), E_2 = ImE\left(z,\overline{z}\right), F_1 = ReF(z,\overline{z}), F_2 = ImF(z,\overline{z}),\\
G_1 &= ReG\left(z,\overline{z}\right), G_2 = ImG\left(z,\overline{z}\right)
\end{aligned}$$

PROOF. Let's write the equation (2.1).

$$A\left(z,\overline{z}\right)\frac{\partial^2 w}{\partial z^2} + B\left(z,\overline{z}\right)\frac{\partial^2 w}{\partial z\partial\overline{z}} + C\left(z,\overline{z}\right)\frac{\partial^2 w}{\partial\overline{z}^2} + D\left(z,\overline{z}\right)\frac{\partial w}{\partial z} + E\left(z,\overline{z}\right)\frac{\partial w}{\partial\overline{z}} + F(z,\overline{z})w = G\left(z,\overline{z}\right)$$

We let's rewrite above equation using the definition of complex derivatives $(1.1, 1.2, 1.3, 1.4, 1.5)$

$$\begin{aligned}
& \left(A_1\left(x,y\right) + iA_2\left(x,y\right)\right)\frac{1}{4}\left[\frac{\partial^2 w}{\partial x^2} - 2i\frac{\partial^2 w}{\partial x\partial y} - \frac{\partial^2 w}{\partial y^2}\right]\\
& + \left(B_1\left(x,y\right) + iB_2\left(x,y\right)\right)\frac{1}{4}\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right]\\
& + \left(C_1\left(x,y\right) + iC_2\left(x,y\right)\right)\frac{1}{4}\left[\frac{\partial^2 w}{\partial x^2} + 2i\frac{\partial^2 w}{\partial x\partial y} - \frac{\partial^2 w}{\partial y^2}\right]\\
& + \left(D_1\left(x,y\right) + iD_2\left(x,y\right)\right)\frac{1}{2}(\frac{\partial w}{\partial x} - i\frac{\partial w}{\partial y})\\
& + \left(E_1\left(x,y\right) + iE_2\left(x,y\right)\right)\frac{1}{2}(\frac{\partial w}{\partial x} + i\frac{\partial w}{\partial y}) + \left(F_1\left(x,y\right) + iF_2\left(x,y\right)\right)w\\
(2.4) \quad = \; & G_1\left(x,y\right) + iG_2\left(x,y\right).
\end{aligned}$$

If we separate real and imaginer parts of equation (2.4), we have obtained following real equation system.

$$\begin{aligned}
& A_1\left(x,y\right)\frac{\partial^2 u}{\partial x^2} + 2A_1\left(x,y\right)\frac{\partial^2 v}{\partial x\partial y} - A_1\left(x,y\right)\frac{\partial^2 u}{\partial y^2} - A_2\left(x,y\right)\frac{\partial^2 v}{\partial x^2}\\
& +2A_2\left(x,y\right)\frac{\partial^2 u}{\partial x\partial y} + A_2\left(x,y\right)\frac{\partial^2 v}{\partial y^2} + B_1\left(x,y\right)\frac{\partial^2 u}{\partial x^2} + B_1\left(x,y\right)\frac{\partial^2 u}{\partial y^2}\\
& -B_2\left(x,y\right)\frac{\partial^2 v}{\partial x^2} - B_2\left(x,y\right)\frac{\partial^2 v}{\partial y^2} + C_1\left(x,y\right)\frac{\partial^2 u}{\partial x^2} - 2C_1\left(x,y\right)\frac{\partial^2 v}{\partial x\partial y}\\
& -C_1\left(x,y\right)\frac{\partial^2 u}{\partial y^2} - C_2\left(x,y\right)\frac{\partial^2 v}{\partial x^2} - 2C_2\left(x,y\right)\frac{\partial^2 u}{\partial x\partial y} + C_2\left(x,y\right)\frac{\partial^2 v}{\partial y^2}\\
& +2D_1\frac{\partial u}{\partial x} + 2D_1\frac{\partial v}{\partial y} - 2D_2\frac{\partial v}{\partial x} + 2D_2\frac{\partial u}{\partial y} + 2E_1\frac{\partial u}{\partial x} - 2E_1\frac{\partial v}{\partial y}\\
(2.5) \quad & -2E_2\frac{\partial v}{\partial x} - 2E_2\frac{\partial u}{\partial y} + 4F_1 u - 4F_2 v = 4G_1.
\end{aligned}$$

$$A_1(x,y)\frac{\partial^2 v}{\partial x^2}-2A_1(x,y)\frac{\partial^2 u}{\partial x\partial y}-A_1(x,y)\frac{\partial^2 v}{\partial y^2}+A_2(x,y)\frac{\partial^2 u}{\partial x^2}$$
$$+2A_2(x,y)\frac{\partial^2 v}{\partial x\partial y}-A_2(x,y)\frac{\partial^2 u}{\partial y^2}+B_1(x,y)\frac{\partial^2 v}{\partial x^2}+B_1(x,y)\frac{\partial^2 v}{\partial y^2}$$
$$+B_2(x,y)\frac{\partial^2 u}{\partial x^2}+B_2(x,y)\frac{\partial^2 u}{\partial y^2}+C_1(x,y)\frac{\partial^2 v}{\partial x^2}+2C_1(x,y)\frac{\partial^2 u}{\partial x\partial y}$$
$$-C_1(x,y)\frac{\partial^2 v}{\partial y^2}+C_2(x,y)\frac{\partial^2 u}{\partial x^2}-2C_2(x,y)\frac{\partial^2 v}{\partial x\partial y}-C_2(x,y)\frac{\partial^2 u}{\partial y^2}$$
$$+2D_1(x,y)\frac{\partial v}{\partial x}-2D_1(x,y)\frac{\partial u}{\partial y}+2D_2(x,y)\frac{\partial u}{\partial x}+2D_2(x,y)\frac{\partial v}{\partial y}$$
$$+2E_1(x,y)\frac{\partial v}{\partial x}+2E_1(x,y)\frac{\partial u}{\partial y}+2E_2(x,y)\frac{\partial u}{\partial x}-2E_2(x,y)\frac{\partial v}{\partial y}$$
$$+4F_1v+4F_2u=4G_2. \tag{2.6}$$

If these equations (2.5) and (2.6) are taken on brackets of derivatives, also it is obtained following equalities.

$$(B_1-A_1-C_1)\frac{\partial^2 u}{\partial y^2}+(B_1+A_1+C_1)\frac{\partial^2 u}{\partial x^2}+(2A_2-2C_2)\frac{\partial^2 u}{\partial x\partial y}$$
$$+(A_2-B_2+C_2)\frac{\partial^2 v}{\partial y^2}+(-A_2-B_2-C_2)\frac{\partial^2 v}{\partial x^2}+(2A_1-2C_1)\frac{\partial^2 v}{\partial x\partial y}$$
$$+(2D_1+2E_1)\frac{\partial u}{\partial x}+(2D_2-2E_2)\frac{\partial u}{\partial y}+(-2D_2-2E_2)\frac{\partial v}{\partial x}+(2D_1-2E_1)\frac{\partial v}{\partial y}$$
$$+4F_1u-4F_2v=4G_1. \tag{2.7}$$

$$(B_1-A_1-C_1)\frac{\partial^2 v}{\partial y^2}+(B_1+A_1+C_1)\frac{\partial^2 v}{\partial x^2}+(2A_2-2C_2)\frac{\partial^2 v}{\partial x\partial y}$$
$$+(-A_2+B_2-C_2)\frac{\partial^2 u}{\partial y^2}+(A_2+B_2+C_2)\frac{\partial^2 u}{\partial x^2}+(-2A_1+2C_1)\frac{\partial^2 u}{\partial x\partial y}$$
$$+(2D_2-2E_2)\frac{\partial v}{\partial y}+(2D_1+2E_1)\frac{\partial v}{\partial x}+(-2D_1+2E_1)\frac{\partial u}{\partial y}$$
$$+(2D_2+2E_2)\frac{\partial u}{\partial x}+4F_1v+4F_2u=4G_2. \tag{2.8}$$

If $B_1-A_1-C_1\neq 0$ then

$$\begin{aligned}L_y^2u \;=\; &-\frac{(B_1+A_1+C_1)L_x^2u+(2A_2-2C_2)L_xL_yu+(A_2-B_2+C_2)L_y^2v}{B_1-A_1-C_1}\\&-\frac{(-A_2-B_2-C_2)L_x^2v+(2A_1-2C_1)L_xL_yv+(2D_1+2E_1)L_xu}{B_1-A_1-C_1}\\&-\frac{(2D_2-2E_2)L_yu+(-2D_2-2E_2)L_xv+(2D_1-2E_1)L_yv}{B_1-A_1-C_1}\\&-\frac{4F_1u-4F_2v-4G_1}{B_1-A_1-C_1}.\end{aligned}$$

$$
\begin{aligned}
L_y^2 v \;=\; & -\frac{(B_1+A_1+C_1)\,L_x^2 v+(2A_2-2C_2)\,L_xL_y v+(-A_2+B_2-C_2)\,L_y^2 u}{B_1-A_1-C_1} \\
& -\frac{(A_2+B_2+C_2)\,L_x^2 u+(-2A_1+2C_1)\,L_xL_y u+(2D_2-2E_2)\,L_y v}{B_1-A_1-C_1} \\
& -\frac{(2D_1+2E_1)\,L_x v+(-2D_1+2E_1)\,L_y u+(2D_2+2E_2)\,L_x u}{B_1-A_1-C_1} \\
& -\frac{4F_1 v+4F_2 u-4G_2}{B_1-A_1-C_1}.
\end{aligned}
$$

If we apply L_y^{-2} inverse operator, then we get from ADM following inequalities

$$
\begin{aligned}
u_{n+1} \;=\; & L_y^{-2}\left(\frac{B_1+A_1+C_1}{A_1-B_1+C_1}L_x^2 u_n\right)+L_y^{-2}\left(\frac{2A_2-2C_2}{A_1-B_1+C_1}L_xL_y u_n\right) \\
& +L_y^{-2}\left(\frac{A_2-B_2+C_2}{A_1-B_1+C_1}L_y^2 v_n\right)+L_y^{-2}\left(\frac{-A_2-B_2-C_2}{A_1-B_1+C_1}L_x^2 v_n\right) \\
& +L_y^{-2}\left(\frac{2A_1-2C_1}{A_1-B_1+C_1}L_xL_y v_n\right)+L_y^{-2}\left(\frac{2D_1+2E_1}{A_1-B_1+C_1}L_x u_n\right) \\
& +L_y^{-2}\left(\frac{2D_2-2E_2}{A_1-B_1+C_1}L_y u_n\right)+L_y^{-2}\left(\frac{-2D_2-2E_2}{A_1-B_1+C_1}L_x v_n\right) \\
& +L_y^{-2}\left(\frac{2D_1-2E_1}{A_1-B_1+C_1}L_y v_n\right)+L_y^{-2}\left(\frac{4F_1u_n-4F_2v_n}{A_1-B_1+C_1}\right).
\end{aligned}
$$

$$
\begin{aligned}
v_{n+1} \;=\; & L_y^{-2}\left(\frac{B_1+A_1+C_1}{A_1-B_1+C_1}L_x^2 v_n\right)+L_y^{-2}\left(\frac{2A_2-2C_2}{A_1-B_1+C_1}L_xL_y v_n\right) \\
& +L_y^{-2}\left(\frac{-A_2+B_2-C_2}{A_1-B_1+C_1}L_y^2 u_n\right)+L_y^{-2}\left(\frac{A_2+B_2+C_2}{A_1-B_1+C_1}L_x^2 u_n\right) \\
& +L_y^{-2}\left(\frac{-2A_1+2C_1}{A_1-B_1+C_1}L_xL_y u_n\right)+L_y^{-2}\left(\frac{2D_2-2E_2}{A_1-B_1+C_1}L_y v_n\right) \\
& +L_y^{-2}\left(\frac{2D_1+2E_1}{A_1-B_1+C_1}L_x v_n\right)+L_y^{-2}\left(\frac{-2D_1+2E_1}{A_1-B_1+C_1}L_y u_n\right) \\
& +L_y^{-2}\left(\frac{2D_2+2E_2}{A_1-B_1+C_1}L_x u_n\right)+L_y^{-2}\left(\frac{4F_1v_n+4F_2u_n}{A_1-B_1+C_1}\right).
\end{aligned}
$$

By conditions (2.2) and (2.3), values of $u_0\,(x,y)\,,v_0\,(x,y)$ are as follows:

$$
\begin{aligned}
u_0\,(x,y) \;=\; & L_y^{-2}\left(\frac{4G_1}{B_1-A_1-C_1}\right)+L_y^{-1}\left(Reg(x)\right)+Ref(x), \\
v_0\,(x,y) \;=\; & L_y^{-2}\left(\frac{4G_2}{B_1-A_1-C_1}\right)+L_y^{-1}\left(Img(x)\right)+Imf(x).
\end{aligned}
$$

Similarly , if $A_2 - B_2 + C_2 \neq 0$, then by (2.7) and (2.8) we get those:

$$\begin{aligned} L_y^2 v &= -\frac{(B_1 - A_1 - C_1)\, L_y^2 u + (2A_2 - 2C_2)\, L_x L_y u + (A_1 + B_1 + C_1)\, L_x^2 u}{A_2 - B_2 + C_2} \\ &\quad -\frac{(-A_2 - B_2 - C_2)\, L_x^2 v + (2A_1 - 2C_1)\, L_x L_y v + (2D_1 + 2E_1)\, L_x u +}{A_2 - B_2 + C_2} \\ &\quad -\frac{(2D_2 - 2E_2)\, L_y u + (-2D_2 - 2E_2)\, L_x v + (2D_1 - 2E_1)\, L_y v +}{A_2 - B_2 + C_2} \\ &\quad -\frac{4F_1 u - 4F_2 v - 4G_1}{A_2 - B_2 + C_2}. \end{aligned}$$

$$\begin{aligned} L_y^2 u &= \frac{(B_1 + A_1 + C_1)\, L_x^2 v + (2A_2 - 2C_2)\, L_x L_y v + (B_1 - A_1 - C_1)\, L_y^2 v}{A_2 - B_2 + C_2} \\ &\quad +\frac{(A_2 + B_2 + C_2)\, L_x^2 u + (-2A_1 + 2C_1)\, L_x L_y u + (2D_2 - 2E_2)\, L_y v +}{A_2 - B_2 + C_2} \\ &\quad +\frac{(2D_1 + 2E_1)\, L_x v + (-2D_1 + 2E_1)\, L_y u + (2D_2 + 2E_2)\, L_x u +}{A_2 - B_2 + C_2} \\ &\quad +\frac{4F_1 v + 4F_2 u - 4G_2}{A_2 - B_2 + C_2}. \end{aligned}$$

If we apply L_y^{-2} inverse operator, then we get from ADM following inequalities

$$\begin{aligned} v_{n+1} &= L_y^{-2}\left(\frac{-B_1 + A_1 + C_1}{A_2 - B_2 + C_2} L_y^2 u_n\right) + L_y^{-2}\left(-\frac{A_1 + B_1 + C_1}{A_2 - B_2 + C_2} L_x^2 u_n\right) \\ &\quad + L_y^{-2}\left(\frac{-2A_2 + 2C_2}{A_2 - B_2 + C_2} L_x L_y u_n\right) + L_y^{-2}\left(\frac{A_2 + B_2 + C_2}{A_2 - B_2 + C_2} L_x^2 v_n\right) \\ &\quad + L_y^{-2}\left(\frac{2C_1 - 2A_1}{A_2 - B_2 + C_2} L_x L_y v_n\right) + L_y^{-2}\left(\frac{-2D_1 - 2E_1}{A_2 - B_2 + C_2} L_x u_n\right) \\ &\quad + L_y^{-2}\left(\frac{-2D_2 + 2E_2}{A_2 - B_2 + C_2} L_y u_n\right) + L_y^{-2}\left(\frac{2D_2 + 2E_2}{A_2 - B_2 + C_2} L_x v_n\right) \\ &\quad + L_y^{-2}\left(\frac{-2D_1 + 2E_1}{A_2 - B_2 + C_2} L_y v_n\right) + L_y^{-2}\left(\frac{-4F_1 u_n + 4F_2 v_n}{A_2 - B_2 + C_2}\right). \end{aligned}$$

$$\begin{aligned} u_{n+1} &= L_y^{-2}\left(\frac{B_1 - A_1 - C_1}{A_2 - B_2 + C_2} L_y^2 v_n\right) + L_y^{-2}\left(\frac{A_1 + B_1 + C_1}{A_2 - B_2 + C_2} L_x^2 v_n\right) \\ &\quad + L_y^{-2}\left(\frac{2A_2 - 2C_2}{A_2 - B_2 + C_2} L_x L_y v_n\right) + L_y^{-2}\left(\frac{A_2 + B_2 + C_2}{A_2 - B_2 + C_2} L_x^2 u_n\right) \\ &\quad + L_y^{-2}\left(\frac{-2A_1 + 2C_1}{A_2 - B_2 + C_2} L_x L_y u_n\right) + L_y^{-2}\left(\frac{2D_2 - 2E_2}{A_2 - B_2 + C_2} L_y v_n\right) \\ &\quad + L_y^{-2}\left(\frac{2D_1 + 2E_1}{A_2 - B_2 + C_2} L_x v_n\right) + L_y^{-2}\left(\frac{-2D_1 + 2E_1}{A_2 - B_2 + C_2} L_y u_n\right) \\ &\quad + L_y^{-2}\left(\frac{2D_2 + 2E_2}{A_2 - B_2 + C_2} L_x u_n\right) + L_y^{-2}\left(\frac{4F_1 v_n + 4F_2 u_n}{A_2 - B_2 + C_2}\right). \end{aligned}$$

By conditions (2.2) and (2.3) values of $u_0(x,y), v_0(x,y)$ are as follows:

$$\begin{aligned} u_0(x,y) &= L_y^{-2}\left(\frac{-4G_2}{A_2-B_2+C_2}\right)+L_y^{-1}(Reg(x))+Ref(x), \\ v_0(x,y) &= L_y^{-2}\left(\frac{4G_1}{A_2-B_2+C_2}\right)+L_y^{-1}(Img(x))+Imf(x) \end{aligned}$$

□

Example 2.1. ([**13**]) Solve the following differential equation

$$\frac{\partial^2 w}{\partial z \partial \overline{z}} = 4$$

with the conditions

$$w(x,0) = 9x^2, \ \frac{\partial w}{\partial y}(x,0) = 10ix.$$

Solution 2.1. By the Theorem 2.1 the coeffients of equation are as follows:

$$A = 0, B = 1, C = D = E = F = 0, G = 4.$$

Therefore

$$A_1 = A_2 = B_2 = C_1 = C_2 = D_1 = D_2 = E_1 = E_2 = F_1 = F_2 = G_2 = 0,$$
$$B_1 = 1, G_1 = 4$$
$$u_0(x,y) = L_y^{-2}(16) + 9x^2 = 8y^2 + 9x^2, v_0(x,y) = L_y^{-1}(10x) = 10xy$$
$$u_1 = L_y^{-2}(-18) = -9y^2, u_2 = u_3 = u_4 = ... = 0, v_2 = v_3 = v_4 = ... = 0$$
$$u = u_0 + u_1 + u_2 + ... = 8y^2 + 9x^2 - 9y^2 = 9x^2 - y^2$$
$$v = v_0 + v_1 + v_2 + ... = 10xy$$
$$w = u + iv = 9x^2 - y^2 + 10ixy = 4z\overline{z} + 5z^2.$$

Example 2.2. Solve the following differential equation

$$\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial z \partial \overline{z}} + \frac{\partial^2 w}{\partial \overline{z}^2} = 0$$

with the conditions

$$w(x,0) = x^2, \ \frac{\partial w}{\partial y}(x,0) = 18ix$$

Solution 2.2. From Theorem 2.1 the coeffients of equation are as follows:

$$A = 1, B = 1, C = 1, D = E = F = 0 = G = 0.$$

Therefore

$$A_1 = B_1 = C_1 = 1, A_2 = B_2 = C_2 = D_1 = D_2 = E_1 = E_2 = F_1 = F_2$$
$$= G_1 = G_2 = 0$$
$$u_0(x,y) = x^2, v_0(x,y) = L_y^{-1}(18x) = 18xy$$
$$u_1 = 3y^2, u_2 = u_3 = u_4 = ... = 0, v_1 = v_2 = v_3 = v_4 = ... = 0$$
$$u = u_0 + u_1 + u_2 + ... = x^2 + 3y^2$$

$$v = v_0 + v_1 + v_2 + ... = 18xy$$

$$w = u + iv = x^2 + 3y^2 + 18ixy =$$

$$\left(\frac{z+\overline{z}}{2}\right)^2 + \left(\frac{z-\overline{z}}{2i}\right)^2 + 18i\left(\frac{z+\overline{z}}{2}\right)\left(\frac{z-\overline{z}}{2i}\right) = 4z^2 + 2z\overline{z} - 5\overline{z}^2.$$

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