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TOTAL EQUITABLE DOMINATION IN FUZZY GRAPHS

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Abstract. Let G be a fuzzy graph. Let D be a subset of V. D is said to be a fuzzy total equitable dominating set if for all $x \in V$, there exists $y \in D$ such that y is adjacent to x and y is fuzzy degree equitable with x. If G has no fuzzy equitable isolated point then V is fuzzy total equitable dominating set. For such graphs, the minimum cardinality of a fuzzy total equitable dominating set is called the fuzzy total equitable domination number and is denoted by $\gamma_t^{ef}(G)$. In this paper we introduce the concept of a fuzzy total equitable dominating set, minimal fuzzy total equitable dominating set and obtain some interesting results for these new parameter in total equitable domination in fuzzy graphs.

1. Introduction

L.A.Zadeh (1965) introduced the concepts of a fuzzy subset of a set as a way for representing uncertainty. His idea have been applied to a wide range of scientific areas.

Fuzzy concepts is also introduced in Graph theory. Formally, a fuzzy graph $G = (V, \sigma, \mu)$ is a non empty set V together with a pair of functions $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that $\mu(uv) \leqslant \sigma(u) \wedge \sigma(v)$ for all u, v in V, σ is called the fuzzy vertex set of G and μ is called the fuzzy edge set of G. The concept of equitable domination [10] in graphs was introduced by Venkatrasubramanian Swaminathan and Kuppusamy Markandan Dharmalingam. The notation of domination in fuzzy graphs [9] was developed by A. Somasundaram and S.Somasundaram. In this paper we introduce the concept of fuzzy total equitable dominating set, minimal fuzzy total equitable dominating set and obtain some interesting result for these new parameter in total equitable domination in fuzzy graphs.

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Preliminaries :

DEFINITION 1.1. A fuzzy graph $G = (\sigma, \mu)$ is a set with two functions σ :V$\rightarrow$ $[0, 1]$ and $\mu : V \times V \rightarrow [0, 1]$ such that $\mu(uv) \leqslant \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

DEFINITION 1.2. Let $G = (\sigma, \mu)$ be a fuzzy graph on V and $V_1 \subseteq V$. Define $\sigma_1$on $V_1$by $\sigma_1(u) = \sigma(u)$ for all $u \in V_1$ and $\mu_1$on the collection $E_1$of two element subset of V_1 by $\mu_1(u, v) = \mu(u, v)$ for all $u, v \in V_1$. Then (σ_1,μ_1) is called the fuzzy subgraph of G induced by V_1 and is denoted by $< V_1 >$.

DEFINITION 1.3. The degree of vertex u is defined as the sum of the weights of the edges incident at u and is denoted by $deg(u)$.

DEFINITION 1.4. Let G be a fuzzy graph. Let u and v be two vertices of G. A subset D of V is called a fuzzy equitable dominating set if for every $v \in V - D$ there exist a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leqslant 1$ and $\mu(uv) \leqslant \sigma(u) \wedge \sigma(v)$. The minimum cardinality of a fuzzy equitable dominating set is denoted by γ^{ef}.

DEFINITION 1.5. A vertex $u \in V$ is said to be degree equitable fuzzy graph with a vertex $v \in V$ if $|deg(u) - deg(v)| \leqslant 1$ and $\mu(uv) \leqslant \sigma(u) \wedge \sigma(v)$.

DEFINITION 1.6. If D is an fuzzy equitable dominating set then any super set of D is a fuzzy equitable dominating set.

2. Fuzzy total equitable domination

DEFINITION 2.1. Let D be a subset of V. D is said to be a fuzzy total equitable dominating set if for all $x \in V$, there exists $y \in D$ such that y is adjacent to x and y is fuzzy degree equitable with x. If G has no fuzzy equitable isolated point then V is fuzzy total equitable dominating set. For such graphs, the minimum cardinality of a fuzzy total equitable dominating set is called the fuzzy total equitable domination number and is denoted by $\gamma_t^{ef}(G)$. It is convention to take $\gamma_t^{ef}(G) = \infty$ if G has an fuzzy equitable isolated point.

THEOREM 2.1. $\gamma_t^{ef}(G) = 1$ *if and only if* $\Delta^{ef}(G) = n - 1$ *and* $\delta^{ef}(G) \geqslant n - 2$.

PROOF. If $\Delta^{ef} = n - 1$ and $\delta^{ef} \geqslant n - 2$, then any point of degree Δ^{ef} will be fuzzy equitable degree and adjacent with all other points. Therefore $\gamma_t^{ef}(G) = 1$.

Conversely, If $\gamma_t^{ef}(G) = 1$, then there exists a point u of degree $n - 1$ which is degree fuzzy equitable with all other points. Therefore $d^{ef}(u) = n - 1$ and $d^{ef}(v) \geqslant n - 2$ for all $v \neq u$, $v \in V$. Therefore $\Delta^{ef} = n - 1$ and $\delta^{ef} \geqslant n - 2$. □

DEFINITION 2.2. Fuzzy total equitable dominating set is said to be minimal fuzzy total equitable dominating set if no proper subset of D is a fuzzy total equitable dominating set.

THEOREM 2.2. *For any fuzzy graph G without isolated vertices a fuzzy total equitable dominating set D is minimal if and only if for every $u \in D$, one of the following two properties holds.*

(i) There exists a vertex $v \in V - D$ *such that* $N(v) \cap D = \{u\}$, $|deg(u) - deg(v)| \leqslant 1$.

(ii) $< D - \{u\} >$ *contains no isolated vertices.*

Proof. Assume that D is a minimal fuzzy total equitable dominating set and (i) and (ii) do not hold, then for some $u \in D$, there exists $v \in V - D$ such that $|deg(u) - deg(v)| \leqslant 1$ and for every $v \in V - D$, either $N(v) \cap D \neq \{u\}$ or $|deg(u) - deg(v)| \geqslant 2$.or both. Therefore $< D - \{u\} >$ contains an isolated vertex, which is contradiction to the minimality of D. Therefore (i) and (ii) holds.

Conversely, if for every vertex $u \in D$, the statement (i) or (ii) holds and D is not minimal. Then there exits $u \in D$, such that $D - \{u\}$ is a fuzzy total equitable dominating set . Therefore there exists $v \in D - \{u\}$ such that v equitable dominates u. that is, $v \in N(u)$ and $|deg(u) - deg(v)| \leqslant 1$. Hence u does not satisfy (i). Then u must satisfy (ii) and there exists $v \in V - D$ such that $N(v) \cap D = \{u\}$, and $|deg(u) - deg(v)| \leqslant 1$. And also there exists $w \in D - \{u\}$ such that w is adjacent to v. Therefore $w \in N(v) \cap D$, $|deg(w) - deg(v)| \leqslant 1$ and $w \neq v$, a contradiction to $N(v) \cap D = \{u\}$. Hence D is a minimal fuzzy total equitable dominating set. □

Theorem 2.3. $\{\gamma_t^{ef}(G)\}^{-1} + \{\gamma_t^{ef}(\bar{G})\}^{-1} \leqslant 1$.

Proof. Case (i): G has a vertex v such that $d^{ef}(v) = n-1$. Then $\gamma_t^{ef}(G) = 1$. v is an isolate in $\bar{G}$ and $\gamma_t^{ef}(\bar{G}) = \infty$. Therefore $\{\gamma_t^{ef}(G)\}^{-1} + \{\gamma_t^{ef}(\bar{G})\}^{-1} = 1$.

Case(ii) $d^{ef}(v) \leqslant n - 2$ for all $v \in V(G)$. Then $\gamma_t^{ef}(G) \geqslant 2$. Also $\gamma_t^{ef}(\bar{G}) \geqslant 2$. Therefore $\{\gamma_t^{ef}(G)\}^{-1} + \{\gamma_t^{ef}(\bar{G})\}^{-1} \leqslant 1$. □

Theorem 2.4. *Let* G *be a fuzzy graph without isolated vertices. Then* $\gamma_t(G) \leqslant \gamma_t^{ef}(G)$.

Proof. Every fuzzy total equitable dominating set is a total dominating set. Thus $\gamma_t(G) \leqslant \gamma_t^{ef}(G)$. □

Definition 2.3. For every $u \in V - D$ there exist a vertex $v \in D$ such that $uv \in E(G)$ also $deg(u) = deg(v) = r$ and in this case G is called regular fuzzy graph of degree r or a r-regular fuzzy graph.

Theorem 2.5. *If* G *is a r-regular fuzzy graph for* $r \geqslant 1$, *then* $\gamma_t^{ef}(G) = \gamma_t(G)$.

Proof. Suppose G is a regular fuzzy graph. Then every vertex of G is of same degree say r. Let D be the minimal total dominating set of G. Then $|D| = \gamma_t(G)$. If $u \in V - D$, then D is a total dominating set, then there exists $v \in D$ and $uv \in E(G)$, also $deg(u) = deg(v) = r$. Therefore, $|deg(u) - deg(v)| = 0 \leqslant 1$. Hence D is an fuzzy total equitable dominating set of G, such that $\gamma_t^{ef}(G) \leqslant |D| = \gamma_t(G)$. And also we have $\gamma_t(G) \leqslant \gamma_t^{ef}(G)$. Therefore $\gamma_t^{ef}(G) = \gamma_t(G)$. □

Definition 2.4. Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be two fuzzy graphs on V_1 and V_2 respectively with $V_1 \cap V_2 = \phi$. The join of G_1 and G_2denoted by G_1+$G_2$is the fuzzy graph on $V_1 \cup V_2$ defined as follows.

$G_1{+}G_2 = (\sigma_1 + \sigma_2, \mu_1 + \mu_2)$ where

$$(\sigma_1 + \sigma_2)(u) = \{ \begin{array}{lll} \sigma_1(u) & if & u \in V_1 \\ \sigma_2(u) & if & u \in V_2 \end{array}$$

$$(\mu_1 + \mu_2)(uv) = \{ \begin{array}{ccc} \mu_1(uv) & if & u, v \in V_1 \\ \mu_2(uv) & if & u, v \in V_2 \\ \sigma_1(u) \wedge \sigma_2(v) & if & u \in V_1 \& v \in V_2 \end{array}$$

THEOREM 2.6. *Let $G_1 = (\sigma_1, \mu_1)$ and $G_2 = (\sigma_2, \mu_2)$ be two fuzzy graphs on V_1 and V_2 respectively with $V_1 \cap V_2 = \phi$. Then*

(i) If G_1 has no isolated vertices and G_2 has no isolated vertices, then
$\gamma_t^{ef}(G_1 + G_2) = min\{\gamma_t^{ef}(G_1), \gamma_t^{ef}(G_2), \sigma_1(u) + \sigma_2(v)\}$ where $u \in V_1$, $v \in V_2$
(ii) If G_1 has no isolated vertices and G_2 has an isolated vertices, then
$\gamma_t^{ef}(G_1 + G_2) = min\{\gamma_t^{ef}(G_1), \sigma_1(u) + \sigma_2(v)\}$ where $u \in V_1$, $v \in V_2$
(iii) If G_1 has an isolated vertices and G_2 has no isolated vertices, then
$\gamma_t^{ef}(G_1 + G_2) = min\{\gamma_t^{ef}(G_2), \sigma_1(u) + \sigma_2(v)\}$ where $u \in V_1$, $v \in V_2$
(iv) If both G_1 and G_2 have isolated vertices, then
$\gamma_t^{ef}(G_1 + G_2) = min\{\sigma_1(u) + \sigma_2(v)\}$ where $u \in V_1$, $v \in V_2$

PROOF. Two fuzzy graphs G_1 and G_2 with join $G_1{+}G_2$is a fuzzy graphs having no isolated vertices and hence
$\gamma_t^{ef}(G_1 + G_2)$ always exists.

(i) Both G_1 and G_2 have no isolated vertices. In this case $\gamma_t^{ef}(G_1)$ and $\gamma_t^{ef}(G_2)$ exist. Minimal fuzzy total equitable dominating set D of $G_1 + G_2$is of one of the following forms.

(a) $D = D_1$ where D_1 is a minimal fuzzy total equitable dominating set of G_1.

(b) $D = D_2$ where D_2 is a minimal fuzzy total equitable dominating set of G_2.

$D = u, v$ where $u \in V_1$, $v \in V_2$

Hence, $\gamma_t^{ef}(G_1 + G_2) = min\{\gamma_t^{ef}(G_1), \gamma_t^{ef}(G_2), \sigma_1(u) + \sigma_2(v)\}$ where $u \in V_1$, $v \in V_2$

(ii) G_1 has no isolated vertices and G_2 have isolated vertices. In this case $\gamma_t^{ef}(G_2)$ does not exist. Let, D be minimal fuzzy total equitable dominating set D of $G_1 + G_{2.}$. Then D is one of the following forms.

$D = D_1$ where D_1 is a minimal fuzzy total equitable dominating set of G_1.

$D = u, v$ where $u \in V_1$, $v \in V_2$

Hence, $\gamma_t^{ef}(G_1 + G_2) = min\{\gamma_t^{ef}(G_1), \sigma_1(u) + \sigma_2(v)\}$ where $u \in V_1$, $v \in V_2$

(iii) G_1 has isolated vertices and G_2 has no isolated vertices. Proof is similar to (ii).

(iv) Both G_1 and G_2 have isolated vertices. In this case $\gamma_t^{ef}(G_1)$ and $\gamma_t^{ef}(G_2)$ does not exist. Now, minimal fuzzy total equitable dominating set of $G_1 + G_2$is one of the following form u, v where $u \in V_1$, $v \in V_2$

Hence, $\gamma_t^{ef}(G_1 + G_2) = min\{\sigma_1(u) + \sigma_2(v)\}$ where $u \in V_1$, $v \in V_2$ □

THEOREM 2.7. *Let G be a any connected fuzzy graph contain without isolated vertices and $\Delta(G) < n - 1$. Then $\gamma_t^{ef}(G) \leqslant n - \Delta(G)$.*

PROOF. Let v be a vertex with maximum degree $\Delta(G)$ in G and $X = V - N[v]$. If $X = \phi$, then $\Delta(G) = n - 1$ and $\gamma_t^{ef}(G) = n - \Delta(G) - 1$. If $X \neq \phi$, then $\Delta(G) < n - 1$. Let $x \in X$ is adjacent to $y \in N(v)$. Let Δ'.be the vertex set and maximum degree of the component of $G[x]$ which contains x. The component of $G[S]$ has a fuzzy total equitable dominating set Y of cardinality atmost $|S| - \Delta' + 1$.

If $\Delta' = 1$, .then the set $\{v, y, x\} \cup (X - S)$ is fuzzy total equitable dominating in G and $\gamma_t^{ef}(G) \leqslant 3 + (n - \Delta(G) - 1) - 2 = n - \Delta(G)$.

If $\Delta' = 2$ then the set $\{v, y\} \cup Y \cup (X - S)$ is fuzzy total equitable dominating in G and $\gamma_t^{ef}(G) \leqslant 2 + |S| - \Delta' + 1 + n - \Delta(G) - 1 - |S| = n - \Delta(G)$. Therefore $\gamma_t^{ef}(G) \leqslant n - \Delta(G)$. □

DEFINITION 2.5. For every $u \in V - D$ there exist a vertex $v \in D$ such that $uv \in E(G)$ and one of the vertex u or v is with degree k and other is with degree $k + 1$ and in this case G is called a bi-regular fuzzy graph.

THEOREM 2.8. *If G is a $(k, k+1)$ bi-regular fuzzy graph for any positive integer k, then $\gamma_t^{ef}(G) = \gamma_t(G)$.*

PROOF. Suppose G is a $(k, k + 1)$ bi-regular fuzzy graph then the degree of each vertex in G is either k or $k + 1$, where k is a positive integer. Let D be a minimal total dominating set of G. If $u \in V - D$ there exist a vertex $v \in D$ such that $uv \in E(G)$ and one of the vertex u or v is with degree k and other is with degree $k + 1$. Therefore $|deg(u) - deg(v)| = 1$. Hence D is a fuzzy total equitable dominating set of G such that $\gamma_t^{ef}(G) \leqslant \gamma_t(G)$. And also we have $\gamma_t(G) \leqslant \gamma_t^{ef}(G)$.. Therefore $\gamma_t^{ef}(G) = \gamma_t(G)$. □

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