# INTEGRITY OF FUZZY GRAPHS 

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#### Abstract

In this paper, integrity of fuzzy graph is defined. Integrity of special types of graphs with constant node strength and node strength sequences as $\left(p_{1}^{n-1}, p_{2}\right)$ is discussed. Also, integrity of regular fuzzy graph and complete fuzzy graph is presented.


## 1. INTRODUCTION

Graph theory has wide range of applications in the field of computer networks, chemical structures, biological models, and real life problems. Connectivity plays a vital role in all these models. The connectivity parameter only discusses the number of sub graphs. If a graph is designed for a communication network, then breakage is not the only problem, still there is need to know how many sub networks are working, how many are not working, what is the largest working group, and what are the vulnerabilities that break the network and so on. So the connectivity parameter is not enough to measure the efficiency of a graph in the real life problems. In an undirected graph, the connectivity, the number of components, order of the components are inter related to each other. Using these parameters other vulnerability measures, such as integrity, toughness, rupture degree, tenacity and more, are defined. These measures give the amount of damage caused by the vulnerabilities. Integrity concepts in graph theory are introduced by Barefoot, Entringer, Swart, K.S. Bagga $[\mathbf{1 , 2 , 3}]$.

In general, all the nodes and edges do not have equal importance, especially in practical applications. The network processing units represented by nodes need not have equal capacity similarly, the links represented by edges need not have equal capacity. The capacity of processing units and its links may be independent, but in

[^0]certain cases, they are dependent on each other. In such cases, vulnerabilities are needed for weighted graphs. The capacity of processing units and links are noted as weights of nodes and edges. If the weighted values lies between $[0,1]$ then given graph turns to fuzzy graph whose nodes are processing units and edges are links between processing units and capacities as membership values.

If the target is to break a network, it is not easy to break all processing units. So it is necessary to break minimum number of nodes and it results in reducing the working group to a minimum. This measure is calculated by integrity. A good service provider wants to give a big integrity to the network system. If all the processing units and links do not have equal importance the working model turns to fuzzy graph whose membership value is its percentage of importance. To calculate the vulnerability of these models, vulnerable parameters such as integrity, toughness, tenacity, rupture, scattering number and binding number for fuzzy graphs are needed.

In 1975, Rosenfeld [4] introduced the concept of fuzzy graphs from the fuzzy relations, defined and studied by L.A.Zadeh in 1965 [5]. Basic notations and definitions are introduced by Rosenfeld [4] and developed by Bhattacharya and Bhutani $[6,7]$. In 2008, Nagoor Gani and Radha [8] introduced and studied the properties of regular fuzzy graphs, total degree and totally regular fuzzy graphs. In this paper, we have defined integrity of fuzzy graphs and derived the integrity of some special type of fuzzy graphs.

## 2. PRELIMINARIES

2.1. Basic definitions in graph theory. A graph $G$ is a pair of sets $(V, E)$, where $V$ is the set of vertices and $E$ is the set of edges, formed by pairs of vertices. A path of length $n$ is a sequence of distinct nodes $u_{0}, u_{1}, u_{2}, \cdots, u_{n}$. A graph is connected if every pair of vertices is joined by a path. Closed path is known as cycle. A cut vertex or a cut point of a graph is one whose removal increase the number of components. An edge whose removal increases the number of components is called bridge. A connected acyclic graph is tree. A graph with no parallel edges and loops is simple graph. A simple graph that contains every possible edge between all the vertices is called a complete graph. A complete graph with $n$ vertices is denoted by $K_{n}$. A Bipartite graph is a graph whose vertex set $V$ can be divided in to two sets, whose union is $V$ and their intersection is empty. Let $S$ be any sub set of $V$. The number of vertices in $S$ is known as cardinality of $S$ and it denoted as $|\mathrm{S}|$. The degree of the vertex $v$, written as $d(v)$, is the number of edges incident with $v$. The minimum degree of $G$ is defined as $\min \{d(v) / v \in V\}$ and it denoted as $\delta(G)$. The maximum degree of $G$ is defined as $\max \{d(v) / v \in V\}$ and it denoted by $\Delta(G)$. The number of vertices in a graph is known as order of the graph and the number of edges is known as size of the graph.

Integrity of graphs was introduced by Barefoot, Entringer and Swart [1, 2] as a measure of the vulnerability of graphs. This parameter is used to measure the
stability of a graph, the number of vertices not in link and the size of the largest sub network is still in connection. The definition is given as follows: Integrity of a graph $G=(V, E)$ is, $I(G)=\min \{|S|+m(G-S) / S \in V\}$, where $m(G-S)$ denotes the order of the largest component in the graph $G-S$. An I-set of $G$ is any (strict) subset $S$ of $V(G)$ for which $I(G)=|S|+m(G-S)$.

Theorem 2.1. The integrity of
(1) the complete graph $K_{n}$ is $n$
(2) the null graph $\overline{K_{n}}$ is 1
(3) the star $K_{1, n}$ is 2
(4) the path $P_{n}$ is $\lceil 2 \sqrt{n+1}\rceil-2$
(5) the cycle $C_{n}$ is $\lceil 2 \sqrt{n}\rceil-1$
(6) the complete bipartite graph $K_{m, n}$ is $1+\min \{m, n\}$
(7) any complete multipartite graph of order $p$ and largest partite set of order $r$ is $p-r+1$.
2.2. Integrity of Weighted Graphs. Sibabrata Ray, Jitender S Deogun, [9] introduced the concept of integrity in weighted graphs. They discussed the NPCompleteness of integrity of weighted graphs. They defined the weighted integrity as follows: The integrity of a weighted graph $G=(V, E)$ is $\mathrm{I} \omega(\mathrm{G})=\min _{S \subset V}\{\omega(\mathrm{~S})$ $\left.+m_{\omega}(G-S) / S \subseteq V\right\}$ where $\omega: V \rightarrow \mathbb{R} \geqslant 0$ is vertex weight function, $S \subseteq V$. $m_{\omega}(\mathrm{G}-\mathrm{S})$ is the maximum sum of the vertex weights of the connected components $(G-S)$ with $\omega(S)=\Sigma_{v \in S} \omega(v)$.

For fuzzy graph, the weight function in section 2.2 is restricted to the interval $(0,1]$ instead of $\mathbb{R}$, the set of real numbers.
2.3. Fuzzy Graphs. A fuzzy graph $G$ is a pair of functions $G=(\sigma, \mu)$ where $\sigma$ is a fuzzy subset of non empty vertex set $V$ and $\mu$ is a symmetric fuzzy relation on $\sigma$. The underlying crisp graph of the fuzzy graph $G:(\sigma, \mu)$ is denoted as $G^{*}=(V, E)$, where $E \subseteq V \times V$. The crisp graph $G^{*}=(V, E)$ is a special case of fuzzy graph $G$ with each vertex and edge having degree of membership 1. The order $p$ and size $q$ of a fuzzy graph $G=(\sigma, \mu)$ are defined to be $p=\Sigma_{x \in V} \sigma(x)$ and $q=\Sigma_{x y \in E} \mu(x y)$. The degree of a vertex $u$ is defined as $\sum_{u \neq v} \mu$ (uv) and denoted by $d_{G}(\mathrm{u})$ or simply $d(u)$. The total degree of a vertex $v \in V$ is defined as $d(u)+\sigma(u)$. An edge $\mu(x, y)$ is an effective edge if $\mu(x, y)=\sigma(x) \wedge \sigma(y)$. An arc of an fuzzy graph is called strong edge if its weight is at least as great as the connectedness of its end vertices when it is deleted. An arc is strong if and only if its weight is equal to the strength of connectedness of its end nodes. A fuzzy graph is said to be a strong fuzzy graph if all edges are effective edges. The minimum degree of $G$ is $\delta(G)=\min \{d(v) / v \in V\}$. The maximum degree of $G$ is $\Delta(G)=\max \{d(v) / v \in V\}$. The fuzzy cardinality of $S \subset V$ is defined to be sum of all $\sigma(v)$, for all $v \in S$. The strong degree of a vertex $v \in V$ is defined as the sum of membership values of all strong arcs incident at v and it is denoted by $d_{s}(v)$. The minimum strong degree of $G$ is $\delta_{s}(\mathrm{G})=\min \left\{d_{s}(v) / v \in V\right\}$. The maximum strong degree of $G$ is $\Delta_{s}(G)=\max \left\{d_{s}(v) / v \in V\right\}$.

A path $P_{n}$ in $G$ of length $n$ is a sequence of distinct nodes $u_{0}, u_{1}, u_{2}, \cdots, u_{n}$ such that $\sigma\left(u_{i-1}, u_{i}\right)>0,1 \leqslant i \leqslant n$. The number of edges in the path is called the length of the path and the degree of membership of a weakest arc is defined as its strength. A closed path is called a fuzzy cycle if it contains more than one weakest arc. A fuzzy graph is said to be regular fuzzy graph if all the vertex is of same degree. A complete fuzzy graph (CFG) is a fuzzy graph $G=(\sigma, \mu)$ such that $\mu(x, y)=\sigma(x) \wedge \sigma(y)$. The complement of a fuzzy graph $G=(\sigma, \mu)$ is $\bar{G}=(\sigma, \bar{\mu})$, where $\bar{\mu}=\sigma(x) \wedge \sigma(y)-\mu(x, y)$ for $x, y \in V$.

## 3. INTEGRITY OF FUZZY GRAPHS

For fuzzy graph, the weight function in Section 2.2 is restricted to the interval $(0,1]$ instead of $\mathbb{R}$, the set of real numbers. In this section, the definition of integrity of weighted graphs are modified to fit for fuzzy graphs.

Definition 3.1. Let $G=(\sigma, \mu)$ be a fuzzy graph. The integrity of $G$, denoted by $\widetilde{I}(G)$, is defined as $\widetilde{I}(G)=\min \{|S|+m(G-S) / S \in V\}$, where $|S|$ denotes the cardinality of $S, m(G-S)$ is maximum order of the component of $G-S$.

Definition 3.2. An $\widetilde{I}$-set of $G$ is any (strict) subset $S$ of $V(G)$ for which $\widetilde{I}(G)=\{|S|+m(G-S) / S \subset V\}$.

Example 3.1. Consider the fuzzy graph $G=(\sigma, \mu)$ with $\sigma^{*}=\{a, b, c, d\}$, with $\sigma(\mathrm{a})=0.3, \sigma(\mathrm{~b})=0.2, \sigma(\mathrm{c})=0.2, \sigma(\mathrm{~d})=0.5, \mu(\mathrm{a}, \mathrm{b})=\mu(\mathrm{a}, \mathrm{c})=\mu(\mathrm{b}, \mathrm{c})=\mu(\mathrm{a}$, $\mathrm{d})=\mu(\mathrm{c}, \mathrm{d})=1$. For this graph, $\widetilde{I}$-sets are $\{\mathrm{a}, \mathrm{c}\}$ and $\{\mathrm{b}, \mathrm{d}\}$ and $\widetilde{I}(\mathrm{G})=1$.

Theorem 3.1. Let $G=(\sigma, \mu)$ be a fuzzy graph with under lying graph $G^{*}$. If $\sigma$ is constant, then $\widetilde{I}$ - set of $G$ and $I$ - set of $G^{*}$ are same subset $S$ of $V(G)$.

Proof. Given that $\sigma$ is constant. The adjacent matrix for both $G$ and $G^{*}$ are same and then are isomorphic to each other. Also under lying graph $G^{*}$ is a special case of fuzzy graph $G$ with $\sigma=1$ and $\mu=1$. Therefore the I-set, $S$, of $G^{*}$, forms same sub graph in $G^{*}-S$ and $G-S$. This implies the maximum component in $G^{*}$ and $G$ are same. Since order and cardinality of $G$ depends only on vertex membership value and here it is constant, any individual vertex or any other component does have more order value other than this maximum component. Also $S$ have same number of vertex in $G^{*}$ and $G$. Hence the subset $S$ of $V$ is the $\widetilde{I}$ - set of $G$ and I - set of $G^{*}$.

Theorem 3.2. Let $G=(\sigma, \mu)$ be a fuzzy graph with the under lying graph $G^{*}$. If $\sigma$ is a constant then integrity $\widetilde{I}(G)$, for
(1) null graph is $\sigma$
(2) complete graph $K_{n}$ is $n \sigma$
(3) star $K_{1, n}$ is $2 \sigma$
(4) path $P_{n}$ of length $n$ is $(\lceil 2 \sqrt{n+1}\rceil$ - 2) $\sigma$
(5) cycle $C_{n}$ is ( $\left.\lceil 2 \sqrt{n}\rceil-1\right) \sigma$
(6) complete bipartite graph $K_{m, n}$ is $(1+\min \{m, n\}) \sigma$
(7) complete multipartite graph of order $p$ and largest partite set of order $r$ is $(p-r+1) \sigma$

From the theorem 3.1, any $\widetilde{I}$-set of $G$ and I-set of $G^{*}$ have same number of vertices. Therefore the number of vertices for each special graphs $G^{*}$, discussed in theorem 2.1, are same as for $G$ in the above theorem. Thus if $\sigma$ is constant then $\widetilde{I}(G)$ is multiple of $I\left(G^{*}\right)$, that is $\widetilde{I}(G)=\sigma I\left(G^{*}\right)$.

Definition $3.3([\mathbf{1 0}])$. Let $G=(\sigma, \mu)$ be a fuzzy graph with n vertices. The node-strength sequence ( $n-s$ sequence) of $G$ is ( $p_{1}, p_{2}, \cdots, p_{n}$ ) where $p_{i}, 0<p_{i} \leqslant 1$ is the strength of node $i$ when nodes are arranged so that their strengths are non decreasing. In particular, $p_{1}$ is the smallest node strength and $p_{n}$ is the largest node strength.

Theorem 3.3. Let $P_{n}$ be a fuzzy path having $n$ vertices with node strength sequence $\left(p_{1}^{n-1}, p_{2}\right)$. Then

$$
\widetilde{I}\left(P_{n}\right)= \begin{cases}p_{1}(\lceil 2 \sqrt{n+1}\rceil-2)+\left(p_{2}-p_{1}\right), & \text { if } v \in S \text { or } v \in m(G-S) \\ p_{1}(\lceil 2 \sqrt{n+1}\rceil-2), & \text { with } \sigma(v)=p_{2}\end{cases}
$$

Proof. Given $P_{n}$ is a fuzzy path with $n$ vertices such that $(n-1)$ vertices of node strength $p_{1}$ and the remaining one vertex have node strength as $p_{2}$, say $\sigma$ (v) $=p_{2}$. Let $S$ be any strict subset of $V$ having $x$ vertices then $G-S$ have $x+1$ or fewer components and one of them must have at least $\frac{n-x}{x+1}$ vertices.

Let $H$ denotes the maximum order connected component of $(G-S)$.
The following cases are possible.
Case 1: If $v \in S$, then $(x-1)$ vertices have node strength as $p_{1}$ and $v$ has node strength as $p_{2}$ in $S$. Also every vertex in $G-S$ have node strength $p_{1}$. Therefore, $|S|=(x-1) p_{1}+p_{2}$ and $m(G-S)=\left(\frac{n-x}{x+1}\right) p_{1}$.

Hence $\widetilde{I}\left(P_{n}\right)=\min \left\{(x-1) p_{1}+p_{2}+\left(\frac{n-x}{x+1}\right) p_{1}\right\}=\min \left\{p_{1}\left(\frac{x^{2}+n}{x+1}\right)+\left(p_{2}\right.\right.$ - $p_{1}$ ) \}. Minimum of this function attained at $x=\sqrt{n+1}-1$ and the minimum value is $2 \sqrt{n+1}-2$. For integer, by rounding up the value, the result is $\widetilde{I}\left(P_{n}\right)=$ $p_{1}(\lceil 2 \sqrt{n+1}\rceil-2)+\left(p_{2}-p_{1}\right)$.

Case 2: If $v$ not in $S$ but $v \in H$, then $S$ have $x$ vertices with node strength $p_{1}$ and H have $\left(\frac{n-x}{x+1}\right)-1$ vertices of node strength $p_{1}$ and one vertex with strength $p_{2}$.

Then $|S|=x p_{1}$ and $m(G-S)=\left(\frac{n-x}{x+1}-1\right) p_{1}+p_{2}$.
Therefore $\widetilde{I}\left(P_{n}\right)=\min \left\{\mathrm{xp}_{1}+\left(\frac{n-x}{x+1}-1\right) p_{1}+p_{2}\right\}$

$$
\begin{aligned}
& =\min \left\{\left(\frac{x^{2}+n}{x+1}\right) p_{1}+\left(p_{2}-p_{1}\right)\right\} \\
& =p_{1}(\lceil 2 \sqrt{n+1}\rceil-2)+\left(p_{2}-p_{1}\right)
\end{aligned}
$$

Case 3: If $v$ is not in both $S$ and $H$ then $|\mathrm{S}|=\mathrm{x} p_{1}$ and $\mathrm{m}(\mathrm{G}-\mathrm{S})=\left(\frac{n-x}{x+1}\right) p_{1}$.
Then $\widetilde{I}\left(P_{n}\right)=\min \left\{p_{1}\left(\frac{n-x}{x+1}+x\right)\right\}$
$=\min \left\{\left(\frac{x^{2}+n}{x+1}\right) p_{1}\right\}$

$$
=p_{1}(\lceil 2 \sqrt{n+1}\rceil-2)
$$

Thus from case 1,2 and 3 , it is concluded that,
$\widetilde{I}\left(P_{n}\right)= \begin{cases}p_{1}(\lceil 2 \sqrt{n+1}\rceil-2)+\left(p_{2}-p_{1}\right) & , \text { if } v \in S \text { or } v \in m(G-S) \\ & \text { with } \sigma(\mathrm{v})=p_{2} \\ p_{1}(\lceil 2 \sqrt{n+1}\rceil-2) & , \text { otherwise }\end{cases}$

Example 3.2. Example for cases $1 \& 2$ : Consider the fuzzy graph G: $(\sigma, \mu)$ with $\sigma *=\{a, b, c, d\}$, with $\sigma(\mathrm{a})=0.2, \sigma(\mathrm{~b})=0.3, \sigma(c)=\sigma(d)=0.2$, $\mu(a, b)=\mu(b, c)=\mu(c, d)=1$. For this graph, $\widetilde{I}$-sets are $\{b\}$ and $\{a, c\}$ with $\widetilde{I}(\mathrm{G})$ $=0.7$.

Example 3.3. Example for case 3: Consider the fuzzy graph $G=(\sigma, \mu)$ with $\sigma *=\{a, b, c, d\}$, with $\sigma(\mathrm{a})=0.3, \sigma(\mathrm{~b})=\sigma(\underset{\sim}{\mathrm{c}})=\sigma(\mathrm{d})=0.2, \mu(\mathrm{a}, \mathrm{b})=\mu(\mathrm{b}$, $\mathrm{c})=\mu(\mathrm{c}, \mathrm{d})=1$. For this graph, $\widetilde{I}$-set, $\mathrm{S}=\{\mathrm{b}\}$ and $\widetilde{I}(\mathrm{G})=0.6$.

Theorem 3.4. Let $G$ be a non-trivial connected fuzzy graph. Then $\widetilde{I}(G)=$ $\sigma(v)+\min _{v \in V} \widetilde{I}(G-v)$

Proof. Clearly if $\widetilde{I}$-set is $\emptyset$, then $\widetilde{I}(G)=m(G)$, otherwise $\widetilde{I}(G)<m(G)$.
Therefore $\widetilde{I}(G)=\min _{\emptyset \subset S \subset V}\{|S|+m(G-S)\}$.

$$
\begin{aligned}
& =\min _{v \in V} \min _{T \subset V-v}\{|\mathrm{~T} \cup\{\mathrm{v}\}|+\mathrm{m}(\mathrm{G}-(\mathrm{T} \cup\{\mathrm{v}\}))\} \\
& =\min _{v \in V}\left\{\sigma(v)+\min _{T \subset V-v}\{|T|+m((G-v)-T)\}\right\} \\
& =\sigma(v)+\min _{v \in V} \widetilde{I}(G-v)
\end{aligned}
$$

TheOrem 3.5. Let $C_{n}$ be a fuzzy cycle having $n$ vertices with node strength sequence $\left(p_{1}^{n-1}, p_{2}\right)$. Then
$\widetilde{I}\left(C_{n}\right)= \begin{cases}p_{1}(\lceil 2 \sqrt{n}\rceil-1)+\left(p_{2}-p_{1}\right) & , \text { if } v \in S \text { or } v \in m(G-S) \\ p_{1}(\lceil 2 \sqrt{n}\rceil-1) & \text { with } \sigma(v)=p_{2}\end{cases}$
Proof. Let $C_{n}$ be a fuzzy cycle having n vertices with $\sigma(\mathrm{v})=p_{2}$ and $\sigma(\mathrm{u})$ $=p_{1}$ for all $\mathrm{u} \in \mathrm{V}$ other than v . Let $P_{n-1}$ be a path defined by $C_{n}-w$, where $w \in V$.

Case 1: If $w=v$ then every vertex in $P_{n-1}$ has degree strength as $p_{1}$. Then by previous theorem 3.3, $\widetilde{I}\left(P_{n-1}\right)=p_{1}(\lceil 2 \sqrt{n}\rceil-2)$ and by above theorem 3.4 $\widetilde{I}\left(C_{n}\right)=(\lceil 2 \sqrt{n}\rceil-2) p_{1}+p_{2}=(\lceil 2 \sqrt{n}\rceil-1) p_{1}+\left(p_{2}-p_{1}\right)$

Case 2: If $\mathrm{w} \neq \mathrm{v}$ then node strength of $P_{n-1}$ is $\left(P_{1}^{n-2}, p_{2}\right)$, then by theorem 3.3, $\widetilde{I}\left(P_{n-2}\right)$ is either $p_{1}(\lceil 2 \sqrt{n}\rceil-2)$ or $p_{1}(\lceil 2 \sqrt{n}\rceil-2)+\left(p_{2}-p_{1}\right)$. Now $\widetilde{I}$ $\left(C_{n}\right)=\widetilde{I}\left(P_{n-1}\right)+\sigma(\mathrm{v})$ implies $p_{1}(\lceil 2 \sqrt{n}\rceil-1)$ or $p_{1}(\lceil 2 \sqrt{n}\rceil-1)+\left(p_{2}-\right.$ $p_{1}$ ).

Hence
$\widetilde{I}\left(C_{n}\right)= \begin{cases}p_{1}(\lceil 2 \sqrt{n}\rceil-1)+\left(p_{2}-p_{1}\right) & , \text { if } \mathrm{v} \in \mathrm{S} \text { or } \mathrm{v} \in \mathrm{m}(\mathrm{G}-\mathrm{S}) \\ p_{1}(\lceil 2 \sqrt{n}\rceil-1) & \text { with } \sigma(\mathrm{v})=p_{2}\end{cases}$

Theorem 3.6. Let $G$ be a complete fuzzy graph having $n$ vertices with node strength sequence $\left(p_{1}^{n-1}, p_{2}\right)$. Then $\widetilde{I}(G)=(n-1) p_{1}+p_{2}$.

Proof. For any strict subset $S \subset V$, the complete fuzzy graph $G$ with $n$ vertices is divided into two connected components $S$ and $G-S$ having $|S|+m(G-$ $S)=(n-1) p_{1}+p_{2}$. Thus $\widetilde{I}(G)=(n-1) p_{1}+p_{2}$.

Corollary 3.1. (S.Mathew and M.S. Sunitha proved in [10]), Complete fuzzy graphs having node strength sequence $\left(p_{1}^{n-1}, p_{2}\right)$ have $\delta(G)=\Delta(G)=\delta_{s}(G)=$ $\Delta_{s}(G)=(n-1) p_{1}$. The above theorem results in, $\delta(G)=\Delta(G)=\widetilde{I}(G)-p_{2}$.

Corollary 3.2. In a complete fuzzy graph $G$, if the node strength sequence is $\left(p_{1}^{n-1}, p_{2}\right)$, then $\mu(x, y)=p_{1}$ for all $x, y$. This implies $G$ is a regular fuzzy graph with $d(u)=(n-1) p_{1}$. Thus the theorem can be restated as if $G$ is a regular fuzzy graph having $n$ vertices with node strength sequence $\left(p_{1}^{n-1}, p_{2}\right)$ then, $\widetilde{I}(G)=(n-1) \mu$ $+p_{2}$.

Corollary 3.3. If $G$ is a regular fuzzy graph having $n$ vertices with node strength sequence $\left(p_{1}^{k}, p_{2}^{n-k}\right)$ then, $\widetilde{I}(G)=k p_{1}+(n-k) p_{2}=k \mu+(n-k) p_{2}$.

Theorem 3.7. Let $G$ be a fuzzy graph with node strength sequence ( $p_{1}^{m+n-1}$, $p_{2}$ )where the under lying graph $G^{*}$ is $K_{m, n}$, then
$\widetilde{I}(G)= \begin{cases}m p_{1}+p_{2} & \text { if } m \leqslant n \\ n p_{1}+p_{2} & m>n\end{cases}$
Proof. Bipartite the vertex set V as $V_{1}$ and $V_{2}$ such that $\left|V_{1}\right|=\mathrm{m}$ and $\mid V_{2}$ $\mid=\mathrm{n}$ and $\sigma(\mathrm{v})=p_{2}$. If $v \in V_{1}$ and $m \leqslant n$, then $\widetilde{I}(\mathrm{G})=\mathrm{m} p_{1}+p_{2}$. If $\mathrm{v} \in V_{1}$ and $\mathrm{m}>\mathrm{n}$, then $\widetilde{I}(\mathrm{G})=\mathrm{n} p_{1}+p_{2}$. Similarly If $\mathrm{v} \in V_{2}$ and $\mathrm{m}<\mathrm{n}$, then $\widetilde{I}(\mathrm{G})=\mathrm{m} p_{1}+p_{2}$. If $v \in V_{2}$ and $m>n$, then $\widetilde{I}(\mathrm{G})=\mathrm{n} p_{1}+p_{2}$. Consolidating all the above possibilities, we get, $\widetilde{I}(\mathrm{G})= \begin{cases}\mathrm{m} p_{1}+p_{2} & \text { if } \mathrm{m} \leqslant \mathrm{n} \\ \mathrm{n} p_{1}+p_{2} & \mathrm{~m}>\mathrm{n}\end{cases}$

Corollary 3.4. Let $G$ be a fuzzy graph with node strength sequence ( $p_{1}^{n-1}$, $p_{2}$ ) where the under lying graph $G^{*}$ is a star $K_{1, n}$, then $\widetilde{I}(G)=p_{1}+p_{2}$.

Theorem 3.8. Let $G$ be a fuzzy null graph $\overline{K_{n}}$ with node strength sequence $\left(p_{1}, p_{2}, \cdots, p_{n}\right)$. Then $\widetilde{I}\left(\overline{K_{p}}\right)=p_{n}$.

## 4. Conclusion

This paper attempts in developing the integrity concepts in fuzzy graphs. For the node strength sequence $\left(p_{1}^{n-1}, p_{2}\right)$ and for constant vertex degree, integrity of
special types of fuzzy graphs are developed. For complete fuzzy graph, this integrity parameter is compared with other connectivity parameters.

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