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NEIGHBOR RUPTURE DEGREE OF TOTAL GRAPHS AND THEIR COMPLEMENTS

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Abstract. A vertex subversion strategy of a graph G, say S, is a set of vertices in G whose closed neighborhood is removed from G. The survival subgraph is denoted by G/S. The neighbor rupture degree of G is defined to be $Nr(G) = max\{w(G/S) - |S| - c(G/S) : S \subset V(G), w(G/S) \geqslant 1\}$ where S is any vertex subversion strategy of G and $c(G/S)$ is the maximum order of the components of G/S. This paper includes results on the neighbor rupture degree of total graphs and their complements.

1. Introduction

In a communication network, the connection between the nodes can be broken for many unexpected reasons. The vulnerability parameters measure the resistance of the network after a failure. Some vulnerability parameters such as connectivity [**9**], integrity [**2**] and rupture degree [**11**] measure only the damaged nodes. However, some vulnerability parameters, such as neighbor connectivity [**5**], neighbor integrity [**8**] and neighbor rupture degree [**3**] can be used to assess not only damaged nodes but also neighboring ones. When we compare the vulnerability parameters, neighbor rupture degree which has recently been defined by Bacak-Turan and Kirlangic in [**4**] is an effective parameter. In the study of Bacak-Turan and Kirlangic [**4**], spy network was used as an example in order to evaluate neighbor rupture degree and the importance of neighborhood was stressed. Following this, some results for the neighbor rupture degree of the graphs obtained by some graph operations were given by Kandilci, Bacak-Turan and Polat [**10**]. In this study, a procedure is described for obtaining the total graph of a graph and for assessing the neighbor rupture degree for this total graph. Neighbor integrity of some total graphs were also studied by Kirlangic and Ozan in [**1**].

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Terminology and notation not defined in this paper can be found in [**3**]. Let G be a simple graph and let u be any vertex of G. The set $N(u) = \{u \in V(G) | v \neq u; v$ and u are adjacent$\}$ is the open neighborhood of u, and $N[u] = \{u\} \cup N(u)$ is the closed neighborhood of u. A vertex u in G is said to be subverted if the closed neighborhood of u is removed from G. A set of vertices $S = \{u_1, u_2, \cdots, u_m\}$ is called a vertex subversion strategy of G if each of the vertices in S has been subverted from G. If S has been subverted from the graph G, then the remaining graph is called survival subgraph, denoted by G/S.

The concept of neighbor rupture degree was introduced by Bacak-Turan and Kirlangic in 2011 [**4**].

Definition 1.1. [**4**] The neighbor rupture degree of a non-complete connected graph G is defined to be

$$Nr(G) = max\{w(G/S) - |S| - c(G/S) : S \subset V(G), w(G/S) \geqslant 1\}$$

where S is any vertex subversion strategy of G, $w(G/S)$ is the number of connected components in G/S and $c(G/S)$ is the maximum order of the components of G/S.

2. Neighbor Rupture Degree of Total Graphs

In this section the total graphs of some special graphs are examined and their neighbor rupture degrees are obtained.

Definition 2.1. [**6**] The vertices and edges of a graph are called its elements. Two elements of a graph are neighbors if they are either incident or adjacent. The total graph has vertex set $V(G) \cup E(G)$ and two vertices of a total graph of G are adjacent whenever they are neighbors in G, denoted by $T(G)$ or G^{+++}.

In other words, the total graph $T(G)$ of a graph G is a graph such that the vertex set of $T(G)$ corresponds to the vertices and edges of G and two vertices are adjacent in $T(G)$ if and only if their corresponding elements are either adjacent or incident in G.

Let $K_{m,n}$ be a complete bipartite graph then the total graph of a complete bipartite graph $K^{+++}_{m,n}$ is given in Figure 1.

Let a_i and b_j be the vertices of $K_{m,n}$ and let (a_i, b_j) be the vertices of $L(K_{m,n})$. In total graph edges between the vertices of $K_{m,n}$ and $L(K_{m,n})$ are as follows; a_i is joined to all (a_i, b_j) by an edge, and b_j is joined to all (a_i, b_j) by an edge where $i = 1, 2, \cdots, m$ and $j = 1, 2, \cdots, n$.

Theorem 2.1. *Neighbor rupture degree of total graph of $K_{m,n}$ with $m \leqslant n$ is*

$$Nr(K^{+++}_{m,n}) = n - m - 2.$$

Proof. Let S be a subversion strategy of $K^{+++}_{m,n}$ and let $|S| = x$. We have two cases according to the cardinality of S.

Case 1: If $1 \leqslant x \leqslant m - 1$, then $w(K^{+++}_{m,n}/S) = 1$ and $c(K^{+++}_{m,n}/S) \geqslant (m - x)(n - x) + m + n - 2x$. Thus we have

$$\begin{aligned} w(K^{+++}_{m,n}/S) - |S| - c(K^{+++}_{m,n}/S) &\leqslant 1 - x - (m - x)(n - x) - m - n + 2x \\ &\leqslant 1 - nm + nx + mx - m - n + x - x^2 \end{aligned}$$

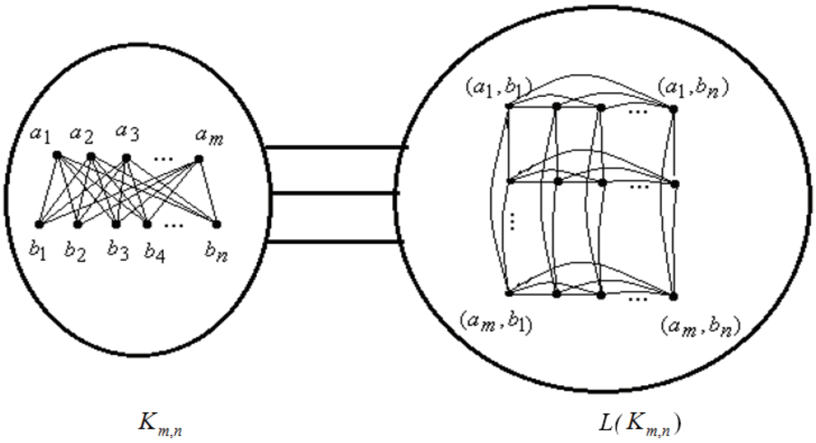

FIGURE 1. Total graph of a complete bipartite graph

Let $f(x) = 1 - nm + nx + xm - m - n + x - x^2$. Since $f(x)$ is an increasing function for $1 \leqslant x \leqslant m - 1$, it takes its maximum value at $x = m - 1$.

$f(m-1) = 1 - nm + n(m-1) + m(m-1) - m - n + m - 1 - (m-1)^2 = m - 2n - 1$

Therefore

$$Nr(K_{m,n}^{+++}) \leqslant m - 2n - 1. \tag{2.1}$$

Case 2: If $x \geqslant m$, then $w(K_{m,n}^{+++}/S) \leqslant n - 1$ and $c(K_{m,n}^{+++}/S) \geqslant 1$. Thus we have

$$w(K_{m,n}^{+++}/S) - |S| - c(K_{m,n}^{+++}/S) \leqslant n - 1 - x - 1.$$

Let $f(x) = n - x - 2$. Since $f(x)$ is a decreasing function then it takes its maximum value at $x = m$. Therefore

$$Nr(K_{m,n}^{+++}) \leqslant n - m - 2. \tag{2.2}$$

By (2.1) and (2.2) we get

$$Nr(K_{m,n}^{+++}) \leqslant n - m - 2 \tag{2.3}$$

There exist S^* such that $|S^*| = m$, $w(K_{m,n}^{+++}/S^*) = n - 1$ and $c(K_{m,n}^{+++}/S^*) = 1$ then we get

$$Nr(K_{m,n}^{+++}) \geqslant n - m - 2. \tag{2.4}$$

From (2.3) and (2.4) we get $Nr(K_{m,n}^{+++}) = n - m - 2$. □

COROLLARY 2.1. *Let $K_{1,n}$ be a star graph. Then*

$$Nr(K_{1,n}^{+++}) = n - 3.$$

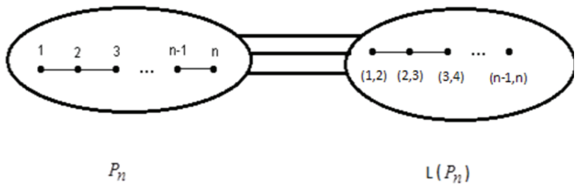

FIGURE 2. Total graph of a path graph

The total graph of a path graph P_n, P_n^{+++}, is given in Figure (2). Let i be the vertices of P_n and $(i, i+1)$ be the vertices of $L(P_n)$ where $i = 1, 2, \cdots, n$. In this total graph, the edges between the vertices of P_n and $L(P_n)$ are as follows:
i is joined to all $(i, i+1)$ and $(i-1, i)$ by an edge where $i = 1, 2, \cdots, n-1, n$.

THEOREM 2.2. *Neighbor rupture degree of total graph of P_n is*

$$Nr(P_n^{+++}) = \begin{cases} 0, & n \equiv 1 \pmod 3 \\ -1, & \textit{otherwise.} \end{cases}$$

PROOF. Let S be a subversion strategy of P_n^{+++} and let $|S| = r$. There are two cases according to the number of elements of S.
Case 1: If $0 \leqslant r \leqslant \lceil \frac{2n-1}{6} \rceil - 1$, then $w(P_n^{+++}/S) \leqslant r+1, c(P_n^{+++}/S) \geqslant \frac{2n-1-5r}{r+1}$ then we get

$$w(P_n^{+++}/S) - |S| - c(P_n^{+++}/S) \leqslant r + 1 - r - \frac{2n-1-5r}{r+1} = 6 - \frac{2n+4}{r+1}$$

Let $f(r) = 6 - \frac{2n+4}{r+1}$. Since $f'(r) > 0$ the function $f(r)$ is an increasing function so it takes its maximum value at $r = \lceil \frac{2n-1}{6} \rceil - 1$. Thus, if $n \equiv 1 \pmod 3$, then

$$f(r) = 0$$

otherwise

$$f(r) < 0 \tag{2.5}$$

Case 2: If $r = \lceil \frac{2n-1}{6} \rceil$, then $w(P_n^{+++}/S) \leqslant r, c(P_n^{+++}/S) \geqslant 1$ then we get

$$w(P_n^{+++}/S) - |S| - c(P_n^{+++}/S) \leqslant r - r - 1 = -1$$

$$Nr(P_n^{+++}) \leqslant -1 \tag{2.6}$$

Case 3: If $\lceil \frac{2n-1}{6} \rceil + 1 \leqslant r \leqslant n$, then $w(P_n^{+++}/S) \leqslant r-1, c(P_n^{+++}/S) \geqslant 1$ then we get

$$w(P_n^{+++}/S) - |S| - c(P_n^{+++}/S) \leqslant r - 1 - r - 1 = -2$$

$$Nr(P_n^{+++}) \leqslant -2 \tag{2.7}$$

According to (2.5), (2.6), (2.7) we have

$$Nr(P_n^{+++}) \leqslant \begin{cases} 0, & n \equiv 1 \pmod 3 \\ -1, & \text{otherwise.} \end{cases} \tag{2.8}$$

There exist S^* such that $|S^*| = \lceil \frac{2n-2}{6} \rceil, w(P_n^{+++}/S^*) = \lceil \frac{2n-1}{6} \rceil, c(P_n^{+++}/S^*) = 1$ then

$$w(P_n^{+++}/S^*) - |S| - c(P_n^{+++}/S^*) = \begin{cases} 0, & n \equiv 1 \pmod 3 \\ -1, & \text{otherwise} \end{cases}$$

$$Nr(P_n^{+++}) \geqslant \begin{cases} 0, & n \equiv 1 \pmod 3 \\ -1, & \text{otherwise} \end{cases} \tag{2.9}$$

By (2.8) and (2.9) we get the result. □

Let $C_{t,r}$ be comet graph. Then the total graph of comet $C_{t,r}(C_{t,r}^{+++})$ is given in Figure (3).

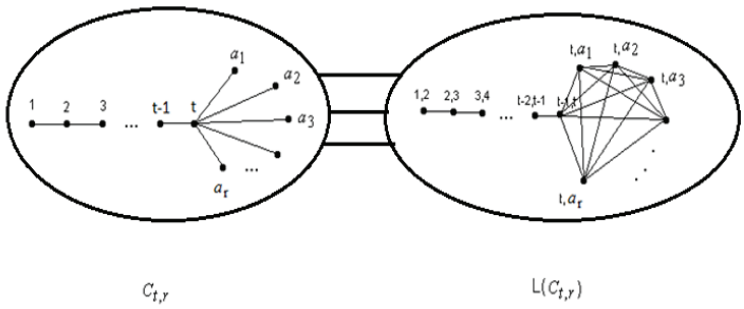

FIGURE 3. Total graph of a comet graph

Let $i = \{1, 2, \cdots, t-1, t, a_1, a_2, \cdots, a_r\}$ be the vertices of $C_{t,r}$ and let $(i, i+1), (t, a_i)$ be the vertices of $L(C_{t,r})$. In the total graph, the edges between the vertices of $C_{t,r}$ and $L(C_{t,r})$ are follows;
i is joined to all $(i, i+1)$ and a_i is joined to all (t, a_i) by an edge for $i = 1, 2, \cdots, t$.

THEOREM 2.3. *Let $C_{t,r}$ is comet graph with $r \geqslant 2$. Then the neighbor rupture degree of total graph of $C_{t,r}$ is*

$$Nr(C_{t,r}^{+++}) = \begin{cases} r-1, & t \equiv 2 \pmod 3 \\ r-2, & \textit{otherwise.} \end{cases}$$

PROOF. Let S be a subversion strategy of $C_{t,r}^{+++}$. There are three cases according to the elements of S.
Case 1: Let $|S| = x$. If $S \subset V(L(C_{t,r}))$ or $S \subset V(C_{t,r})$, then $w(C_n^{+++}/S) \leqslant x+1$ and $c(G/S) \geqslant r$. Therefore we get

$$w(C_n^{+++}/S) - |S| - c(C_n^{+++}/S) \leqslant x + 1 - x - r = 1 - r$$

$$Nr(C_{t,r}^{+++}) \leqslant 1 - r \tag{2.10}$$

Case 2: If $S = \{t\}$ where $t \epsilon V(C_{t,r})$, then $C_{t,r}^{+++}/S \cong P_{t-2}^{+++}$. Thus we have $Nr(C_{t,r}^{+++}) = Nr(P_{t-2}^{+++}) - 1$ and since the neighbor rupture degree of P_{t-2}^2 is

$$Nr(P_{t-2}^{+++}) = \begin{cases} 0, & t \equiv 3 \pmod 3 \\ -1, & \text{otherwise.} \end{cases}$$

We get

$$Nr(C_{t,r}^{+++}) = \begin{cases} -1, & t \equiv 2 \pmod 3 \\ -2, & \text{otherwise.} \end{cases} \tag{2.11}$$

Case 3: If $S = \{(t, a_i)\}$ where $(t, a_i) \epsilon V(L(C_{t,r}))$, then $C_{t,r}^{+++}/S \cong P_{t-1}^{+++} \cup (r-1)K_1$. Thus we have $Nr(C_{t,r}^{+++}) = Nr(P_{t-1}^{+++}) + r - 1 - 1$ and since the neighbor rupture degree of (P_{t-1}^{+++}) is

$$Nr(P_{t-1}^{+++}) = \begin{cases} 0, & t \equiv 2 \pmod 3 \\ -1, & \text{otherwise.} \end{cases}$$

We obtain

$$Nr(C_{t,r}^{+++}) = \begin{cases} r-2, & t \equiv 2 \pmod 3 \\ r-3, & \text{otherwise.} \end{cases} \tag{2.12}$$

According to (2.10), (2.11), (2.12) we get the result. □

3. Neighbor Rupture Degree of Complement of Total Graphs

In this section complement of total graphs are operated on graphs and their neighbor rupture degrees are evaluated.

Definition 3.1. [**9**] The complement of a simple graph G is obtained by taking the vertices of G and joining two of them whenever they are not joined in G and denoted by G^c. The complement of a total graph G^{+++} is denoted by G^{---}.

Let $K_{m,n}$ be a complete bipartite graph. Then the complement of total graph of a complete bipartite graph is denoted by $K_{m,n}^{---}$ and is given in Figure (4).

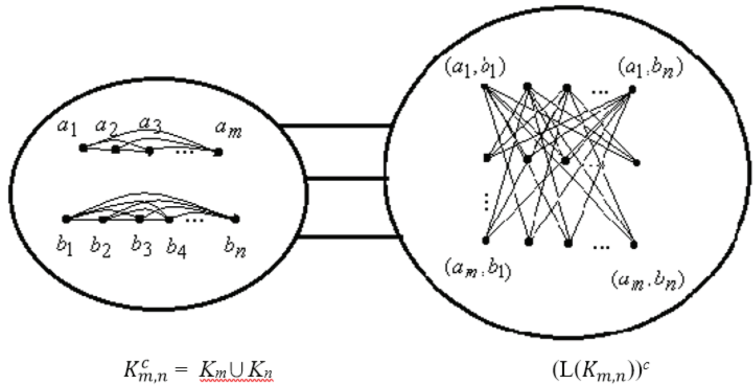

Figure 4. Complement of total graph of a complete bipartite graph

Let a_i and b_j be the vertices of $K^c_{m,n}$ and let (a_i, b_j) be the vertices of $L(K^c_{m,n})$. In the complement of the total graph, edges between the vertices of $K^c_{m,n}$ and $L(K^c_{m,n})$ are as follows;
a_i is joined to all (a_k, b_l) by an edge where $k \neq i$ and $i = 1, 2, \cdots, m$,
b_j is joined to all (a_k, b_l) by an edge where $j \neq l$ and $j = 1, 2, \cdots, n$.

THEOREM 3.1. *Neighbor rupture degree of the complement of a total graph of $K_{m,n}$ with $m \leqslant n$ is*

$$Nr(K^{---}_{m,n}) = n - 4.$$

PROOF. Let S be a subversion strategy of $K^{---}_{m,n}$ and let $|S| = x$. We have two cases according to the cardinality of S.
Case 1: If $x = 1$, then $w(K^{---}_{m,n}/S) = 1$ and $c(K^{---}_{m,n}/S) \geqslant 2m$. Thus we have

$$w(K^{---}_{m,n}/S) - |S| - c(K^{---}_{m,n}/S) \leqslant 1 - 1 - 2m = -2m$$

$$Nr(K^{---}_{m,n}) \leqslant -2m \tag{3.1}$$

Case 2: If $2 \leqslant x \leqslant m + n$, then $w(K^{---}_{m,n}/S) \leqslant n - 1$ and $c(K^{---}_{m,n}/S) \geqslant 1$. Thus we have

$$w(K^{---}_{m,n}/S) - |S| - c(K^{---}_{m,n}/S) \leqslant n - 1 - x - 1$$
$$Nr(K^{---}_{m,n}) \leqslant n - x - 2.$$

Let $f(x) = n - x - 2$. Since $f(x)$ is a decreasing function then it takes its maximum value at $x = 2$

$$Nr(K^{---}_{m,n}) \leqslant n - 4. \tag{3.2}$$

By (3.1) and (3.2) we have

$$Nr(K^{---}_{m,n}) \leqslant n - 4. \tag{3.3}$$

There exist S^* such that $|S^*| = 2$, $w(K^{---}_{m,n}/S^*) = n - 1$ and $c(K^{---}_{m,n}/S^*) = 1$ then we get

$$Nr(K^{---}_{m,n}) \geqslant n - 4. \tag{3.4}$$

From (3.3) and (3.4) we get $Nr(K^{---}_{m,n}) = n - 4$. □

COROLLARY 3.1. *Let $K_{1,n}$ be a star graph. Then the neighbor rupture degree is*

$$Nr(K^{---}_{1,n}) = n - 4.$$

Let P_n be a path graph. Then the complement of a total graph of path graph P^{---}_n is given in Figure (5).

Let i be the vertices of P^c_n and $(i, i + 1)$ be the vertices of $L(P^c_n)$ for $i = 1, 2, \cdots, n - 1, n$. In the complement of the total graph, the edges between the vertices of P^c_n and $L(P^c_n)$ are as follows;
i is joined to all $(j, j + 1)$ by an edge where $i \neq j + 1$ and $i = 1, 2, \cdots, n$.

THEOREM 3.2. *Neighbor rupture degree of the complement of the total graph of a path graph P_n is*

$$Nr(P^{---}_n) = 0.$$

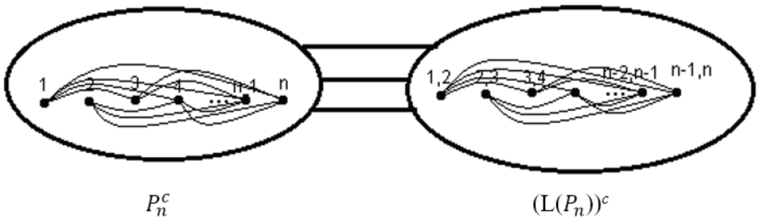

FIGURE 5. Complement of a total graph of path graph

PROOF. Let S be a subversion strategy of P_n^{---}. There are two cases according to the elements of S.

Case 1: Let $i \in V(P_n^c)$ and $S = \{i\}$ for $i = 1, 2, \cdots, n$.

If $deg(i) = 1$, then $|S| = 1, w(P_n^{---}/S) = 2$ and $c(P_n^{---}/S) = 1$. Thus we get

$$w(P_n^{---}/S) - |S| - c(P_n^{---}/S) = 0. \tag{3.5}$$

If $deg(i) = 2$, then $|S| = 1, w(P_n^{---}/S) = 2$ and $c(P_n^{---}/S) = 2$. Thus we have

$$w(P_n^{---}/S) - |S| - c(P_n^{---}/S) = -1. \tag{3.6}$$

Case 2: Let $(i, i+1) \in V(L(P_n)^c)$ in P_n^{---} and let $S = \{(i, i+1)\}$.

If $deg(i, i+1) = 1$, then $|S| = 1, w(P_n^{---}/S) = 2$ and $c(P_n^{---}/S) = 2$. Thus we get

$$w(P_n^{---}/S) - |S| - c(P_n^{---}/S) = -1. \tag{3.7}$$

If $deg(i, i+1) = 2$, then $|S| = 1, w(P_n^{---}/S) = 2$ and $c(P_n^{---}/S) = 2$. Thus we have

$$w(P_n^{---}/S) - |S| - c(P_n^{---}/S) = -1. \tag{3.8}$$

From (3.5), (3.6), (3.7), (3.8) we have $Nr(P_n^{---}) = 0$. □

Let $C_{t,r}$ be a complete bipartite graph. Then the complement of the total graph of a comet graph $C_{t,r}^{---}$ is given in Figure (6).

Let $i = \{1, 2, \cdots, t-1, t, a_1, a_2, \cdots, a_r\}$ be the vertices of $C_{t,r}^c$ and let $(i, i+1), (t, a_i)$ be the vertices of $L(C_{t,r}^c)$. In the total graph, the edges between the vertices of $C_{t,r}^c$ and $L(C_{t,r}^c)$ are as follows;
i is joined by an edge to all vertices of $L(C_{t,r}^c)$ except $(i-1, i)$ and $(i, i+1)$ and a_i is joined by an edge to all vertices of $L(C_{t,r}^c)$ except (t, a_i) where $i = 1, 2, \cdots, t$.

THEOREM 3.3. *Let $C_{t,r}$ is a comet graph with $r \geqslant 4$. Then the neighbor rupture degree of complement of the total graph of a comet graph is*

$$Nr(C_{t,r}^{---}) = r - 4.$$

PROOF. Let S be a subversion strategy of $C_{t,r}^{---}$. We have four cases according to elements of S.

Case 1: Let $i \in V(C_{t,r}^c)$ and $S = \{i\}$ for $i = 1, 2, \cdots, n$.

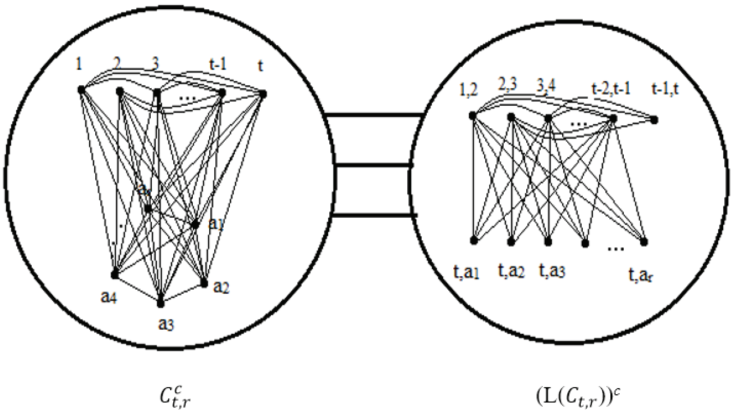

FIGURE 6. Complement of total graph of comet graph

If $deg(i) = 1$, then $|S| = 1, w(C_{t,r}^{---}/S) = 2$ and $c(C_{t,r}^{---}/S) = 1$. Thus we get

$$w(C_{t,r}^{---}/S) - |S| - c(C_{t,r}^{---}/S) = 0. \tag{3.9}$$

If $deg(i) = 2$, then $|S| = 1, w(C_{t,r}^{---}/S) = 1$ and $c(C_{t,r}^{---}/S) = 4$. Thus we have

$$w(C_{t,r}^{---}/S) - |S| - c(C_{t,r}^{---}/S) = -4. \tag{3.10}$$

Case 2: Let $a_i \in V(C^c_{t,r})$ and $S = \{a_i\}$ for $i = 1, 2, \cdots, r$. Since $S = \{a_i\}$ we get $w(C_{t,r}^{---}/S) = 1$ and $c(C_{t,r}^{---}/S) = 1$. Therefore we have

$$w(C_{t,r}^{---}/S) - |S| - c(C_{t,r}^{---}/S) = -1. \tag{3.11}$$

Case 3: Let $(i, i+1) \in V(L(C^c_{t,r}))$ and $S = \{(i, i+1)\}$.
If $deg(i, i+1) = 1$, then $|S| = 1, w(C_{t,r}^{---}/S) = 2$ and $c(C_{t,r}^{---}/S) = 2$. Thus we have

$$w(C_{t,r}^{---}/S) - |S| - c(C_{t,r}^{---}/S) = -1. \tag{3.12}$$

If $deg(i, i+1) = 2$, then $|S| = 1, w(C_{t,r}^{---}/S) = 1$ and $c(C_{t,r}^{---}/S) = 4$. Thus we obtain

$$w(C_{t,r}^{---}/S) - |S| - c(C_{t,r}^{---}/S) = -4. \tag{3.13}$$

Case 4: Let $(a_t, b_i), (a_t, b_j) \in V(L(C^c_{t,r}))$.
If $S = \{(a_t, b_i)\}$, then we have

$$w(C_{t,r}^{---}/S) - |S| - c(C_{t,r}^{---}/S) = -r. \tag{3.14}$$

If $S = \{(a_t, b_i), (a_t, b_j)\}$, then we have

$$w(C_{t,r}^{---}/S) - |S| - c(C_{t,r}^{---}/S) = r - 4. \tag{3.15}$$

From (3.9),(3.10), (3.11),(3.12),(3.13), (3.14),(3.15) we have $Nr(C_{t,r}^{---}) = r-4$. $\square$

4. Conclusion

In this study, the total graphs and complements of total graphs for some special graphs are studied and their neighbor rupture degrees are determined. Total graphs are unary operations which create a new graph from the old one.

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