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# WEIGHTED SZEGED INDEX OF GRAPHS

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Abstract. The weighted Szeged index of a connected graph $G$ is defined as $Sz_w(G) = \sum\limits_{e = uv \in E(G)} \Big(d_G(u) + d_G(v)\Big) n_u^G(e)\, n_v^G(e)$, where $n_u^G(e)$ is the number of vertices of $G$ whose distance to the vertex $u$ is less than the distance to the vertex $v$ in $G$. In this paper, we have obtained the weighted Szeged index $Sz_w(G)$ of the splice graph $S(G_1, G_2, y, z)$ and link graph $L(G_1, G_2, y, z)$.

## 1. Introduction

Let $G = G(V, E)$, be the graph, where $V = V(G)$ and $E = E(G)$ denotes the vertex set and edge set of the graph $G$, respectively. All the graphs considered in this paper are simple. Graph theory has successfully provided chemists with a variety of useful tools [**3, 5, 6, 7**], among which are the topological indices. A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. In theoretical chemistry, assigning a numerical value to the molecular structure that will closely correlate with the physical quantities and activities. Molecular structure descriptors (also called topological indices) are used for modeling physicochemical, pharmacologic, toxicologic, biological and other properties of chemical compounds. There exist several types of such indices, especially those based on degree and distances. The degree of a vertex $x \in V(G)$ is denoted by $d_G(x)$.

A vertex $x \in V(G)$ is said to be *equidistant* from the edge $e = uv$ of $G$ if $d_G(u, x) = d_G(v, x)$, where $d_G(u, x)$ denotes the distance between $u$ and $x$ in $G$; otherwise, $x$ is a nonequidistant vertex analogously, $d_G(v, x)$. For an edge $uv = e \in E(G)$, the number of vertices of $G$ whose distance to the vertex $u$ is less than the distance to the vertex $v$ in $G$ is denoted by $n_u^G(e)$ ( or $n_u(e, G)$;) analogously,$n_v^G(e)$ ( or $n_v(e, G)$) is the number of vertices of $G$ whose distance to the vertex $v$ in $G$ is

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less than the distance to the vertex $u$; the vertices equidistant from both the ends of the edge $e = uv$ are not counted. The Szeged index of a connected graph $G$ is defined as $Sz(G) = \sum\limits_{e = uv \in E(G)} n_u^G(e)\, n_v^G(e)$.

Similarly, the weighted Szeged index of a connected graph $G$ which is introduced by Ilić and Milosavljević [**9**], defined as
$Sz_w(G) = \sum\limits_{e = uv \in E(G)} \Big(d_G(u) + d_G(v)\Big) n_u^G(e)\, n_v^G(e)$.

For more recent results on the weighted Szeged index and the weighted PI index refer[**10, 11, 13**]. A variety of topological indices of splice graphs and link graphs have been computed already in [**1, 2, 4, 8, 14**]. Weighted Szeged index of generalized hierarchical product of two graphs is ontained in [**12**]. In this paper, we aim at continuing work along the same lines, for finding the exact value of the weighted Szeged index of splice and link graphs.

## 2. Weighted Szeged index of splice of graphs

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs with disjoint vertex sets $V_1$ and $V_2$. Let $y \in V_1$ and $z \in V_2$. A splice $S = S(G_1, G_2, y, z)$ of $G_1$ and $G_2$ by vertices $y$ and $z$, is defined by identifying the vertices $y$ and $z$ in the union of $G_1$ and $G_2$ (see Fig. 1), introduced by Došlić [**4**]. In this section, we compute the Weighted Szeged index of splice of two given graphs.

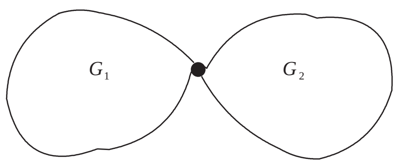


Fig. 1 Splice graph $S(G_1,G_2,y,z)$

We define the set $N_u^G(e) = \{x \in V(G) \mid d_G(x,u) < d_G(x,v)\}$. The proof of the following lemma is follows from the structure of splice of two connected graphs.

Lemma 2.1. *Let $G_i$ be the graphs with $n_i$ vertices and $|E_i|$ edges, $i = 1, 2$, then for the splice graph $S$ we have the following. For $e = uv \in E_1$.*

*(i) If $y \in N_u^{G_1}(e)$ and $u \neq y$, then*

$$\begin{aligned} n_u^S(e) &= n_u^{G_1}(e) - 1 + n_2 \quad , \quad d_S(u) = d_{G_1}(u) \\ n_v^S(e) &= n_v^{G_1}(e) \quad , \quad d_S(v) = d_{G_1}(v) \end{aligned}$$

*(ii) If $y \in N_u^{G_1}(e)$ and $u = y$, then*

$$\begin{aligned} n_u^S(e) &= n_u^{G_1}(e) - 1 + n_2 \quad , \quad d_S(u) = d_{G_1}(u) + d_{G_2}(z) \\ n_v^S(e) &= n_v^{G_1}(e) \quad , \quad d_S(v) = d_{G_1}(v) \end{aligned}$$

*(iii)* *If* $d_{G_1}(y,u) = d_{G_1}(y,v)$, *then*

$$\begin{aligned} n_u^S(e) &= n_u^{G_1}(e), \quad d_S(u) = d_{G_1}(u) \\ n_v^S(e) &= n_v^{G_1}(e), \quad d_S(v) = d_{G_1}(v) \end{aligned}$$

*Analogous relations hold if* $e = uv \in E_2$.

THEOREM 2.1. *Let* $G_i$ *be the graphs with* $n_i$ *vertices,* $i = 1, 2$ *then the weighted Szeged index of the Splice graphs is*

$$Sz_w(S(G_1,G_2,y,z)) = Sz_w(G_1) + (n_2-1)\sum_{\substack{uv\in E_1\\ u \neq y}} (d_e^{G_1}(u) + d_e^{G_1}(v))(n_v^{G_1}(e)) + \sum_{\substack{uv\in E_1\\ u = y}} (d_e^{G_2}(z))(n_u^{G_1}(e))(n_v^{G_1}(e)) + (n_2-1)\sum_{\substack{uv\in E_1\\ u = y}} (d_e^{G_1}(u)+d_e^{G_1}(v)+d_e^{G_2}(z))(n_v^{G_1}(e)) + Sz_w(G_2) + (n_1-1)\sum_{\substack{uv\in E_2\\ u \neq z}} (d_e^{G_2}(u)+d_e^{G_2}(v))(n_v^{G_2}(e)) + \sum_{\substack{uv\in E_2\\ u = z}} (d_e^{G_1}(z))(n_u^{G_2}(e)) (n_v^{G_2}(e)) + (n_1-1)\sum_{\substack{uv\in E_2\\ u = z}} (d_e^{G_2}(u)+d_e^{G_2}(v)+d_e^{G_1}(y))(n_v^{G_2}(e)).$$

PROOF. For a splice graph $S = S(G_1, G_2, y, z)$, let the edge set can be partitioned as $E_1 = E(G_1)$ and $E_2 = E(G_2)$, by the definition of $Sz_w$

$$Sz_w(S) = \sum_{uv\in E(S)} (d_S(u) + d_S(v))n_u^S(e)n_v^S(e)$$

By Lemma 2.1, we have

$$\begin{aligned} Sz_w(S) &= \sum_{uv\in E_1} (d_{G_1}(u) + d_{G_1}(v))n_u^{G_1}(e)n_v^{G_1}(e) \\ &\quad + \sum_{uv\in E_2} (d_{G_2}(u) + d_{G_2}(v))n_u^{G_2}(e)n_v^{G_2}(e) \\ &= \sum_{\substack{uv\in E_1\\ u \neq y}} (d_e^{G_1}(u) + d_e^{G_1}(v))(n_u^{G_1}(e) - 1 + n_2)(n_v^{G_1}(e)) \\ &\quad + \sum_{\substack{uv\in E_1\\ u = y}} (d_e^{G_1}(u) + d_e^{G_1}(v) + d_e^{G_2}(z))(n_u^{G_1}(e) - 1 + n_2)(n_v^{G_1}(e)) \\ &\quad + \sum_{\substack{uv\in E_1\\ d_{G_1}(y,u) = d_{G_1}(y,v)}} (d_e^{G_1}(u) + d_e^{G_1}(v))(n_u^{G_1}(e))(n_v^{G_1}(e)) \\ &\quad + \sum_{\substack{uv\in E_2\\ u \neq z}} (d_e^{G_2}(u) + d_e^{G_2}(v))(n_u^{G_2}(e) - 1 + n_1)(n_v^{G_2}(e)) \\ &\quad + \sum_{\substack{uv\in E_2\\ u = z}} (d_e^{G_2}(u) + d_e^{G_2}(v) + d_e^{G_1}(y))(n_u^{G_2}(e) - 1 + n_1)(n_v^{G_2}(e)) \\ &\quad + \sum_{\substack{uv\in E_2\\ d_{G_2}(z,u) = d_{G_2}(z,v)}} (d_e^{G_2}(u) + d_e^{G_2}(v))(n_u^{G_2}(e))(n_v^{G_2}(e)) \end{aligned}$$

$$
\begin{aligned}
&= \sum_{\substack{uv\in E_1 \\ u \neq y}} (d_e^{G_1}(u)+d_e^{G_1}(v))(n_u^{G_1}(e))(n_v^{G_1}(e)) \\
&+ \sum_{\substack{uv\in E_1 \\ u = y}} (d_e^{G_1}(u)+d_e^{G_1}(v))(n_u^{G_1}(e))(n_v^{G_1}(e)) \\
&+ \sum_{\substack{uv\in E_1 \\ d_{G_1}(y,u) = d_{G_1}(y,v)}} (d_e^{G_1}(u)+d_e^{G_1}(v))(n_u^{G_1}(e))(n_v^{G_1}(e)) \\
&+ (n_2-1)\sum_{\substack{uv\in E_1 \\ u \neq y}} (d_e^{G_1}(u)+d_e^{G_1}(v))(n_v^{G_1}(e)) \\
&+ \sum_{\substack{uv\in E_1 \\ u = y}} (d_e^{G_2}(z))(n_u^{G_1}(e))(n_v^{G_1}(e)) \\
&+ (n_2-1)\sum_{\substack{uv\in E_1 \\ u = y}} (d_e^{G_1}(u)+d_e^{G_1}(v)+d_e^{G_2}(z))(n_v^{G_1}(e)) \\
&+ \sum_{\substack{uv\in E_2 \\ u \neq z}} (d_e^{G_2}(u)+d_e^{G_2}(v))(n_u^{G_2}(e))(n_v^{G_2}(e)) \\
&+ \sum_{\substack{uv\in E_2 \\ u = z}} (d_e^{G_2}(u)+d_e^{G_2}(v))(n_u^{G_2}(e))(n_v^{G_2}(e)) \\
&+ \sum_{\substack{uv\in E_2 \\ d_{G_2}(z,u) = d_{G_2}(z,v)}} (d_e^{G_2}(u)+d_e^{G_2}(v))(n_u^{G_2}(e))(n_v^{G_2}(e)) \\
&+ (n_1-1)\sum_{\substack{uv\in E_2 \\ u \neq z}} (d_e^{G_2}(u)+d_e^{G_2}(v))(n_v^{G_2}(e)) \\
&+ \sum_{\substack{uv\in E_2 \\ u = z}} (d_e^{G_1}(y))(n_u^{G_2}(e))(n_v^{G_2}(e)) \\
&+ (n_1-1)\sum_{\substack{uv\in E_2 \\ u = z}} (d_e^{G_2}(u)+d_e^{G_2}(v)+d_e^{G_1}(y))(n_v^{G_2}(e))
\end{aligned}
$$

By the definition of $Sz_w$, for $G_1$ and $G_2$ we have

$$
\begin{aligned}
Sz_w(S) &= Sz_w(G_1) + (n_2-1)\sum_{\substack{uv\in E_1 \\ u \neq y}} (d_e^{G_1}(u)+d_e^{G_1}(v))(n_v^{G_1}(e)) \\
&\quad + \sum_{\substack{uv\in E_1 \\ u = y}} (d_e^{G_2}(z))(n_u^{G_1}(e))(n_v^{G_1}(e)) \\
&\quad + (n_2-1)\sum_{\substack{uv\in E_1 \\ u = y}} (d_e^{G_1}(u)+d_e^{G_1}(v)+d_e^{G_2}(z))(n_v^{G_1}(e))
\end{aligned}
$$

$$+ \quad Sz_w(G_2) + (n_1 - 1) \sum_{\substack{uv \in E_2 \\ u \neq z}} (d_e^{G_2}(u) + d_e^{G_2}(v))(n_v^{G_2}(e))$$

$$+ \quad \sum_{\substack{uv \in E_2 \\ u = z}} (d_e^{G_1}(y))(n_u^{G_2}(e))(n_v^{G_2}(e))$$

$$+(n_1 - 1) \sum_{\substack{uv \in E_2 \\ u = z}} (d_e^{G_2}(u) + d_e^{G_2}(v) + d_e^{G_1}(y))(n_v^{G_2}(e)).$$

□

Using the Theorem 2.1, we have the following examples.

EXAMPLE 2.2. *For the cycles, we have,* $Sz_w(S(C_n, C_m, y, z))$

$$= \begin{cases} n(n+1)(n-2) + m(m+1)(m-2) + 2nm(m+n+2), \\ \text{ if } n \text{ is even } m \text{ is even,} \\ n^2(n+1) + (m-1)(m+1)(2n+m-3) + 2n(m-1)(n+1), \\ \text{ if } n \text{ is even } m \text{ is odd,} \\ (n-1)^2(n+1) + (m-1)^2(m+1) + 2(n-1)(m-1)(m+n+2), \\ \text{ if } n \text{ is odd } m \text{ is odd.} \end{cases}.$$

EXAMPLE 2.3. *For the cycle and path, we have,*$Sz_w(S(C_n, P_m, y, z))$

$$= \begin{cases} n^3 + \frac{n^2}{2} + (n-1)(2m^2 - m - 2) + (m-1)(2n^2 + n + 2) \\ + \frac{2(m-1)(m^2+m-3)}{3}, \text{ if } n \text{ is even }, \\ \frac{(n-1)^2(2n+1)}{2} + \frac{2m(m-1)(m+1)}{3} + 2(n-1)(n+3)(m-1) \\ +(n-1)(2m^2 - 6m + 3), \text{ if } n \text{ is odd }. \end{cases}$$

A *broom* $T$ is a tree which is union of the path and the star, plus one edge joining the center of the star to an endpoint of the path. Clearly, $T \cong S(K_{1,n}, P_m, y, z)$.

EXAMPLE 2.4. *For broom* $T$, $Sz_w(T) = n^2(n+2) + 2n(n+3)(m-1) + n(2m^2 - 6m + 3) + \frac{2(m-1)(m^2+m-3)}{3}$.

## 3. Weighted Szeged index of link of graphs

The *link* $L = L(G_1, G_2, y, z)$ of $G_1$ and $G_2$ by vertices $y$ and $z$ is defined as the graph, obtained by joining $y$ and $z$ by an edge in the union of $G_1$ and $G_2$ (see Fig. 2). In this section, we obtain the weighted Szeged index of link of the given two graphs.

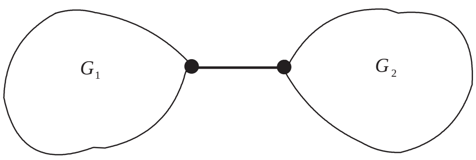


Fig. 2 Link graph $L(G_1, G_2, y, z)$

The proof of the following lemma is follows from the structure of link of two connected graphs.

Lemma 3.1. *Let $G_i$ be the graphs with $n_i$ vertices and $|E_i|$ edges, $i = 1, 2$, then for the link graph $L$ we have the following. For $e = uv \in E_1$.*
*(i) If $y \in N_u^{G_1}(e)$ and $u \neq y$, then*

$$\begin{aligned} n_u^L(e) &= n_u^{G_1}(e) + n_2 \quad , \quad d_L(u) = d_{G_1}(u) \\ n_v^L(e) &= n_v^{G_1}(e) \quad , \quad d_L(v) = d_{G_1}(v) \end{aligned}$$

*(ii) If $y \in N_u^{G_1}(e)$ and $u = y$, then*

$$\begin{aligned} n_u^L(e) &= n_u^{G_1}(e) + n_2 \quad , \quad d_L(u) = d_{G_1}(u) + 1 \\ n_v^L(e) &= n_v^{G_1}(e) \quad , \quad d_L(v) = d_{G_1}(v) \end{aligned}$$

*(iii) If $d_{G_1}(y, u) = d_{G_1}(y, v)$, then*

$$\begin{aligned} n_u^L(e) &= n_u^{G_1}(e) \quad , \quad d_L(u) = d_{G_1}(u) \\ n_v^L(e) &= n_v^{G_1}(e) \quad , \quad d_L(v) = d_{G_1}(v) \end{aligned}$$

*Analogous relations hold if $e = uv \in E_2$.*

Theorem 3.1. *Let $G_i$ be the graphs with $n_i$ vertices, $i = 1, 2$ then the weighted Szeged index of the link of $G_1$ and $G_2$ graphs is*
$Sz_w(L(G_1, G_2, y, z)) = Sz_w(G_1) + n_2 \sum\limits_{\substack{uv \in E_1 \\ u \neq y}} (d_e^{G_1}(u) + d_e^{G_1}(v))(n_v^{G_1}(e))$
$+ n_2 \sum\limits_{\substack{uv \in E_1 \\ u = y}} (d_e^{G_1}(u) + d_e^{G_1}(v))(n_v^{G_1}(e)) + \sum\limits_{\substack{uv \in E_1 \\ u = y}} (n_u^{G_1}(e) n_v^{G_1}(e) + n_2 n_v^{G_1}(e)) + Sz_w(G_2) +$
$n_1 \sum\limits_{\substack{uv \in E_2 \\ u \neq z}} (d_e^{G_2}(u) + d_e^{G_2}(v))(n_v^{G_2}(e)) + n_1 \sum\limits_{\substack{uv \in E_2 \\ u = z}} (d_e^{G_2}(u) + d_e^{G_2}(v))(n_v^{G_2}(e))$
$+ \sum\limits_{\substack{uv \in E_2 \\ u = z}} (n_u^{G_2}(e) n_v^{G_2}(e) + n_1 n_v^{G_2}(e)) + (d_l^{G_1}(y) + d_l^{G_2}(z) + 2) n_1 n_2.$

Proof. For a link $L = L(G_1, G_2, y, z)$, of $G_1$ and $G_2$ graphs, let the edge set can be partitioned as $E_1 = E(G_1), E_2 = E(G_2)$ and the link edge $yz$, then by the definition of $Sz_w$ $Sz_w(L) = \sum\limits_{uv \in E(L)} (d_L(u) + d_L(v)) n_u^L(e) n_v^L(e)$ By Lemma 3.1, we have

$$\begin{aligned} Sz_w(L) &= \sum_{uv \in E_1} (d_{G_1}(u) + d_{G_1}(v)) n_u^{G_1}(e) n_v^{G_1}(e) \\ &\quad + \sum_{uv \in E_2} (d_{G_2}(u) + d_{G_2}(v)) n_u^{G_2}(e) n_v^{G_2}(e) \\ &\quad + (d_L(y) + d_L(z)) n_1 n_2 \\ &= \sum_{\substack{uv \in E_1 \\ u \neq y}} (d_e^{G_1}(u) + d_e^{G_1}(v))(n_u^{G_1}(e) + n_2)(n_v^{G_1}(e)) \\ &\quad + \sum_{\substack{uv \in E_1 \\ u = y}} (d_e^{G_1}(y) + 1 + d_e^{G_1}(v))(n_u^{G_1}(e) + n_2)(n_v^{G_1}(e)) \end{aligned}$$

$$
\begin{aligned}
&+ \sum_{\substack{uv\in E_1\\ d_{G_1}(y,u) \;=\; d_{G_1}(y,v)}} (d_e^{G_1}(u)+d_e^{G_1}(v))(n_u^{G_1}(e))(n_v^{G_1}(e))\\
&+ \sum_{\substack{uv\in E_2\\ u \;\neq\; z}} (d_e^{G_2}(u)+d_e^{G_2}(v))(n_u^{G_2}(e)+n_1)(n_v^{G_2}(e))\\
&+ \sum_{\substack{uv\in E_2\\ u \;=\; z}} (d_e^{G_2}(z)+1+d_e^{G_2}(v))(n_u^{G_2}(e)+n_1)(n_v^{G_2}(e))\\
&+ \sum_{\substack{uv\in E_2\\ d_{G_2}(z,u) \;=\; d_{G_2}(z,v)}} (d_e^{G_2}(u)+d_e^{G_2}(v))(n_u^{G_2}(e))(n_v^{G_2}(e))\\
&+(d_l^{G_1}(y)+1+d_l^{G_2}(z)+1)n_1n_2\\
= & \sum_{\substack{uv\in E_1\\ u \;\neq\; y}} (d_e^{G_1}(u)+d_e^{G_1}(v))(n_u^{G_1}(e))(n_v^{G_1}(e))\\
&+ \sum_{\substack{uv\in E_1\\ u \;=\; y}} (d_e^{G_1}(u)+d_e^{G_1}(v))(n_u^{G_1}(e))(n_v^{G_1}(e))\\
&+ \sum_{\substack{uv\in E_1\\ d_{G_1}(y,u) \;=\; d_{G_1}(y,v)}} (d_e^{G_1}(u)+d_e^{G_1}(v))(n_u^{G_1}(e))(n_v^{G_1}(e))\\
&+(n_2) \sum_{\substack{uv\in E_1\\ u \;\neq\; y}} (d_e^{G_1}(u)+d_e^{G_1}(v))(n_v^{G_1}(e))\\
&+(n_2) \sum_{\substack{uv\in E_1\\ u \;=\; y}} (d_e^{G_1}(u)+d_e^{G_1}(v))(n_v^{G_1}(e))\\
&+ \sum_{\substack{uv\in E_1\\ u \;=\; y}} (n_u^{G_1}(e)n_v^{G_1}(e)+n_2n_v^{G_1}(e))\\
&+ \sum_{\substack{uv\in E_2\\ u \;\neq\; z}} (d_e^{G_2}(u)+d_e^{G_2}(v))(n_u^{G_2}(e))(n_v^{G_2}(e))\\
&+ \sum_{\substack{uv\in E_2\\ u \;=\; z}} (d_e^{G_2}(u)+d_e^{G_2}(v))(n_u^{G_2}(e))(n_v^{G_2}(e))\\
&+ \sum_{\substack{uv\in E_2\\ d_{G_2}(z,u) \;=\; d_{G_2}(z,v)}} (d_e^{G_2}(u)+d_e^{G_2}(v))(n_u^{G_2}(e))(n_v^{G_2}(e))\\
&+(n_1) \sum_{\substack{uv\in E_2\\ u \;\neq\; z}} (d_e^{G_2}(u)+d_e^{G_2}(v))(n_v^{G_2}(e))\\
&+(n_1) \sum_{\substack{uv\in E_2\\ u \;=\; z}} (d_e^{G_2}(u)+d_e^{G_2}(v))(n_v^{G_2}(e))\\
&+ \sum_{\substack{uv\in E_2\\ u \;=\; z}} (n_u^{G_2}(e)n_v^{G_2}(e)+n_1n_v^{G_2}(e))+(d_l^{G_1}(y)+1+d_l^{G_2}(z)+1)n_1n_2.
\end{aligned}
$$

By the definitions of $Sz_w$, for $G_1$ and $G_2$ we have

$$\begin{aligned} Sz_w(L) &= Sz_w(G_1) + (n_2) \sum_{\substack{uv \in E_1 \\ u \neq y}} (d_e^{G_1}(u) + d_e^{G_1}(v))(n_v^{G_1}(e)) \\ & +(n_2) \sum_{\substack{uv \in E_1 \\ u = y}} (d_e^{G_1}(u) + d_e^{G_1}(v))(n_v^{G_1}(e)) \\ & + \sum_{\substack{uv \in E_1 \\ u = y}} (n_u^{G_1}(e) n_v^{G_1}(e) + n_2 n_v^{G_1}(e)) \\ & +Sz_w(G_2) + (n_1) \sum_{\substack{uv \in E_2 \\ u \neq z}} (d_e^{G_2}(u) + d_e^{G_2}(v))(n_v^{G_2}(e)) \\ & +(n_1) \sum_{\substack{uv \in E_2 \\ u = z}} (d_e^{G_2}(u) + d_e^{G_2}(v))(n_v^{G_2}(e)) \\ & + \sum_{\substack{uv \in E_2 \\ u = z}} (n_u^{G_2}(e) n_v^{G_2}(e) + n_1 n_v^{G_2}(e)) \\ & +(d_l^{G_1}(y) + d_l^{G_2}(z) + 2) n_1 n_2. \end{aligned}$$

□

A *double broom* $T_1$ is a tree consisting of two stars, whose centers are joined by a path. Clearly, $T_1 \cong L(K_{1,n}, K_{1,m}, y, z)$, thus by using Theorem 3.1, we have the following example.

EXAMPLE 3.2. *For a double broom* $T_1$, $Sz_w(T_1) = (n+m+1)(2(n+1)(m+1) + n + m) + n^2(n+1) + m^2(m+1)$.

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