EDGE PAIR SUM LABELING OF SOME SUBDIVISION OF GRAPHS

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Abstract. An injective map \( f : E(G) \rightarrow \{ \pm 1, \pm 2, \cdots, \pm q \} \) is said to be an edge pair sum labeling of a graph \( G(p, q) \) if the induced vertex function \( f^*: V(G) \rightarrow \mathbb{Z} - \{0\} \) defined by \( f^*(v) = \sum_{e \in E_v} f(e) \) is one-one, where \( E_v \) denotes the set of edges in \( G \) that are incident with a vertex \( v \) and \( f^*(V(G)) \) is either of the form \( \{ \pm k_1, \pm k_2, \cdots, \pm k_{\frac{p}{2}} \} \) or \( \{ \pm k_1, \pm k_2, \cdots, \pm k_{\frac{p-1}{2}} \} \cup \{ \pm k_{\frac{p+1}{2}} \} \) according as \( p \) is even or odd. A graph which admits edge pair sum labeling is called an edge pair sum graph. In this paper, we prove that the subdivision of graph such as bistar \( S(B_{m,n}) \), path \( P_n \odot k_1 \), triangular snake \( S(T_n) \) if \( n \) is odd, double triangular snake \( D(T_n) \), double quadrilateral snake \( D(Q_n) \), double alternative triangular snake \( DA(T_n) \) and double alternative quadrilateral snake \( DA(Q_n) \) are edge pair sum graph.

1. preliminaries

A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and for a dynamic survey of various graph labeling problems along with extensive bibliography we refer to Gallian [1]. The concept of edge pair sum labeling has been introduced in [3] and further studied in [4-12]. This is the further extension work on edge pair sum labeling. Through out this paper we consider finite, simple and undirected graph \( G = (V(G), E(G)) \) with \( p \) vertices and \( q \) edges. \( G \) is also called a \((p, q)\)

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The double triangular snake $D(T_n)$ is a graph obtained from a path $P_n$ with vertices $v_1, v_2, ..., v_n$ by joining $v_i$ and $v_{i+1}$ to the new vertices $w_i$ and $u_i$ for $i = 1, 2, ..., n-1$.

The double quadrilateral snake $D(Q_n)$ is a graph obtained from a path $P_n$ with vertices $u_1, u_2, ..., u_n$ by joining $u_i$ and $u_{i+1}$ to the new vertices $v_i, x_i$ and $w_i, y_i$ respectively and then joining $v_i, w_i$ and $x_i, y_i$ for $i = 1, 2, ..., n-1$.

A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path. That is, a double alternate triangular snake is obtained from a path $u_1, u_2, ..., u_n$ by joining $u_i$ and $u_{i+1}$ (alternatively) to the two new vertices $v_i$ and $w_i$ for $i = 1, 2, ..., n-1$.

A double alternate quadrilateral snake $DA(Q_n)$ consists of two alternate quadrilateral snakes that have a common path. That is, a double alternate quadrilateral snake is obtained from a path $u_1, u_2, ..., u_n$ by joining $u_i$ and $u_{i+1}$ (alternatively) to the new vertices $v_i, x_i$ and $w_i, y_i$ respectively and adding the edges $v_i w_i$ and $x_i y_i$ for $i = 1, 2, ..., n-1$.

Let $G$ be a graph. The subdivision graph $S(G)$ is obtained from $G$ by subdividing each edge of $G$ with a vertex.

2. Main Results

In this section, we prove that subdivision of bistar $S(B_{m,n})$, $P_n \odot k_1$, triangular snake $S(T_n)$ if $n$ is odd, double triangular snake $D(T_n)$, double quadrilateral snake $D(Q_n)$, double alternative triangular snake $DA(T_n)$ and double alternative quadrilateral snake $DA(Q_n)$ are edge pair sum graph.

**Theorem 2.1.** The subdivision of bistar graph $S(B_{m,n})$ is an edge pair sum graph.

**Proof.** Let

$$V(S(B_{m,n})) = \{u, v, w, u_i : 1 \leq i \leq 2m, v_i : 1 \leq i \leq 2n\}$$

and

$$E(S(B_{m,n})) = \{e_{1}'' = uw, e_2'' = vw, e_{2i-1} = uv_{2i-1}, e_{2i} = u_{2i-1}v_{2i} : 1 \leq i \leq m, e_{2i-1} = v_{2i-1}, e_{2i} = v_{2i-1}v_{2i} : 1 \leq i \leq n\}$$

are the vertices and edges of the graph $S(B_{m,n})$.

Define

$$f : E(S(B_{m,n})) \rightarrow \{\pm 1, \pm 2, \pm 3, ..., \pm 2(m + n + 1)\}$$
by considering the following three cases:

**Case (i).**  \( m \) and \( n \) are even.

\[
f(e_1^{'}) = -2, \quad f(e_2^{'}) = 1,
\]

for \( 1 \leq i \leq \frac{m}{2} \)

\[
f(e_{2i-1}) = (2i + 1) = -f(e_{m-1+2i}), \quad f(e_{2i}) = (m + 1 + 2i) = -f(e_{m+2i});
\]

for \( 1 \leq i \leq \frac{n}{2} \)

\[
f(e_{2i-1}^{'}) = (2m + 1 + 2i) = -f(e_{n-1+2i}^{'}), \quad f(e_{2i}^{'}) = (2m + n + 1 + 2i) = -f(e_{n+2i}^{'}).\]

For each edge label \( f \) the induced vertex label \( f^* \) is defined as follows:

\[
f^*(u) = -2, \quad f^*(w) = -1 = -f^*(v),
\]

for \( 1 \leq i \leq \frac{m}{2} \)

\[
f^*(u_{2i-1}) = (m + 2 + 4i) = -f^*(u_{m-1+2i}),
\]

\[
f^*(u_{2i}) = (m + 1 + 2i) = -f^*(u_{m+2i}),
\]

for \( 1 \leq i \leq \frac{n}{2} \)

\[
f^*(v_{2i-1}) = (4m + n + 2 + 4i) = -f^*(v_{n-1+2i}),
\]

\[
f^*(v_{2i}) = (2m + n + 1 + 2i) = -f^*(v_{n+2i}).
\]

Then

\[
f^*(V(B_{m,n})) = \{ \pm 1, \pm (m + 6), \pm (m + 10), \pm (m + 14), ..., \pm (3m + 2), \pm (m + 3), \pm (m + 5), \pm (m + 7), ..., \pm (2m + 1), \pm (4m + n + 6), \pm (4m + n + 10), \pm (4m + n + 14), ..., \pm (4m + 3n + 2), \pm (2m + n + 3), \pm (2m + n + 5), \pm (2m + n + 7), ..., \pm (2m + 2n + 1) \} \cup \{-2\}.
\]

It can be verified that \( f \) is an edge pair sum labeling of \( S(B_{m,n}) \) if \( m \) and \( n \) are even. Hence \( S(B_{m,n}) \) is an edge pair sum graph if \( m \) and \( n \) are even.

**Case (ii).**  \( m \) and \( n \) are odd.

\[
f(e_1) = -3 = -f(e_1^{'}), \quad f(e_2) = -5 = -f(e_2^{'}), \quad f(e_1^{'}) = 2, \quad f(e_2^{'}) = -1,
\]

for \( 1 \leq i \leq \frac{m-1}{2} \)

\[
f(e_{2i+1}) = (2i + 5) = -f(e_{m+2i}),
\]

\[
f(e_{2i+2}) = (m + 4 + 2i) = -f(e_{m+1+2i}),
\]

for \( 1 \leq i \leq \frac{n-1}{2} \)

\[
f(e_{2i+1}^{'}) = (2m + 3 + 2i) = -f(e_{n+2i}^{'},)
\]

\[
f(e_{2i+2}^{'}) = (2m + n + 2 + 2i) = -f(e_{n+1+2i}).
\]

For each edge label \( f \) the induced vertex label \( f^* \) is defined as follows:

\[
f^*(v) = 2, \quad f^*(w) = 1 = -f^*(u),
\]

\[
f^*(u_1) = -8 = -f^*(v_1), \quad f^*(u_2) = -5 = -f^*(v_2),
\]
for $1 \leq i \leq \frac{m-1}{2}$

$$f^*(u_{2i+1}) = (m + 9 + 4i) = -f^*(u_{m+2i}),$$

$$f^*(u_{2i+2}) = (m + 4 + 2i) = -f^*(u_{m+1+2i}),$$

for $1 \leq i \leq \frac{n-1}{2}$

$$f^*(v_{2i+1}) = (4m + n + 5 + 4i) = -f^*(v_{n+2i}),$$

$$f^*(v_{2i+2}) = (2m + n + 2 + 2i) = -f^*(v_{n+1+2i}).$$

Then

$$f^*(V(B_{m,n})) =$$

$$\{\pm 1, \pm 5, \pm 8, \pm (m + 13), \pm (m + 17), \pm (m + 21), ..., \pm (3m + 7), \pm (m + 6), \pm (m + 8), \pm (m + 10), ..., \pm (2m + 3), \pm (4m + n + 9), \pm (4m + n + 13), \pm (4m + n + 17), ..., \pm (4m + 3n + 3), \pm (2m + n + 4), \pm (2m + n + 6), \pm (2m + n + 8), ..., \pm (2m + 2n + 1) \} \cup \{2\}.$$  

It can be verified that $f$ is an edge pair sum labeling of $S(B_{m,n})$ if $m$ and $n$ are odd. Hence $S(B_{m,n})$ is an edge pair sum graph if $m$ and $n$ are odd.

Case (iii). $m$ is odd and $n$ is even.

$$f(e_1) = -1 = -f(e_2''), f(e_2) = 3, f(e_1'') = -2,$$

for $1 \leq i \leq \frac{m-1}{2}$

$$f(e_{2i+1}) = (2i + 3) = -f(e_{m+2i}),$$

$$f(e_{2i+2}) = (m + 2 + 2i) = -f(e_{m+1+2i}),$$

for $1 \leq i \leq \frac{n}{2}$

$$f(e_{2i-1}') = (2m + 1 + 2i) = -f(e_{n-1+2i}'),$$

$$f(e_{2i}') = (2m + n + 1 + 2i) = -f(e_{n+2i}').$$

For each edge label $f$ the induced vertex label $f^*$ is defined as follows:

$$f^*(u_1) = 2, f^*(w) = -1 = -f^*(v), f^*(u_2) = 3 = -f^*(u),$$

for $1 \leq i \leq \frac{m-1}{2}$

$$f^*(u_{2i+1}) = (m + 5 + 4i) = -f^*(u_{m+2i}),$$

$$f^*(u_{2i+2}) = (m + 2 + 2i) = -f^*(u_{m+1+2i}),$$

for $1 \leq i \leq \frac{n}{2}$

$$f^*(v_{2i-1}) = (4m + n + 2 + 4i) = -f^*(v_{n-1+2i}),$$

$$f^*(v_{2i}) = (2m + n + 1 + 2i) = -f^*(v_{n+2i}).$$

Then

$$f^*(V(B_{m,n})) =$$

$$\{\pm 1, \pm 3, \pm (m + 9), \pm (m + 13), \pm (m + 17), ..., \pm (3m + 3), \pm (m + 4), \pm (m + 6), \pm (m + 8), ..., \pm (2m + 1), \pm (4m + n + 6), \pm (4m + n + 10), \pm (4m + n + 14), ..., \pm (4m + 3n + 2), \pm (2m + n + 3), \pm (2m + n + 5), \pm (2m + n + 7), ..., \pm (2m + 2n + 1) \} \cup \{2\}.$$
It can be verified that $f$ is an edge pair sum labeling of $S(B_{m,n})$ if $m$ is odd and $n$ is even. Hence $S(B_{m,n})$ is an edge pair sum graph if $m$ is odd and $n$ is even.

An example for the edge pair sum labeling of $S(B_{3,4})$ is given in Figure 1.

![Figure 1: Edge pair sum labeling of $S(B_{3,4})$](image)

**Theorem 2.2.** The subdivision of $(P_n \circ K_1)$ graph is an edge pair sum graph.

**Proof.** Let

$$V(S(P_n \circ K_1)) = \{v_i, w_i : 1 \leq i \leq n, u_i : 1 \leq i \leq 2n - 1\}$$

and

$$E(S(P_n \circ K_1)) = \{e_i = v_iu_{2i-1}, e''_i = v_iw_i : 1 \leq i \leq n, e_i = u_iu_{i+1} : 1 \leq i \leq 2n-2\}$$

are the vertices and edges of the graph $S(P_n \circ K_1)$.

Define an edge labeling $f : E((P_n \circ K_1)) \rightarrow \{\pm 1, \pm 2, \pm 3, ..., \pm (4n-2)\}$ by considering the following two cases:

**Case (i).** $n$ is odd.

$$f(e'_{\frac{n+1}{2}}) = 2, \quad f(e''_{\frac{n+1}{2}}) = -4,$$

for $1 \leq i \leq \frac{n-1}{2}$

$$f(e_i) = (2i - 1), \quad f(e'_{\frac{n+1}{2}+i}) = -(n - 2i), \quad f(e''_{\frac{n+1}{2}+i}) = (4 + 2i)$$

and

$$f(e''_{\frac{n+1}{2}+i}) = -(n + 5 - 2i),$$

for $1 \leq i \leq (n - 1)$

$$f(e_i) = (n + 2 + 2i)$$

and

$$f(e_{n-1+i}) = -(3n + 2 - 2i).$$

For each edge label $f$ the induced vertex label $f^*$ is defined as follows:
Then the induced vertex labeling is as follows:

\[ f^*(w_{\frac{2i+1}{2}}) = -4, \quad f^*(v_{\frac{2i+1}{2}}) = -2 = -f^*(u_n), \quad f^*(u_1) = (n+5) = -f^*(u_{2n-1}), \]

for \( 1 \leq i \leq \frac{n-1}{2} \)

\[ f^*(w_i) = (4+2i), \quad f^*(v_{\frac{2i+1}{2}+1}) = -(n+5-2i), \quad f^*(v_i) = (4i+3), \]

\[ f^*(v_{\frac{2i+1}{2}+1}) = -(2n+5-4i), \quad f^*(u_{2i}) = (2n+2+8i) \text{ and} \]

\[ f^*(u_{n-1+2i}) = -(6n+6-8i), \]

for \( 1 \leq i \leq \frac{n-3}{2} \)

\[ f^*(u_{2i+1}) = (2n+7+10i) \text{ and} \quad f^*(u_{n+2i}) = -(7n+2-10i). \]

Hence we get

\[ f^*(V(P_n \circ K_1)) = \{ \pm 2, \pm 6, \pm 8, \pm 10, \ldots, \pm (n+3), \pm 7, \pm 11, \pm 15, \ldots, \pm (2n+1), \pm (2n+17), \pm (2n+27), \pm (2n+37), \ldots, \pm (7n-8), \pm (2n+10), \pm (2n+18), \pm (2n+26), \ldots, \pm (6n-2), \pm (n+5) \} \bigcup \{-4\}. \]

It can be verified that \( f \) is an edge pair sum labeling of \((P_n \circ K_1)\) if \( n \) is odd. Hence \((P_n \circ K_1)\) is an edge pair sum graph if \( n \) is odd.

**Case (ii).** \( n \) is even.

Subcase (a). \( n = 2 \).

\[ f(e'_1) = 4 = -f(e'_2), \quad f(e'_2) = 2 = -f(e_1), \quad f(e'_1) = -3 \text{ and} \quad f(e_2) = 1. \]

Then the induced vertex labeling is as follows:

\[ f^*(u_1) = 2 = -f^*(v_2), \quad f^*(v_2) = -1 = -f^*(v_1), \quad f^*(v_3) = 3 = -f^*(w_1) \text{ and} \]

\[ f^*(w_2) = -4. \]

Then \( f^*(V(P_n \circ K_1)) = \{ \pm 1, \pm 2, \pm 3 \} \bigcup \{-4\}. \)

Hence \( f \) is an edge pair sum labeling if \( n = 2 \).

Subcase (b). \( n \geq 4 \).

\[ f(e_{\frac{2i+1}{4}}) = 3n, \quad f(e_{\frac{2i+2}{4}}) = -(3n-3), \]

for \( 1 \leq i \leq n-2 \)

\[ f(e_i) = -(2n+1-2i) \text{ and} \quad f(e_{\frac{2i+4}{4}+1}) = (3+2i), \]

for \( 1 \leq i \leq \frac{n}{2} \)

\[ f(e'_i) = -(2n-2+2i), \quad f(e_{\frac{2i+1}{4}+i}) = (3n-2i), \quad f(e''_i) = -(3n-3+2i) \text{ and} \]

\[ f(e_{\frac{2i+1}{4}+1}) = (4n-1-2i). \]

For each edge label \( f \) the induced vertex label \( f^* \) is defined as follows:

for \( 1 \leq i \leq \frac{n-1}{2} \)

\[ f^*(w_i) = -(3n-3+2i), \quad f^*(v_{\frac{2i+1}{4}+1}) = (4n-1-2i), \quad f^*(v_i) = -(5n-5+4i), \]

\[ f^*(v_{\frac{2i+1}{4}+1}) = (7n-1-4i), \quad f^*(u_i) = -(4n-1) = -f^*(u_n), \]

\[ f^*(u_n) = 3 = -f^*(u_{n-1}), \]

for \( 1 \leq i \leq \frac{n-2}{2} \)
The subdivision of triangular snake graph

Let \( f(u_{2i}) = -(4n + 4 - 8i) \) and \( f(u_{n+2i}) = (4 + 8i) \),
for \( 1 \leq i \leq \frac{n-4}{2} \).

\( f(u_{2i+1}) = -6(n - 8) \) and \( f(u_{n+2i+1}) = (3n + 6 + 6i) \) and \( f(u_{n+1}) = 6 \).

We get \( f(V(P_n \odot K_1)) = \{ \pm 3, \pm (3n - 1), \pm (3n + 1), \pm (3n + 3), \ldots, \pm (4n - 3), \pm (5n - 1), \pm (5n+3), \pm (5n+7), \ldots, \pm (7n-5), \pm (4n-1), \pm 12, \pm 20, \pm 28, \ldots, \pm (4n-4), \pm (5n+12), \pm (3n + 18), \pm (3n + 24), \ldots, \pm (6n - 6) \} \cup \{6\} \).

It can be verified that \( f \) is an edge pair sum labeling of \((P_n \odot K_1)\) if \( n \geq 4 \).
Hence \((P_n \odot K_1)\) is an edge pair sum graph if \( n \geq 4 \).

The example for the edge pair sum labeling of \((P_3 \odot K_1)\) and \((P_2 \odot K_1)\) are shown in Figure 2.

\[
\begin{array}{c}
1 & 2 & 3 & 4 & 5 & 6 \\
7 & 9 & -9 & -7 & -2 & 1 \\
6 & 4 & -6 & -3 & -4 & 2 \\
\end{array}
\]

Figure 2: Edge pair sum labeling of \((P_3 \odot K_1)\) and \((P_2 \odot K_1)\)

**Theorem 2.3.** The subdivision of triangular snake graph \(S(T_n)\) is an edge pair sum graph if \( n \) is odd.

**Proof.** Let \( V(S(T_n)) = \{ w_i : 1 \leq i \leq (n-1), v_i : 1 \leq i \leq 2(n-1), u_i : 1 \leq i \leq (2n-1) \} \) and \( E(S(T_n)) = \{ e_{2i-1} = e_{2i-1}'w_i, e_{2i} = e_{2i}'v_i, e_{2i-1}' = u_{2i-1}v_{2i-1}, e_{2i}' = u_{2i+1}v_{2i} : 1 \leq i \leq n-1, e_i = u_iu_{i+1} : 1 \leq i \leq 2(n-1) \} \) are the vertices and edges of the graph \(S(T_n)\).

Define an edge labeling \( f : E(S(T_n)) \to \{ \pm 1, \pm 2, \pm 3, \ldots, \pm 6(n-1) \} \).

\[
f(e_{n-2}') = -6 = f(e_{n+1}''), f(e_{n+1}'') = 1, f(e''_n) = 2, f(e_{n-2}') = -5 = f(e''_{n+1}),\\ f(e_{n-1}') = -4 = f(e''_n), f(e_{n-2}) = -7 = f(e_{n+1}), f(e_{n-1}) = -8 = f(e_n),
\]

for \( 1 \leq i \leq \frac{n-3}{2} \).
The subdivision of double triangular snake graph

For each edge label $f$ the induced vertex label $f^*$ is defined as follows:

$$f^*(w_{n-1}) = -9 = f^*(w_{n+1}), \quad f^*(v_{n-2}) = -11 = f^*(v_{n+1}),$$
$$f^*(v_{n-1}) = -3 = f^*(u_n), \quad f^*(v_n) = 6, \quad f^*(u_1) = -(6n - 5) = f^*(u_{2n-1}),$$
$$f^*(u_{2n-2}) = (6n - 3),$$

for $1 \leq i \leq \frac{n-3}{2}$

$$f^*(w_i) = -(6n + 3 - 12i), \quad f^*(w_{i+1}) = (9 + 12i), \quad f^*(v_{i-1}) = -(6n + 5 - 12i),$$
$$f^*(v_i) = -(6n + 1 - 12i), \quad f^*(v_{i+1}) = (7 + 12i), \quad f^*(v_{n+1}) = (11 + 12i),$$
$$f^*(u_{i+1}) = -(12n - 24i), \quad f^*(u_{n+1}) = (3 + 12i) \text{ and } f^*(u_{n+2}) = (12 + 24i),$$

for $1 \leq i \leq \frac{n-1}{2}$

$$f^*(u_{2i}) = -(6n + 9 - 12i).$$

Then

$$f^*(V(S(T_n))) = \{\pm 3, \pm 11, \pm 9, \pm 21, \pm 33, \pm 45, \ldots, \pm (6n - 9), \pm 23, \pm 35, \pm 47, \ldots, \pm (6n - 7), \pm 19, \pm 31, \pm 43, \ldots, \pm (6n - 11), \pm (6n - 5), \pm 15, \pm 27, \pm 39, \ldots, \pm (6n - 15), \pm 36, \pm 60, \pm 84, \ldots, \pm (12n - 24), \pm (6n - 3)\} \bigcup \{6\}.$$

It can be verified that $f$ is an edge pair sum labeling of $S(T_n)$ if $n$ is odd. Hence $S(T_n)$ is an edge pair sum graph if $n$ is odd.

**Theorem 2.4.** The subdivision of double triangular snake graph $S(D(T_n))$ is an edge pair sum graph.

**Proof.** Let

$$V(S(D(T_n))) = \{u_i : 1 \leq i \leq (2n - 1), v_i, v_i' : 1 \leq i \leq 2(n - 1), w_i w_i' : 1 \leq i \leq (n - 1)\}$$

and

$$E(S(D(T_n))) = \{e_i = u_i u_{i+1} : 1 \leq i \leq 2(n - 1), e_i = v_{i-1} v_{i-1}, e_i' = v_{i-1} v_{i-1}, e_i'' = v_{i-1} w_i, e_i''' = w_i w_i' : 1 \leq i \leq (n - 1)\}$$

are the vertices and edges of the graph $S(D(T_n))$.

Define $f : E(S(D(T_n))) \rightarrow \{\pm 1, \pm 2, \pm 3, \ldots, \pm 10(n - 1)\}$ by considering the following two cases:

**Case (i).** $n = 2$.

$$f(e_1) = -2, \quad f(e_2) = 1, \quad f(e'_1) = 4 = -f(e'_2), \quad f(e'_2) = 5 = -f(e'_3),$$
$$f(e'_3) = 6 = -f(e'_4) \text{ and } f(e'_4) = 7 = -f(e'_5).$$

For each edge label $f$ the induced vertex label $f^*$ is defined as follows:
For each edge label $f$ the induced vertex label $f^*$ is defined as follows:

- For $1 \leq i \leq (n-1)$:
  \[ f^*(w_i) = (4n - 5 + 8i) = -f^*(w'_i), \]
  \[ f^*(v_{2i-1}) = (4n - 7 + 8i) = -f^*(v'_{2i-1}), \]
  and
  \[ f^*(v_{2i}) = (4n - 3 + 8i) = -f^*(v'_{2i}). \]

- For $1 \leq i \leq (n-3)$:
  \[ f^*(u_{1+i}) = -4(n - i) \]
  and
  \[ f^*(u_{n+1+i}) = (8 + 4i). \]

Then

\[ f^*(V(S(D(T_n)))) = \{-1, \pm 9, \pm 11, \pm 13\} \cup \{-2\}. \]

**Case (ii).** $n \geq 3$.

For $1 \leq i \leq (n-2)$

- $f(e_i) = -(2n + 1 - 2i)$ and $f(e_{n+i}) = (3 + 2i)$,
- $f(e_{n-1}) = 2$,
- $f(e_n) = 1$,
- for $1 \leq i \leq (n-1)$ $f(e_{4i-3}) = (2n - 4 + 4i) = -f(e'_{4i-3}),$
- $f(e'_{4i-2}) = (2n - 3 + 4i) = -f(e''_{4i-2}),$
- $f(e'_{4i-1}) = (2n - 2 + 4i) = -f(e'''_{4i-1})$ and
- $f(e'_{4i}) = (2n - 1 + 4i) = -f(e''_{4i}).$

An example for the edge pair sum labeling of subdivision of double triangular snake graph $S(D(T_5))$ is shown in Figure 3.

**Figure 3.** Edge pair sum labeling of $S(D(T_5))$
Corollary 2.1. The subdivision of double alternative triangular snake graph \( S(DA(T_n)) \) is an edge pair sum graph.

Proof. The proof follows from the Theorem 2.4. \( \square \)

Theorem 2.5. The subdivision of double quadrilateral snake graph \( S(D(Q_n)) \) is an edge pair sum graph.

Proof. Let
\[
V(S(D(Q_n))) = \{ u_i : 1 \leq i \leq (2n - 1), v_{ij}, v'_{ij} : 1 \leq i \leq (n - 1), 1 \leq j \leq 5 \}
\]

and
\[
E(S(D(Q_n))) = \{ e_i = u_i u_{i+1} : 1 \leq i \leq (2n - 2), e_i = u_{2i-1} v_{i1}, e_i' = u_{2i-1} v'_{i1}, e_i'' = v_5 u_{2i+1}, e_i'' = v_{ij} e_{i1+j}, e_i'' = v_j e_{i1+j} : 1 \leq i \leq (n - 1), 1 \leq j \leq 4 \}
\]
are the vertices and edges of the graph \( S(D(Q_n)) \).

Define
\[
f : E(S(D(Q_n))) \to \{ \pm 1, \pm 2, \pm 3, ..., \pm 14(n - 1) \}
\]
by considering the following two cases:

Case (i). \( n = 2 \).
\[
f(e_1) = -2, f(e_2) = 1, f(e_{11}) = 3 = -f(e'_{11}), f(e_{16}) = 8 = -f(e'_{16}),
\]
for \( 1 \leq j \leq 4 \)
\[
f(e_{11+j}) = (3 + j) = -f(e'_{11+j})
\]
For each edge label \( f \) the induced vertex label \( f^* \) is defined as follows:
\[
f^*(u_1) = -2, f^*(u_2) = -1 = -f^*(u_3)
\]
for \( 1 \leq j \leq 5 \)
\[
f^*(v_{ij}) = (5 + 2j) = -f^*(v'_{ij}).
\]
Then
\[
f^*(V(S(D(Q_n)))) = \{ \pm 1, \pm 7, \pm 9, \pm 11, \pm 13, \pm 15 \} \cup \{ -2 \}.
\]

Case (ii). \( n \geq 3 \).
\[
f(e_{n-1}) = 2, f(e_n) = 1,
\]
for \( 1 \leq i \leq (n - 2) \)
\[
f(e_i) = -(2n - 2i + 1) \text{ and } f(e_{n+i}) = (3 + 2i),
\]
for \( 1 \leq i \leq (n - 1) \),
\[
1 \leq j \leq 6, f(e_{ij}) = (2n - 7 + 6i + j) = -f(e'_{ij}).
\]
For each edge label \( f \) the induced vertex label \( f^* \) is defined as follows:
\[
f^*(u_1) = -(2n - 1) = -f^*(u_{2n-1}), f^*(u_{n-1}) = -3 = -f^*(u_n), f^*(u_{n+1}) = 6,
\]
for $1 \leq i \leq (n - 3)$,
\[
f^*(u_{1+i}) = -2(2n - 2i)\text{and} f^*(u_{n+1+i}) = (8 + 4i),
\]
for $1 \leq i \leq (n - 1)$ and $1 \leq j \leq 5$,
\[
f^*(v_{ij}) = (4n - 13 + 12i + 2j) = -f^*(v'_{ij}).
\]
Then $f^*(V(S(D(Q_n)))) = \{ \pm 3, \pm (2n-1), \pm 12, \pm 16, \pm 20, \ldots, \pm 4(n-1), \pm (4n-13+12i+2j) : 1 \leq i \leq (n-1), 1 \leq j \leq 5 \} \cup \{6\}$.

It can be verified that $f$ is an edge pair sum labeling of $S(D(Q_n))$. Hence $S(D(Q_n))$ is an edge pair sum graph.

An example for the edge pair sum labeling of subdivision of double quadrilateral snake graph of $S(D(Q_3))$ is shown in Figure 4.

**Figure 4:**

**Figure 4.** Edge pair sum labeling of $S(D(Q_3))$

**Corollary 2.2.** The subdivision of double alternative quadrilateral snake graph $S(DA(Q_n))$ is an edge pair sum graph.

**Proof.** The proof follows from the Theorem 2.5.

**Theorem 2.6.** The subdivision of slanting graph $S(Sl_n)$ is an edge pair sum graph.

**Proof.** Let
\[
V(S(Sl_n)) = \{u_i, w_i : 1 \leq i \leq (2n - 1), v_i : 1 \leq i \leq (n - 1)\}
\]
and
\[
E(S(Sl_n)) = \{e_i = u_iu_{i+1}, e''_i = w_iw_{i+1} : 1 \leq i \leq (2n - 2), e'_i = u_{2i-1}v_i, e''_i = w_{2i+1}v_i : 1 \leq i \leq (n - 1)\}
\]
are the vertices and edges of the graph $S(Sl_n)$.

Define $f : E(S(Sl_n)) \rightarrow \{\pm 1, \pm 2, \pm 3, \ldots, \pm (6n-6)\}$ by considering the following two cases:

**Case (i).** $n$ is odd.

For $1 \leq i \leq (2n-2)$
\[ f(e_i) = (2i-1), \quad f(e_i'') = -(4n-3-2i), \]

for $1 \leq i \leq (n-1)$
\[ f(e_i') = 4n+2i-5, \quad f(e_i') = -(6n-5-2i). \]

For each edge label $f$ the induced vertex label $f^*$ is defined as follows:
\[ f^*(u_1) = (4n-2) = -f^*(w_{2n-1}), \]

for $1 \leq i \leq (n-1)$
\[ f^*(u_2i) = (8i - 4), \]

for $1 \leq i \leq (n-2)$
\[ f^*(u_{2i+1}) = (4n-3+10i), \]

for $1 \leq i \leq \frac{n-1}{2}$
\[ f^*(u_{\frac{n-1}{2}+1}) = (4i - 2) \text{ and } f^*(v_i) = -(2n-4i), \quad f^*(u_{2n-1}) = (4n-5) = -f^*(w_1), \]

for $1 \leq i \leq (n-1)$
\[ f^*(w_2i) = -(8n-4-8i) \]

and for $1 \leq i \leq (n-2)$
\[ f^*(w_{2i+1}) = -(14n-13-10i). \]

Then
\[ f^*(V(S(Sl_n))) = \{ \pm 2, \pm 6, \pm 10, \ldots, \pm (2n-4), \pm 4, \pm 12, \pm 20, \ldots, \pm (8n-12), \pm (4n-2), \pm (4n-5), \pm (4n+7), \pm (4n+17), \ldots, \pm (14n-23) \}. \]

It can be verified that $f$ is an edge pair sum labeling of $S(Sl_n)$ if $n$ is odd.

Hence $S(Sl_n)$ is an edge pair sum graph if $n$ is odd.

**Case (ii).** $n$ is even.

Subcase (a). $n = 2$.

\[ f(e_1) = 3 = -f(e_2), \quad f(e_2) = 4 = -f(e_1''), \quad f(e_1') = 1 \text{ and } f(e_1') = -2. \]

For each edge label $f$ the induced vertex label $f^*$ is defined as follows:
\[ f^*(u_1) = 1 = -f^*(v_1), \quad f^*(u_3) = 4 = -f^*(w_1), \quad f^*(u_2) = 7 = -f^*(w_2) \text{ and } \]
\[ f^*(w_3) = -2. \]

Then
\[ f^*(V(S(DQ_n))) = \{ \pm 1, \pm 4, \pm 7 \} \bigcup \{-2\}. \]

Hence $f$ is an edge pair sum labeling if $n = 2$.

Subcase (b). $n \geq 4$.

for $1 \leq i \leq (2n-2)$
\( f(e_i) = (2i - 1) \) and \( f(e''_i) = -(4n - 3 - 2i), f(e'_2) = -(4n - 3), f(e''_2) = (4n + 2), \)
for \( 1 \leq i \leq n - 2 \)
\[
\begin{align*}
  f(e'_i) & = (4n - 3 + 2i), \\
  f(e''_{i+1}) & = (5n - 5 + 2i), \\
  f(e''_i) & = -(6n - 5 - 2i), \\
  f(e''_{2i}) & = -(5n - 3 - 2i).
\end{align*}
\]
For each edge label \( f \) the induced vertex label \( f^* \) is defined as follows:
\[
\begin{align*}
  f^*(u_1) & = 4n = -f^*(w_{2n-1}), \\
  f^*(u_{n-1}) & = (4n - 5) = -f^*(w_1), \\
  f^*(u_2i) & = (8i - 4), \\
  f^*(u_{n-1}) & = -5 = -f^*(v_2), \\
  f^*(u_{2i+1}) & = (4n - 1 + 10i), \\
  f^*(v_{2i}) & = -(8n - 4 - 8i), \\
  f^*(v_{2i+1}) & = (-14n + 13 + 10i), \\
  f^*(w_{n+1+2i}) & = (9n + 11 + 10i), \\
  f^*(w_{n+1}) & = 10.
\end{align*}
\]
Then
\[
f^*(V(S(Sl_n))) = \\
\{ \pm 2, \pm 6, \pm 10, \ldots, \pm (2n - 6), \pm 4, \pm 12, \pm 20, \ldots, \pm (8n - 12), \pm 5, \pm 4n, \pm (4n - 5), \pm (4n + 9), \pm (4n + 19), \pm (4n + 29), \ldots, \pm (9n - 21), \pm (9n - 3), \pm (9n + 7), \pm (9n + 17), \ldots, \pm (14n - 23) \} \cup \{10\}.
\]
It can be verified that \( f \) is an edge pair sum labeling of \( S(Sl_n) \) if \( n \) is even. Hence \( S(Sl_n) \) is an edge pair sum graph if \( n \) is even.

The subdivision of slanting of \( S(Sl_4) \) and \( S(Sl_3) \) are shown in Figure 5.
References


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