

WEAK SUB SEQUENTIAL CONTINUOUS MAPS IN NON ARCHIMEDEAN MENGER PM SPACE

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ABSTRACT. In this paper , we established some common fixed point theorems for two pairs of self maps by using the more weaker notion of weak sub sequential continuity (wsc) with compatibility of type (E) in non Archimedean Menger PM space. We improve some earlier results in this line.

1. Introduction

Last more than half a century saw a tremendous growth in the field of fixed point theory and its applications to study the existence and establishment of common fixed point for different metric structure spaces especially where the probabilistic situations arises such as probabilistic metric spaces. It plays a very important role where the distance between the two points are unknown but the probabilities of the possible values of the distance are known. Menger [21] introduced the notion of probabilistic metric space (briefly PM space) as generalization of metric space. In his work he emphasized on the use of distribution function in lieu of non negative real numbers as values of metric. Schweizer and Sklar [24, 26] stimulated the study further with their pioneering article on statistical metric spaces. Working on the same line, Sehgal and Bharucha-Reid [25] studied some fixed points of contraction mappings on probabilistic metric spaces. Istratescu and Crivat [18] introduced the notion of non-Archimedean PM-space and gave some basic topological preliminaries on it. Further, Istratescu [16, 17] generalized the results of Sehgal and Bharucha [25] to NA Menger PM space where as Achari [1] generalized the results of Istratescu [16, 17] by establishing common fixed point theorems for qausi-contraction type of mappings in non-Archimedean PM - space. Bouhadjera et. al. [2] proved Common fixed point theorems for pairs of sub compatible maps.

2010 *Mathematics Subject Classification.* 47H10; 54H25.

Key words and phrases. fixed point, non Archimedean Menger PM-Space, compatible maps of type (E), weak sub sequential continuous maps (wsc).

Recently, Beloul [3] established some fixed point theorems for two pairs of self mappings satisfying contractive conditions by using the weak sub-sequential mappings with compatibility of type (E). In the present paper, we introduce the concept of weak sub-sequential mappings with compatibility of type (E) in N.A. Menger P.M. space. For more details, we refer to [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [18], [19], [20], [22], [23], [27], [28], [29], [30], [31], [32] and [33].

2. Preliminaries

DEFINITION 2.1. ([16], [18]) Let X be any nonempty set and D be the set of all left-continuous distribution functions. An ordered pair (X, F) is called a non-Archimedean probabilistic metric space (briefly, a N.A. PM-space) if F is a mapping from $X \times X$ into mapping D satisfying the following conditions (we shall denote the distribution function $F(x, y)$ by $F_{x,y}, \forall x, y \in X$):

$$(2.1) \quad F_{x,y}(t) = 1, \forall t > 0 \text{ if and only if } x = y$$

$$(2.2) \quad F_{x,y}(0) = 0, \forall x, y \in X$$

$$(2.3) \quad F_{x,y}(t) = F_{y,x}(t), \forall x, y \in X$$

$$(2.4) \quad F_{x,y}(t_1) = 1 \text{ and } F_{y,z}(t_2) = 1, \text{ then } F_{x,z}\{\max(t_1, t_2)\} = 1, \forall x, y, z \in X$$

DEFINITION 2.2. ([21]) A t - norm is a function $\Delta : [0, 1] \times [0, 1] \rightarrow [0, 1]$ which is associative, commutative, non-decreasing in each coordinate and $\Delta(a, 1) = a, \forall a \in [0, 1]$.

DEFINITION 2.3. ([19]) A N.A. Menger PM-space is an ordered triplet (X, F, Δ) , where Δ is a t - norm and (X, F) is a non-Archimedean PM-space satisfying the following condition:

$$(2.5) \quad F_{(x,z)}(\max\{t_1, t_2\}) \geq \Delta(F_{(x,y)}(t_1), F_{(y,z)}(t_2)), \forall x, y, z \in X \text{ and } t_1, t_2 \geq 0.$$

For more details we refer to [18]

DEFINITION 2.4. ([7], [12]) A N.A. Menger PM-space (X, F, Δ) , is said to be of type $(C)_g$ if there exists a $g \in \Omega$ such that $g(F_{(x,z)}(t)) \leq g(F_{(x,y)}(t)) + g(F_{(y,z)}(t)), \forall x, y, z \in X$ and $t \geq 0$, where $\Omega = \{g|g : [0, 1] \rightarrow [0, \infty)$ is continuous, strictly decreasing with $g(1) = 0$ and $g(0) < \infty\}$.

DEFINITION 2.5. ([7], [12]) A N.A. Menger PM-space (X, F, Δ) is said to be of type $(D)_g$ if there exists a $g \in \Omega$ such that $g(\Delta(s, t)) \leq g(s) + g(t)$ for all $s, t \in (0, 1)$.

REMARK 2.1. ([12])

(2.6)

A N.A. Menger PM - space (X, F, Δ) is of type $(D)_g$, then it is of type $(C)_g$.

(2.7) *If (X, F, Δ) is a N.A. Menger PM - space and $\Delta \geq \Delta_m$, where $\Delta_m(s, t) = \max\{s + t - 1, 1\}$, then (X, F, Δ) is of type $(D)_g$ for $g \in \Omega$ defined by $g(t) = 1 - t$.*

Throughout this paper, let (X, F, Δ) be a complete N.A. Menger PM-space of type $(D)_g$ with a continuous strictly increasing t - norm Δ .

Let $\phi : [0, \infty) \rightarrow [0, \infty)$ be a function satisfying the following condition (Φ) :

(Φ) ϕ is a upper semi continuous from the right and $\phi(t) < t$ for all $t > 0$.

DEFINITION 2.6. ([7], [12]) A sequence $\{x_n\}$ in a N.A. Menger PM space (X, F, Δ) converges to a point x if and only if for each $\epsilon > 0, \lambda > 0$ there exists an integer $M(\epsilon, \lambda)$ such that $g(F(x_n, x; \epsilon)) < g(1 - \lambda)$ for all $n > M$.

DEFINITION 2.7. ([7], [12]) A sequence $\{x_n\}$ in a N.A. Menger PM space is a Cauchy sequence if and only if for each $\epsilon > 0, \lambda > 0$ there exists an integer $M(\epsilon, \lambda)$ such that $g(F(x_n, x_{n+p}; \epsilon)) < g(1 - \lambda)$ for all $n > M$ and $p \geq 1$.

LEMMA 2.1. ([12]) If a function $\phi : [0, \infty) \rightarrow [0, \infty)$ satisfies the condition (Φ) , then we have

(2.8) For all $t \geq 0, \lim_{n \rightarrow \infty} \phi^n(t) = 0$, where $\phi^n(t)$ is the n th iteration of t .

(2.9) If $\{t_n\}$ is a non - decreasing sequence of real numbers and $\{t_{n+1}\} \leq \phi(t_n)$, $n = 1, 2, \dots$, then $\lim_{n \rightarrow \infty} t_n = 0$. In particular, if $t \leq \phi(t)$ for all $t \geq 0$, then $t = 0$. Singh et al. [32, 33] introduced the notion of compatibility of type (E), in the setting of the N.A. Menger PM spaces, it becomes.

DEFINITION 2.8. Two self maps A and S on a N. A. Menger PM space (X, F, Δ) are said to be compatible of type (E), if $\lim_{n \rightarrow \infty} S^2 x_n = \lim_{n \rightarrow \infty} S A x_n = A z$ and $\lim_{n \rightarrow \infty} A^2 x_n = \lim_{n \rightarrow \infty} A S x_n = S z$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} S x_n = z$ for some $z \in X$.

DEFINITION 2.9. Two self maps A and S on a N. A. Menger PM space (X, F, Δ) are said to be A -compatible of type (E), if $\lim_{n \rightarrow \infty} A^2 x_n = \lim_{n \rightarrow \infty} A S x_n = S z$ for some $z \in X$. Pair A and S are said to be S -compatible of type (E), if $\lim_{n \rightarrow \infty} S^2 x_n = \lim_{n \rightarrow \infty} S A x_n = A z$ for some $z \in X$.

REMARK 2.2. It is also interesting to see that if A and S are compatible of type (E), then they are A -Compatible and S -Compatible of type (E), but the converse is not true (see example 1 in [3]).

Bouhadjera and Godet Thobie [2] introduced the concept of sub-sequential continuity as follows:

DEFINITION 2.10. Two self maps A and S of a metric space (X, d) is said to be sub-sequentially continuous, if there exists a sequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} S x_n = t$, for some $t \in X$ and $\lim_{n \rightarrow \infty} A S x_n = A t$, or $\lim_{n \rightarrow \infty} S A x_n = S t$.

Motivated by the definition (2.10) and [3], we define the following.

DEFINITION 2.11. The pair $\{A, S\}$ defined on a N.A. Menger PM space (X, F, Δ) is said to be weakly sub-sequentially continuous (in short wsc), if there exists a sequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} A x_n = \lim_{n \rightarrow \infty} S x_n = z$, for some $z \in X$ and $\lim_{n \rightarrow \infty} A S x_n = A z$, or $\lim_{n \rightarrow \infty} S A x_n = S z$

DEFINITION 2.12. The pair $\{A, S\}$ defined on a N.A. Menger PM space (X, F, Δ) is said to be S sub-sequentially continuous, if there exists a sequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$, for some $z \in X$ and $\lim_{n \rightarrow \infty} SAx_n = Sz$.

DEFINITION 2.13. The pair $\{A, S\}$ defined on a N.A. Menger PM space (X, F, Δ) is said to be A sub-sequentially continuous, if there exists a sequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$, for some $z \in X$ and $\lim_{n \rightarrow \infty} ASx_n = Az$.

REMARK 2.3. If the pair $\{A, S\}$ is A -subsequentially continuous (or S -subsequentially continuous), then it is wsc. (See example 3 in [3])

3. Main Results

THEOREM 3.1. Let A, B, S and T be four self maps of a N. A. Menger PM-space (X, F, Δ) such that for all $x, y \in X$ and $t > 0$, we have:

$$(3.1) \quad g(F(Ax, By, t)) \leq \phi[\max\{g(F(Sx, Ty, t)), g(F(Ax, Sx, t)), g(F(By, Ty, t)), g(F(Sx, By, t)), g(F(Ty, Ax, t))\}].$$

where $\phi \in \Phi$ such that $\phi : [0, \infty) \rightarrow [0, \infty)$. If the pairs $\{A, S\}$ and $\{B, T\}$ are weakly sub sequentially continuous and compatible of type (E), then A, B, S and T have a unique common fixed point in X .

PROOF. Since the pair $\{A, S\}$ is wsc (Suppose that it is A -sub-sequentially continuous) and compatible of type (E), therefore there exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Sx_n = z$, for some $z \in X$ and $\lim_{n \rightarrow \infty} ASx_n = Az$. The compatibility of type (E) implies that

$$\lim_{n \rightarrow \infty} A^2x_n = \lim_{n \rightarrow \infty} ASx_n = Sz$$

and

$$\lim_{n \rightarrow \infty} S^2x_n = \lim_{n \rightarrow \infty} SAx_n = Az.$$

Therefore $Az = Sz$, whereas in respect of the pair $\{B, S\}$ (Suppose that it is B -sub-sequentially continuous), there exists a sequence $\{y_n\}$ in X such that $\lim_{n \rightarrow \infty} By_n = \lim_{n \rightarrow \infty} Ty_n = w$, for some $w \in X$ and $\lim_{n \rightarrow \infty} BTy_n = Bw$. The pair $\{B, T\}$ is compatible of type (E), then so $\lim_{n \rightarrow \infty} B^2y_n = \lim_{n \rightarrow \infty} BTy_n = Tw$ and $\lim_{n \rightarrow \infty} T^2y_n = \lim_{n \rightarrow \infty} TBy_n = Bw$, for some $w \in X$, then $Bw = Tw$. Hence z is a coincidence point of the pair $\{A, S\}$ whereas w is a coincidence point of the pair $\{B, T\}$. Now we prove that $z = w$. By putting $x = x_n$ and $y = y_n$ in inequality (3.1.), we have

$$g(F(Ax_n, By_n, t)) \leq \phi[\max\{g(F(Sx_n, Ty_n, t)), g(F(Ax_n, Sx_n, t)), g(F(By_n, Ty_n, t)), g(F(Sx_n, By_n, t)), g(F(Ty_n, Ax_n, t))\}].$$

Taking the limit as $n \rightarrow \infty$, we get

$$g(F(z, w, t)) \leq \phi[\max\{g(F(z, w, t)), g(F(z, z, t)), g(F(w, w, t)), g(F(z, w, t)), g(F(w, z, t))\}]$$

and

$$g(F(z, w, t)) \leq \phi[\max\{g(F(z, w, t)), 0, 0, g(F(z, w, t)), g(F(w, z, t))\}]$$

i.e.

$$g(F(z, w, t)) \leq \phi[g(F(z, w, t))] < g(F(z, w, t)),$$

a contradiction. Thus, we have $z = w$. Now we prove that $Az = z$. By putting $x = z$ and $y = y_n$ in the inequality (3.1.), we get

$$\begin{aligned} g(F(Az, By_n, t)) &\leq \\ \phi[\max\{g(F(Sz, Ty_n, t)), g(F(Az, Sz, t)), g(F(By_n, Ty_n, t)), \\ &g(F(Sz, By_n, t)), g(F(Ty_n, Az, t))\}]. \end{aligned}$$

Taking the limit as $n \rightarrow \infty$, we get

$$\begin{aligned} g(F(Az, w, t)) &\leq \\ \phi[\max\{g(F(Sz, w, t)), g(F(Az, Sz, t)), g(F(w, w, t)), \\ &g(F(Sz, w, t)), g(F(w, Az, t))\}] \end{aligned}$$

i.e.

$$g(F(Az, w, t)) \leq \phi[\max\{g(F(Sz, w, t)), 0, 0, g(F(Sz, w, t)), g(F(w, Az, t))\}]$$

and

$$g(F(Az, w, t)) \leq \phi[g(F(Az, w, t))] < g(F(Az, w, t))$$

which yields $Az = w$. Since $Az = Sz$. Therefore $Az = Sz = w = z$.

Now we prove that $Bz = z$. By putting $x = \{x_n\}$ and $y = z$ in the inequality (3.1.), we get

$$\begin{aligned} g(F(Ax_n, Bz, t)) &\leq \\ \phi[\max\{g(F(Sx_n, Tz, t)), g(F(Ax_n, Sx_n, t)), g(F(Bz, Tz, t)), \\ &g(F(Sx_n, Bz, t)), g(F(Tz, Ax_n, t))\}]. \end{aligned}$$

Taking the limit as $n \rightarrow \infty$, we get

$$\begin{aligned} g(F(z, Bz, t)) &\leq \\ \phi[\max\{g(F(z, Tz, t)), g(F(z, z, t)), g(F(Bz, Tz, t)), g(F(z, Bz, t)), \\ &g(F(Tz, z, t))\}] \end{aligned}$$

i.e.

$$g(F(z, Bz, t)) \leq \phi[\max\{g(F(z, Tz, t)), 0, 0, g(F(z, Bz, t)), g(F(Tz, z, t))\}]$$

and

$$g(F(z, Bz, t)) \leq \phi[g(F(z, Bz, t))] < g(F(z, Bz, t)),$$

which yields $Bz = z$. Since $Bz = Tz$. Therefore, $Bz = Tz = z$. Therefore in all $z = Az = Bz = Sz = Tz$, i.e. z is a common fixed point of A, B, S and T . The uniqueness of common fixed point is an easy consequence of inequality (3.1). \square

If we put $A = B$ in Theorem 3.1 we have the following corollary for three mappings:

COROLLARY 3.1. *Let A, S and T be three self maps of a N. A. Menger PM-space (X, F, Δ) such that for all $x, y \in X$ and $t > 0$, we have:*

$$(3.2) \quad g(F(Ax, Ay, t)) \leq \phi[\max\{g(F(Sx, Ty, t)), g(F(Ax, Sx, t)), g(F(Ay, Ty, t)), g(F(Sx, Ay, t)), g(F(Ty, Ax, t))\}]$$

where $\phi \in \Phi$ such that $\phi : [0, \infty) \rightarrow [0, \infty)$. If the pairs $\{A, S\}$ and $\{A, T\}$ are weakly sub sequentially continuous and compatible of type (E), then A, S and T have a unique common fixed point in X .

Alternatively, if we set $S = T$ in Theorem 3.1, we'll have the following corollary for three self mappings:

COROLLARY 3.2. *Let A, B and S be three self maps of a N. A. Menger PM-space (X, F, Δ) such that for all $x, y \in X$ and $t > 0$, we have:*

$$(3.3) \quad g(F(Ax, By, t)) \leq \phi[\max\{g(F(Sx, Sy, t)), g(F(Ax, Sx, t)), g(F(By, Sy, t)), g(F(Sx, By, t)), g(F(Sy, Ax, t))\}]$$

where $\phi \in \Phi$ such that $\phi : [0, \infty) \rightarrow [0, \infty)$. If the pairs $\{A, S\}$ and $\{B, S\}$ are weakly sub sequentially continuous and compatible of type (E), then A, B and S have a unique common fixed point in X .

If we put $S = T$ in corollary 3.1, we have the following result for two self mappings:

COROLLARY 3.3. *Let A and S be two self maps of a N. A. Menger PM-space (X, F, Δ) such that for all $x, y \in X$ and $t > 0$, we have:*

$$(3.4) \quad g(F(Ax, Ay, t)) \leq \phi[\max\{g(F(Sx, Sy, t)), g(F(Ax, Sx, t)), g(F(Ay, Sy, t)), g(F(Sx, Ay, t)), g(F(Sy, Ax, t))\}]$$

where $\phi \in \Phi$ such that $\phi : [0, \infty) \rightarrow [0, \infty)$. If the pair $\{A, S\}$ is weakly sub sequentially continuous and compatible of type (E), then A and S have a unique common fixed point in X .

THEOREM 3.2. *Let A, B, S and T be four self maps of a N. A. Menger PM-space (X, F, Δ) satisfying (3.1.). Assume that*

- (i) *the pair $\{A, S\}$ is A -compatible of type (E) and A -sub sequentially continuous.*
 - (ii) *the pair $\{B, T\}$ is B -compatible of type (E) and B -sub sequentially continuous.*
- Then A, B, S and T have a unique common fixed point in X .*

PROOF. The proof is obvious as on the lines of theorem 3.1. □

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Received by editors 01.02.2016; Available online 26.09.2016.

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