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ON THE NEIGHBOURHOOD POLYNOMIAL OF GRAPHS

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Abstract. Graph polynomials are polynomials associated to graphs that encode the number of subgraphs with given properties. In this paper we introduce a new type of graph polynomial called neighbourhood polynomial. We obtained the neighbourhood polynomial of some interested standard graphs like complete graph, complete bipartite graph, bi-star graph, spider, wounded spider graph and for some corona product graphs.

1. Introduction

All the graphs considered here are finite and undirected with no loops and multiple edges. Let $G = (V, E)$ be a graph. As usual $p = |V|$ and $q = |E|$ denote the number of vertices and edges of a graph G, respectively. In general, we use $\langle X \rangle$ to denote the subgraph induced by the set of vertices X and $N(v)$ and $N[v]$ denote the open neighbourhood and closed neighbourhood of a vertex v, respectively. A set D of vertices in a graph G is a dominating set if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G.

The corona $G_1 \circ G_2$ of two graphs G_1 and G_2 is the graph obtained by taking one copy of G_1 (which has n_1 vertices) and n_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

A set $S \subseteq V(G)$ is called a neighbourhood set of G, if $G = \cup_{v \in S} \langle N[v] \rangle$, where $\langle N[v] \rangle$ is the subgraph of G induced by v and the vertices adjacent to v. The neighbourhood number of G is the minimum cardinality of a neighbourhood set of G, and it is denoted by $\eta(G)$.

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For terminology and notations not specifically defined here we refer reader to [**2**] and [**4**] . For more details about neighbourhood number, we refer to [**6**], and [**7**].

2. Neighbourhood polynomial of a graph

Definition 2.1. Let $G = (V, E)$ be a graph. The neighbourhood polynomial of G is defined as $N(G, x) = \sum_{i=1}^{n} n(G, i)x^i$, where $n(G, i)$ is the number of neighbourhood set in G of size i.

Example 2.1. Let $G \cong P_5$ as in Figure 3. There is only one neighbourhood set of size two namely $\{v_2, v_4\}$.
There are six neighbourhood sets of size 3 are :
$\{v_2, v_3, v_4\}, \{v_1, v_3, v_4\}, \{v_1, v_3, v_5\}, \{v_1, v_2, v_4\}, \{v_2, v_4, v_5\}, \{v_2, v_3, v_5\}$.
There are five neighbourhood of size four which they are:
$\{v_2, v_3, v_4, v_5\}, \{v_1, v_3, v_4, v_5\}, \{v_1, v_2, v_4, v_5\}, \{v_1, v_2, v_3, v_5\}, \{v_1, v_2, v_3, v_4\}$
Also there is one neighbourhood set of size five.
Hence, $N(G, x) = x^5 + 5x^4 + 6x^3 + x^2 = x^2(x^3 + 5x^2 + 6x + 1)$.

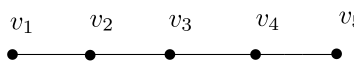

Figure 1. Path P_5

Proposition 2.1. *For any complete graph* K_n, $N(G, x) = (1 + x)^n - 1$.

Proof. Let G be a complete graph of n vertices. Then there are n neighbourhood sets of size one and $\binom{n}{2}$ neighbourhood sets of size two and so on.
$N(G, x) = \binom{n}{1}x + \binom{n}{2}x^2 + ... + x^n = \sum_{i=1}^{n} \binom{n}{i}x^i$
$= \sum_{i=0}^{n} \binom{n}{i}x^i - 1 = (1 + x)^n - 1.$ □

Proposition 2.2. *For any graph the neighbourhood polynomial of* G *is*

$$N(G, x) = \sum_{i=\eta(G)}^{n} n(G, i)x^i$$

where $n(G,i)$ is the number of neighbourhood sets in G of size i and $\eta(G)$ is the neighbourhood number of G.

Proposition 2.3. *For any graph $G = G_1 \cup G_2$,*

$$N(G,x) = N(G_1,x)N(G_2,x).$$

Proof. Any neighbourhood set of size k in G is arising by select the number of neighbourhood sets of size i in G_1, and the vertices of neighbourhood set of size $k-j$ in G_2. And the number of ways of selecting this vertices is equal to the coefficient of the term x^k in the polynomial $N(G_1,x)N(G_2,x)$. Hence $N(G,x) = N(G_1,x)N(G_2,x)$. □

Also by mathematical induction we can generalize Proposition 2.3.

Proposition 2.4. *Let $G = \bigcup_{i=1}^{k} G_i$. Then $N(G,x) = \prod_{i=1}^{k} N(G_i,x)$.*

Corollary 2.1. *For any totally disconnected graph $\overline{K_n}$, $N(\overline{K_n},x) = x^n$.*

Theorem 2.1. *Let G_1, G_2 be graphs of order n_1 and n_2 respectively. Then* $N(G_1+G_2,x) = \big((1+x)^{n_1}-1\big)\big((1+x)^{n_2}-1\big) + N(G_1,x) + N(G_2,x)$.

Proof. First, for any neighbourhood set of G_1 or G_2 is also neighbourhood set of G_1+G_2 that means $N(G_1,x)$ and $N(G_2,x)$ contains in $N(G_1+G_2,x)$. Second, Any neighbourhood set of G_1+G_2 of size k contains j vertices from G_1 and any $k-j$ vertices of G_2 that means

$$\left[\binom{n_1}{1}x + \binom{n_1}{2}x^2 + \binom{n_1}{3}x^3 + ... + \binom{n_1}{n_1}x^{n_1}\right]\cdot$$
$$\left[\binom{n_2}{1}x + \binom{n_2}{2}x^2 + \binom{n_2}{3}x^3 + ... + \binom{n_2}{n_2}x^{n_2}\right],$$

contains in $N(G_1+G_2,x)$. That means

$$N(G_1+G_2,x) = \sum_{i=1}^{n_1}\binom{n_1}{i}x^i \sum_{i=1}^{n_2}\binom{n_2}{i}x^i + N(G_1,x) + N(G_2,x).$$

Therefore,

$$N(G_1+G_2,x) = \left[\sum_{i=0}^{n_1}\binom{n_1}{i}x^i - 1\right]\left[\sum_{i=0}^{n_2}\binom{n_2}{i}x^i - 1\right] + N(G_1,x) + N(G_2,x).$$

Hence, $N(G_1+G_2,x) = \big((1+x)^{n_1}-1\big)\big((1+x)^{n_2}-1\big)+N(G_1,x)+N(G_2,x)$. □

Corollary 2.2. *Let G be complete bipartite graph $K_{m,n}$. Then*

$$N(G,x) = \big((1+x)^m-1\big)\big((1+x)^n-1\big) + N(G_1,x) + N(G_2,x).$$

Proof. Let $G \cong K_{m,n}$ and can be construct $K_{m,n}$ by joining the graph $G_1 = \overline{K_m}$ with $G_2 = \overline{K_n}, i,e., K_{m,n} = \overline{K_m} + \overline{K_n}$. Now by using Theorem 2.1 we get $N(G,x) = ((1+x)^m-1)((1+x)^n-1\big) + N(G_1,x) + N(G_2,x)$. □

COROLLARY 2.3. *For any star graph $K_{1,n}$, then*

$$N(K_{1,n}, x) = x(1+x)^n + x^n.$$

DEFINITION 2.2. The bistar graph is constructed from K_2 by attaching m edges in one vertex and n edges in the other vertex, and is denoted by $B(m,n)$.

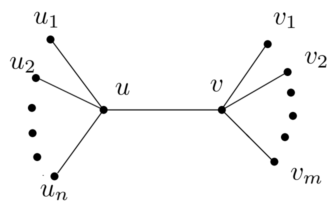

FIGURE 2. Bistar $B(m,n)$

THEOREM 2.2. *For any bistar graph $B(m,n)$, $m,n \geqslant 3$, then*

$$N(B(m,n),x) = x^{m+1} + x^{n+1} + 2x^{m+n+1} + x^2(1+x)^{m+n}.$$

PROOF. Let $G \cong B(m,n)$, where $m,n \geqslant 3$. Clearly the neighbourhood number of $B(m,n)$ is two,there is only one minimum neighbourhood set $\{u,v\}$ see Figure 2. Any neighbourhood set for $B(m,n)$ either contains the two vertices u and v and any other vertices or contains the vertices of the set $v_1, v_2, ..., v_m$ and the vertex u, similarly may be contains the vertex v and the set $u_1, u_2, ..., u_n$ or the neighbourhood set contains all the vertices except u and v. So there is one neighbourhood set of size $m+1$, similarly there is one neighbourhood set of size $n+1$. Now to select neighbourhood set contains three vertices, we have to select two vertices u and v and to select the third vertex there are $\binom{m+n}{1}$ ways. Similarly to select neighbourhood set of four vertices we have $\binom{m+n}{2}$ ways, in the same way we can get the number of ways of select neighbourhood set of five vertices. In general the number of ways to select neighbourhood set of size i, where $i \geqslant 3$ is $\binom{m+n}{i-2}$. So there are $\binom{m+n}{m+n-1}$ ways to select neighbourhood set of size $m+n+1$ and there is only one neighbourhood set of size $m+n+2$.

Therefore,

$$n(G, m+n+1) = \binom{m+n}{m+n-1}$$

and $n(G, m+n+2) = 1$. Hence,

$$N(G,x) = \\ x^2 + x^{m+1} + x^{n+1} + 2x^{m+n+1} + \binom{m+n}{1}x^3 + \binom{m+n}{2}x^4 + ... + \binom{m+n}{m+n}x^{m+n+2}.$$

Thus,

$$N(G,x) = x^2 + x^{m+1} + x^{n+1} + 2x^{m+n+1} + x^2 \sum_{i=1}^{m+n} \binom{m+n}{i} x^i.$$

Hence, $N(G,x) = x^{m+1} + x^{n+1} + 2x^{m+n+1} + x^2(1+x)^{m+n}$. □

THEOREM 2.3. *Let $G \cong C_n \circ K_1$. Then $N(G,x) = x^n(1+x)^n$.*

PROOF. Let G be a corona graph $C_n \circ K_1$as in Figure 3. There is only one min-

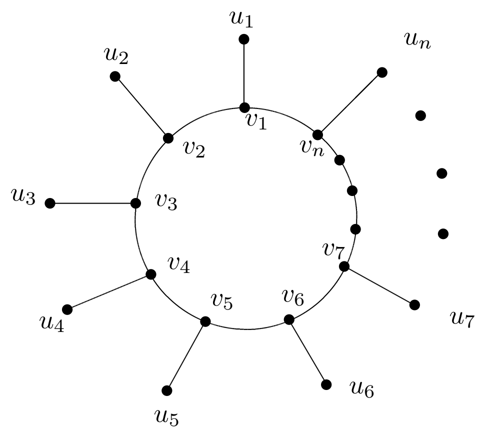

FIGURE 3. Corona graph $C_n \circ K_1$

imum neighbourhood set of G, which is the set $\{v_1, v_2, ..., v_n\}$. Therefore $n(G,n) = 1$. To get neighbourhood sets of size $n+1$, we have $\binom{n}{1}$ ways .That means there are $\binom{n}{1}$ neighbourhood sets of G of size $n+1$.Therefore $n(G, n+2) = \binom{n}{2}$ and $n(G, n+1) = \binom{n}{2}$, similarly to get the number of neighbourhood sets of size $n+i$, there are $\binom{n}{i}$ ways.

Hence,

$$\begin{aligned} N(G,x) &= x^n + \binom{n}{1}x^{n+1} + \binom{n}{2}x^{n+2} + \binom{n}{3}x^{n+3} + ... \\ &\quad + \binom{n}{n}x^{2n} \\ &= x^n + x^n \sum_{i=1}^{n} \binom{n}{i} x^i \\ &= x^n + x^n \left(\sum_{i=0}^{n} (\binom{n}{i} x^i) - 1 \right) \\ &= x^n (1+x)^n. \end{aligned}$$

$$\begin{aligned} D_c(G,x) &= (n_1-2)(n_2-2)x^{n_1+n_2-4} \\ &+[(n_1-2)(n_2-1)+(n_1-1)(n_2-2)]x^{n_1+n_2-3} \\ &+[n_2(n_1-2)n_1(n_2-2)+(n_1-1)(n_2-1)]x^{n_1+n_2-2} \\ &+[n_2(n_1-1)+n_1(n_2-1)]x^{n_1+n_2-1} \\ &+x^{n_1+n_2}. \end{aligned}$$

□

Now, We can generalize Theorem 2.3 as follows:

THEOREM 2.4. *For any corona graph $G \cong C_n \circ \overline{K_m}$, we have*

$$N(G,x) = x^n(1+x)^{mn}.$$

PROOF. There is only one minimum neighbourhood set of G which is of size n, that means $n(G,n)=1$, similarly $n(G,n+1) = \binom{mn}{1}$ and $n(G,n+2) = \binom{mn}{2}$. In general, It is easy to see that $n(G,n+j) = \binom{mn}{j}$, where $j = 1,2,..,mn$.

Therefore,

$$N(G,x) = x^n + \binom{(}{mn} 1x^{n+1} + \binom{mn}{2}x^{n+2} + ... + \binom{mn}{mn}x^{n+mn}.$$

Hence

$$\begin{aligned} N(G,x) &= x^n + x^n \left[\binom{mn}{1}x + \binom{mn}{2}x^2 + ... + \binom{mn}{mn}x^{mn} \right] \\ &= x^n + x^n [\sum_{i=0}^{mn} \binom{mn}{i} - 1] \\ &= x^n + x^n [(1+x)^{mn} - 1] \\ &= x^n(1+x)^{mn}. \end{aligned}$$

□

THEOREM 2.5. *For any corona graph $G \cong C_n \circ K_m$, we have*

$$N(G,x) = x^n\left(\sqrt{1+x}\right)^{nm(m-1)}.$$

PROOF. There is only one minimum neighbourhood set of G of size n, that means $n(G,n) = 1$. Similarly it easy to see that,

$$n(G,n+1) = \binom{\frac{nm(m-1)}{2}}{1},$$

$$n(G,n+2) = \binom{\frac{nm(m-1)}{2}}{2}.$$

In general,

$$n(G,n+i) = \binom{\frac{nm(m-1)}{2}}{i},$$

where $i = 1,2,..,\frac{nm(m-1)}{2}$

Let $\frac{nm(m-1)}{2} = \alpha$. Then

$$N(G,x) = x^n + (\binom{\alpha}{1}x^{n+1} + \binom{\alpha}{2}x^{n+2} + ... + \binom{\alpha}{\alpha}x^{n+\alpha}$$

$$= x^n + x^n\sum_{i=1}^{\alpha}\binom{\alpha}{i}x^i$$

$$= x^n + x^n[(1+x)^\alpha - 1]$$

$$= x^n(1+x)^\alpha$$

Hence, $N(G,x) = x^n(1+x)^{\frac{nm(m-1)}{2}} = x^n\left(\sqrt{1+x}\right)^{nm(m-1)}$. □

THEOREM 2.6. *Let $K_{1,p}$ be a star with $p \geqslant 2$ and let G be a spider graph which is constructed by subdivide each edges once in $k_{1,p}$ as in Figure 4. Then*

$$N(G,x) = x^p(1+x)^{p+1} + x^{p+1}(1+x)^p + 2(2^{p-1}-1)x^{p+1} - x^{2p+1}.$$

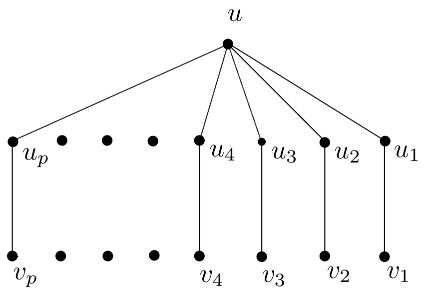

FIGURE 4. Spider graph with $2p+1$ vertices

PROOF. Let $A = \{u_1, u_2, ..., u_p\}$ and $B = \{v_1, v_2, ..., v_p\}$. There is one minimum neighbourhood set of size n which is A. For any neighbourhood set of G there are three cases:

Case 1. The neighbourhood set contains all the vertices $\{u_1, u_2, ..., u_p\}$.

Case 2. The neighbourhood set contains all the vertices of $B = \{v_1, v_2, ..., v_p\}$ and the vertex u.

Case 3. The neighbourhood set contains some vertices from A and some vertices from B and vertex the vertex u.

In **case 1.**There is only one neighbourhood set of size p which is $\{u_1, u_2, ..., u_p\}$ i.e., $n(G, p) = 1$ and there are $\binom{p+1}{1}$ ways to extend the neighbourhood set of size p to size $p+1$, similarly there are $\binom{p+1}{2}$ neighbourhood sets of size $p+2$. So in this case $n(G, p) = 1$, $n(G, p+1) = \binom{p+1}{1}$, $n(G, p+2) = \binom{p+1}{2}$, in general $n(G, p+i) = \binom{p+1}{i}$, where $1 \leqslant i \leqslant p+1$.

In **case 2.** There is one neighbourhood set of size $p+1$ which is $\{v_1, v_2, ..., v_p, u\}$, i.e., $n(G, p+1) = 1$ and there are $\binom{p}{1}$ ways to select neighbourhood set of size $p+2$. Similarly $\binom{p}{2}$ ways to select neighbourhood set of size $p+3$. In general there are $\binom{p}{i}$ ways to select neighbourhood set of size $p+i+1$, where $1 \leqslant i \leqslant p-1$. So in this case $n(G, p+1) = 1$, $n(G, p+2) = \binom{p}{1}$, $n(G, p+3) = \binom{p}{2}$, ... ,$n(G, 2p-1) = \binom{p}{p-1}$.

In **case 3.** To select some vertices from A and other vertices from B, we have to select the vertex u in every selected neighbourhood set. Therefore, we select one vertex from A and $p-1$ vertices from B and the vertex u, then two vertices from A and $p-2$ vertices from B and the vertex u and so on to get the neighbourhood sets of size $p+1$.

That means there are $\binom{p}{1} + \binom{p}{2} + \binom{p}{3} + ... + \binom{p}{p-1}$ ways to select neighbourhood set of size $p+1$ in this case. Also $\binom{p}{1} + \binom{p}{2} + \binom{p}{3} + ... + \binom{p}{p-1} = \left(\sum_{i=0}^{p} \binom{p}{i}\right) - 2 = 2^p - 2$.

Hence, from all cases we can get $\eta(G, p) = 1$, $n(G, p+1) = \binom{p+1}{1} + 1 + 2^p - 2$, $n(G, p+2) = \binom{p+1}{2} + \binom{p}{1}$, $n(G, p+3) = \binom{p+1}{3} + \binom{p}{2}$,... , $n(G, 2p+1) = \binom{p+1}{p+1}$.

Therefore,

$$N(G,x) = x^p + \binom{p+1}{1}x^{p+1} + \binom{p+1}{1}x^{p+2} + \ldots + \binom{p+1}{p+1}x^{2p+1} + x^{p+1}$$

$$+\binom{p}{1}x^{p+2} + \binom{p}{2}x^{p+3} + \ldots + \binom{p}{p-1}x^{2p} + (2^p-2)x^{p+1}.$$

$= x^p + x^p\big[\sum_{i=1}^{p+1}\binom{p+1}{i}x^i\big] + x^{p+1} + x^{p+1}\big[\sum_{i=1}^{p-1}\binom{p}{i}x^i\big] + 2(2^{p-1}-1)x^{p+1}$

$= x^p + x^p\big[\big(\sum_{i=0}^{p+1}\binom{p+1}{i}x^i\big) - 1\big] + x^{p+1} + x^{p+1}\big[\sum_{i=1}^{p}\binom{p}{i}x^i - x^p\big] + 2(2^{p-1}-1)x^{p+1}$

$= x^p \sum_{i=0}^{p+1}\binom{p+1}{i})x^p + x^{p+1} - x^{2p+1} + x^{p+1}\big[\sum_{i=0}^{p}\binom{p}{i}\big] - 1 + 2(2^{p-1}-1)x^{p+1}$

$= x^p(1+x)^{p+1} + x^{p+1}(1+x)^p + 2(2^{p-1}-1)x^{p+1} - x^{2p+1}.$ □

THEOREM 2.7. *Let $G \cong F_m$ be a friendship graph with $2m+1$ vertices as in Figure 5. Then*

$$N(G,x) = x(1+x)^{2m} + \sum_{i=0}^{m}\tbinom{m}{i}2^{m-i}x^{m+i}.$$

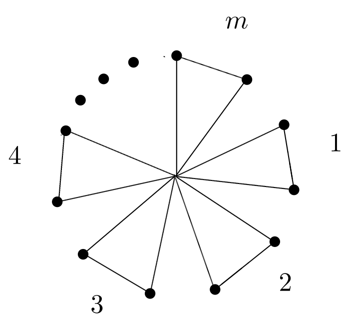

FIGURE 5. Friendship graph

PROOF. Let G be a friendship graph of $2m+1$ vertices, where $m \geqslant 2$. Then $\eta(G) = 1$, $n(G,1) = 1$,$n(G,2) = \binom{2m}{1}$, $n(G,3) = \binom{2m}{2}$, in general the number of ways of selecting neighbourhood set of size i which containing the center is $\binom{2m}{i-1}$. Also there are 2^m number of neighbourhood sets of size m which does not contain the center vertex. Similarly there are $\binom{m}{1}2^{m-1}$ ways to select neighbourhood set of size $m+1$ which does not contain the center. In general there are $\binom{m}{i}2^{m-i}$ ways to select a neighbourhood set of size $m+i$ which does not contain the center vertex.

Therefore,
$N(G,x) = x + \binom{2m}{1}x^2 + \binom{2m}{2}x^3 +$

$... + \binom{2m}{2m}x^{2m+1} + 2^m x^m + \binom{m}{1}2^{m-1}x^{m+1} + \binom{m}{2}2^{m-2}x^{m+2} + ... + \binom{m}{m}x^{2m}$

$= x + x\sum_{i=1}^{2m}\binom{2m}{i}x^i + \sum_{i=0}^{m}\binom{m}{i}2^{m-i}x^{m+i}$

$= x + x\big(\sum_{i=0}^{2m}\binom{2m}{i}x^i - 1\big) + \sum_{i=0}^{m}\binom{m}{i}2^{m-i}x^{m+i}$

$= x(1+x)^{2m} + \sum_{i=0}^{m}\binom{m}{i}2^{m-i}x^{m+i}.$ □

THEOREM 2.8. *Let $K_{1,p}$ be a star with $p \geqslant 3$ and let G be a wounded spider graph which is constructed by subdivided s edges, where $1 \leqslant s \leqslant p-1$, from $k_{1,p}$. Then $N(G,x) = (1+x)^p + (2^s-1)x^p + (1+x)^s x^p + x^{p+1}(1+x)^s - x^{p+1} - x^{p+s+1}$.*

PROOF. Let G be a wounded spider graph as in Figure 6 clearly the neighbourhood number of G is $s+1$ the set $\{u, v_1, v_2, ..., v_s\}$ is minimum dominating set of G that means $\eta(G) = s+1$.
There are four cases to construct a neighbourhood set for G.

Case 1. The neighbourhood set contains the set $\{u, v_1, v_2, ..., v_s\}$. So to extend this neighbourhood set to neighbourhood set of size $s+2$ there are $\begin{pmatrix} p \\ 1 \end{pmatrix}$ ways and to extend to neighbourhood set of size $s+3$ there are $\begin{pmatrix} p \\ 2 \end{pmatrix}$ ways. In general to extend the neighbourhood set $\{u, v_1, v_2, ..., v_s\}$ to neighbourhood set of G of size $s+1+i$ there are $\begin{pmatrix} p \\ i \end{pmatrix}$ ways.
Therefore,

$$x^{s+1} + \begin{pmatrix} p \\ 1 \end{pmatrix}x^{s+2} + \begin{pmatrix} p \\ 2 \end{pmatrix}x^{s+3} + ... + \begin{pmatrix} p \\ p \end{pmatrix}x^{s+p+1},$$

contained in the neighbourhood polynomial of G.

Case 2. The neigbourhoods set of G contains some vertices from $\{v_1, v_2, ..., v_s\}$ and some vertices from $\{u_1, u_2, ..., u_s\}$ and the vertex u and there are

$$\begin{pmatrix} s \\ 1 \end{pmatrix} + \begin{pmatrix} s \\ 2 \end{pmatrix} + \begin{pmatrix} s \\ 3 \end{pmatrix} + ... + \begin{pmatrix} s \\ s-1 \end{pmatrix} = 2^s - 2$$

ways to select this neighbourhood set of size $s+1$. Therefore, the neighbourhood polynomial of G contains the polynomial $(2^s - 2)x^{s+1}$.

Case 3. The neighbourhood set contains all the vertices of the set

$$\{u_1, u_2, ..., u_p\}$$

. So to extend this neighbourhood set to neighbourhood set of size $p+1$ there are $\begin{pmatrix} s+1 \\ 1 \end{pmatrix}$ ways and to extend to neighbourhood set of size $p+2$ there are $\begin{pmatrix} s+1 \\ 2 \end{pmatrix}$ ways. In general to extend the neighbourhood set $\{u_1, u_2, ..., u_p\}$ to neighbourhood set of G of size $p+i$ there are $\binom{s+1}{i}$ ways.

Therefore,

$$x^p + \binom{s+1}{1}x^{p+1} + \binom{s+1}{2}x^{s+2} + ... + \binom{s+1}{s}x^{s+p},$$

contained in the neighbourhood polynomial of G.

Case 4. The neighbourhood set of G contains some vertices from the set $\{v_1, v_2, ..., v_s\}$ and some vertices from the set $\{u_1, u_2, ..., u_s\}$ and the vertex u and some vertices from the set $\{u_s+1, u_s+2, ..., u_p\}$. Similarly as the previous cases we get that the polynomial

$$(2^s \ -2)\binom{p-s}{1}x^{s+2} + 2^s \ -2)\binom{p-s}{2}x^{s+3} + ... + 2^s \ -2)\binom{p-s}{p-s}x^{p+1}$$

is contained in the neighbourhood polynomial of G. Now by calculation the polynomials in all the cases, we get:

$$D(G,x) = (2^s-2)x^{s+1}(1+x)^{p-s} + x^{s+1}(1+x)^p + x^p(1+x)^{s+1} - x^{s+p+1}.$$

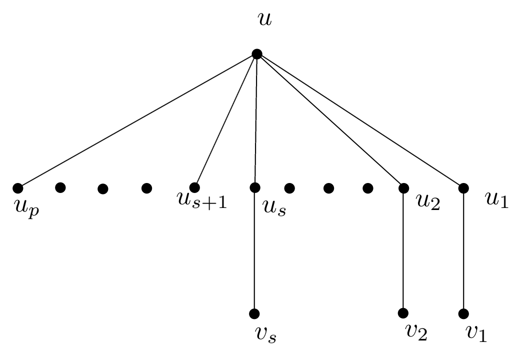

FIGURE 6. Wounded spider

□

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